Chiral Effective Theory with a scalar field

Joan Soto

Departament d'Estructura i Constituents de la Matèria

and

Institut de Ciències del Cosmos

Universitat de Barcelona

(with Pere Talavera and Jaume Tarrús, in progress)

Introduction

Introduction

- QCD developes an intrinsic scale Λ_χ .

Introduction

- QCD developes an intrinsic scale Λ_χ .
- $\Lambda_\chi \gg m_q$, m_q being the light quark masses.

Introduction

- QCD developes an intrinsic scale Λ_χ .
- $\Lambda_\chi \gg m_q$, m_q being the light quark masses.
- $\mathcal{L}_{QCD}$ enjoys an approximate $SU_L(n_f) \otimes SU_R(n_f)$ chiral symmetry.

Introduction

- QCD developes an intrinsic scale Λ_χ .
- $\Lambda_\chi \gg m_q$, m_q being the light quark masses.
- $\mathcal{L}_{QCD}$ enjoys an approximate $SU_L(n_f) \otimes SU_R(n_f)$ chiral symmetry.
- It is spontaneously broken to $SU_V(n_f)$.

Introduction

- QCD developes an intrinsic scale Λ_χ .
- $\Lambda_\chi \gg m_q$, m_q being the light quark masses.
- $\mathcal{L}_{QCD}$ enjoys an approximate $SU_L(n_f) \otimes SU_R(n_f)$ chiral symmetry.
- It is spontaneously broken to $SU_V(n_f)$.
- There are quasi Goldstone bosons with masses $m_\pi \ll \Lambda_\chi$.

Introduction

- QCD developes an intrinsic scale Λ_χ .
- $\Lambda_\chi \gg m_q$, m_q being the light quark masses.
- $\mathcal{L}_{QCD}$ enjoys an approximate $SU_L(n_f) \otimes SU_R(n_f)$ chiral symmetry.
- It is spontaneously broken to $SU_V(n_f)$.
- There are quasi Goldstone bosons with masses $m_\pi \ll \Lambda_\chi$.
- An effective Lagrangia for $p \sim m_\pi \ll \Lambda_\chi$ can be use to describe their physics, the Chiral Lagrangian.

Introduction

- QCD developes an intrinsic scale Λ_χ .
- $\Lambda_\chi \gg m_q$, m_q being the light quark masses.
- $\mathcal{L}_{QCD}$ enjoys an approximate $SU_L(n_f) \otimes SU_R(n_f)$ chiral symmetry.
- It is spontaneously broken to $SU_V(n_f)$.
- There are quasi Goldstone bosons with masses $m_\pi \ll \Lambda_\chi$.
- An effective Lagrangia for $p \sim m_\pi \ll \Lambda_\chi$ can be use to describe their physics, the Chiral Lagrangian.
  - The calculations can be systematically organized in powers of p^2/Λ_χ^2 and m_π^2/Λ_χ^2.

The Chiral Lagrangian ($n_f = 2$)

$$\mathcal{L}_{\chi\mathrm{PT}} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \cdots$$

$$\mathcal{L}^{(2)} = \frac{F^2}{4} \langle D^\mu U^+ D_\mu U + \chi^+ U + U^+ \chi \rangle , \quad \chi = 2B_0 m_q$$

$$\begin{aligned} \mathcal{L}^{(4)} = & \frac{l_1}{4} \langle D^\mu U^+ D_\mu U \rangle^2 + \frac{l_2}{4} \langle D^\mu U^+ D^\nu U \rangle \langle D_\mu U^+ D_\nu U \rangle \\ & + \frac{l_3}{16} \langle \chi^+ U + U^+ \chi \rangle^2 + \frac{l_4}{4} \langle D^\mu U^+ D_\mu \chi + D^\mu \chi^+ D_\mu U \rangle \end{aligned}$$

Weinberg (79)

Gasser, Leutwyler (84)

Aim

Aim

- Generalize the Chiral Lagragian to include a light scalar with mass $m_\sigma \ll \Lambda_\chi$

Aim

- Generalize the Chiral Lagragian to include a light scalar with mass $m_\sigma \ll \Lambda_\chi$
- The calculations can be systematically organized in powers of p^2/Λ_χ^2, m_π^2/Λ_χ^2, and $m_\sigma^2/\Lambda_\chi^2$

Aim

- Generalize the Chiral Lagragian to include a light scalar with mass $m_\sigma \ll \Lambda_\chi$
- The calculations can be systematically organized in powers of p^2/Λ_χ^2, m_π^2/Λ_χ^2, and $m_\sigma^2/\Lambda_\chi^2$
- Explore the physical consequences of such generalization

Motivation

- A resonance exists at a relatively low mass,

$$\sqrt{s_\sigma} \sim 441 - i272$$

Caprini, Colangelo, Leutwyler (05); Leutwyler (08)

Motivation

- A resonance exists at a relatively low mass,

$$\sqrt{s_\sigma} \sim 441 - i272$$

  Caprini, Colangelo, Leutwyler (05); Leutwyler (08)

- χPT breaks down earlier than expected, at least in the scalar chanel.

Motivation

- A resonance exists at a relatively low mass,

$$\sqrt{s_\sigma} \sim 441 - i272$$

  Caprini, Colangelo, Leutwyler (05); Leutwyler (08)

- χPT breaks down earlier than expected, at least in the scalar chanel.
- Proposed solutions typically involve resummations (unitarization methods), e.g. Oller, Oset, Pelaez (98).

Motivation

- A resonance exists at a relatively low mass,

$$\sqrt{s_\sigma} \sim 441 - i272$$

  Caprini, Colangelo, Leutwyler (05); Leutwyler (08)

- χPT breaks down earlier than expected, at least in the scalar chanel.
- Proposed solutions typically involve resummations (unitarization methods), e.g. Oller, Oset, Pelaez (98).
- Can one carry out systematically these resummations in an EFT framework ?

Motivation

- A resonance exists at a relatively low mass,

$$\sqrt{s_\sigma} \sim 441 - i272$$

  Caprini, Colangelo, Leutwyler (05); Leutwyler (08)

- χPT breaks down earlier than expected, at least in the scalar chanel.
- Proposed solutions typically involve resummations (unitarization methods), e.g. Oller, Oset, Pelaez (98).
- Can one carry out systematically these resummations in an EFT framework ?
- Try including a scalar in the Chiral Lagrangian

The Lagrangian

$$\mathcal{L}_{\chi\mathrm{PT}+\sigma} = \mathcal{L}^{\pi}_{\chi\mathrm{PT}+\sigma} + \mathcal{L}^{\sigma}_{\chi\mathrm{PT}+\sigma} + \mathcal{L}^{\pi\sigma}_{\chi\mathrm{PT}+\sigma}$$

$$\mathcal{L}^{\pi}_{\chi\mathrm{PT}+\sigma} = \mathcal{L}_{\chi\mathrm{PT}} + \mathcal{L}_{CT}$$

- $\mathcal{L}_{CT}$ is obtained from $\mathcal{L}_{\chi\mathrm{PT}}$ by multiplying the terms in the latter by powers of $m_\sigma^2/\Lambda_\chi^2$.

e.g.

$$\mathcal{L}^{(4)}_{CT} = Z_1 m_\sigma^2 \langle \chi^\dagger U + U^\dagger \chi \rangle + Z_2 m_\sigma^2 \langle D^\mu U^\dagger D_\mu U \rangle$$

The Lagrangian

$$\mathcal{L}^{\sigma}_{\chi\mathrm{PT}+\sigma} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma\sigma - \frac{\lambda_3}{3!}\sigma^3 - \frac{\lambda_4}{4!}\sigma^4 + \cdots$$

- $\lambda_3 \sim O(\Lambda_\chi)$, $\lambda_4 \sim O(1)$

The Lagrangian

$$\mathcal{L}^{\sigma}_{\chi\mathrm{PT}+\sigma} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma\sigma - \frac{\lambda_3}{3!}\sigma^3 - \frac{\lambda_4}{4!}\sigma^4 + \cdots$$

- $\lambda_3 \sim O(\Lambda_\chi)$, $\lambda_4 \sim O(1)$
- Hence, strongly coupled (!)

The Lagrangian

$$\mathcal{L}^{\sigma}_{\chi\mathrm{PT}+\sigma} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma\sigma - \frac{\lambda_3}{3!}\sigma^3 - \frac{\lambda_4}{4!}\sigma^4 + \cdots$$

- $\lambda_3 \sim O(\Lambda_\chi)$, $\lambda_4 \sim O(1)$
- Hence, strongly coupled (!)
- But scalar theories beyond perturbation theory are believed to be trivial

The Lagrangian

$$\mathcal{L}^{\sigma}_{\chi\mathrm{PT}+\sigma} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma\sigma - \frac{\lambda_3}{3!}\sigma^3 - \frac{\lambda_4}{4!}\sigma^4 + \cdots$$

- $\lambda_3 \sim O(\Lambda_\chi)$, $\lambda_4 \sim O(1)$
- Hence, strongly coupled (!)
- But scalar theories beyond perturbation theory are believed to be trivial
  - Take $\lambda_3 = \lambda_4 = 0$

The Lagrangian

$$\mathcal{L}^{\sigma}_{\chi\mathrm{PT}+\sigma} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma\sigma - \frac{\lambda_3}{3!}\sigma^3 - \frac{\lambda_4}{4!}\sigma^4 + \cdots$$

- $\lambda_3 \sim O(\Lambda_\chi)$, $\lambda_4 \sim O(1)$
- Hence, strongly coupled (!)
- But scalar theories beyond perturbation theory are believed to be trivial
  - Take $\lambda_3 = \lambda_4 = 0$
  - It is enough to assume them suppressed by powers of $m_\sigma^2/\Lambda_\chi^2$

The Lagrangian

$$\mathcal{L}^{\pi\sigma}_{\chi\mathrm{PT}+\sigma} = \left(F_1\sigma + c_1\sigma\sigma + \zeta_1\sigma^3 + \cdots\right)\langle D^\mu U^\dagger D_\mu U\rangle + \left(F_2\sigma + c_2\sigma\sigma + \zeta_2\sigma^3 + \cdots\right)\langle \chi^\dagger U + U^\dagger\chi\rangle$$

- $F_i \sim O(\Lambda_\chi)$, $c_i \sim O(1)$, $\zeta_i \sim O(\frac{1}{\Lambda_\chi})$, $i = 1, 2$
- $\delta m^2_\sigma \sim c_2 B_0 m_q$ is induced.
- In the linear sigma model $F_1 = F/2$, $c_1 = 1/4$, $\zeta_1 = 0$.

The calculations

- A considerable number of new LECs appear

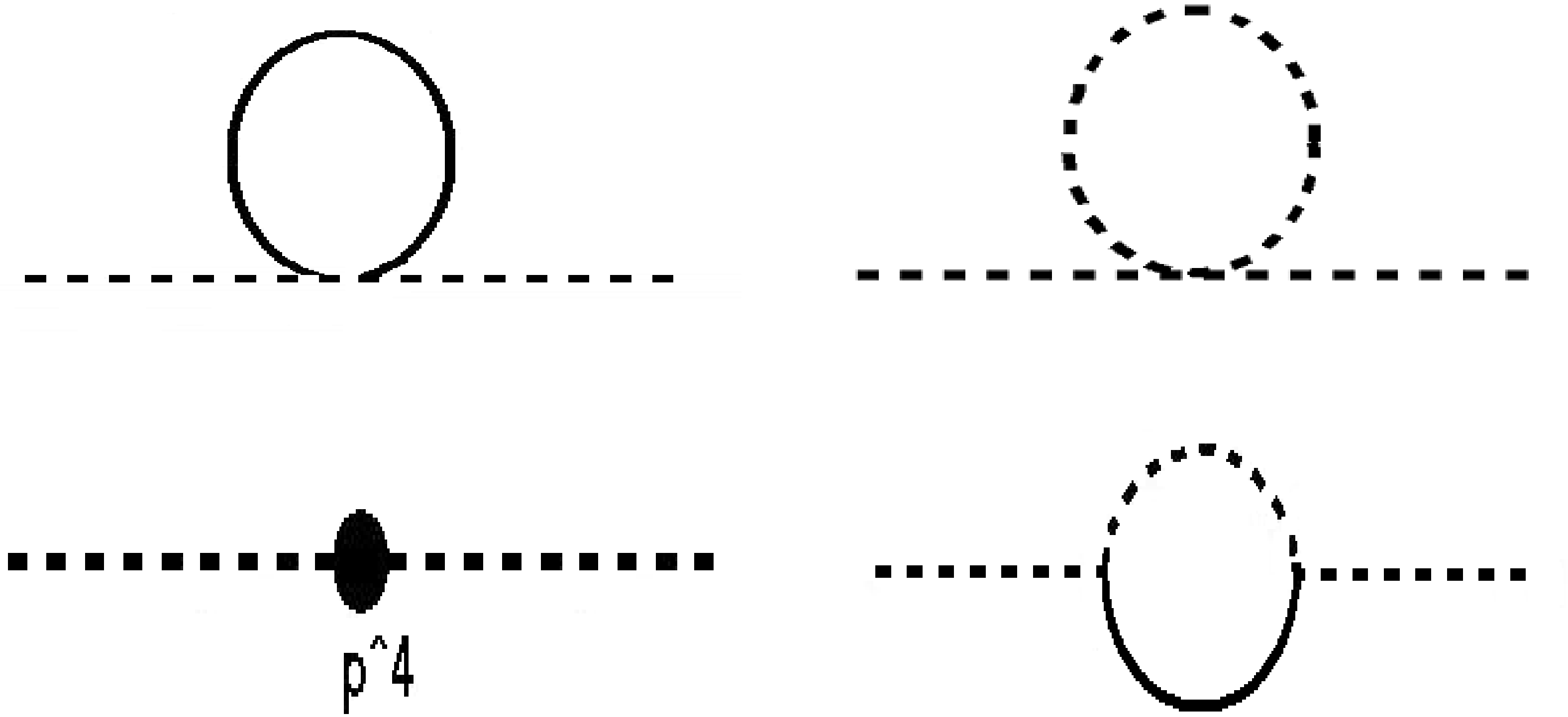
The calculations

- A considerable number of new LECs appear
- Using lattice data will be necessary to fix them up

The calculations

- A considerable number of new LECs appear
- Using lattice data will be necessary to fix them up
- Plan:
  - Calculate $m_\pi(m_q)$
  - Calculate $f_\pi(m_q)$
  - Compare with lattice data

$m_\pi(m_q)$



Dotted line: pion propagator , Solid line: scalar propagator

$m_\pi(m_q)$

$$
\begin{aligned}
m^2_{\pi,phys} \quad = \quad & m^2_\pi \left(1 - \frac{A[m^2_\pi]}{2F^2} + \frac{2m^2_\pi}{F^2} l_3 + (c_1 - c_2)\, \frac{4A[m^2_\sigma]}{F^2} \right) \\
& + \frac{4F_1}{F^4} \left(\left(m^2_\sigma - 4m^2_\pi \right) F_1 + 4m^2_\pi F_2 \right) A[m^2_\sigma] \\
& - \frac{4}{F^4} \left(\left(m^2_\sigma - 2m^2_\pi \right) F_1 + 2m^2_\pi F_2 \right)^2 B[m^2_\pi, m^2_\sigma, m^2_\pi] \\
& - \frac{4F_1}{F^4} \left(\left(m^2_\sigma - 2m^2_\pi \right) F_1 + 4m^2_\pi F_2 \right) A[m^2_\pi] + Z_1 \frac{m^2_\pi m^2_\sigma}{F^2}
\end{aligned}
$$

$$
A[m^2] = \frac{m^2}{16\pi} \left(\lambda_0 - \ln\left(\frac{m^2}{\mu^2} \right) \right) \;, \quad \lambda_0 = \frac{1}{\epsilon} - \gamma_e + \ln(4\pi) + 1
$$

$$
\begin{aligned}
B[m^2_1, m^2_2, m^2_1] = & \frac{1}{16\pi^2} \left(\lambda_0 + 1 - \ln\left(\frac{m^2_1}{\mu^2} \right) - \frac{m^2_2}{2m^2_1} \ln\left(\frac{m^2_2}{m^2_1} \right) \right. \\
& \left. + \frac{m^2_2}{m^2_1} \sqrt{1 - \frac{4m^2_1}{m^2_2}} ArcTanh\left(\sqrt{1 - \frac{4m^2_1}{m^2_2}} \right) \right)
\end{aligned}
$$

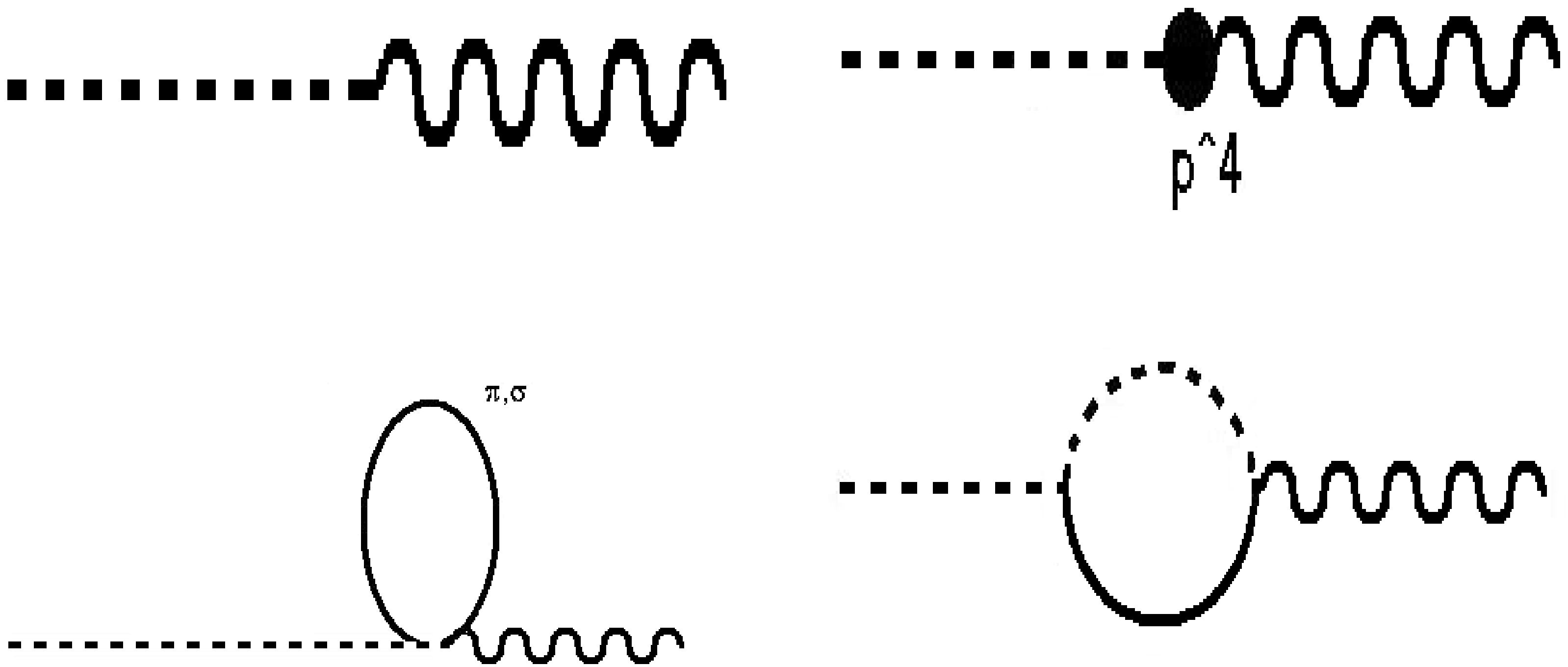
$m_\pi(m_q)$

Comments:

- l_3 absorbs new divergences from loops involving the scalar particle.
- A couterterm proportional to B_0 (Z_1) must be included.
- In the chiral limit, $m^2_{\pi,phys} \to 0$.

$$m^2_{\pi,phys} \to \frac{-4F_1^2 m_\sigma^4}{F^4} B[0, m_\sigma^2, 0] + \frac{4F_1^2 m_\sigma^2}{F^4} A[m_\sigma^2] = 0$$

$f_\pi(m_q)$



Wavy line: axial-vector current

$f_\pi(m_q)$

$$
\begin{aligned}
F_\pi \quad = \quad & F\Bigg(1+\frac{l_4 m_\pi^2}{F^2}+\frac{A[m_\pi^2]}{F^2}+\frac{4m_\sigma^2 Z_2}{F^2}\\
&+\frac{4F_1}{F^4 m_\pi^2}(m_\pi^2-m_\sigma^2)(F_1(2m_\pi^2-m_\sigma^2)-2F_2 m_\pi^2)B[m_\pi^2,m_\sigma^2,m_\pi^2]\\
&-\frac{2}{F^4 m_\pi^2}\left(c_1 F^2 m_\pi^2+F_1(F_1(2m_\sigma^2-5m_\pi^2)+4F_2 m_\pi^2)\right)A[m_\sigma^2]\\
&+\frac{1}{F^4 m_\pi^2}\left(2F_1(F_1(2m_\sigma^2-3m_\pi^2)+4F_2 m_\pi^2)\right)A[m_\pi^2]\\
&-\frac{2}{F^4}(2F_2 m_\pi^2+F_1(m_\sigma^2-2m_\pi^2))^2\left.\frac{dB[m_\pi^2,m_\sigma^2,p^2]}{dp^2}\right|_{p^2=m_\pi^2}\Bigg)
\end{aligned}
$$

$$
\left.\frac{dB[m_1^2,m_2^2,p^2]}{dp^2}\right|_{p^2=m_1^2}=-\frac{1}{16\pi^2 m_1^2}\Bigg(1+\frac{(m_1^2-m_2^2)}{2m_1^2}\ln\left(\frac{m_2^2}{m_1^2}\right)+
$$

$$
+\frac{m_2^2}{m_1^2}\sqrt{1-\frac{4m_1^2}{m_2^2}}\,ArcTanh\left(\sqrt{1-\frac{4m_1^2}{m_2^2}}\right)+\frac{1}{\sqrt{1-\frac{4m_1^2}{m_2^2}}}ArcTanh\left(\sqrt{1-\frac{4m_1^2}{m_2^2}}\right)\Bigg)
$$

$f_\pi(m_q)$

Comments:

- l_4 absorbs new divergences from loops involving the scalar particle.
- A couterterm proportional to F^2 (Z_2) must be included.
- In the chiral limit, F_π^2 remains finite.

Decoupling

- When $m_\sigma \gg m_\pi$ one should recover the standard χPT results

Decoupling

- When $m_\sigma \gg m_\pi$ one should recover the standard χPT results
- One actually does, with the following redefinitions:

$$B_0 \to B_0 \left(1 - 4\left(\frac{c_2 - c_1}{F^2} + \frac{F_1^2}{F^4}\right) A[m_\sigma] + \frac{4 Z_1 m_\sigma^2}{F^2} - \frac{F_1^2 m_\sigma^2}{8\pi^2 F^4}\right)$$

$$F \to F \left(1 - 2\left(\frac{c_1}{F^2} - \frac{F_1^2}{F^4}\right) A[m_\sigma] + \frac{4 Z_2 m_\sigma^2}{F^2} - \frac{F_1^2 m_\sigma^2}{16\pi^2 F^4}\right)$$

What does lattice data say?

ETM Collaboration, R. Baron *et al.*, arXiv:0911.5061

	$a\mu_q$	$am_{\rm PS}$	$af_{\rm PS}$	$am_{\rm PCAC}$	r_0/a	L/a
A_1	0.0060	$0.1852(9)$	$0.0770(8)$	$+0.0019(4)$	$4.321(32)$	24
A_2	0.0080	$0.2085(8)$	$0.0835(4)$	$+0.0008(3)$	$4.440(34)$	24
A_3	0.0110	$0.2424(5)$	$0.0892(3)$	$-0.0002(5)$	$4.362(21)$	24
A_4	0.0165	$0.2957(5)$	$0.0969(2)$	$-0.0017(2)$	$4.264(14)$	24
A_5	0.0060	$0.1831(6)$	$0.0784(4)$	$+0.0005(4)$	NA	20
B_1	0.0040	$0.1362(7)$	$0.0646(4)$	$+0.00017(17)$	$5.196(28)$	24
B_2	0.0064	$0.1694(4)$	$0.0705(4)$	$-0.00009(17)$	$5.216(27)$	24
B_3	0.0085	$0.1940(5)$	$0.0742(2)$	$-0.00052(17)$	$5.130(28)$	24
B_4	0.0100	$0.2100(5)$	$0.0759(4)$	$-0.00097(26)$	$5.143(25)$	24
B_5	0.0150	$0.2586(7)$	$0.0830(3)$	$-0.00145(42)$	$5.039(24)$	24
B_6	0.0040	$0.1338(2)$	$0.0663(2)$	$+0.00022(11)$	$5.259(21)$	32
B_7	0.0030	$0.1167(4)$	$0.0633(3)$	$+0.00030(14)$	NA	32

$$A_i\text{: } \beta = 3.8 \quad , \quad B_i\text{: } \beta = 3.9$$

What does lattice data say?

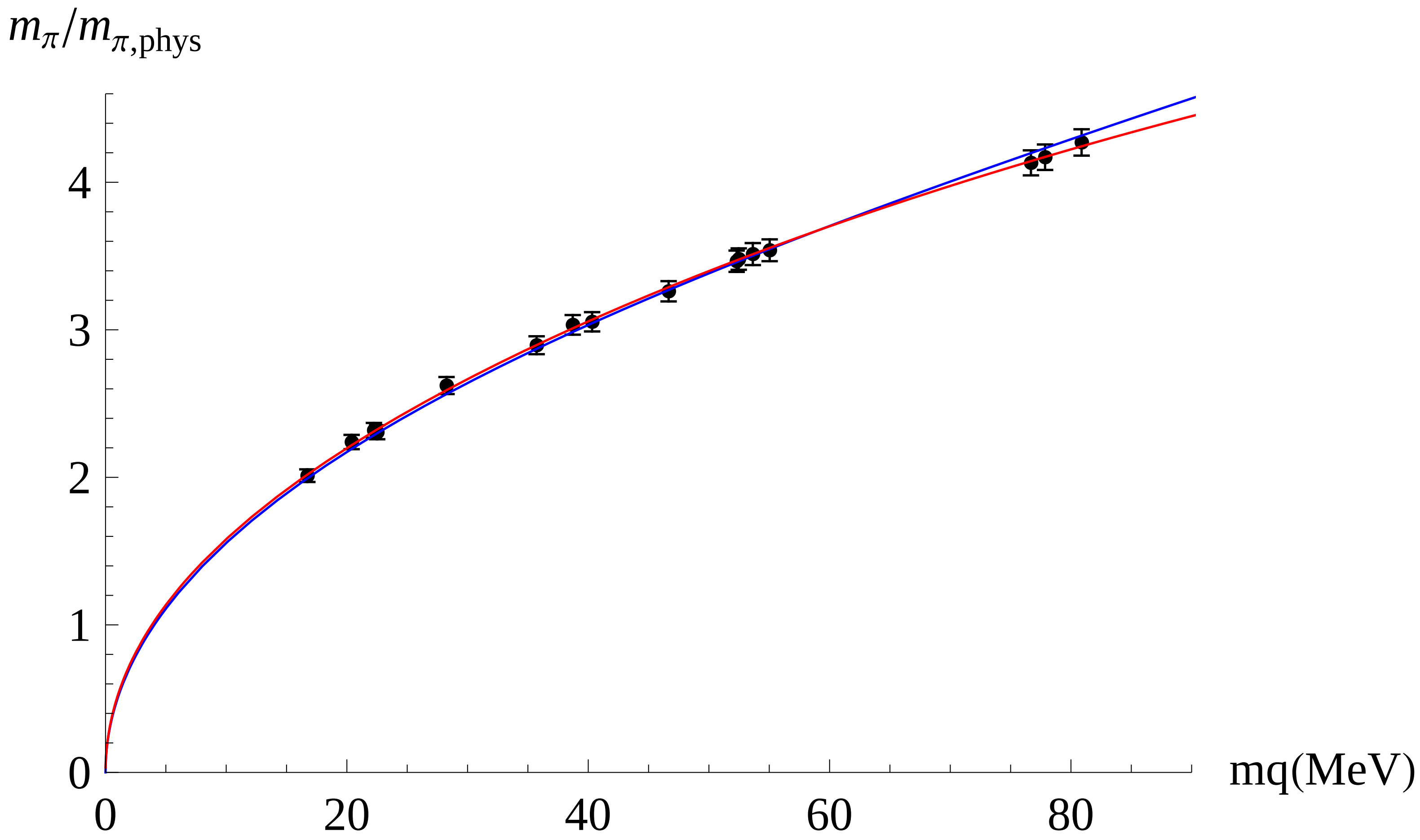
ETM Collaboration, R. Baron *et al.*, arXiv:0911.5061

	$a\mu_q$	$am_{\rm PS}$	$af_{\rm PS}$	$am_{\rm PCAC}$	r_0/a	L/a
C_1	0.003	0.1038(6)	0.0500(4)	+0.00036(14)	6.584(34)	32
C_2	0.006	0.1432(6)	0.0569(2)	−0.00004(14)	6.509(38)	32
C_3	0.008	0.1651(5)	0.0595(2)	−0.00065(13)	6.494(36)	32
C_4	0.012	0.2025(6)	0.0644(2)	−0.00092(14)	6.284(22)	32
C_5	0.006	0.1448(11)	0.0558(5)	−0.00027(19)	NA	24
C_6	0.006	0.1520(15)	0.0508(5)	+0.00002(20)	NA	20
D_1	0.0020	0.0740(3)	0.0398(2)	+0.00006(6)	8.295(45)	48
D_2	0.0065	0.1326(5)	0.0465(3)	−0.00032(11)	8.008(29)	32

C_i: $\beta = 4.05$, D_i: $\beta = 4.20$

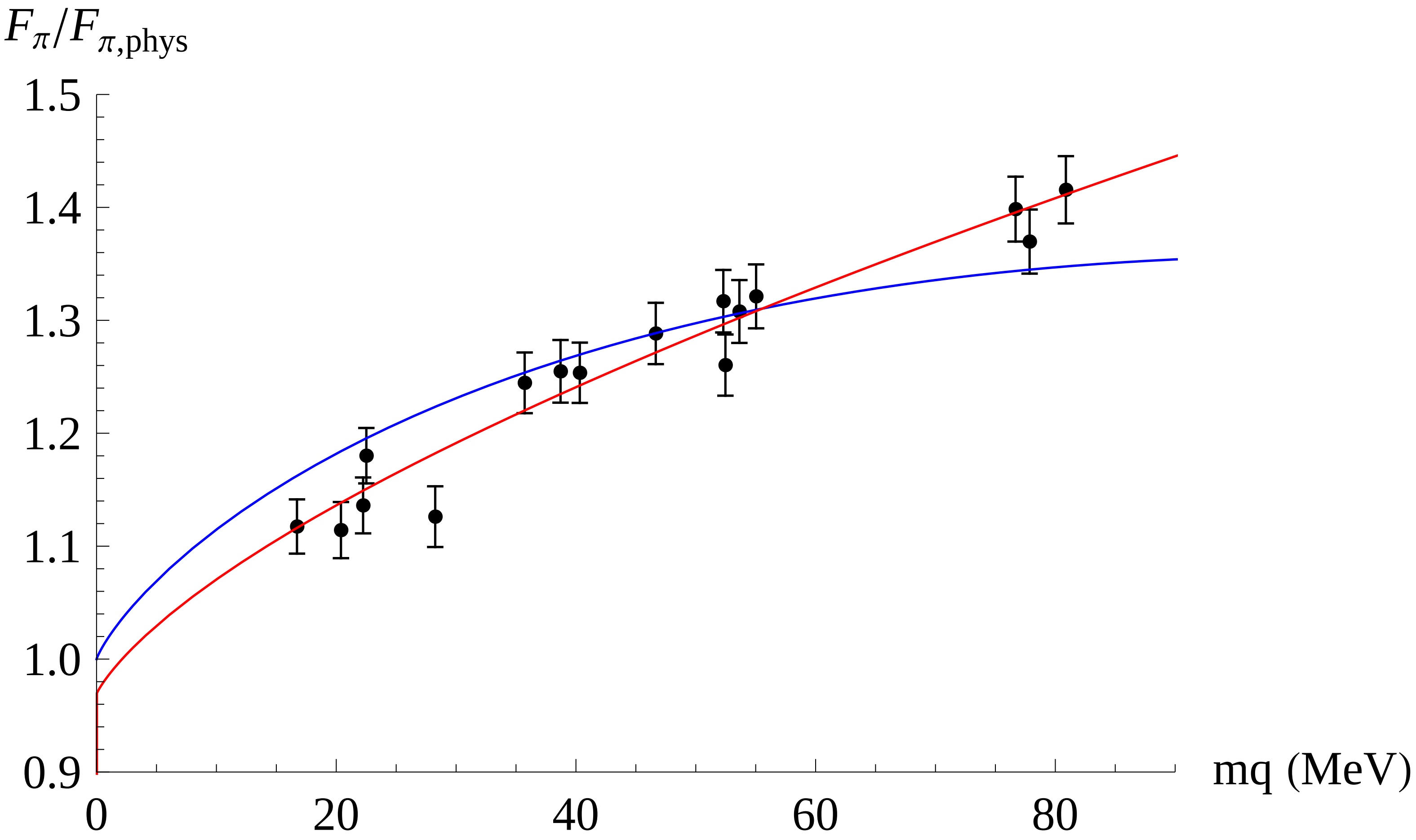
- Chiral extrapolations in χPT and χPT+σ, (Schindler, Phillips (08)).

$m_\pi(m_q)$



Blue line: χPT Red line: χPT+σ

$f_\pi(m_q)$



Blue line: χPT Red line: χPT+σ

Results from the fit

	χPT	χPT+σ
χ^2/dof	3.01	0.8733
F	$92MeV$	$92MeV$
m_σ^{phys}		$400MeV$
B_0	$2313MeV$	$1683MeV$
$l_3(770)$	$-0.329*10^{-3}$	$9.51*10^{-4}$
$l_4(770)$	$5.02*10^{-3}$	$8.97*10^{-4}$
F_1		$-0.560F$
F_2		$0.460F$
c_1		0.519
c_2		-0.777

Conclusions

Conclusions

- A consistent chiral effective theory including a light scalar can be built.

Conclusions

- A consistent chiral effective theory including a light scalar can be built.
- It leads to distinct quark mass dependence of physical observables, which for m_π and f_π is:

Conclusions

- A consistent chiral effective theory including a light scalar can be built.
- It leads to distinct quark mass dependence of physical observables, which for m_π and f_π is:
  - Compatible with lattice data,

Conclusions

- A consistent chiral effective theory including a light scalar can be built.
- It leads to distinct quark mass dependence of physical observables, which for m_π and f_π is:
  - Compatible with lattice data,
  - Favoured with respect to standard χPT.

Conclusions

- A consistent chiral effective theory including a light scalar can be built.
- It leads to distinct quark mass dependence of physical observables, which for m_π and f_π is:
  - Compatible with lattice data,
  - Favoured with respect to standard χPT.
- It may lead, in combination with lattice data, to an improved description of low energy QCD, in particular in the scalar sector.