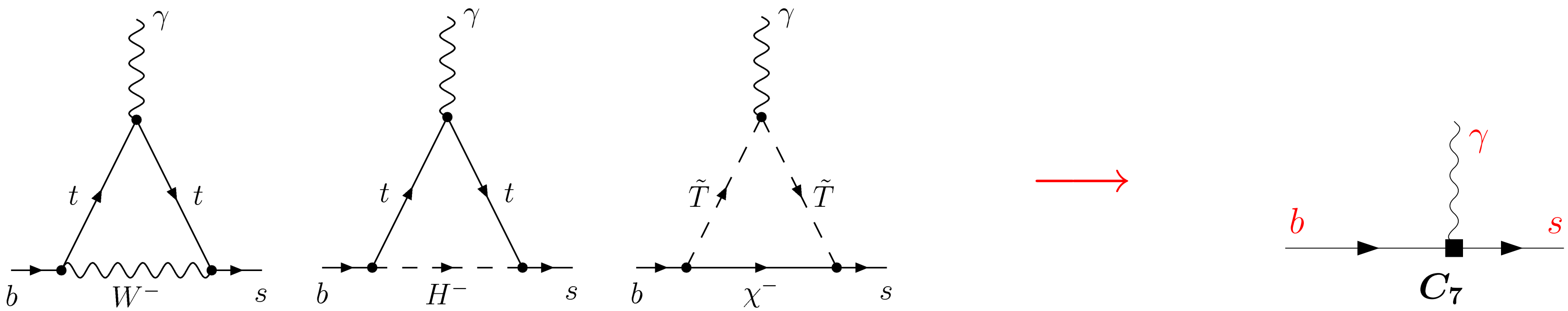
QCD challenges in radiative B decays

Mikołaj Misiak

(University of Warsaw and Karlsruhe Institute of Technology)

1. **Motivation**
2. **Perturbative calculations**
3. **Non-perturbative effects**
4. **Summary**

Information on electroweak-scale physics in the $b \to s\gamma$ transition is encoded in an effective low-energy local interaction:



Basic properties:

- **Sensitivity to new physics at scales (a few)$\times\mathcal{O}(100\,\text{GeV})$ with the present th/exp accuracy, even in models with Minimal Flavour Violation (MFV). For instance, charged Higgs (THDM-II) effects for $M_H = 500$ (1000) GeV exceed 35% (14%) in the electroweak-scale coupling $C_7(\mu_0)$, and 18% (7%) in the inclusive $\bar{B} \to X_s\gamma$ decay rate.**

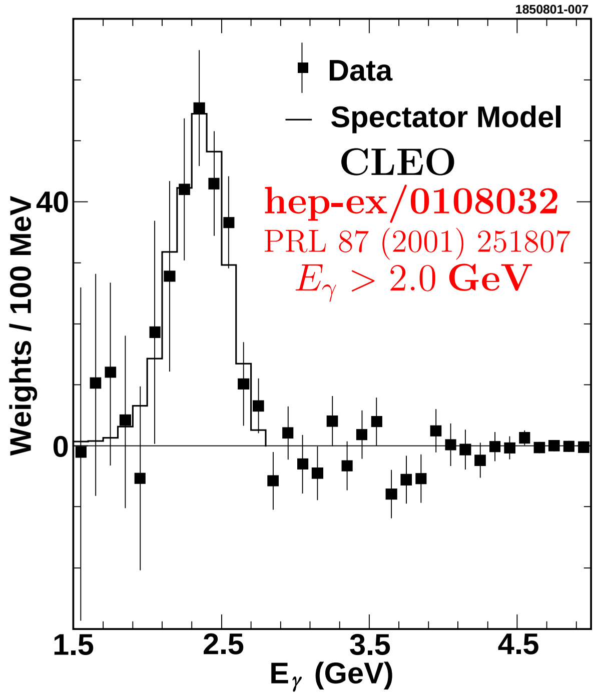
- **Perturbative calculability of the inclusive rate**

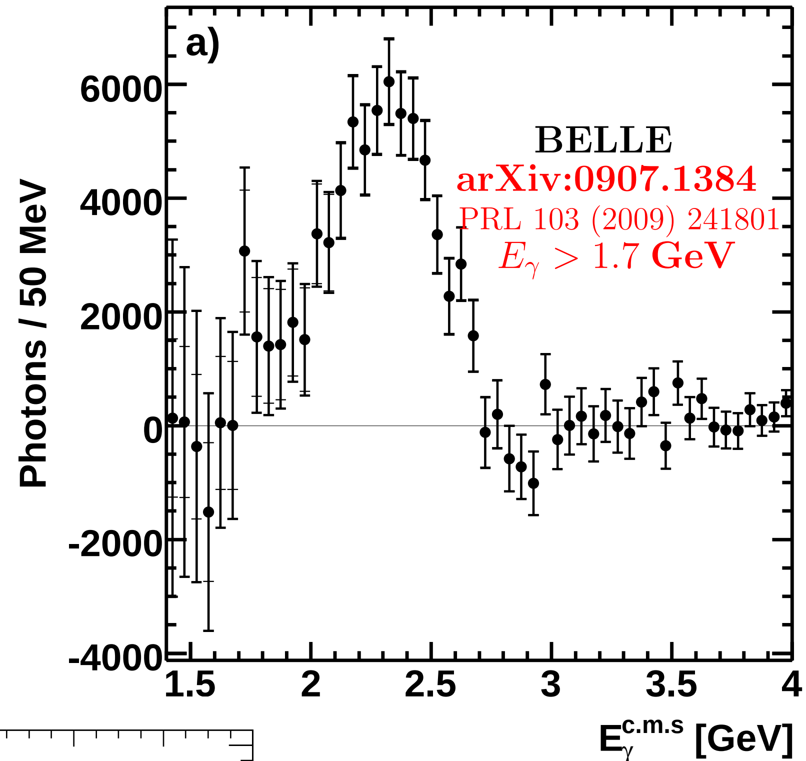
$$\Gamma(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \Gamma(b \to X_s^p\gamma)_{E_\gamma > E_0} + \left(\begin{array}{c}\textbf{non-perturbative effects}\\ \sim\ (2 \pm 5)\%\\ \textbf{see Benzke et al., arXiv:1003.5012}\end{array}\right)$$

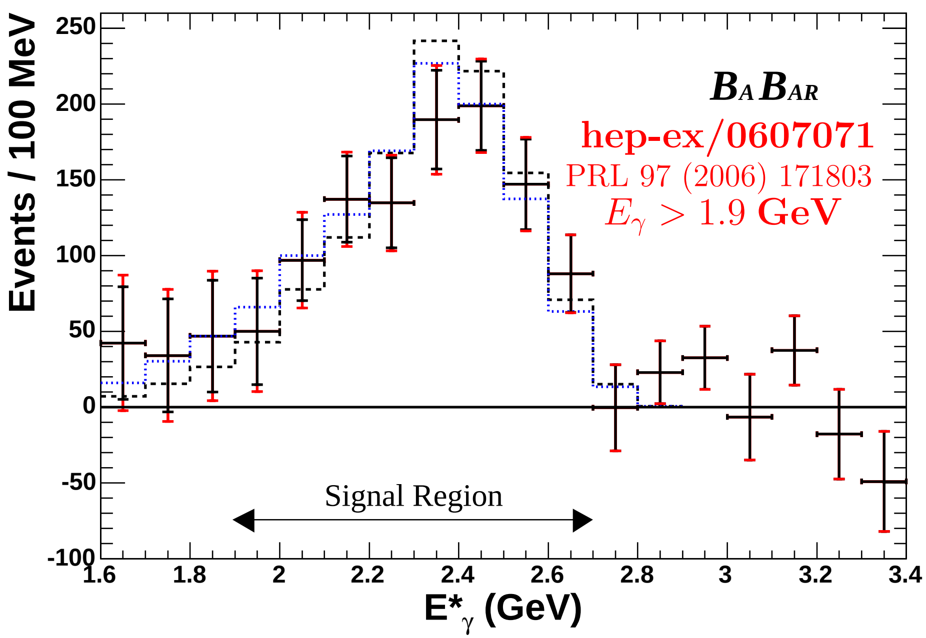
provided E_0 **is large** $(E_0 \sim m_b/2)$ **but not too close to the endpoint** $(m_b - 2E_0 \gg \Lambda_{\rm QCD})$.

- **The known/estimated NNLO $\mathcal{O}(\alpha_s^2)$ contributions to the partonic rate are $\sim\mathcal{O}(10\%)$. An uncertainty of $\pm 3\%$ is assumed for the unknown part.**

The measured photon energy spectra:







Combination of the results and extrapolation of the integrated rates to $E_0 = 1.6$ GeV are performed in the same step, to minimize model-dependence.

Non-perturbative cutoff-related TH uncertainties in the integrated rate become small for $E_0 \sim 1.6$ GeV.

Shape functions (e.g.): [A. Kagan, M. Neubert, hep-ph/9805303], [Z. Ligeti, I. W. Stewart, F.J. Tackmann, arXiv:0807.1926].

DGE model: [J.R. Andersen, E. Gardi, hep-ph/0609250].

Fixed-order $\mathcal{O}\left(\frac{\alpha_s \Lambda^2}{m_b^2}\right)$**:** [T. Ewerth, P. Gambino, S. Nandi, arXiv:0911.2175].

Results of the SM calculations:

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm NNLO}_{E_\gamma > 1.6\ {\rm GeV}} = \begin{cases} (3.15 \pm 0.23) \times 10^{-4}, & \text{hep-ph/0609232, using the 1S scheme,} \\ (3.26 \pm 0.24) \times 10^{-4}, & \text{following the kin scheme analysis of arXiv:0805.0271, but } \overline{m}_c(\overline{m}_c)^{\rm 2loop} \text{ rather than } \overline{m}_c(\overline{m}_c)^{\rm 1loop}. \end{cases}$$

Experimental world averages:

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm EXP}_{E_\gamma > 1.6\ {\rm GeV}} = \begin{cases} (3.55 \pm 0.26) \times 10^{-4}, & \text{[HFAG, winter 2010],} \\ (3.50 \pm 0.17) \times 10^{-4}, & \text{[Artuso, Barberio, Stone, arXiv:0902.3743].} \end{cases}$$

$\Rightarrow$ **Clean signals of new physics — unlikely.**

(even after reducing the uncertainties by factors of 2 on both sides)

Constraints on new physics — certainly.

Inclusive $\bar{B} \to X_s\gamma$ in the SM:

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > 1.6\,\mathrm{GeV}} = \begin{cases} (3.15 \pm 0.23) \times 10^{-4}, \\ (3.26 \pm 0.24) \times 10^{-4}, \end{cases}$$

- $(3.15 \pm 0.23) \times 10^{-4}$: **MM *et al.*, hep-ph/0609232, using the 1S scheme.**
- $(3.26 \pm 0.24) \times 10^{-4}$: **following the kinetic scheme analysis of P. Gambino and P. Giordano in arXiv:0805.0271.**

Contributions to the total TH uncertainty:

5% **non-perturbative** $\mathcal{O}\left(\frac{\alpha_s \Lambda}{m_b}\right)$, $\underbrace{\mathcal{O}\left(\frac{m_b^2 \Lambda^4}{m_c^6}\right), \mathcal{O}\left(\frac{\Lambda^3}{m_b m_c^2}\right)}_{?}$,

3% **parametric** $(\alpha_s(M_Z), \mathcal{B}^{\mathrm{exp}}_{\mathrm{semileptonic}}, m_c \;\&\; C, \ldots)$.

- $\alpha_s(M_Z)$: **2.0%**
- $\mathcal{B}^{\mathrm{exp}}_{\mathrm{semileptonic}}$: **1.6%**
- m_c & C: **1.1% (1S)**, **2.5% (kin)**

3% m_c**-interpolation ambiguity (to be reduced soon),**

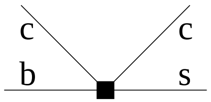
3% **higher order** $\mathcal{O}(\alpha_s^3)$,

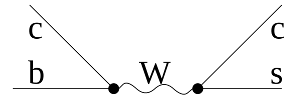
Their sum in quadrature ($\sim 7\%$) is close to the current experimental error.

Decoupling of W, Z, t, H^0 $\Rightarrow$ effective weak interaction Lagrangian:

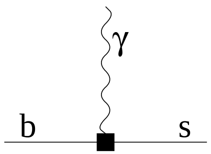
$$L_{\rm weak} \sim \sum C_i(\mu_b)\, Q_i$$

where

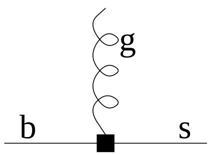
$Q_2 =$ 

$= (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)$, **from** 

, $\quad C_2(\mu_b) \simeq 1$

$Q_7 =$ 

$\sim (\bar{s}_L \sigma^{\mu\nu} b_R)\, F_{\mu\nu}$, $\quad C_7(\mu_b) \simeq -0.3$

$Q_8 =$ 

$\sim (\bar{s}_L \sigma^{\mu\nu} T^a b_R)\, G^a_{\mu\nu}$, $\quad C_8(\mu_b) \simeq -0.2$

Q_1 **differs from** Q_2 **only by color structure.**

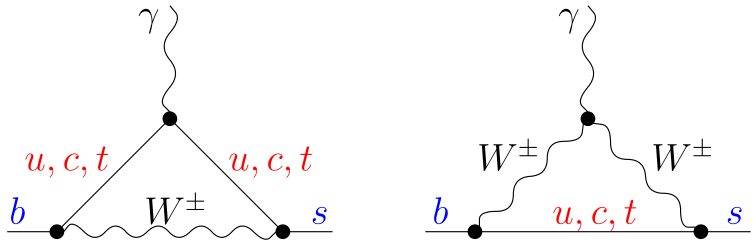
$Q_3, \ldots, Q_6$ – **other 4-quark operators with small Wilson coefficients** $C_i(\mu)$.

All the $C_i(\mu)$ are known up to $\mathcal{O}(\alpha_s^2)$ (NNLO) in the SM.

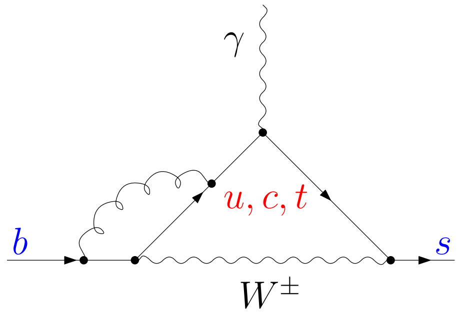
[Bobeth, MM, Urban, 2000], [MM, Steinhauser, 2004], [Gorban, Haisch, 2005], [Gorban, Haisch, MM, 2005], [Czakon, Haisch, MM, 2007].

Examples of SM diagrams for the matching of $C_7(\mu_0)$

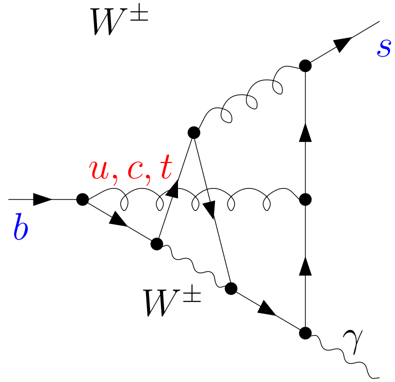
LO:
[Inami, Lim, 1981]



NLO:
[Adel, Yao, 1993]



NNLO:
[Steinhauser, MM, 2004]



NNLO method:

- Taylor expansion in the off-shell external momenta is applied before integration.
- The UV and spurious IR divergences are regulated dimensionally.
- $\Rightarrow$ In the effective theory, only tree-level diagrams survive (tree vertices and UV counterterms). The UV renormalization constants are known from former anomalous-dimension calculations.
- All the $1/\epsilon$ poles cancel in the matching equation, i.e. in the difference between the effective theory and the full SM Green functions.
- At the 3-loop level, the difference $m_t - M_W$ is taken into account with the help of expansions in y^n and $(1-y^2)^n$ up to $n = 8$, where $y = M_W/m_t$.

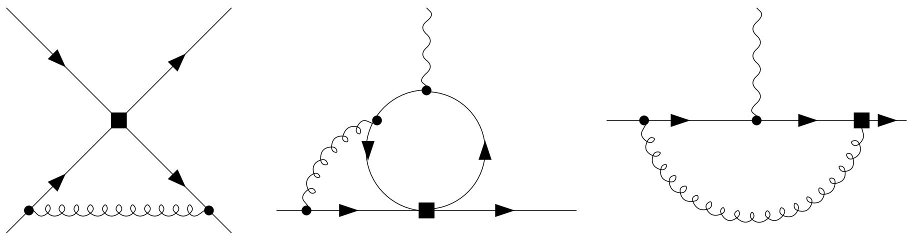
Resummation of large logarithms $\left(\alpha_s \ln \frac{M_W^2}{m_b^2}\right)^n$ in the $b \to s\gamma$ amplitude.

RGE for the Wilson coefficients: $$\mu \frac{d}{d\mu} C_j(\mu) = C_i(\mu)\gamma_{ij}(\mu)$$

The anomalous dimension matrix γ_{ij} is found from the effective theory renormalization constants, e.g.:

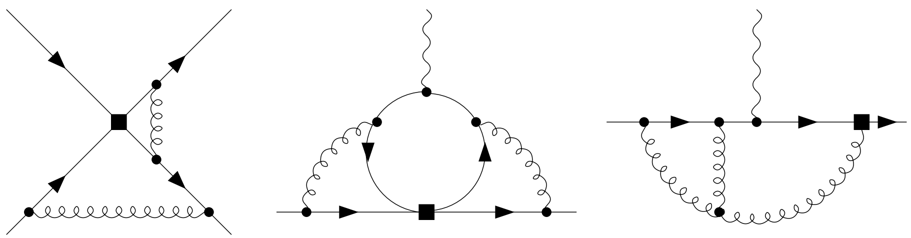
Z_{22} Z_{27} Z_{87}

LO



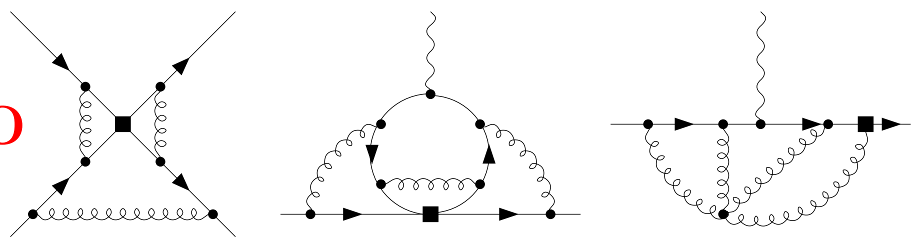
[Gaillard, Lee, 1974] [Altarelli, Maiani, 1974] — [Grinstein *et al.*, 1990] — [Shifman *et al.*, 1978] [Grigjanis *et al.*, 1988]

NLO



[Altarelli *et al.*, 1981] [Buras, Weisz, 1990] — [Chetyrkin, MM, Münz, 1997] — [MM, Münz, 1995]

NNLO



[Gorbahn, Haisch, 2004] — [Czakon, Haisch, MM, 2006] — [Gorbahn, Haisch, MM, 2005]

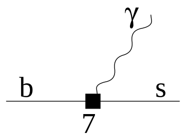
$\sim 2 \times 10^4$ **diagrams,**
-4% **effect in the BR**

All the Wilson coefficients $C_1(\mu_b), \ldots, C_8(\mu_b)$ are now known at the NNLO in the SM.

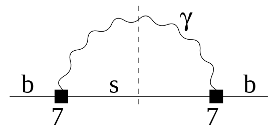
Perturbative evaluation of $\Gamma(b \to X_s^{\rm p}\gamma)$ at $\mu_b \sim \frac{m_b}{2}$.

$$\Gamma(b \to X_s^{\rm p}\gamma)_{E_\gamma > E_0} = \frac{G_F^2 m_b^5 \alpha_{\rm em}}{32\pi^4} |V_{ts}^* V_{tb}|^2 \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}(E_0, \mu_b)$$

LO: $G_{ij} = \delta_{i7}\delta_{j7}$



$\Leftrightarrow$



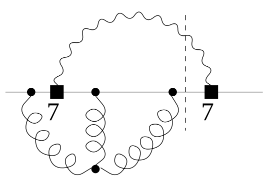
$|C_{1,2}(\mu_b)| \sim 1$, $|C_{3,4,5,6}(\mu_b)| < 0.07$,
$C_7(\mu_b) \sim -0.3$, $C_8(\mu_b) \sim -0.15$.

NLO: The most important G_{ij} $(i,j = 1,2,7,8)$ are known since 1996.
[Greub, Hurth, Wyler, 1996]
[Ali, Greub, 1991-1995]

The remaining G_{ij} are known since 2002.
[Buras, Czarnecki, MM, Urban, 2002]
[Pott, 1995]

NNLO: Only $i,j = 1,2,7,8$ have been considered so far.

G_{77} is fully known:



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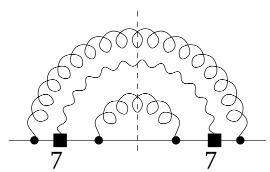
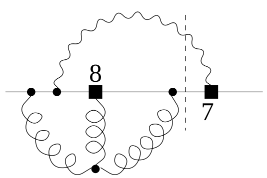


$+ \ldots$

[Blokland et al., 2005]
[Melnikov, Mitov, 2005]
[Asatrian et al., 2006-2007]

G_{78} is fully known:



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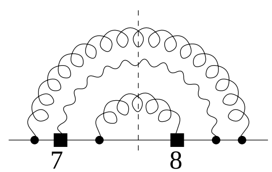
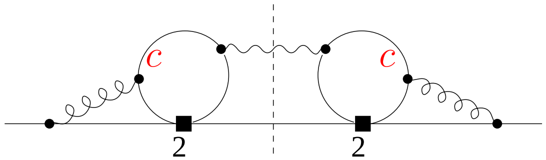


$+ \ldots$

[Asatrian et al., arXiv:1005.5587]

G_{88} and G_{28} (G_{18}): Two-particle cuts are known (just $|{\rm NLO}|^2$).
Three- and four-particle cuts are known in the BLM approximation only:
[Ferroglia, Haisch, 2007, unpublished]
[Poradziński, MM, 2010, to be published]

G_{22}**:**
(and analogous G_{11} & G_{12})



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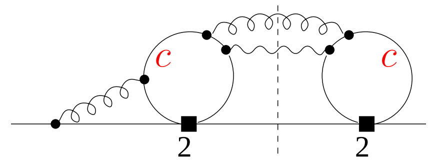

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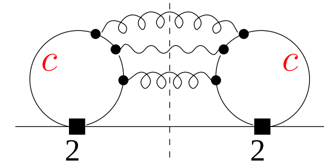
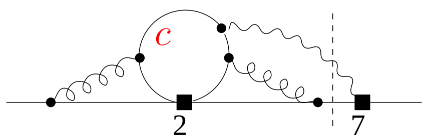


$+ \ldots$

Two-particle cuts are known (just $|\mathrm{NLO}|^2$).

Three- and four-particle cuts are known in the BLM approximation only. [Ligeti, Luke, Manohar, Wise, 1999]
No singularity at the endpoint $E_\gamma = m_b/2$ appears.
Analogous NLO corrections are not big $(+3.6\%)$.

G_{27}**:**
(and analogous G_{17})



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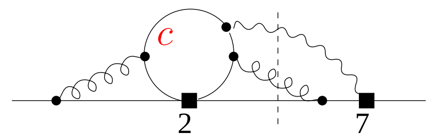

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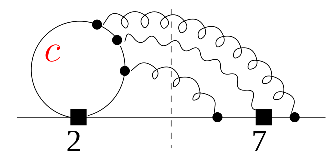


$+ \ldots$

$m_c = 0$**:** [Boughezal, Czakon, Schutzmeier, to be published]
[T. Schutzmeier, Ph.D. thesis, 2010]
$\mathcal{O}(200)$ **massive 4-loop on-shell master integrals.**

$m_c = 0$**:** [Czakon, Huber, Schutzmeier]
in progress...

The $m_c \gg m_b/2$ limit is known [MM, Steinhauser, 2006]
The BLM approximation is known for arbitrary m_c [Bieri, Greub, Steinhauser, 2003] [Ligeti, Luke, Manohar, Wise, 1999]

Non-BLM correction to G_{27} needs to be interpolated in m_c.
Its vanishing at $m_c = 0$ has been assumed so far.

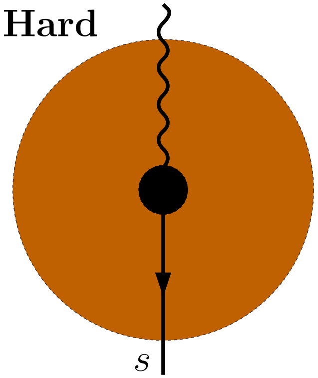
Beyond BLM, diagrams with massive quark loops on gluon lines are known for all the relevant G_{ij}
[Boughezal, Czakon, Schutzmeier, 2007], [Asatrian, Ewerth, Gabrielyan, Greub, 2007], [Ewerth, 2008].

Energetic photon production in charmless decays of the $\bar{B}$-meson

$(E_\gamma \gtrsim \frac{m_b}{3} \simeq 1.6\,\mathrm{GeV})$ [see MM, arXiv:0911.1651]

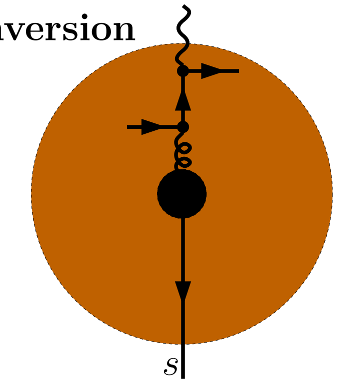
A. Without long-distance charm loops:

1. **Hard**



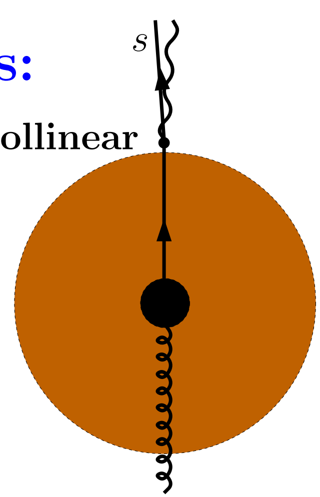
Dominant, well-controlled.

2. **Conversion**



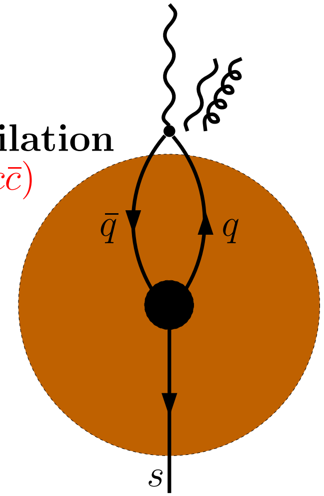
$\mathcal{O}(\alpha_s \Lambda/m_b)$, $(-1.6 \pm 1.2)\%$.
[Benzke, Lee, Neubert, Paz, 2010]

3. **Collinear**



$\sim -0.2\%$ or $(+0.8 \pm 1.1)\%$.
[Kapustin,Ligeti,Politzer, 1995]
[Benzke, Lee, Neubert, Paz, 2010]

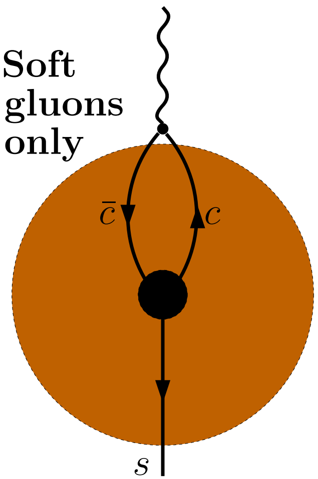
4. **Annihilation** $(q\bar{q} \neq c\bar{c})$



Exp. π^0, η, η', ω subtracted.
Perturbatively $\sim 0.1\%$.

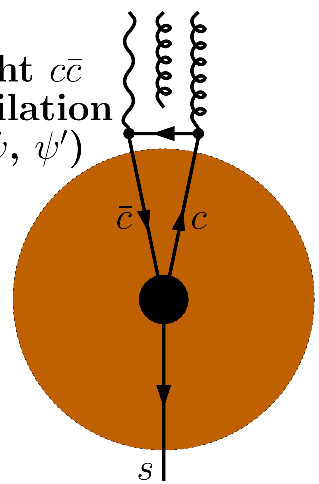
B. With long-distance charm loops:

5. **Soft gluons only**



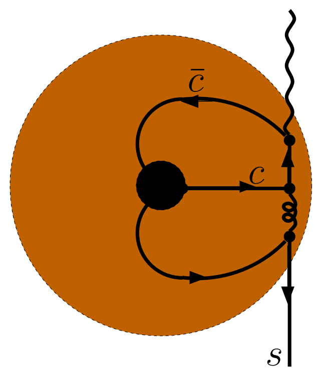
$\mathcal{O}(\Lambda^2/m_c^2)$, $\sim +3.1\%$.
[Voloshin, 1996], [...],
[Buchalla, Isidori, Rey, 1997]
[Benzke, Lee, Neubert, Paz, 2010]: add $(+1.1 \pm 2.9)\%$

6. **Boosted light $c\bar{c}$ state annihilation (e.g. η_c, J/ψ, ψ')**

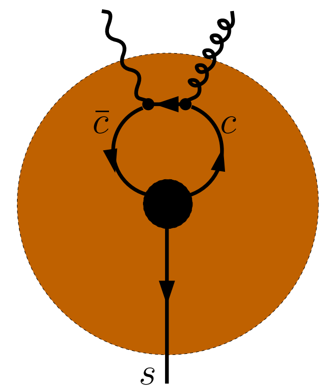


Exp. J/ψ subtracted ($< 1\%$).
Perturbatively (including hard): $\sim +3.6\%$.

7. **Annihilation of $c\bar{c}$ in a heavy $(\bar{c}s)(\bar{q}c)$ state**



$\mathcal{O}(\alpha_s(\Lambda/M)^2)$



$\mathcal{O}(\alpha_s \Lambda/M)$

$M \sim 2m_c, 2E_\gamma, m_b$.

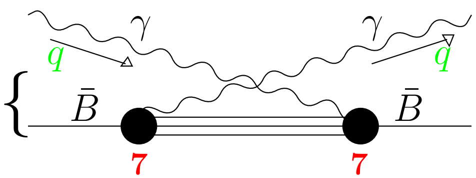
e.g. $\mathcal{B}[B^- \to D_{sJ}(2457)^- \, D^*(2007)^0 \,] \simeq 1.2\%$,
$\mathcal{B}[B^0 \to D^*(2010)^+ \, \bar{D}^*(2007)^0 K^-] \simeq 1.2\%$.

The "hard" contribution to $\bar{B} \to X_s \gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

Goal: calculate the inclusive sum $\sum_{X_s} \left| C_7(\mu_b) \langle X_s \gamma | O_7 | \bar{B} \rangle + C_2(\mu_b) \langle X_s \gamma | O_2 | \bar{B} \rangle + \ldots \right|^2$

The "77" term in this sum is purely "hard". It is related via the optical theorem to the imaginary part of the elastic forward scattering amplitude $\bar{B}(\vec{p}=0)\gamma(\vec{q}) \to \bar{B}(\vec{p}=0)\gamma(\vec{q})$:

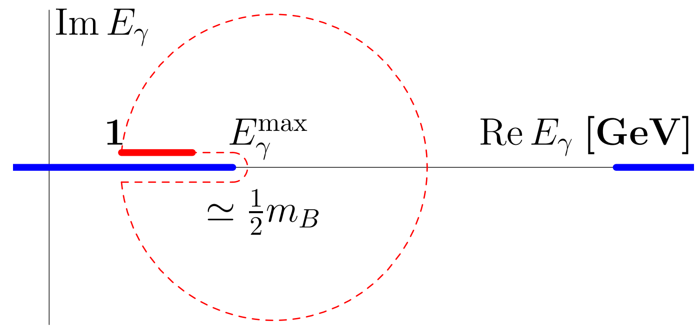
$\mathrm{Im}\{$ 

$\} \equiv \mathrm{Im}\, A$

When the photons are soft enough, $m_{X_s}^2 = |m_B(m_B - 2E_\gamma)| \gg \Lambda^2 \Rightarrow$ **Short-distance dominance** $\Rightarrow$ **OPE.**
However, the $\bar{B} \to X_s \gamma$ photon spectrum is dominated by hard photons $E_\gamma \sim m_b/2$.

Once $A(E_\gamma)$ is considered as a function of arbitrary complex E_γ, $\mathrm{Im}A$ turns out to be proportional to the discontinuity of A at the physical cut. Consequently,

$$\int_{1\ \mathrm{GeV}}^{E_\gamma^{\max}} dE_\gamma\ \mathrm{Im}\, A(E_\gamma) \sim \oint_{\mathrm{circle}} dE_\gamma\ A(E_\gamma).$$



Since the condition $|m_B(m_B - 2E_\gamma)| \gg \Lambda^2$ is fulfilled along the circle, the OPE coefficients can be calculated perturbatively, which gives

$$A(E_\gamma)\big|_{\mathrm{circle}} \simeq \sum_j \left[\frac{F^{(j)}_{\mathrm{polynomial}}(2E_\gamma/m_b)}{m_b^{n_j}(1-2E_\gamma/m_b)^{k_j}} + \mathcal{O}\left(\alpha_s(\mu_{\mathrm{hard}})\right) \right] \langle \bar{B}(\vec{p}=0) | Q^{(j)}_{\mathrm{local\ operator}} | \bar{B}(\vec{p}=0) \rangle.$$

Thus, contributions from higher-dimensional operators are suppressed by powers of Λ/m_b.

At $(\Lambda/m_b)^0$: $\langle \bar{B}(\vec{p}) | \bar{b}\gamma^\mu b | \bar{B}(\vec{p}) \rangle = 2p^\mu \quad \Rightarrow \quad \Gamma(\bar{B} \to X_s\gamma) = \Gamma(b \to X_s^{\mathrm{parton}}\gamma) + \mathcal{O}(\Lambda/m_b).$

At $(\Lambda/m_b)^1$: **Nothing! All the possible operators vanish by the equations of motion.**

At $(\Lambda/m_b)^2$: $\langle \bar{B}(\vec{p}) | \bar{h} D^\mu D_\mu h | \bar{B}(\vec{p}) \rangle = -2m_B\lambda_1,$ $\quad \lambda_1 = (-0.27 \pm 0.04)\mathrm{GeV}^2$ **from $\bar{B} \to X\ell^-\nu$ spectrum.**

$\langle \bar{B}(\vec{p}) | \bar{h} \sigma^{\mu\nu} G_{\mu\nu} h | \bar{B}(\vec{p}) \rangle = 6m_B\lambda_2,$ $\quad \lambda_2 \simeq \frac{1}{4}\left(m_{B^*}^2 - m_B^2\right) \simeq 0.12\,\mathrm{GeV}^2.$

The HQET heavy-quark field $h(x)$ is defined by $h(x) = \frac{1}{2}(1+\not{v})b(x)\exp(im_b\, v\cdot x)$ **with** $v = p/m_B$.

The $\bar{B} \to X_s \gamma$ photon spectrum for $E_\gamma \sim E_\gamma^{\max} \simeq \frac{M_B}{2}$ is dominated by contributions from "hard" radiative decays of the b-quark

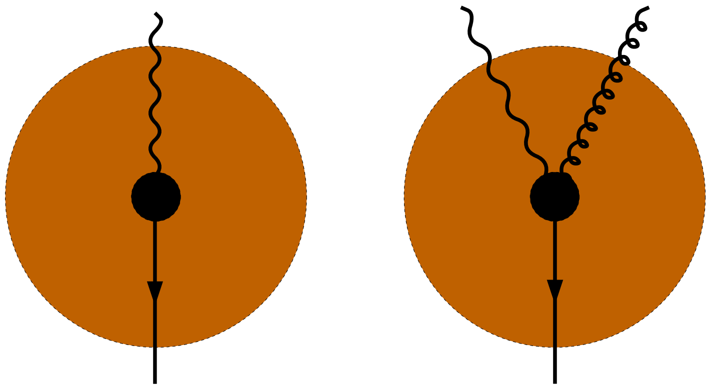
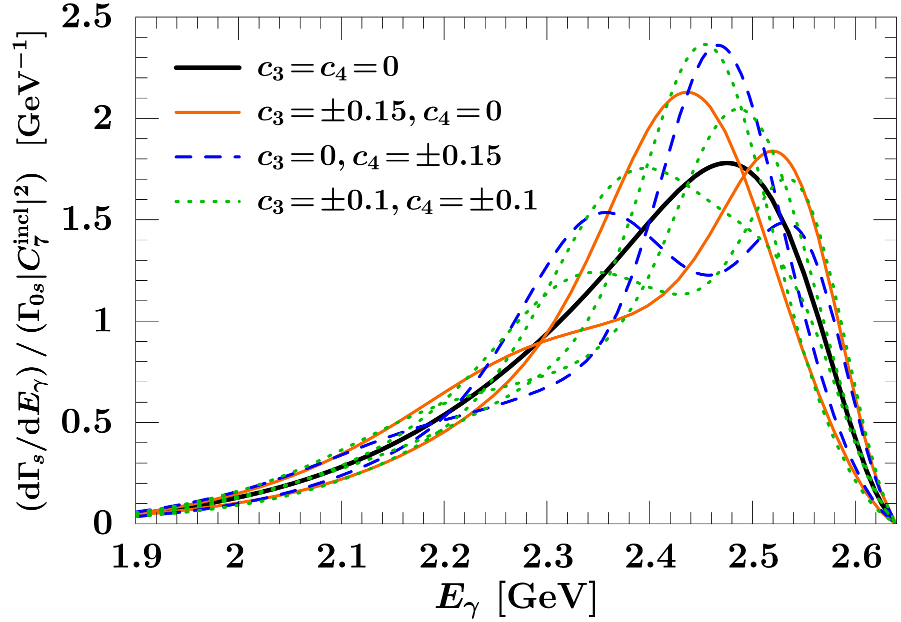


Fig. 13 from arXiv:0807.1926 by Z. Ligeti, I. Stewart and F. Tackmann.

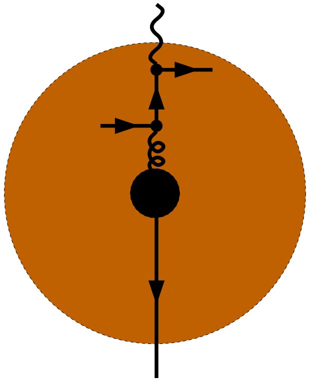


The integrated branching ratio with a lower cut E_0 on the photon energy $\mathcal{B}(\bar{B} \to X_s \gamma)_{E_\gamma > E_0}$ becomes very uncertain when E_0 is too large $(m_b - 2E_0 \sim \Lambda)$ or too small (when other than "hard" mechanisms of the photon production dominate). In a certain intermediate range of E_0:

$$\Gamma(\bar{B} \to X_s \gamma)_{E_\gamma > E_0} = \Gamma(b \to X_s^{\rm parton} \gamma)_{E_\gamma > E_0} + \mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right) + \left(\begin{array}{c}\text{small corrections due to}\\ \text{other than "hard" photons}\end{array}\right).$$

$E_0 = 1.6\,\text{GeV} \simeq \frac{m_b}{3}$ is usually chosen as default.

Gluon-to-photon conversion in the QCD medium



This is hard gluon scattering on the valence quark or a "sea" quark that produces an energetic photon. The quark that undergoes this Compton-like scattering is assumed to remain soft in the $\bar{B}$-meson rest frame to ensure effective interference with the leading "hard" amplitude. Without interference the contribution would be negligible ($\mathcal{O}(\alpha_s^2\Lambda^2/m_b^2)$).

Suppression by Λ can be understood as originating from dilution of the target (size of the $\bar{B}$-meson $\sim \Lambda^{-1}$).

A rough estimate using vacuum insertion approximation gives

$$\Delta\Gamma/\Gamma \in [-2.8\%, -0.3\%] \qquad (\mathcal{O}(\alpha_s\Lambda/m_b)).$$

[Lee, Neubert, Paz, hep-ph/0609224]

[Benzke, Lee, Neubert, Paz, arXiv:1003.5012]

However:

1. Contribution to the interference from scattering on the "sea" quarks vanishes in the $SU(3)_{\text{flavour}}$ limit because $Q_u + Q_d + Q_s = 0$.

2. If the valence quark dominates, then the isospin-averaged $\Delta\Gamma/\Gamma$ is given by:

$$\frac{\Delta\Gamma}{\Gamma} \simeq \frac{Q_d+Q_u}{Q_d-Q_u}\,\Delta_{0-} = -\frac{1}{3}\Delta_{0-} = \left(+0.2 \pm 1.9_{\text{stat}} \pm 0.3_{\text{sys}} \pm 0.8_{\text{ident}}\right)\%,$$

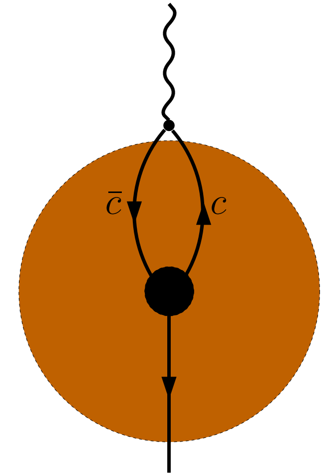
using the BABAR measurement (hep-ex/0508004) of the isospin asymmetry

$$\Delta_{0-} = [\Gamma(\bar{B}^0 \to X_s\gamma) - \Gamma(B^- \to X_s\gamma)]/[\Gamma(\bar{B}^0 \to X_s\gamma) + \Gamma(B^- \to X_s\gamma)],$$

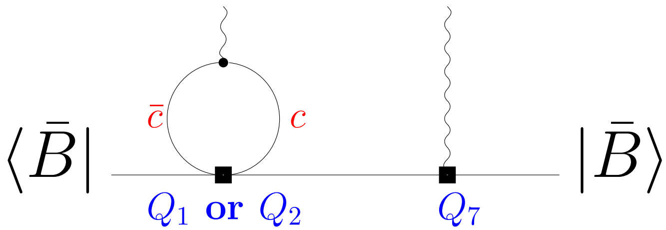
for $E_\gamma > 1.9$ GeV.

Quark-to-photon conversion gives a soft s-quark and poorly interferes with the "hard" $b \to s\gamma g$ amplitude.

Charm loops with soft gluons only



M.B. Voloshin, Phys. Lett. B **397** (1997) 275 [hep-ph/9612483];
A. Khodjamirian, R. Rückl, G. Stoll and D. Wyler, Phys. Lett. B **402** (1997) 167 [hep-ph/9702318];
Z. Ligeti, L. Randall and M.B. Wise, Phys. Lett. B **402** (1997) 178 [hep-ph/9702322];
A.K. Grant, A.G. Morgan, S. Nussinov and R.D. Peccei, Phys. Rev. D **56** (1997) 3151 [hep-ph/9702380];
G. Buchalla, G. Isidori and S.J. Rey, Nucl. Phys. B **511** (1998) 594 [hep-ph/9705253];
M. Benzke, S.J. Lee, M. Neubert and G. Paz, arXiv:1003.5012.



$$\frac{\Delta\Gamma}{\Gamma} = \frac{-C_7(\mu_b)}{C_2(\mu_b) - \frac{1}{6}C_1(\mu_b)} \left\{ \frac{\lambda_2}{9m_c^2} - \frac{2\rho_{LS}^3}{27 m_b m_c^2} + \mathcal{O}\left[\frac{\Lambda}{m_b} \sum_{n=3}^{\infty} b_n^{(1)} \left(\frac{m_b \Lambda}{m_c^2} \right)^n \right] + \mathcal{O}\left[\frac{\Lambda^2}{m_b^2} \sum_{n=2}^{\infty} b_n^{(2)} \left(\frac{m_b \Lambda}{m_c^2} \right)^n \right] + \mathcal{O}\left[\frac{\Lambda^3}{m_b^3} \right] + R \right\}.$$

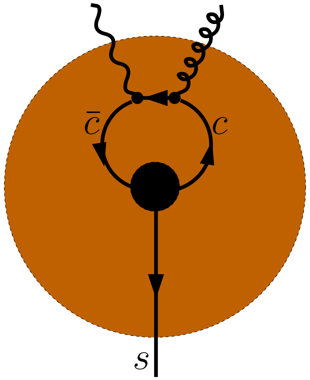
Λ **is related to the soft gluon energy in the** $\bar{B}$ **rest frame.**

$R =$ **(contribution from soft gluons with energies too high for convergence of the** $b_n^{(k)}$**-series).**

The first $(\lambda_2/9m_c^2)$ **term enhances** $\mathcal{B}[\bar{B} \to X_s\gamma]$ **by around 3%.**

The remaining terms are estimated to give a contribution in the range $[-1.7, +4.0]\%$ **(arXiv:1003.5012).**

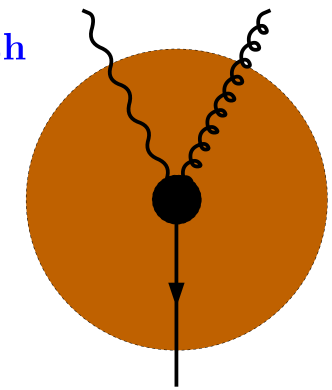
Annihilation of $c\bar{c}$ in a heavy $(\bar{c}s)(\bar{q}c)$ state



Heavy $\Leftrightarrow$ **Above the $D\bar{D}$ production threshold**

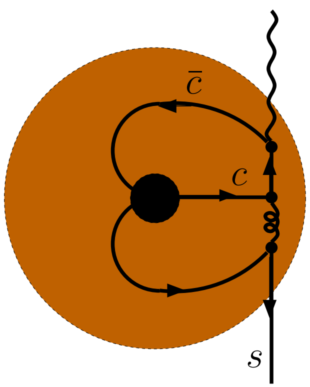
Long-distance $\Rightarrow$ **Annihilation amplitude is suppressed with respect to the open-charm decay due to the order Λ^{-1} distance between c and $\bar{c}$. By analogy to the $\mathbf{B}$-meson decay constant $f_B \sim \Lambda(\Lambda/m_b)^{1/2}$, we may expect that the suppression factor scales like $(\Lambda/M)^{3/2}$, where $M \sim 2m_c, 2E_\gamma, m_b$.**

Hard gluon $\Leftrightarrow$ **Suppression by α_s of the interference with**
(non-soft)



Altogether: $\mathcal{O}\left(\alpha_s(\Lambda/M)^{3/2}\right)$.

To stay on the safe side, assume $\mathcal{O}\left(\alpha_s\Lambda/m_b\right)$ for numerical error estimates.



This type of amplitude interferes with the leading term but receives an additional Λ/M suppression (at least) due to participation of the s-quark in the hard annihilation.

Summary

- **Given the present consistency of measurements and SM calculations, observing clean signals of new physics in $\bar{B} \to X_s \gamma$ is unlikely, even if the uncertainties were reduced by factors of 2 on both sides. However, achieving such a reduction is worth an effort, as it would lead to strengthening constraints on most popular beyond-SM theories (e.g. MSSM with MFV).**

- **New perturbative NNLO results are coming soon. This is going to improve the m_c-interpolation. No BLM approximation at $m_c = 0$ will be necessary any more.**

- **Non-perturbative uncertainty remains at the 5% level. However, making use of the $m_b \Lambda / m_c^2$ expansions whenever possible may lead to a reduction of this error.**

BACKUP SLIDES

Interpolation in m_c

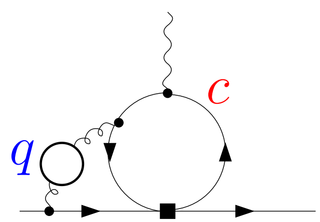
$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \underbrace{X}_{\text{normalization}} \left[\underbrace{P(E_0)}_{\text{perturbative}} + \underbrace{N(E_0)}_{\text{non-perturbative}} \right]$$

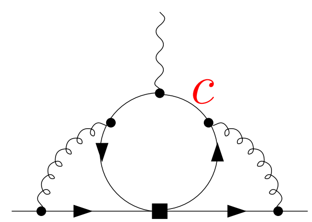
Expansion of $P(E_0)$:

$$P = \underbrace{P^{(0)} + \frac{\alpha_s(\mu_b)}{4\pi}\left(P_1^{(1)} + P_2^{(1)}(r)\right) + \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^2 \left(P_1^{(2)}\right.}_{\textbf{known}} + P_2^{(2)}(r) + \left.\underbrace{P_3^{(2)}(r)}_{\textbf{known}}\right)$$

$$P_1^{(1)}, P_3^{(2)} \sim C_i^{(0)} C_j^{(1)}, \qquad P_2^{(1)}, P_2^{(2)} \sim C_i^{(0)} C_j^{(0)}, \qquad P_1^{(2)} \sim \left(C_i^{(0)} C_j^{(2)},\ C_i^{(1)} C_j^{(1)}\right)$$

Moreover: $P_2^{(2)} = A\, n_f + B = -\frac{3}{2}(11 - 2/3 n_f)A + \frac{33}{2}A + B = P_2^{(2)\beta_0} + P_2^{(2)\text{rem}}$



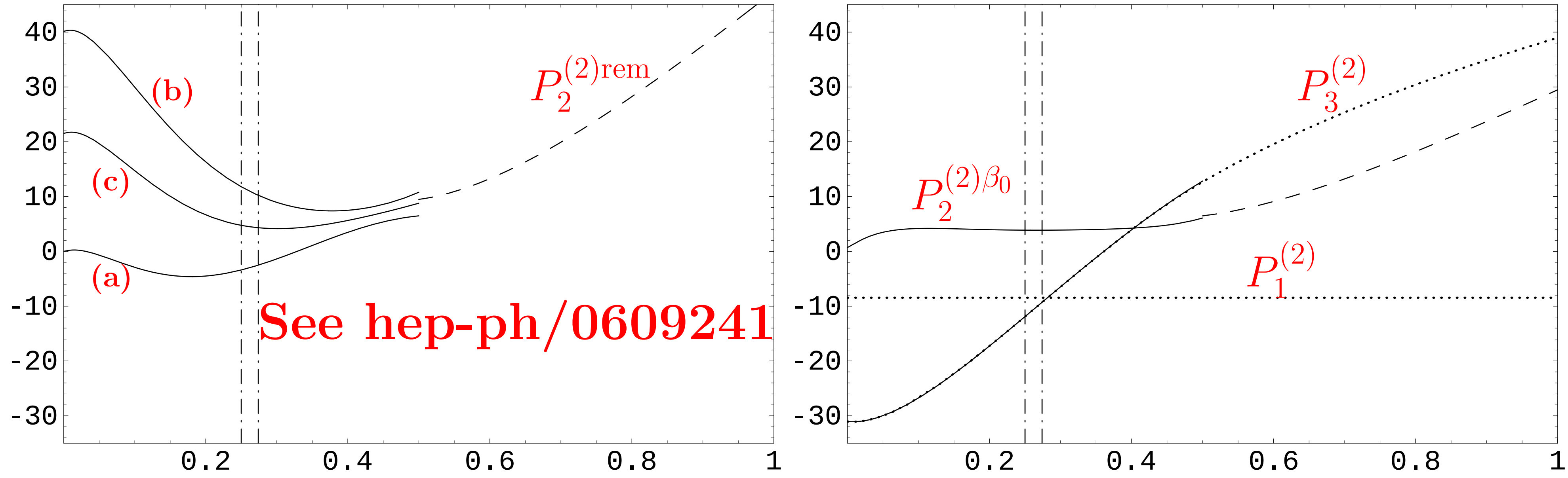


$$r = \frac{m_c(m_c)}{m_b^{1S}}$$

$P_2^{(2)\beta_0}$ **known for all r**

The complete $P_2^{(2)}$ has been calculated only for $r \gg \frac{1}{2}$.

The NNLO corrections $P_k^{(2)}$ as functions of $r = m_c(m_c)/m_b^{1S}$



Dotted: exact, Solid: small-r expansions, Dashed: leading large-r asymptotics.

Interpolation:

$$P_2^{(2)\text{rem}}(r) = x_1 + x_2\, P_2^{(1)}(r) + x_3\, r\, \frac{d}{dr} P_2^{(1)}(r) + x_4\, P_2^{(2)\beta_0}(r) + x_5 |A_{\text{NLO}}(r)|^2$$

The coefficients x_k are determined from the asymptotic behaviour at large r

and from the requirement that either (a) $P_2^{(2)\text{rem}}(0) = 0$,

or (b) $P_1^{(2)} + P_2^{(2)\text{rem}}(0) + P_3^{(2)}(0) = 0$,

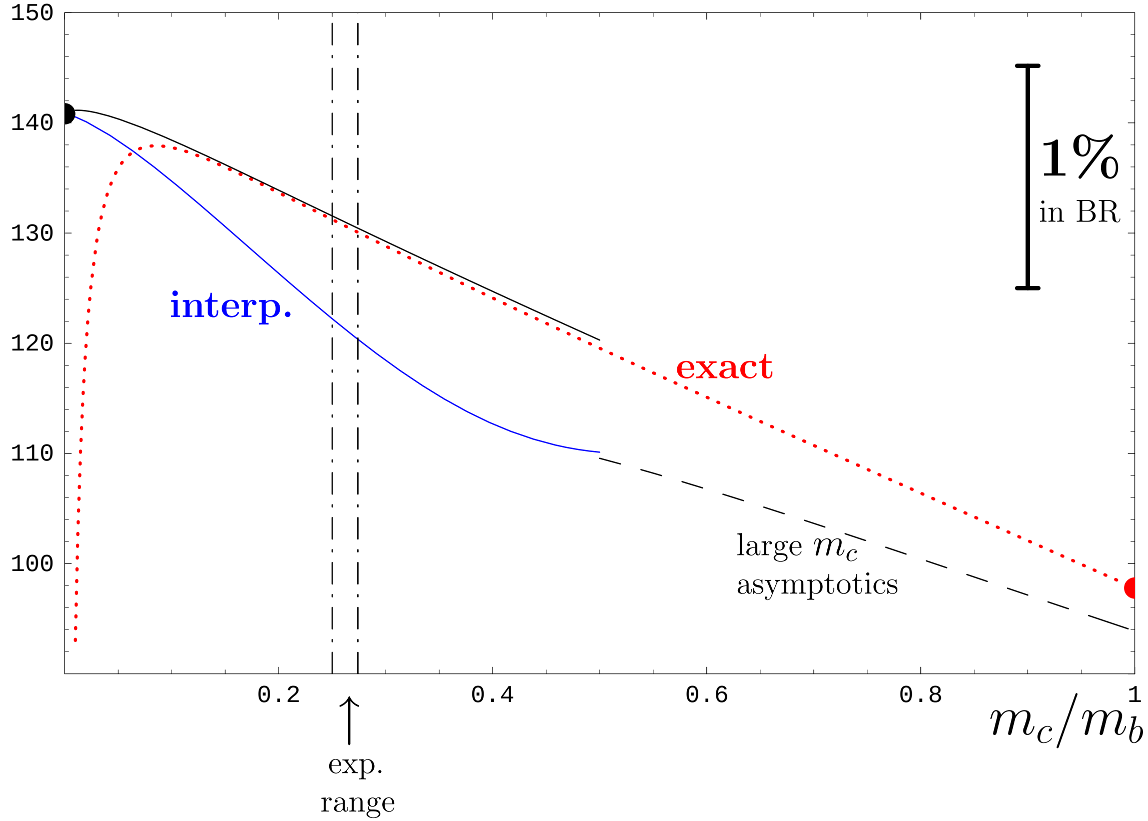
or (c) $P_2^{(2)\text{rem}}(0) = \left[P_2^{(2)\text{rem}}(0)\right]_{77}$.

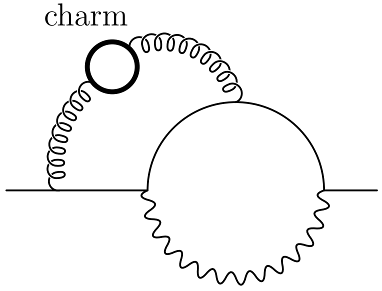
The average of (a) and (b) is chosen to determine the central value of the NNLO branching ratio.

The difference between these two cases is used to estimate the interpolation ambiguity.

The m_c-dependence of $P_2^{(2)\text{rem}} = C_i^{(0)}(\mu_b) C_j^{(0)}(\mu_b) K_{ij}^{(2)\text{rem}}(\mu_b, E_0)$.

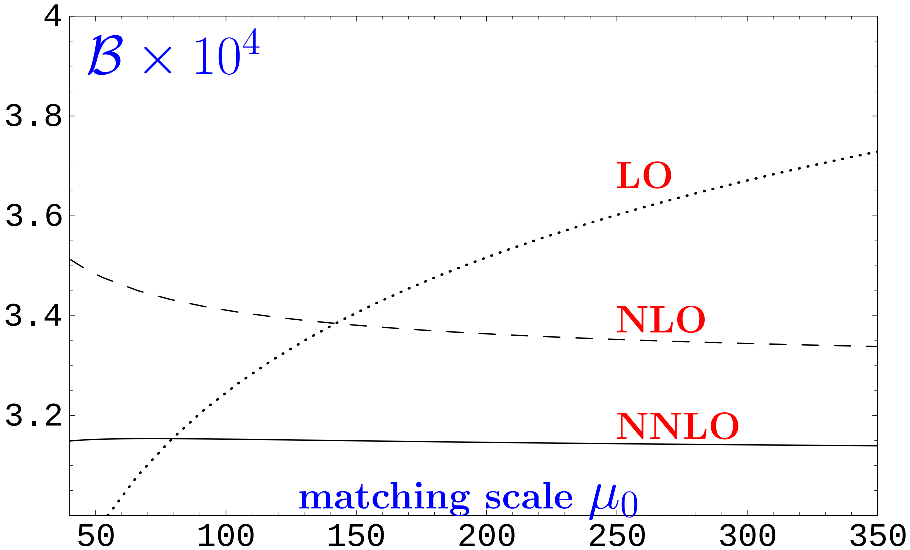
Example: $K_{77}^{(2)\text{rem}}(2.5\ \text{GeV}, 1.6\ \text{GeV})$ **as a function of** m_c/m_b:

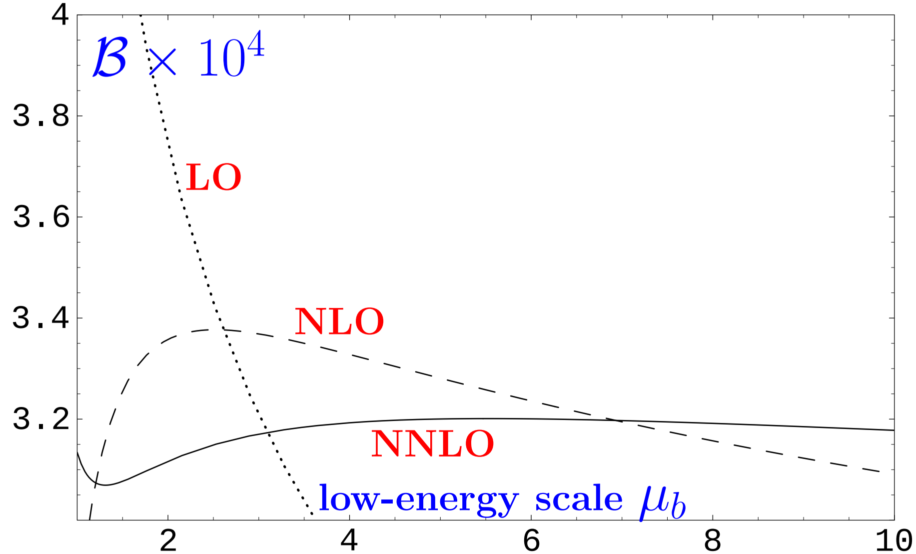


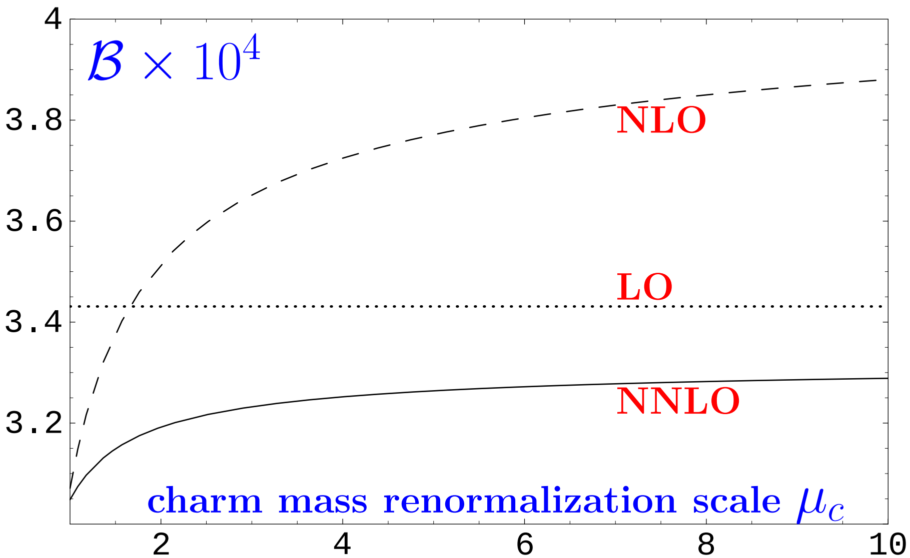


Value at $m_c = 0$: **Blokland et al., hep-ph/0506055** ($c\bar{c}$ production included).
Large-m_c **asymptotics: Steinhauser, MM, hep-ph/0609241.**
Interpolation: " " " ($c\bar{c}$ production included).
Exact $b \to X_s\gamma$: **Asatrian et al, hep-ph/0611123** ($c\bar{c}$ production excluded).
Exact $b \to X_u e\bar{\nu}$: **Pak, Czarnecki, arXiv:0803.0960** ($c\bar{c}$ production included).

Renormalization scale dependence of $\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > 1.6\ \mathrm{GeV}}$





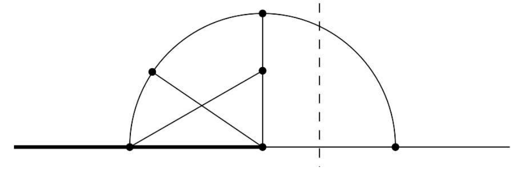


"Central" values:

$\mu_0 = 160$ **GeV**

$\mu_b = 2.5$ **GeV**

$\mu_c = 1.5$ **GeV**

Evaluation of the master integrals I_k. (e.g.: )

(from the Ph.D. thesis of T. Schutzmeier)

(i) Generalization to the off-shell case $z \equiv \frac{p^2}{m_b^2} \neq 1$

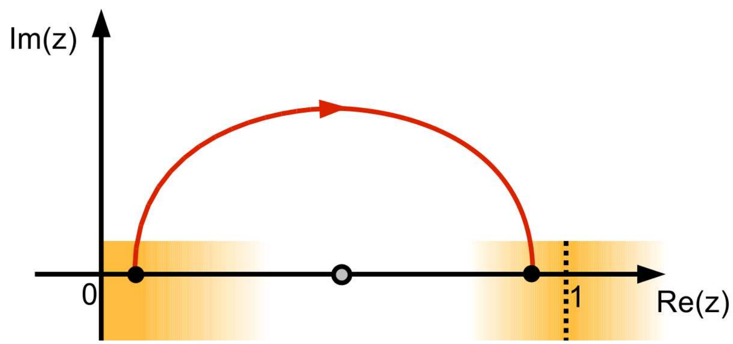
(ii) Automatic derivation (with the help of IBP) of differential equations of the form:

$$\frac{d}{dz} I_n = \Sigma_k \, w_{nk}(z, \epsilon) \, I_k$$

where w_{nk} are rational functions of their arguments.

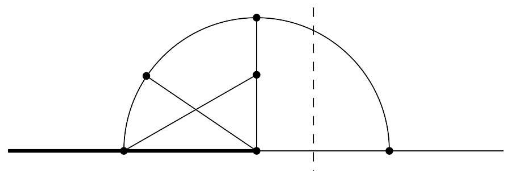
(iii) Establishing initial conditions from expansions around $z = 0$ that involve massless integrals only (apart from massive tadpoles).

(iv) Evolving to the vicinity of $z = 1$ using precise numerical solutions to the differential equations.



The evolution goes either in the upper or in the lower part of the complex z-plane to bypass spurious singularities of w_{nk} on the real axis. Path-independence of the final results serves as a test.

(v) Matching with expansions around $z = 1$, assuming their form $\sum c_{pq} \, (1-z)^p \, \ln^q(1-z)$ (with unknown coefficients c_{pq}). This is necessary only if numerical instabilities occur at $z = 1$.

Example of z-dependence (the $\mathcal{O}(\epsilon^0)$ part of ):