The trace anomaly and the coupling of QCD to gravity

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Papers and collaborators

- R. A., C. Corianò, L. Delle Rose
"Conformal Anomalies and the Gravitational Effective Action: The TJJ Correlator for a Dirac Fermion"
Phys.Rev.D81:085001,2010, arXiv:0910.3381 [hep-ph]

- R. A., C. Corianò, L. Delle Rose
"Anomaly Poles as Common Signatures of Chiral and Conformal Anomalies"
Phys.Lett.B682:322-327,2009, arXiv:0909.4522 [hep-ph]

- R. A., C. Corianò, L. Delle Rose
"Trace Anomaly, Massless Scalars and the Gravitational Coupling of QCD"
arXiv:1005.4173 [hep-ph]

- R. A., C. Corianò, L. Delle Rose, L. Manni
"The Trace Anomaly and the Gravitational Coupling of an Anomalous U(1)"
arXiv:1003.3930 [hep-ph]

- R. A., C. Corianò, L. Delle Rose, M. Guzzi, A. Mariano
"The Effective Actions of Pseudoscalar and Scalar Particles in Theories with Gauge and Conformal Anomalies"
arXiv:1001.5240 [hep-ph] – to be published in Fortschritte der Physik

Outline

The context is the one of an EFT of gravity

Trace anomaly

Evidence of new massless degreees of freedom, not contained in the Einstein-Hilbert action

Explicit analytic perturbative computations on specific correlators

The trace anomaly

The trace of the stress tensor obeys at the classical level

$$T^{\mu\ (cl)}_{\ \mu} \equiv g_{\mu\nu} T^{\mu\nu\ (cl)} = -m\bar{\psi}\psi$$

At the quantum level this symmetry under global scale transformations cannot be maintained

[Duff, NPB 125 (1977), Capper, Duff, Halpern, PRD 10 (1974)]

We end up with a well defined TRACE ANOMALY

also for a vanishing mass fermion

$$\langle T^{\mu}_{\ \mu} \rangle_A \big|_{m=0} = -\frac{e^2}{24\pi^2} F_{\mu\nu} F^{\mu\nu}$$

The role of the trace anomaly

The trace anomaly provides additional terms to be included in the classical Einstein-Hilbert classical action

$$S_{\rm eff}[g] = S_{\rm EH}[g] + S_{\rm anom}[g]$$

The trace anomaly implies the existence of

new massless scalar degrees of freedom

[Antoniadis, Mazur, Mottola, New J.Phys.9:11,2007
Giannotti, Mottola, PRD79 (2009),
R.A., Corianò, Delle Rose, Phys.Lett.B682:322-327,2009,
R.A., Corianò, Delle Rose, arXiv:0910.3381 to appear in PRD]

The non local effective action

Starting from the trace anomaly

$$T^{\mu}_{\mu} = b\,F + b'\left(E - \frac{2}{3}\Box R\right) + b''\Box R + c\,F^{a\,\mu\nu}F^{a}_{\mu\nu}$$

Riegert found this non local gravitational effective action

$$S_{anom}[g,A] = \frac{1}{8}\int d^4x\sqrt{-g}\int d^4x'\sqrt{-g'}\left(E - \frac{2}{3}\Box R\right)_x \Delta_4^{-1}(x,x')\left[2b\,F + b'\left(E - \frac{2}{3}\Box R\right) + 2\,c\,F_{\mu\nu}F^{\mu\nu}\right]_{x'}$$

[Riegert, PLB 134 (1984)]

by solving the variational equation satisfied by the trace of the energy momentum tensor

$$\Delta_4 \equiv \nabla_\mu\left(\nabla^\mu\nabla^\nu + 2R^{\mu\nu} - \frac{2}{3}Rg^{\mu\nu}\right)\nabla_\nu = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu + \frac{1}{3}(\nabla^\mu R)\nabla_\mu - \frac{2}{3}R\Box$$

The local formulation

The non local effective action can be recast in a local form after the introduction of two auxiliary fields

$$S_{anom}[g, A; \varphi, \psi'] = \int d^4x \sqrt{-g} \left[-\psi' \Box \varphi - \frac{R}{3} \psi' + \frac{c}{2} F_{\alpha\beta} F^{\alpha\beta} \varphi \right]$$

satisfying the equations

$$\psi' \equiv b \Box \psi \,,$$
$$\Box \psi' = \frac{c}{2} F_{\alpha\beta} F^{\alpha\beta}$$
$$\Box \varphi = -\frac{R}{3} \,.$$

[Antoniadis, Mazur, Mottola, New J.Phys.9:11 (2007)
Giannotti, Mottola, PRD79 (2009)]

The local formulation

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$$S_{anom}[g, A; \varphi, \psi'] = \int d^4x \sqrt{-g} \left[-\psi' \Box \varphi - \frac{R}{3} \psi' + \frac{c}{2} F_{\alpha\beta} F^{\alpha\beta} \varphi \right]$$

Expanding around flat space the local formulation of Riegert's action can be

$$S_{anom}[g, A] \to -\frac{c}{6} \int d^4x \sqrt{-g} \int d^4x' \sqrt{-g'}\, R_x \Box^{-1}_{x,x'} [F_{\alpha\beta} F^{\alpha\beta}]_{x'}$$

Given a solution of a variational equation it is mandatory to check

whether the solution is indeed justified by a perturbative expansion

The coupling of QCD to gravity

In the weak gravitational field limit

$$\mathcal{L}_{int} = -\frac{1}{2}\kappa\, h^{\mu\nu} T_{\mu\nu}$$

where

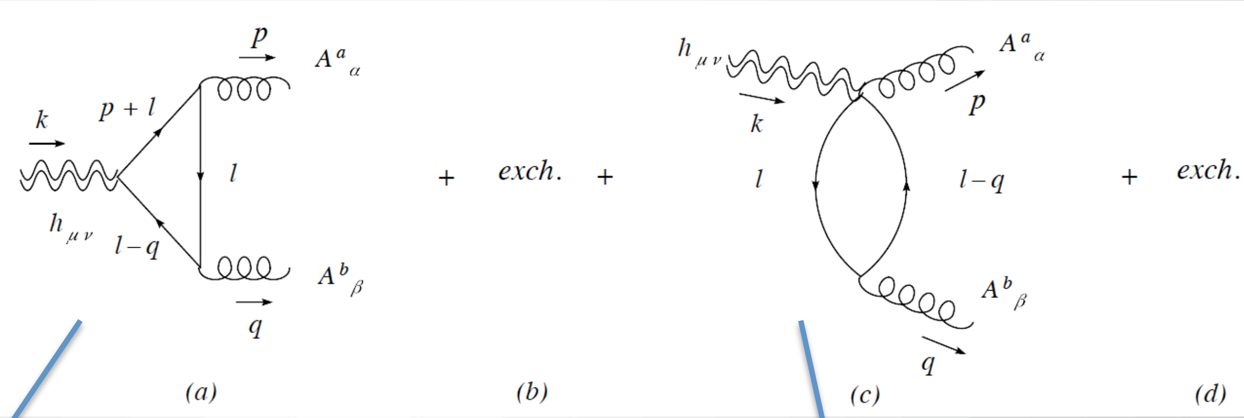
$$\begin{aligned} T_{\mu\nu} &= -g_{\mu\nu}\mathcal{L}_{QCD} - F^a_{\mu\rho}F^{a\rho}_{\nu} - \frac{1}{\xi} g_{\mu\nu}\partial^\rho(A^a_\rho \partial^\sigma A^a_\sigma) + \frac{1}{\xi}(A^a_\nu \partial_\mu(\partial^\sigma A^a_\sigma) + A^a_\mu \partial_\nu(\partial^\sigma A^a_\sigma)) \\ &+ \frac{i}{4}\Big[\overline{\psi}\gamma_\mu(\overrightarrow{\partial}_\nu - igT^a A^a_\nu)\psi - \overline{\psi}(\overleftarrow{\partial}_\nu + igT^a A^a_\nu)\gamma_\mu\psi + \overline{\psi}\gamma_\nu(\overrightarrow{\partial}_\mu - igT^a A^a_\mu)\psi \\ &- \overline{\psi}(\overleftarrow{\partial}_\mu + igT^a A^a_\mu)\gamma_\nu\psi\Big] + \partial_\mu\overline{\omega}^a(\partial_\nu\omega^a - gf^{abc}A^c_\nu\omega^b) + \partial_\nu\overline{\omega}^a(\partial_\mu\omega^a - gf^{abc}A^c_\mu\omega^b) \end{aligned}$$

The <Tgg> correlator is the first (leading) order contribution to the infinite sum of the <T^ngg> correlators in which the anomalous effective action is expanded

The perturbative expansion 1.

The perturbative expansion is obtained by taking into account all the possible diagrams in which a graviton (initial state) and two gluons (final state) appear.

QUARK SECTOR



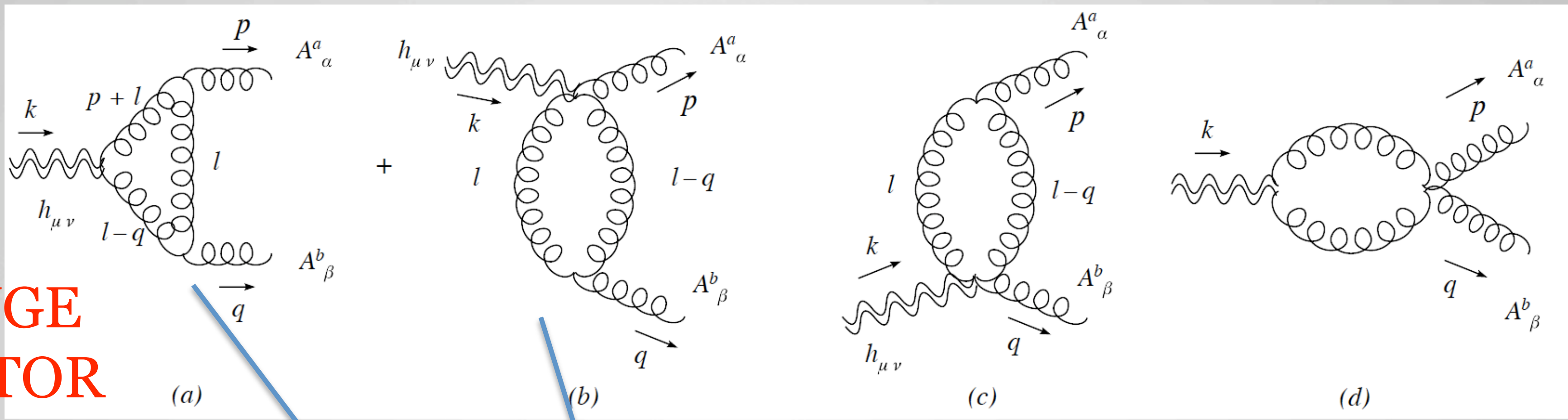
$$-i\frac{\kappa}{2}\Gamma^{[\mathbf{2a}]\,ab}_{\mu\nu\alpha\beta}(p,q) = -\frac{\kappa}{2}g^2\,\mathrm{Tr}(T^bT^a)\int\frac{d^4l}{(2\pi)^4}\mathrm{tr}\left\{V'_{\mu\nu}(l-q,l+p)\frac{1}{\not l-\not q-m}\gamma_\beta\frac{1}{\not l-m}\gamma_\alpha\frac{1}{\not l+\not p-m}\right\}$$

$$-i\frac{\kappa}{2}\Gamma^{[\mathbf{2c}]\,ab}_{\mu\nu\alpha\beta}(p,q) = -\frac{\kappa}{2}g^2\,\mathrm{Tr}(T^aT^b)\int\frac{d^4l}{(2\pi)^4}\mathrm{tr}\left\{W'_{\mu\nu\alpha}\frac{1}{\not l-\not q-m}\gamma_\beta\frac{1}{\not l-m}\right\}$$

[R.A., Corianò, Delle Rose, arXiv: 1005.4173]

The perturbative expansion 2.

GAUGE SECTOR



$$-i\frac{\kappa}{2}\,\Gamma^{[\mathbf{3a}]\,ab}_{\mu\nu\alpha\beta}(p,q) = -\frac{\kappa}{2} g^2 f^{ade} f^{bde} \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2\,(l+p)^2\,(l-q)^2} \Big[V^{Ggg}_{\mu\nu\rho\sigma}(l-q,-l-p)\, \times \\ V^3_{\tau\sigma\alpha}(-l,l+p,-p)\; V^3_{\rho\tau\beta}(-l+q,l,-q) \Big],$$

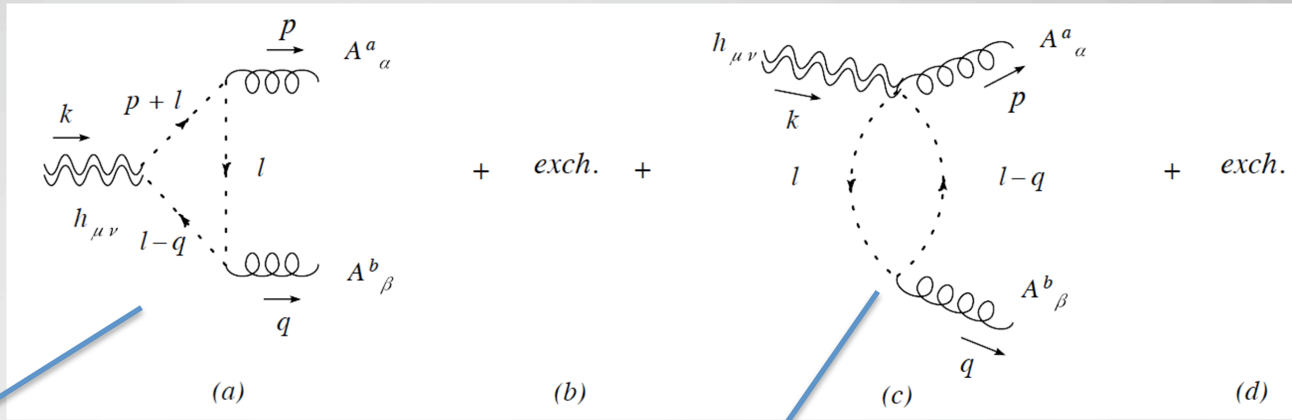
$$-i\frac{\kappa}{2}\,\Gamma^{[\mathbf{3b}]\,ab}_{\mu\nu\alpha\beta}(p,q) = -\frac{1}{2}\frac{\kappa}{2} g^2 f^{ade} f^{bde} \int \frac{d^4 l}{(2\pi)^4} \frac{V^{Gggg}_{\mu\nu\rho\sigma\beta}(-l,l-p,-q)\; V^3_{\rho\alpha\sigma}(k,-p,-l+p)}{l^2\,(l-p)^2}$$

[R.A., Corianò, Delle Rose, arXiv: 1005.4173]

The perturbative expansion 3.

GHOST SECTOR

[R.A., Corianò, Delle Rose, arXiv: 1005.4173]



$$-i\frac{\kappa}{2}\Gamma^{[\mathbf{4a}]\,ab}_{\mu\nu\alpha\beta}(p,q) = -\frac{\kappa}{2}g^2 f^{ade} f^{bde}\int\frac{d^4l}{(2\pi)^4}\frac{C_{\mu\nu\rho\sigma}(l-q)^\rho(l+p)^\sigma l_\alpha (l-q)_\beta}{l^2\,(l+p)^2\,(l-q)^2}$$

$$-i\frac{\kappa}{2}\Gamma^{[\mathbf{4b}]\,ab}_{\mu\nu\alpha\beta}(p,q) = \frac{\kappa}{2}g^2 f^{ade} f^{bde}\int\frac{d^4l}{(2\pi)^4}\frac{C_{\mu\nu\alpha\sigma}l^\sigma (l-q)_\beta}{l^2\,(l-q)^2}$$

All the diagrams are computed analytically by means

of tensor reduction in dimensional regularization

Ward identities for the correlator

1) for the conservation of the energy-momentum tensor

$$k^\mu \langle T_{\mu\nu}(k) A^a_\alpha(p) A^b_\beta(q)\rangle_{trunc} = q_\mu D^{-1}_{\alpha\mu}(p) g_{\beta\nu} + p_\mu D^{-1}_{\beta\mu}(q) g_{\alpha\nu} - q_\nu D^{-1}_{\alpha\beta}(p) - p_\nu D^{-1}_{\alpha\beta}(q)$$

$$D^{-1}_{\alpha\beta}(p) = (p^2 g_{\alpha\beta} - p_\alpha q_\beta)\Pi(p^2)$$

inverse gluon propagator

Ward identities for the correlator

1) for the conservation of the energy-momentum tensor

$$k^\mu \langle T_{\mu\nu}(k) A^a_\alpha(p) A^b_\beta(q)\rangle_{trunc} = q_\mu D^{-1}_{\alpha\mu}(p) g_{\beta\nu} + p_\mu D^{-1}_{\beta\mu}(q) g_{\alpha\nu} - q_\nu D^{-1}_{\alpha\beta}(p) - p_\nu D^{-1}_{\alpha\beta}(q)$$

2) for the trace of the energy-momentum tensor at zero momentum transfer

$$\langle T^\mu_\mu(0) A^a_\alpha(p) A^b_\beta(-p)\rangle_{trunc} = \left(2 - d + p\cdot\frac{\partial}{\partial p}\right) D^{-1}_{\alpha\beta}(p)$$

$$\langle T^\mu_\mu(0) A^a_\alpha(p) A^b_\beta(-p)\rangle_{trunc} = \left[\beta(g)\frac{\partial}{\partial g} - 2\gamma(g) + m(\gamma_m(g) - 1)\frac{\partial}{\partial m}\right] D^{-1}_{\alpha\beta}(p)$$

$$\beta(g) = \mu\frac{\partial g}{\partial \mu}, \qquad \gamma(g) = \mu\frac{\partial}{\partial \mu}\log\sqrt{Z_A}, \qquad m\,\gamma_m(g) = \mu\frac{\partial m}{\partial \mu}$$

Ward identities for the correlator

1) for the conservation of the energy-momentum tensor

$$k^{\mu}\langle T_{\mu\nu}(k)A^a_{\alpha}(p)A^b_{\beta}(q)\rangle_{trunc} = q_{\mu}D^{-1}_{\alpha\mu}(p)g_{\beta\nu} + p_{\mu}D^{-1}_{\beta\mu}(q)g_{\alpha\nu} - q_{\nu}D^{-1}_{\alpha\beta}(p) - p_{\nu}D^{-1}_{\alpha\beta}(q)$$

2) for the trace of the energy-momentum tensor at zero momentum transfer

$$\langle T^{\mu}_{\mu}(0)A^a_{\alpha}(p)A^b_{\beta}(-p)\rangle_{trunc} = \left(2 - d + p\cdot\frac{\partial}{\partial p}\right)D^{-1}_{\alpha\beta}(p)$$

3) generalized Ward identity

$$p^{\alpha}q^{\beta}\langle T_{\mu\nu}(k)A^a_{\alpha}(p)A^b_{\beta}(q)\rangle_{trunc} = 0$$

These three Ward identities turn out to be satisfied in dimensional regularization

[R.A., Corianò, Delle Rose, arXiv: 1005.4173]

Expansion of the whole amplitude

The sum of the quark and gluon/ghost contributions is written as

$$\Gamma^{\mu\nu\alpha\beta}(p,q) = \Gamma_g^{\mu\nu\alpha\beta}(p,q) + \Gamma_q^{\mu\nu\alpha\beta}(p,q) = \sum_{i=1}^{3} \Phi_i(s,0,0)\, \delta^{ab}\, \phi_i^{\mu\nu\alpha\beta}(p,q)$$

form factors

tensor structures

The massless pole evidence

The sum of the quark and gluon/ghost contributions is written as

$$\Gamma^{\mu\nu\alpha\beta}(p,q) = \Gamma_g^{\mu\nu\alpha\beta}(p,q) + \Gamma_q^{\mu\nu\alpha\beta}(p,q) = \sum_{i=1}^{3} \Phi_i(s,0,0)\, \delta^{ab}\, \phi_i^{\mu\nu\alpha\beta}(p,q)$$

form factors tensor structures

The contribution of the trace of the energy momentum tensor is given by

$$g_{\mu\nu}\, \Gamma^{\mu\nu\alpha\beta}(p,q) = 3\, s\, \Phi_1(s;0,0,0)\, u^{\alpha\beta}(p,q) = -2\, \frac{\beta(g)}{g}\, u^{\alpha\beta}(p,q)$$

The relevant form factor shows a massless pole

$$\boxed{\Phi_1(s,0,0) = -\frac{g^2}{72\pi^2\, s}(2n_f - 11C_A)}$$

$$\beta(g) = \frac{g^3}{16\pi^2}\left(-\frac{11}{3}C_A + \frac{2}{3}n_f\right)$$

$$u^{\alpha\beta}(p,q) \equiv (p\cdot q)\, g^{\alpha\beta} - q^\alpha\, p^\beta$$

[R.A., Corianò, Delle Rose, arXiv: 1005.4173]

The massless pole in the effective action

The effective action expanded around the flat space is

$$S_{anom}[g,A] \to -\frac{c}{6}\int d^4x\sqrt{-g}\int d^4x'\sqrt{-g'}\,R_x\,\Box^{-1}_{x,x'}\,[F_{\alpha\beta}F^{\alpha\beta}]_{x'}$$

and now the direct computation of the massless pole contribution yields

$$\begin{aligned} S_{pole} &= -\frac{c}{6}\int d^4x\,d^4y\,R^{(1)}(x)\,\Box^{-1}(x,y)\,F^a_{\alpha\beta}\,F^{a\,\alpha\beta} \\ &= \frac{1}{3}\,\frac{g^3}{16\pi^2}\left(-\frac{11}{3}\,C_A+\frac{2}{3}\,n_f\right)\int d^4x\,d^4y\,R^{(1)}(x)\,\Box^{-1}(x,y)\,F_{\alpha\beta}F^{\alpha\beta} \end{aligned}$$

[R.A., Corianò, Delle Rose,

arXiv: 1005.4173]

$$R^{(1)}_x \equiv \partial^x_\mu\,\partial^x_\nu\,h^{\mu\nu} - \Box\,h, \qquad h = \eta_{\mu\nu}\,h^{\mu\nu}$$

linearized expression of the Ricci scalar

Conclusions and …

The non-local gravitational effective action can be rewritten in a local form in terms of two auxiliary fields whose interpretation is still an open issue

The flat metric expansion of the effective action exhibits a massless scalar pole, confirmed by the perturbative expansion

These long-range gravitational interactions modify the IR region of gravity

… perspectives

Further studies on the relation between the cosmological constant value and the trace anomaly

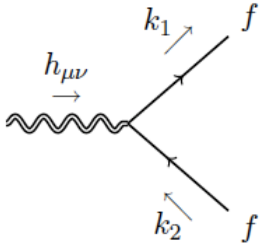
Phenonemological studies in the context of the Arkani-Hamed, Dimopoulos, Dvali model of TeV-scale gravity for collider processes involving gravitons

Extension of the coupling of gravity to the whole SM

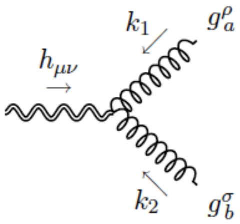
Backup slides

Definitions and conventions

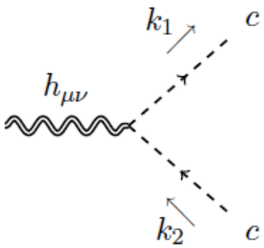
FEYNMAN RULES



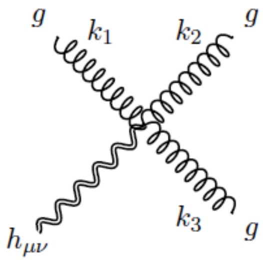
$$= -i\,\frac{\kappa}{2}\,V'_{\mu\nu}(k_1,k_2)$$
$$= -i\,\frac{\kappa}{2}\left\{\frac{1}{4}\left[\gamma_\mu(k_1+k_2)_\nu+\gamma_\nu(k_1+k_2)_\mu\right]-\frac{1}{2}g_{\mu\nu}\left[\gamma^\lambda(k_1+k_2)_\lambda-2m\right]\right\}$$



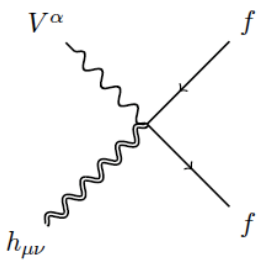
$$= -i\,\frac{\kappa}{2}\,\delta_{ab}\,V^{Ggg}_{\mu\nu\rho\sigma}(k_1,k_2)$$
$$= -i\,\frac{\kappa}{2}\,\delta_{ab}\left\{k_1\cdot k_2\,C_{\mu\nu\rho\sigma}+D_{\mu\nu\rho\sigma}(k_1,k_2)+\frac{1}{\xi}\,E_{\mu\nu\rho\sigma}(k_1,k_2)\right\}$$



$$= -i\,\frac{\kappa}{2}\,\delta^{ab}\,C_{\mu\nu\rho\sigma}\,k_{1\,\rho}\,k_{2\,\sigma}$$



$$= -g\frac{\kappa}{2}f^{abc}V^{Gggg}_{\mu\nu\rho\sigma\lambda}(k_1,k_2,k_3)$$
$$= -g\frac{\kappa}{2}f^{abc}\left\{C_{\mu\nu\rho\sigma}(k_1-k_2)_\lambda+C_{\mu\nu\rho\lambda}(k_3-k_1)_\sigma\right.$$
$$\left.+\,C_{\mu\nu\sigma\lambda}(k_2-k_3)_\rho+F_{\mu\nu\rho\sigma\lambda}(k_1,k_2,k_3)\right\}$$



$$= ig\,\frac{\kappa}{2}\,T^a\,W'_{\mu\nu\alpha} = ig\,\frac{\kappa}{2}\,T^a\left\{-\frac{1}{2}(\gamma_\mu\,g_{\nu\alpha}+\gamma_\nu\,g_{\mu\alpha})+g_{\mu\nu}\,\gamma_\alpha\right\}$$