AdS/QCD and χSB: a view from the tensor side

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Outline

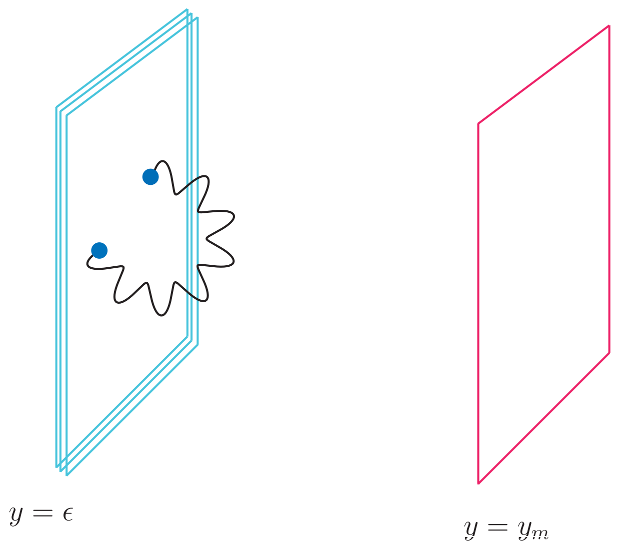
- Motivation
- 2-forms and the AdS/CFT correspondence
- Phenomenological aspects
- Conclusions

The setting

- AdS$_5$ space with metric

$$ds^2 = g_{MN} dx^M dx^N = \frac{1}{y^2}(-dy^2 + \eta_{\mu\nu} dx^\mu dx^\nu)$$

with $\eta_{\mu\nu}$ mostly negative.



- Intuitively, UV boundary captures pQCD (leading term trivial: conformal limit reproduced by AdS metric). IR deals with non-perturbative physics (OPE and non-OPE, confinement scale, SχSB, spectrum patterns)

Gauge/string duality

- Type IIB string theory in a $AdS_5 \times \mathcal{M}_5$ background is dual to $\mathcal{N} = 4$ SYM on a 4-d flat slice. [Maldacena'97]
- Large 't Hooft limit

$$\lambda = g^2 N_c >> 1$$

corrreponds to small curvature in string theory. In this semiclassical limit, supergravity in 10 dimensions, and the duality, in the sense

$$S_{SUGRA}[\phi_0] = \int d^5x\, \mathcal{L}_{5D}(\phi_0) = \int d^4x\, (\mathcal{L}_{4D} + \phi_0 O_i) = S_{4D} + \text{sources}$$

is manageable.

- The dictionary:

gauge theory	gravity
Source J_μ	On-shell $\phi_0(x)$
conformal dimension Δ_ϕ	mass m_ϕ
global symmetry	gauge symmetry
conserved current	gauge field

Gauge/string duality

- **Masses**, p-forms and conformal dimensions are related by

$$m_p^2 = (\Delta - p)(\Delta + p - d)$$

For phenomenological purposes, sources are chosen so one knows Δ from the start. Typical Dirac bilinears: $\Delta = 3$. Then, for instance,

$$\begin{aligned} m_0^2 &= -3\ ; & J &= \bar{q}q \\ m_1^2 &= 0\ ; & J_\mu &= \bar{q}\gamma_\mu q \\ m_2^2 &= 1\ ; & J_{\mu\nu} &= \bar{q}\sigma_{\mu\nu}q \end{aligned}$$

- Well-defined **prescription for correlators**:[Gubser et al'97; Witten'98] Close to the UV boundary

$$\lim_{y\to 0} \phi(x,y) = y^{d-\Delta-p}\phi_0(x)$$

where $\phi_0(x)$ is the gauge theory source. Then,

$$\langle O_i O_j \rangle = -\frac{\delta^2}{\delta\phi_0^{(i)}\delta\phi_0^{(j)}} S$$

where S is the on-shell action (boundary action).

Motivation

- **Gravity side**: AdS/CFT correspondence naturally provides 2-form fields in the gravity side. Impact on holographic QCD unexplored.

- **Gauge theory side**: Rich phenomenology of tensor currents $J_{\mu\nu} = \bar{q}\sigma_{\mu\nu}q$ in QCD for nucleon structure.

  (a) The *quark condensate magnetic susceptibility* χ_0, defined as [Ioffe et al.'84]

$$\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle_F = e_q \chi_0 \langle\bar{q}q\rangle F_{\mu\nu} \quad \text{or} \quad \lim_{q^2\to 0} \Pi_{VT}(q^2) = -\chi_0\langle\bar{q}q\rangle$$

  Relevant in scattering of nucleons off a magnetic field and in $(g-2)_\mu$;

  (b) Also, lattice interest in the ratio [Becirevic et al'03]

$$\xi_n = \frac{f_{Vn}^{\perp}}{f_{Vn}} \qquad \langle\, 0\,|\,\bar{q}\sigma_{\mu\nu}q\,|\rho_n(p,\lambda)\rangle = i f_{Vn}^{\perp}(\epsilon_\mu^{(\lambda)}\, p_\nu - \epsilon_\nu^{(\lambda)}\, p_\mu)$$

  Relevant for precise determinations of $|V_{ub}|$.

Motivation

- Some of the **values/patterns unexplained**. For instance, one can show that asymptotically (in large-N_c) [O.C.,Mateu'08]

$$\lim_{n\to\infty} \xi_n = \frac{(-1)^n}{\sqrt{2}}$$

while the lattice reports $\xi_\rho = 0.72(2)$.

- **Higher level of constraint**: In hadronic QCD models one works at the correlator level. Gauge/string duality works instead at the Lagrangian level. Extremely interesting for the bootstrapped Π_{VV}, Π_{VT} and Π_{TT} correlators.

- **Ideal laboratory for infrared effects**. AdS/CFT is not AdS/QCD. In general the infrared is model-dependent. There exists different models for holographic QCD, dealing differently with issues like confinement or chiral symmetry breaking (SχSB). Bootstrap correlators offer non-trivial consistency checks. In particular, Π_{VT} is very sensitive to chiral symmmetry breaking effects.

Warming up: 1-forms

- **Minimal content**: Proca action in 5d.

$$
\begin{aligned}
S &= \frac{\lambda}{2}\int_{AdS_5}\left[dH\wedge {}^*dH + m^2 H\wedge {}^*H\right] \,, \\
&= \lambda\int d^5x\sqrt{g}\,\mathrm{Tr}\left[-\frac{1}{2}F_{MN}F^{MN} + m^2 V_N V^N\right] \,.
\end{aligned}
$$

- If V_M is to be associated with $J_\mu = \bar{q}\gamma_\mu q$, then $m^2 = 0$. Consequence: V_M is a gauge field and gauge fixing needed. Typically, $V_5 = 0$. For the time being, I'll keep $m^2 \neq 0$ and $V_5 \neq 0$.

- Splitting V_M into V_μ and V_5 the equations of motion read

$$
\begin{aligned}
\left(\Box - \frac{m^2}{y^2}\right)V_5 &= \partial_y\partial_\mu V^\mu \\
\left(\partial_y^2 - \frac{1}{y}\partial_y - \Box + \frac{m^2}{y^2}\right)V_\mu &= y\partial_y\left(\frac{1}{y}\partial_\mu V_5\right) - \partial_\mu\partial_\nu V^\nu
\end{aligned}
$$

- Using the consistency condition $d^*H = 0$ ($m^2 \neq 0!$):

$$\left(\partial_y^2 - \frac{3}{y}\partial_y - \Box + \frac{3+m^2}{y^2}\right) V_5 = 0$$
$$\left(\partial_y^2 - \frac{1}{y}\partial_y - \Box + \frac{m^2}{y^2}\right) V_\mu = \frac{2}{y}\partial_\mu V_5$$

V_5 gets decoupled but adds a particular solution to V_μ.

- Solutions:

$$V_5(y) = y^2\big[AJ_1(py) + BY_1(py)\big]$$
$$V_\lambda(y) = y\big[A_\lambda J_1(py) + B_\lambda Y_1(py)\big] + \bar{v}_\lambda$$

- The study of correlators depends on the on-shell action (boundary term):

$$S = \frac{\lambda}{2}\int d^4x\, \frac{1}{y} V^{\mu(a)} \left[\partial_y V_\mu^{(a)} - \partial_\mu V_5^{(a)}\right]\Bigg|_\epsilon^{y_m},$$

2-forms

- **Minimal content**: Kinetic terms in 5d. [Cappiello, O.C., d'Ambrosio'10]

$$S = \frac{\kappa}{2}\int_{AdS_5}\left[dH\wedge {}^*dH + m^2 H\wedge {}^*H\right] ,$$
$$= \kappa\int d^5x\sqrt{g}\ \mathrm{Tr}\left[-\frac{1}{2}\partial_L H_{MN}\partial^M H^{LN} + \frac{1}{4}\partial_L H_{MN}\partial^L H^{MN} - \frac{m^2}{4}H_{MN}H^{MN}\right]$$

- 5-d mass m^2 related by AdS/CFT to conformal dimension Δ of the 4-d source according to

$$m^2 = (\Delta - p)(\Delta + p - d)$$

  If H_{MN} is to be associated with $J_{\mu\nu} = \bar{q}\sigma_{\mu\nu}q$, then $m^2 = 1$. Consequence: massive 2-forms (10 degrees of freedom) where all components will be physical.

- Kaluza-Klein decomposition:

$$H_{MN} = \left(\begin{array}{c|c} H_{\mu\nu} & H_{5\mu} \\ \hline H_{\mu 5} & 0 \end{array}\right) .$$

- The equations of motion in terms of $H_{\mu\nu}$ and $H_{\mu 5}$, using the consistency condition $d^* H = 0$:

$$\begin{aligned} \left(\partial_y^2 - \frac{1}{y}\partial_y - \Box + \frac{1-m^2}{y^2}\right) H_{5\lambda} &= 0 \\ \left(\partial_y^2 + \frac{1}{y}\partial_y - \Box - \frac{m^2}{y^2}\right) H_{\rho\lambda} &= \frac{2}{y}(\partial_\rho H_{5\lambda} - \partial_\lambda H_{5\rho}) \; . \end{aligned}$$

- Solutions:

$$\begin{aligned} H_{5\lambda}(y) &= y\big[A_\lambda J_1(py) + B_\lambda Y_1(py)\big] \\ H_{\rho\lambda}(q,y) &= \big[\mathcal{A}_{\rho\lambda} J_1(qy) + \mathcal{B}_{\rho\lambda} Y_1(qy)\big] + \bar{h}_{\rho\lambda}(q,y) \end{aligned}$$

- In order to study correlators, on-shell solutions should be plugged back in the action (only boundary terms survive):

$$S = -\frac{\kappa}{4}\int d^4x\, y H^{\mu\nu(a)}(\partial_y H^{(a)}_{\mu\nu} - \partial_\mu H^{(a)}_{5\nu} + \partial_\nu H^{(a)}_{5\mu})\Big|_{\epsilon}^{y_m} \; .$$

Phenomenological aspects

- Set of correlators:

$$\Pi^{TT}_{\mu\nu;\alpha\beta}(q) = i\int d^4x\, e^{iq\cdot x}\langle 0\,|\,T\{\,J_{\mu\nu}(x)\,J^{\dagger}_{\alpha\beta}(0)\,\}|\,0\,\rangle\,,$$
$$= \Pi_1(q^2) I_{\mu\nu;\alpha\beta} + \Pi_2(q^2) F_{\mu\nu;\alpha\beta}$$

$$I_{\mu\nu;\alpha\beta} = q^2(g_{\mu\alpha}g_{\nu\beta} - g_{\nu\alpha}g_{\mu\beta})$$
$$F_{\mu\nu;\alpha\beta} = (q_\mu q_\beta g_{\nu\alpha} + q_\nu q_\alpha g_{\mu\beta} - q_\mu q_\alpha g_{\nu\beta} - q_\nu q_\beta g_{\mu\alpha})$$

- Degrees of freedom: $\sigma_{\mu\nu}$ (6) generates two massive spin-1 fields (3+3) with opposite parity, 1^{+-} and 1^{--}:

$$\langle 0|\,J_{\mu\nu}\,|\,\rho_n(p,\lambda)\rangle = i f^{\perp}_{Vn}(\epsilon^{(\lambda)}_{\mu}\,p_\nu - \epsilon^{(\lambda)}_{\nu}\,p_\mu)$$
$$\langle 0|\,J_{\mu\nu}\,|\,b_n(p,\lambda)\rangle = i f_{Bn}\varepsilon_{\mu\nu}{}^{\eta\rho}\epsilon^{(\lambda)}_{\eta}\,p_\rho$$

Situation similar to electric and magnetic fields in $F_{\mu\nu}$.

- In parity eigenstates,

$$\Pi^{TT}_{\mu\nu;\alpha\beta}(q) = \Pi^{-}_{TT}(q^2)F^{\mu\nu;\alpha\beta}_{-}(q) \;+\; \Pi^{+}_{TT}(q^2)F^{\mu\nu;\alpha\beta}_{+}(q)$$

- For completeness, also vector and mixed correlators:

$$\Pi_{\mu\nu}^{VV}(q) = i\int \mathrm{d}^4x\, e^{iq\cdot x}\langle 0\,|\,T\{\,V_\mu(x)\,V_\nu^\dagger(0)\,\}|\,0\,\rangle$$
$$= (q_\mu q_\nu - q^2 g_{\mu\nu})\,\Pi_{VV}(q^2)$$

$$\Pi_{\mu;\nu\rho}^{VT}(q) = i\int \mathrm{d}^4x\, e^{iq\cdot x}\langle 0\,|\,T\{\,V_\mu(x)\,J_{\nu\rho}^\dagger(0)\,\}|\,0\,\rangle$$
$$= i\,(q_\rho g_{\mu\nu} - q_\nu g_{\mu\rho})\,\Pi_{VT}(q^2)$$

- Structure in the large-N_c limit:

$$\Pi_{VV}(q^2) = \sum_n^\infty \frac{f_{Vn}^2}{-q^2+{m_{Vn}}^2}$$

$$\Pi_{TT}^+(q^2) = \sum_n^\infty \frac{f_{Bn}^2}{-q^2+m_{Bn}^2} + \frac{\Lambda_3}{q^2}$$

$$\Pi_{TT}^-(q^2) = \sum_n^\infty \xi_n^2\frac{f_{Vn}^2}{-q^2+{m_{Vn}}^2} - \frac{\Lambda_3}{q^2}$$

$$\Pi_{VT}(q^2) = \sum_n^\infty \xi_n\frac{f_{Vn}^2\,m_{Vn}}{-q^2+{m_{Vn}}^2}$$

- In the holographic setting, we need to separate field from source, namely $H_{\mu\nu} \equiv \bar{H}(q,y) H^{(0)}_{\mu\nu}(q)$, $H_{5\nu} \equiv \hat{H}(q,y) H^{(0)}_{5\nu}(q)$, $V_\mu \equiv \hat{V}(q,y) V^{(0)}_\mu(q)$

$$\begin{aligned}
\Pi^{VV}_{\mu\nu}(q) &= i \frac{\delta^2}{\delta V^{(0)}_\mu \delta V^{(0)}_\nu} S \\
&= -\frac{2\lambda}{q^2} \left[\frac{1}{y} \hat{V}(q,y) \partial_y \hat{V}(q,y) \right] \Big|_\epsilon^{y_m} (q_\mu q_\nu - q^2 g_{\mu\nu}) \\
\Pi^{TT}_{\mu\nu;\lambda\rho}(q) &= i \frac{\delta^2}{\delta H^{(0)}_{\mu\nu} \delta H^{(0)}_{\lambda\rho}} S \\
&= -\frac{\kappa}{2q^2} \left[y \bar{H}(q,y) \partial_y \bar{H}(q,y) \right] \Big|_\epsilon^{y_m} (g_{\mu\lambda} g_{\nu\rho} - g_{\nu\lambda} g_{\mu\rho})
\end{aligned}$$

Some challenging points:

- For Π_{VV} gauge invariance allows to complete the longitudinal piece. Not the case for Π_{TT}.
- How to generate Π_{VT} is unclear.

Dynamically generated fields

- Remarkable property:

$$\left(\partial_y^2 - \frac{3}{y}\partial_y - \Box + \frac{3+m^2}{y^2}\right)V_5 = 0$$
$$\left(\partial_y^2 - \frac{3}{y}\partial_y - \Box + \frac{3}{y^2}\right)\phi = 0$$

and

$$\left(\partial_y^2 - \frac{1}{y}\partial_y - \Box + \frac{1-m^2}{y^2}\right)H_{5\lambda} = 0$$
$$\left(\partial_y^2 - \frac{1}{y}\partial_y - \Box\right)V_\lambda = 0$$

V_5 satisfies the EOM of a scalar field, while $H_{5\mu}$ the one for a vector field. Right mass assignments come out automatically*. One is tempted to identify the mixed terms with Π_{SV} and Π_{VT}.

- **Consistent picture**: In the chiral limit, QCD tells us that $\Pi_{SV} = 0$ while $\Pi_{VT} \neq 0$. In holography this is achieved as a natural consequence of 5d gauge invariance: V_5 is a gauge artifact but $H_{5\mu}$ not. Moreover, **no double-counting**.

Correlators and the infrared

- **Boundary conditions**: fixed on the UV brane (sources). On the IR brane

$$\begin{aligned} \partial_y \hat{V}(q, y_m) &= 0 \\ \partial_y \hat{H}(q, y_m) &= 0 \\ \bar{H}(q, y_m) &= \rho(q) \end{aligned} \tag{1}$$

  fixed by physical requirements.

- Solutions:

$$\begin{aligned} \Pi_{VV}(q^2) &= -\lambda \left[\log \frac{q^2}{\mu^2} - \pi \frac{Y_0(\zeta)}{J_0(\zeta)} \right] \\ \Pi_{VT}(q^2) &= \frac{\kappa\pi}{4} y_m^2 \rho(q) \left[Y_1(\zeta) - \frac{J_1(\zeta) Y_0(\zeta)}{J_0(\zeta)} \right] \\ \Pi_{TT}^{+} - \Pi_{TT}^{-} \equiv \Pi_{TT}^{\pm}(q^2) &= \frac{\kappa}{4q^2} \left[2 + \zeta \rho^2(q) \frac{J_2(\zeta) - J_0(\zeta)}{J_1(\zeta)} \right] \end{aligned}$$

  where $\zeta = qy$. fixed entirely by matching the large-N_c structure of Π_{VT} (single poles of ρ mesons).

- Consistency of our prescription: no contribution from high energies to Π_{VT} and $\Pi_{TT}^{\pm}$ (order parameters of SχSB).
- Free parameters in the model: λ, κ, y_m, $\rho(q)$ fixed by matching to QCD at the partonic level [λ, κ, $\rho(q)$] and low energies [$\rho(q)$, y_m].

Static properties

- The **decay couplings** are the residues of the correlators: f_{Vn} from Π_{VV}, but $f_{Vn}^{\perp}$ from Π_{TT}^{-} by bootstrapping. Analogously, one can evaluate ξ_n. In holography,

$$\xi_n \equiv \frac{f_{Vn}^{\perp}}{f_{Vn}} = -\frac{3}{N_c}\kappa\rho(\zeta_{0,n})\pi^2 y_m^2 J_1(\zeta_{0,n})$$

  where $\zeta_{0,n}$ is the nth zero of J_0.

- Asymptotic pattern from large-N_c QCD: [O.C.,Mateu'08]

$$\lim_{n\to\infty}\xi_n = \frac{(-1)^{n-1}}{\sqrt{2}}$$

- **Sign pattern** automatic and extends to all energies due to the properties of $J_1(\zeta_{0,n})$.
- **asymptotically constant ratio** $\xi_n \to 1/\sqrt{2}$ requires that

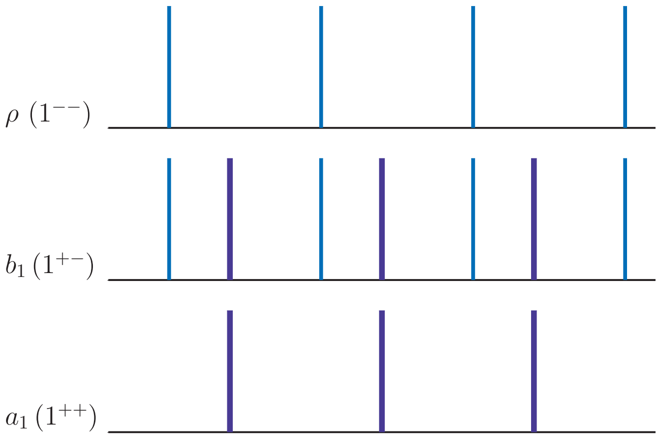
$$\rho(q) = \rho_1\sqrt{q y_m} + \rho_0$$

- The **lattice value** $\xi_\rho = 0.72$ further imposes that $\rho_0 = 0$.

Static properties

- **Mass spectrum**. Since $\Pi^{+}_{TT} = \Pi^{\pm}_{TT} + \Pi^{-}_{TT}$, 1^{+-} mesons are doubly dense with respect to 1^{--} and follow the pattern of degeneracy:

$$m^{(0)}_{Bn} = \frac{\zeta_{0,n}}{y_m} = m_{Vn}$$
$$m^{(1)}_{Bn} = \frac{\zeta_{1,n}}{y_m} = m_{An}$$



- **Appealing**:

$$m_{b1} = 1229 \pm 3 \text{ MeV}; \qquad \Gamma_{b1} = 150 \text{ MeV} \sim \Gamma_\rho \quad (b_1 \to \omega\pi)$$

$$m_{a1} = 1230 \pm 40 \text{ MeV}; \qquad \Gamma_{a1} \sim 250 - 600 \text{ MeV} \quad (a_1 \to 3\pi)$$

- **Predictions**:

$$m_\rho = 775 \text{ MeV } (775); \quad m_{b1} = 1235 \text{ MeV } (1235),$$
$$m_{\rho'} = 1779 \text{ MeV } (1700); \quad m_{b1'} = 2261 \text{ MeV } (2240),$$
$$m_{\rho''} = 2789 \text{ MeV } ; \quad m_{b1''} = 3279 \text{ MeV } .$$

- In a chiral symmetric world (switch off $\rho(q)$) 1^{--} and 1^{++} degenerate but $f_{Vn}^{\perp}, f_{Bn} \to 0$ (decoupling).

Conclusions and future work

- A minimal content approach to 2-forms in AdS/QCD already provides a consistent picture of antisymmetric tensor phenomenology in QCD.
- AdS/CFT correspondence offers a natural mechanism to generate dynamically mixed correlators like Π_{VT}, which are genuine manifestations of χSB. Given a p-form in five dimensions, a $(p-1)$-form is always dynamically generated by dimensional reduction and realized as the fifth component of the p-form.
- Bootstrapping makes tensor currents (and vector mesons) very constrained. IR boundary conditions are linked to χSB and therefore Π_{VT} and Π_{TT} offer an ideal laboratory to explore χSB in gauge/gravity duality. Also, dilaton background mechanisms to describe Regge trajectories fail to give a proper description of Π_{VT}.
- Phenomenological features: consistent predictions for

  (a) the pattern for decay couplings ξ_n.

  (b) Spectrum of b_1 particles.

The fate of linear confinement

- Start from the generic action

$$S = \kappa \int_{AdS_5} e^{-\Phi(y)} \mathrm{Tr}\left[dH \wedge {}^*dH + m^2 H \wedge {}^*H \right] ,$$

with the generalized metric

$$ds^2 = g_{MN} dx^M dx^N = e^{2A(y)}(-dy^2 + \eta_{\mu\nu} dx^\mu dx^\nu) ,$$

- For the vector field, $\Phi(y) = cy^2$ led to Regge trajectories. However, the EOM for $H_{5\mu}$ is dilaton independent, because

$$(\Box + m^2 e^{2A}) H_{5\nu} - \partial_\nu \partial_\alpha H_{5\alpha} - \partial_y \partial^\alpha H_{\alpha\nu} = 0$$

and the consistency condition

$$\begin{aligned} \partial^\mu H_{\mu 5} &= 0 , \\ \partial^\mu H_{\mu\nu} &= e^{-A} \partial_y (e^A H_{5\nu}) , \end{aligned}$$

- Linear confinement through dilatonic backgrounds is not consistent.