NA48/NA62 Results and Perspectives

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NA48/NA62 Results and Perspectives

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Outline

• NA48/2-Experiment.

• $\mathbf{K}^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu$ (K_{e4})

- Form factor and phase shift.
- $\pi\pi$ scattering lengths a_0^0 and a_0^2 .

• $\mathbf{K}^{\pm} \to \pi^{\pm} \pi^0 \gamma \quad (K_{2\pi(\gamma)})$

- DE and INT fraction.
- Test of CPV.
- $\mathbf{K}^{\pm} \rightarrow \pi^{\pm} \gamma \gamma$
 - Branching ratio.
- NA62 Perspectives
- Summary and outlook

NA48/2-Experiment

- NA48/2: is a fixed target experiment in the North Area of the SPS
- 400 GeV protons from the SPS
- Simultaneous K^{\pm} -Beam with $p_{K^{\pm}} = (60 \pm 3) \text{ GeV}$
- 6.3×10^7 particles per pulse





NA48/2-Experiment

Main detector components:

Magnetic Spectrometer

 $\frac{\sigma_p}{p} = 1.02\% \oplus 0.044\% \frac{p}{\text{GeV}/c}$ ~ 1% resolution for charged particles with p=20 GeV/c

Liquid Krypton EM Calorimeter

 $\frac{\sigma_E}{E} = \frac{3.2\%}{\sqrt{E/\text{GeV}}} \oplus \frac{9.0\%}{E/\text{GeV}} \oplus 0.42\%$

1.4% resolution for particles with E=20 GeV

Main trigger modes:

- Charged trigger: 3 charged tracks.
- Neutral trigger: >=2 em cluster in LKr x or y projection



Muon veto sytem

Hadron calorimeter

Liquid krypton calorimeter

Hodoscope

In 2003 and 2004 total of 18×10^9 triggers collected in 110 days

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K_{e4} : Theory

Decay characteristics:

• **4-body decay:** 5 independent parameters Cabibbo-Maksymowicz variables: $S_{\pi}(M_{\pi\pi}^2), S_e(M_{e\nu}^2), \cos \theta_{\pi}, \cos \theta_{e}, \Phi$



$$\frac{\text{Transition amplitude:}}{\langle \pi^{+}\pi^{-}|A^{\lambda}|K^{+}\rangle} = \frac{G_{w}}{\sqrt{2}} V_{us}^{*} \langle \pi^{+}\pi^{-}|V^{\lambda} - A^{\lambda}|K^{+}\rangle \bar{u}_{\nu}\gamma_{\lambda}(1-\gamma_{5})v_{e} \\ \langle \pi^{+}\pi^{-}|A^{\lambda}|K^{+}\rangle = \frac{-i}{\sqrt{m_{K}}} \left(F\left(\mathbf{p}_{\pi^{+}} + \mathbf{p}_{\pi^{-}}\right)^{\lambda} + G\left(\mathbf{p}_{\pi^{+}} - \mathbf{p}_{\pi^{-}}\right)^{\lambda} + R\left(\mathbf{p}_{e} + \mathbf{p}_{\nu}\right)^{\lambda} \right) \\ \langle \pi^{+}\pi^{-}|V^{\lambda}|K^{+}\rangle = \frac{-H}{m_{K}^{3}} \epsilon^{\lambda\mu\rho\sigma} \left(\mathbf{p}_{\pi^{+}} + \mathbf{p}_{\pi^{-}} + \mathbf{p}_{e} + \mathbf{p}_{\nu}\right)_{\mu} \times \left(\mathbf{p}_{\pi^{+}} + \mathbf{p}_{\pi^{-}}\right)_{\rho} \left(\mathbf{p}_{\pi^{+}} - \mathbf{p}_{\pi^{-}}\right)_{\sigma}$$

Axial form factors : F G R (no R contribution in K_{e4} because it is proportional to m_l^2) Vector form factor : H

Partial wave expansion:Expansion in
$$q^2$$
 and S_e/m_{π}^2 : $F = F_s e^{i\delta_s} + F_p e^{i\delta_p} \cos \theta_{\pi} + d$ wave... $F_s = f_s + f'_s q^2 + f''_s q^4 + f'_e S_e/4m_{\pi}^2 + \dots$ $G = G_p e^{i\delta_g} + d$ wave... $F_p = f_p + f'_p q^2 + \dots$ $H = H_p e^{i\delta_h} + d$ wave... $G_p = g_p + g'_n q^2 + \dots$ $H = h_p e^{i\delta_h} + d$ wave... $H_p = h_p + h'_p q^2 + \dots$ Fit parameters: $F_s, F_p, G_p, H_p, \delta$ $\delta(q^2) = \delta_s - \delta_p$.

K_{e4} : Selection and Background rejection

K_{e4} selection:

3 charged tracks and a good vertex
2 opposite sign pions, 1 electron (E/P~1)
missing transverse momentum and energy
reconstructed Kaon energy (E_ν = |**p**_ν|)

Background:

• $K^{\pm} \rightarrow \pi^{\pm} \pi^{+} \pi^{-}$ decay with pion decay, or pion misidentification as an electron.

• $K^{\pm} \rightarrow \pi^{\pm} \pi^{0}(\pi^{0})$ decay followed by the dalitz decay of a pion, a misidentification of the electron as a charged pion and an undetected photons.

ellipse cut in $M_{3\pi} - p_t$ plane.

linear discriminant variables based on shower properties.

 estimation of the background using wrong-sign event rates in the data.

background/signal-ratio: 0.6% (2 WS/RS).



K_{e4} : fitting procedure

full event sample (2003 and 2004): **1.13 million** K_{e4} decays

Using **iso-populated boxes** in the 5-dimensional space of the Ca.Ma. variables, one defines a grid of:

 $10(M_{\pi\pi}) \times 5(M_{e\nu}) \times 5(\cos\theta_e) \times 5(\cos\theta_\pi) \times 12(\phi) = 15000$ Boxes

The set of Form Factor values is used to minimise a <u>log-likelihood estimator</u> well suited for small numbers of data event/bin and taking into account the statistics of the simulation (simulated and expected events/bin)

Assuming constant Form Factors over single boxes, K^+ and K^- samples fitted separately in **10 independent** $M_{\pi\pi}$ **bins/slices** and then combined in each slice according to their statistical error

<u>Data</u>	K^+ sample: (726400 events) K^- sample: (404400 events)	48 events/box 27 events/box
<u>MC</u>	K^+ sample: (17.4 million events) K^- sample: (9.7 million events)	1160 events/box 650 events/box

K_{e4} : Data/MC-compairison after the fit



• Background (in yellow) X 10 to be visible.

• CP symmetry shown in the opposite distribution of $\phi(K^+)$ and $\phi(K^-)$.

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K_{e4} : form factor fit results

• All the form factors are measured relatively to f_s (no measurement of the branching ratio yet).

 Systematics from background, electron-ID and acceptance control and about same size as the statistical errors or smaller.

• first evidence of $f'_e \neq 0$ and $f_p \neq 0$.

 $\begin{aligned} \mathbf{f}_{s}^{'}/f_{s} &= 0.152 \pm 0.007_{\text{stat}} \pm 0.005_{\text{syst}} \\ \mathbf{f}_{s}^{''}/f_{s} &= -0.073 \pm 0.007_{\text{stat}} \pm 0.006_{\text{syst}} \\ \mathbf{f}_{e}^{'}/f_{s} &= 0.068 \pm 0.006_{\text{stat}} \pm 0.007_{\text{syst}} \\ \mathbf{f}_{p}^{'}/f_{s} &= -0.048 \pm 0.003_{\text{stat}} \pm 0.004_{\text{syst}} \\ \mathbf{g}_{p}^{'}/f_{s} &= 0.868 \pm 0.010_{\text{stat}} \pm 0.010_{\text{syst}} \\ \mathbf{g}_{p}^{'}/f_{s} &= 0.089 \pm 0.017_{\text{stat}} \pm 0.013_{\text{syst}} \\ \mathbf{h}_{p}^{'}/f_{s} &= -0.398 \pm 0.015_{\text{stat}} \pm 0.008_{\text{syst}} \end{aligned}$

K_{e4} : δ -dependence and $\pi\pi$ scattering lengths

• The extraction of pion scattering lengths from the fitted $\delta = \delta_s - \delta_p$ phase shift needs external theoretical and experimental inputs:

•The **Roy equations** provide the relation between δ and a_0 and a_2 (as two subtraction const.) the S-wave scattering (isospin sym. assumption).

• Extrapolating experimental data from the $M_{\pi\pi} > 0.8 \text{ GeV/c}^2$ it's possible to fit the result in the threshold region (the uncertainty from the experimental data defines the **Universal Band**)

PRELIMINARY 0.4 0.4 0.3 2 parameters fit 0.3 ð (rad) 0.2 (pg 0.2 0.1 o uncorrected A48/2 (2003 Data) ··· 0.1 isospin corrected 0 E865 Geneva-Saclay -0.1 0 0.36 0.38 0.3 0.32 0.28 0.34 0.4 0.34 0.3 0.32 0.36 5 0.38 Μ_{ππ}(GeV/c²) 0.28 $M_{\pi\pi}(GeV^2/c^2)$

• The fit of the experimental points using the Roy equations allows to extract the a_0 and a_2 values.

• Isospin corrections (generated by the pion and quark mass differeces) result in a 11 to 15 mrad shift in δ can not be ignored. (Gasser et. al. Eur. Phys. J. C59 (2009) 777)

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(Ananthanarayan, Colangelo, Gasser, Leutwyler Phys. Rept. 353 (2001) 207-279) (Descotes-Genon, Fuchs, Girlanda, Stern Eur. Phys. J. C24 (2002) 469-483) (Kaminski, Pelaez, Yndurain Phys. Rev. D77 (2008))

K_{e4} : δ -dependence and $\pi\pi$ scattering lengths

two parameter fit:

correlation 97%)

 $\begin{array}{l} a_0 = & 0.2220 \pm 0.0128_{\rm stat} \pm 0.0050_{\rm syst} \pm 0.0037_{\rm theo} \\ a_2 = & -0.0432 \pm 0.0086_{\rm stat} \pm 0.0034_{\rm syst} \pm 0.0028_{\rm theo} \end{array}$

Single parameter fit:

 $a_2 = -0.0444 \pm 0.0010$

(a_2 constraint to ChPT prediction)

 $a_0 = 0.2206 \pm 0.0049_{\text{stat}} \pm 0.0018_{\text{syst}} \pm 0.0064_{\text{theo}}$

Theory prediction: (CGL NPB603(2001), PRL86(2001)) $a_0 = 0.2220 \pm 0.005$

NA48/2 Ke4 (2003-2004) PRELIMINARY പ്പ^{0.01} universal band -0.02 theory -0.03 -0.04 -0.05 **ChPT** -0.06 two para fit single para fit 0.2 0.22 0.19 0.21 0.23 0.24 0.25 0.26 a0

Systematic errors from background estimation and electron-ID.

 Theoretical error evaluated from control of isospin corrections & inputs to Roy equation numerical solutions.

NA48/2: K_{e4} and Cusp Results combined

Two independent measurements:

- 60 millions $K_{3\pi}$ -events.
- 1.13 millions K_{e4} -events.

Different systematics:

- Cusp: calorimeter and trigger.
- K_{e4} : electron mis-id and background.

Different theoretical inputs:

Cusp: (1 & 2-loop) re-scattering models.

(Cabibbo,Isidori JHEP 0503 (2005) 21) (Bissegger, Fuhrer, Gasser, Kubis, Rusetsky, NPH B806 (2009) 178)

• K_{e4} : Roy equation and isospin breaking corrections.

Combined results:

 $\begin{array}{ll} a_0 = & 0.2210 \pm 0.0047_{\rm stat} \pm 0.0040_{\rm syst} \\ a_2 = & -0.0429 \pm 0.0044_{\rm stat} \pm 0.0028_{\rm syst} \end{array}$

 $a_0 - a_2 = 0.2639 \pm 0.0020_{\text{stat}} \pm 0.0015_{\text{syst}}$ $a_2 = -0.0429 \pm 0.0044_{\text{stat}} \pm 0.0028_{\text{syst}}$





$K_{2\pi\gamma}$: Theory

Decay characteristics: two sources of γ radiation.



$K_{2\pi\gamma}$: Data sample

New NA48/2 measurement:

- Simultaneous K^+/K^- beams ($\Box > CPV$ test possible).
- Larger T_{π}^* region available: $0 \text{ MeV} < T_{\pi}^* < 80 \text{ MeV}$
- Background < 0.01 %(negligible) (mainly from $K^{\pm} \rightarrow \pi^{\pm} \pi^{0} \pi^{0}$)
- γ mistagging probability < 0.1%.

Total $K^{\pm} \rightarrow \pi^{\pm} \pi^{0} \gamma$ data sample:

- more than 1 million events.
- \bullet for the fit: restrict to ~0.2 < W < 0.9~ and $~E_{\gamma} > 5~GeV$
 - \Longrightarrow still 600k $K_{2\pi\gamma}$ candidates.



$K_{2\pi\gamma}$: Fit

Extended Maximum Likelihood fit

in bins of $\,W\,\,$ corrected by acceptance with Monte Carlo

$$Data(i) = N_0 \left[(1 - \alpha - \beta) \cdot IB_{MC}(i) + \alpha \cdot INT_{MC}(i) + \beta \cdot DE_{MC}(i) \right]$$

Frac(DE) =
$$(3.32 \pm 0.15) \times 10^{-2}$$

Frac(INT) = $(-2.35 \pm 0.35) \times 10^{-2}$

$$\operatorname{Frac}(\mathrm{DE}) = \frac{\operatorname{Br}(\mathrm{DE})}{\operatorname{Br}(\mathrm{IB})} \qquad \operatorname{Frac}(\mathrm{INT}) = \frac{\operatorname{Br}(\mathrm{INT})}{\operatorname{Br}(\mathrm{IB})} \quad 0 \ \operatorname{MeV} < \mathrm{T}_{\pi}^* < 80 \ \mathrm{MeV}$$



Polynominal fit

Assumes same acceptance as a function of Wfor IB, DE and INT Fit ratio $W(Data)/W(MC_{IB})$ with: $f(W) = c(1 + aW^2 + bW^4)$

Frac(DE) =
$$(3.19 \pm 0.16) \times 10^{-2}$$

Frac(INT) = $(-2.21 \pm 0.41) \times 10^{-2}$

Fit with no interference term shows disagreement.



W_{data}/W_{MC(IB)}

$K_{2\pi\gamma}$: Final result

Final NA48/2 result for
$$K^{\pm} \to \pi^{\pm} \pi^0 \gamma$$
 fractions:

 $\begin{aligned} \mathbf{Frac}(\mathbf{DE})_{0 < T_{\pi}^* < 80 \text{ MeV}} &= (3.32 \pm 0.15_{\text{stat}} \pm 0.14_{\text{syst}}) \times 10^{-2} \\ \mathbf{Frac}(\mathbf{INT})_{0 < T_{\pi}^* < 80 \text{ MeV}} &= (-2.35 \pm 0.35_{\text{stat}} \pm 0.39_{\text{syst}}) \times 10^{-2} \end{aligned}$

Approximation for extracting X_E and X_M :

•
$$\phi = 0$$

• $\cos(\delta_1^1 - \delta_0^2) = \cos 6.6^\circ \approx 1$
• $X_E = \frac{\operatorname{Frac}(\operatorname{INT})}{2 \cdot 0.105 \cdot m_k^2 m_\pi^2} X_M = \sqrt{\frac{\operatorname{Frac}(\operatorname{DE}) - m_k^4 m_\pi^4 |X_E|^2 0.0227}{0.0227 \cdot m_k^4 m_\pi^4}}$



Magnetic and electric component of DE (first measurement):

$$X_E = (-24 \pm 4_{\text{stat}} \pm 4_{\text{syst}}) \text{GeV}^{-4}$$
$$X_M = (244 \pm 6_{\text{stat}} \pm 6_{\text{syst}}) \text{GeV}^{-4}$$

Wess-Zumino-Witten chiral anomaly prediction: $X_M \approx 270 \text{ GeV}^{-4}$ NA48/2 X_M measurement points to WZW reducible anomaly only

$K_{2\pi\gamma}$: CPV studies

$\begin{aligned} \underline{Asymmetry in the total rate:} \\ A_N &= \frac{\Gamma^+ - \Gamma^-}{\Gamma^+ + \Gamma^-} = \frac{N_{\pi^+ \pi^0 \gamma} - R \cdot N_{\pi^- \pi^0 \gamma}}{N_{\pi^+ \pi^0 \gamma} + R \cdot N_{\pi^- \pi^0 \gamma}} \\ \text{with } R &= N_{K^+} / N_{K^-} = 1.7998(4) \text{ from } K^{\pm} \rightarrow \pi^{\pm} \pi^0 \pi^0 \\ A_N &= (0.0 \pm 1.0_{\text{stat}} \pm 0.6_{\text{syst}}) \times 10^{-3} \\ |A_N| &< 1.5 \times 10^{-3} \quad (90\% \text{ CL}) \end{aligned}$

 \Rightarrow First limit on $\sin \phi$: $\sin \phi = -0.01 \pm 0.43$, $|\sin \phi| < 0.56$ (90%CL)

Asymmetry in the Dalitz plot:

using the W spectrum:
$$\frac{d\Gamma^{\pm}}{dW} = \frac{d\Gamma_{\text{IB}}^{\pm}}{dW}(1 + (a \pm e)W^2 + bW^4)$$

$$A_W = e \int \frac{\text{INT}}{\text{IB}} = (-0.6 \pm 1.0) \times 10^{-3}$$

compatible with A_N

$$\Longrightarrow$$
 no CP asymmetry observed in $K^{\pm} \to \pi^{\pm} \pi^0 \gamma$

$K^{\pm} \rightarrow \pi^{\pm} \gamma \gamma$: Theorie

In the Chiral Perturbation Theory framework the differential rate of the decay $K^{\pm}(p) \rightarrow \pi^{\pm}(p_3)\gamma(q_1)\gamma(q_2)$ (no $\mathcal{O}(p^2)$ contribution) is:

$$\begin{aligned} \frac{\partial^2 \Gamma}{\partial y \partial z} &= \frac{m_{K^{\pm}}}{(8\pi)^3} \cdot \left[z^2 \cdot \left(|A + B|^2 + |C|^2 \right) + \left(y^2 - \frac{1}{4} \lambda \left(1, z, r_\pi^2 \right) \right)^2 \cdot \left(|B|^2 + |D|^2 \right) \right] \\ y &= \frac{p \cdot (q_1 - q_2)}{m_{K^{\pm}}^2} \quad z = \frac{(q_1 + q_2)^2}{m_{K^{\pm}}^2} = \frac{m_{\gamma\gamma}^2}{m_{K^{\pm}}^2} \end{aligned}$$

• The leading contribution at $O(p^4)$ is given by $A(z, \hat{c})$ (loops) which is responsible for a cusp at $m_{\gamma\gamma} = m_{2\pi}$.

• C(z) (generated by Wess-Zumino-Witten funktional) corresponds to ~ 10% of A at $\mathcal{O}(p^4)$.

- B and D vanisch at $\mathcal{O}(p^4)$.
- $\mathcal{O}(p^6)$ unitarity corrections can increase the BR by 30 40%

[D'Ambrosio, Portoles, Nucl. Phys. B386 (1996), 403]



$K^{\pm} \rightarrow \pi^{\pm} \gamma \gamma$: Branching Fraction

1164 $K^{\pm} \rightarrow \pi^{\pm} \gamma \gamma$ candidates in 40% of NA48/2 data.

 Data Data MC $K^{\pm} \rightarrow \pi^{\pm} \gamma$ MC $K^{\pm} \rightarrow \pi^{\pm} \pi^{0}$ MC $K^{\pm} \rightarrow \pi^{\pm} \pi^{0}$ 140 100 120 80 100 60 80 60 40 40 20 20 0.485 0.49 0.495 0.3 0.475 0.5 0.505 0.51 0.26 0.28 0.32 0.48 0.2 0.22 0.34 m_{avy} [GeV/c²] m_{vv} [GeV/c²]

(about 40 times more than previous world statistic)

• **Background:** 3.3 % mainly from $K^{\pm} \rightarrow \pi^{\pm} \pi^{0} \gamma$

• Systematics: mainly from trigger efficiency determination.

Assume ChPT
$$\mathcal{O}(p^6)$$
 and $\hat{c} = 2$ preliminary
 $\mathbf{Br}(K^{\pm} \to \pi^{\pm} \gamma \gamma)_{\hat{c}=2, \mathcal{O}(p^6)} = (1.07 \pm 0.04_{\text{stat}} \pm 0.08_{\text{syst}}) \times 10^{-6}$

• Model independent measurement and \hat{c} extraction in preparation.

0.36

NA62 Perspectives

<u>Main purpose:</u> $K^+ \to \pi^+ \nu \bar{\nu}$

- \bullet Strongly suppressed $(<10^{-10})$
- Theoretically clean calculation.
- almost unexplored by experiment.
- sensitive to new physics.

Experimental aim:

- $\mathcal{O}(100)$ events in two years of data taking.
- 10% background.

Background rejection:

• Kinematical rejection.

Beam tracker: P_K Straw tracker: P_{π}

Partical ID.

Calorimeters: γ

CEDAR: Kaon identification RICH / MUV: $\pi/\mu/e\,$ separation

	$Br(K^+ \to \pi^+ \nu \bar{\nu})[10^{-11}]$
Standard Model	8.0 ± 1.1
MFV (hep-ph/0310208)	19.1
EEWP (NP B697 133)	7.5 ± 2.1
EDSQ (hep-ph/0407021)	15
$MSSM ~({\rm hep-ph}/0408142)$	40
BNL E787/E949 (7 events)	$17.3^{+11.5}_{-10.5}$





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- Distribution defines a signal region. • $K^{\pm} \rightarrow \pi^{\pm} \pi^{0}$ forces to split the region into two parts (**Region I and Region II**).
- Span across the whole signal region.
- Rejection relies on γ veto and PID.

NA62 : prospects for rare decays and other goals



much higher statistics possible.

 \bullet much smaller background because of major detector improvements (γ veto and PID).

Other physics goals:

- Lepton Flavour Violation: measurement of $K_{e2}/K_{\mu 2}$ to ~0.1% precision.
- LFV in forbidden decays: searches for $K^+ \to \pi^- l^+ l^+ \quad K^+ \to \pi^+ e^\pm \mu^\mp$
- Heavy neutrinos (~100 MeV), light sgoldstinos.
- Hadronic K decays and final-state $\pi\pi$ interactions in $K_{3\pi}$ and K_{e4} decays.
- ChPT tests with rare kaon/pion decays.

<u>Trigger issues:</u>

bandwidth for physics still do decide.

Summary and outlook

• NA48/2 results of rare kaon decays:

 $\mathbf{K}^{\pm} \to \pi^{+} \pi^{-} e^{\pm} \nu \quad (K_{e4})$

• form factor measured relatively to f_s .

• first evidence of $f'_{e} \neq 0$ and $f_{p} \neq 0$.

• determination of the $\pi\pi$ scattering length with the combination of K_{e4} and the cusp result. The achieved precision is competitive with the best theoretical prediction.

$\mathbf{K}^{\pm} \to \pi^{\pm} \pi^0 \gamma \ (K_{2\pi(\gamma)})$

• first measurement of the INT component in $K_{2\pi\gamma}$ decays. • compatible with CP conservation.

$\mathbf{K}^{\pm} \to \pi^{\pm} \gamma \gamma$

• presentation of a Branching ratio measurement in $\mathcal{O}(p^6)$ and $\hat{c}=2$.

• Model independent measurement and \hat{c} extraction in preparation.

looking forward to NA62 possibilities in rare decays and new physics.

Spares

Ke4 charged decays : isospin corrections to δ

CGR EPJ C59 (2009) 777 formulation developed in close contact with NA48



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Spares

CUSP effect:

• presence of a structure at the M π + π - threshold value in the M2 π 0 π 0 distribution of K $\rightarrow \pi \pi$ 0 π 0 decay

 NA48/2 data provide an ALTERNATIVE way to estimate pion scattering lengths, to be compared with the result obtained in Ke4

• Effect due to the strong $\pi+\pi-\rightarrow\pi0\pi0$ rescattering in the K $\rightarrow\pi$ $\pi+\pi-$ decay

