A description of the

MEASUREMENT PROCESS

by the

PARAMETRIC REPRESENTATION

with

ENVIRONMENTAL COHERENT STATES

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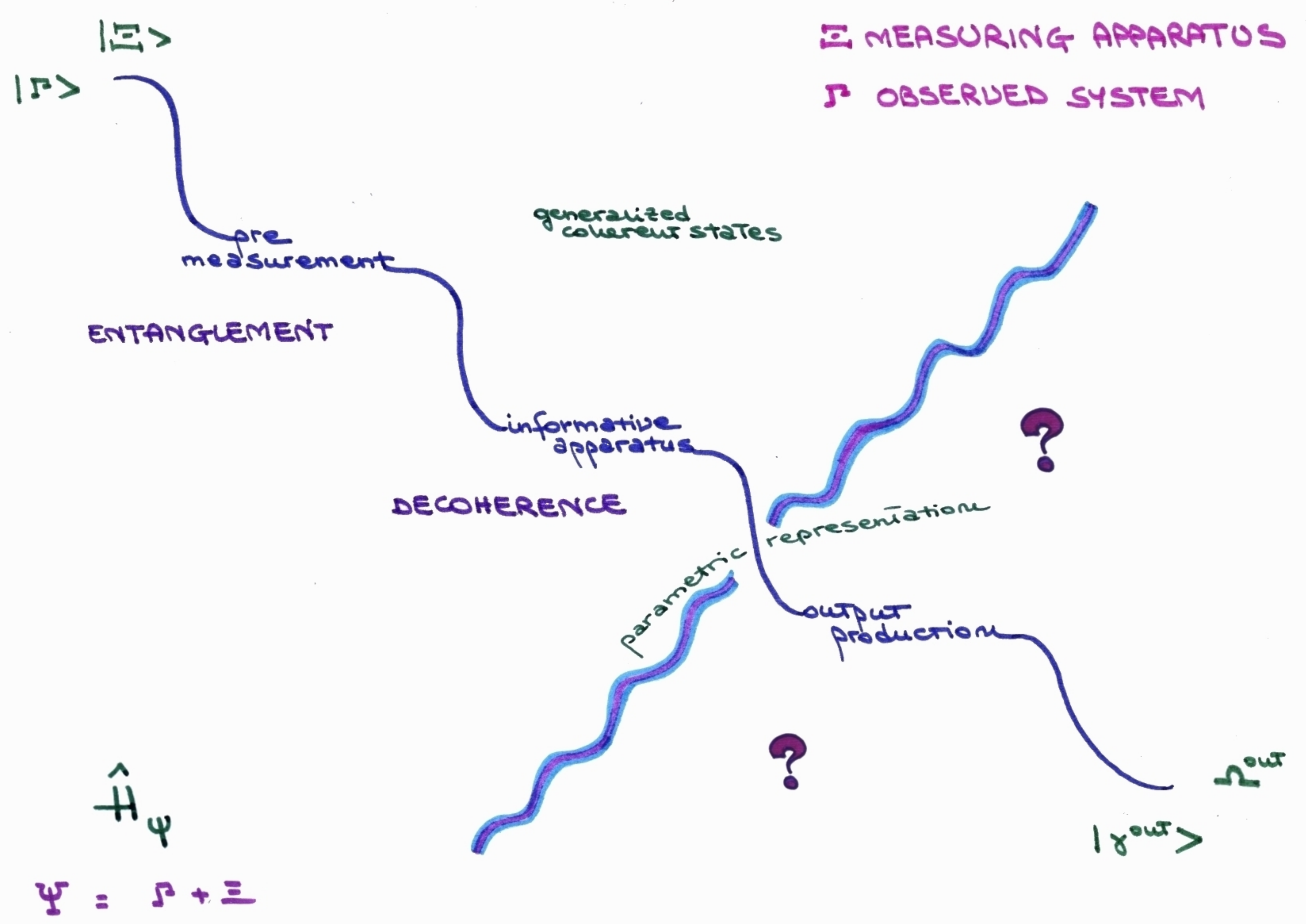
EPL 111, 40008 (2015)



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|ʃ>
Ξ MEASURING APPARATUS
ʃ OBSERVED SYSTEM
generalized
coherent states
pre
measurement
ENTANGLEMENT
informative
apparatus
DECOHERENCE
representation
parametric
output
production
?
?
Ω^out
|γ^out>
Ĥ_Ψ
Ψ = ʃ + Ξ

STANDARD MODEL OF CONVENTIONAL MEASURING PROCESS

$$\Psi = \mathcal{S} + \Xi$$

$$U_t = e^{-i\hat{H}_\Psi t}$$

$$\hat{H}_\Psi = \mu \hat{O}_{\mathcal{S}} \otimes \hat{O}_\Xi + \hat{H}_{\Xi 0}$$

$$\hat{O}_{\mathcal{S}} = \sum_\gamma \omega_\gamma |\gamma\rangle\langle\gamma|$$ no degeneracy

$$|\Psi(0)\rangle = |\mathcal{S}\rangle \otimes |\Xi\rangle = \sum_\gamma c_\gamma |\gamma\rangle \otimes |\Xi\rangle$$

entanglement

$$|\Psi(t)\rangle = \sum_\gamma c_\gamma |\gamma\rangle e^{-i\hat{H}^\gamma t} |\Xi\rangle \equiv \sum_\gamma c_\gamma |\gamma\rangle |\Xi_t^\gamma\rangle$$

$$\hat{H}^\gamma \equiv \mu \omega_\gamma \hat{O}_\Xi + \hat{H}_{\Xi 0}$$

decoherence

$$\rho_{\mathcal{S}}(t) = \sum_\gamma |c_\gamma|^2 |\gamma\rangle\langle\gamma| + \sum_{\gamma\gamma'} c_\gamma c_{\gamma'}^* |\gamma\rangle\langle\gamma'| \langle\Xi_t^{\gamma'}|\Xi_t^\gamma\rangle$$

PARAMETRIC REPRESENTATION WITH ECS

PNAS **110**, 6748 (2013)

$$\Psi = \Omega + \Xi$$

$$|\Psi\rangle = \sum_\gamma c_\gamma |\gamma\rangle \otimes |\Xi^\gamma\rangle = \int d\mu(\Omega)\, \chi(\Omega) |\phi(\Omega)\rangle \otimes |\Omega\rangle$$

$$\int d\mu(\Omega) |\Omega\rangle\langle\Omega| = \mathbb{1}_\Xi$$

$$\langle\Omega|\Omega'\rangle \neq \delta(\Omega - \Omega') \qquad \int d\mu(\Omega)\, \chi^2(\Omega) = 1 \qquad \int_{\mathcal{M}} d\mu(\Omega)$$

$$\rho_\Omega = \int d\mu(\Omega)\, \chi^2(\Omega) |\phi(\Omega)\rangle\langle\phi(\Omega)|$$

once a coherent state, always a coherent state

$$|\Psi(t)\rangle = \int d\mu(\Omega)\, \chi_t(\Omega) |\phi_t(\Omega)\rangle \otimes |\Omega\rangle$$

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$$\int d\mu(\Omega) |\Omega\rangle\langle\Omega| = \mathbb{1}_\Xi$$

$$\langle\Omega|\Omega'\rangle \neq \delta(\Omega - \Omega') \qquad \int d\mu(\Omega)\, \chi^2(\Omega) = 1 \qquad \int_{\mathcal{M}} d\mu(\Omega)$$

$$\rho_\Omega(t) = \int d\mu(\Omega)\, \chi_t^2(\Omega) |\phi_t(\Omega)\rangle\langle\phi_t(\Omega)|$$

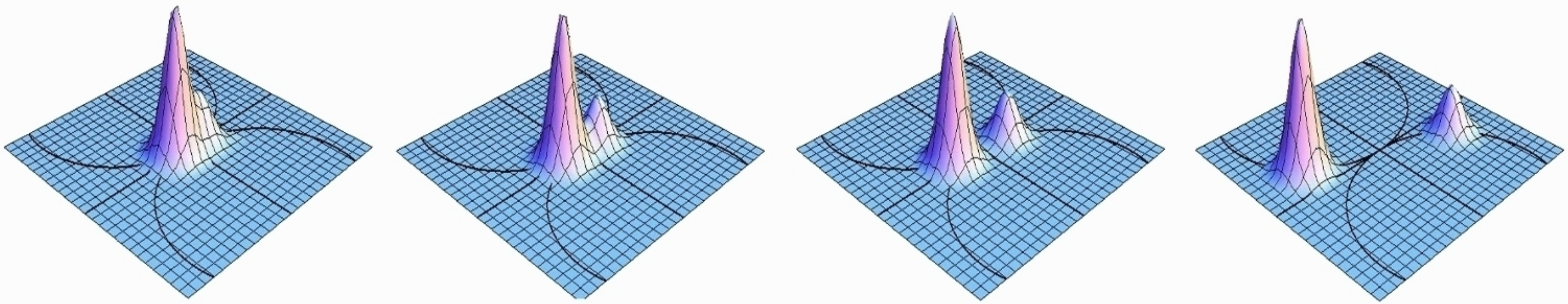
once a coherent state, always a coherent state

$$|\Psi(t)\rangle = \int d\mu(\Omega)\, \chi_t(\Omega) |\phi_t(\Omega)\rangle \otimes |\Omega\rangle$$

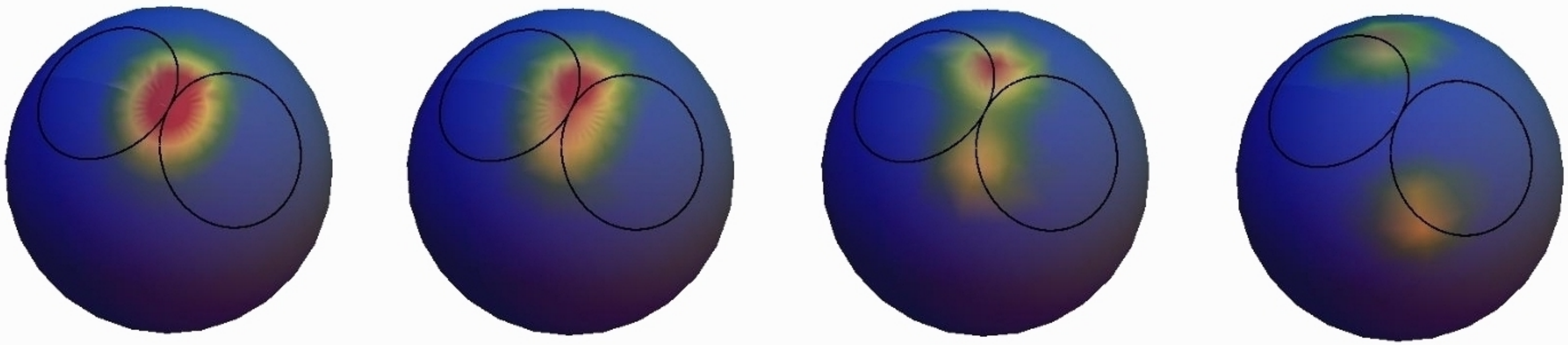
EXAMPLE

$\hat{\sigma}^z$ qubit

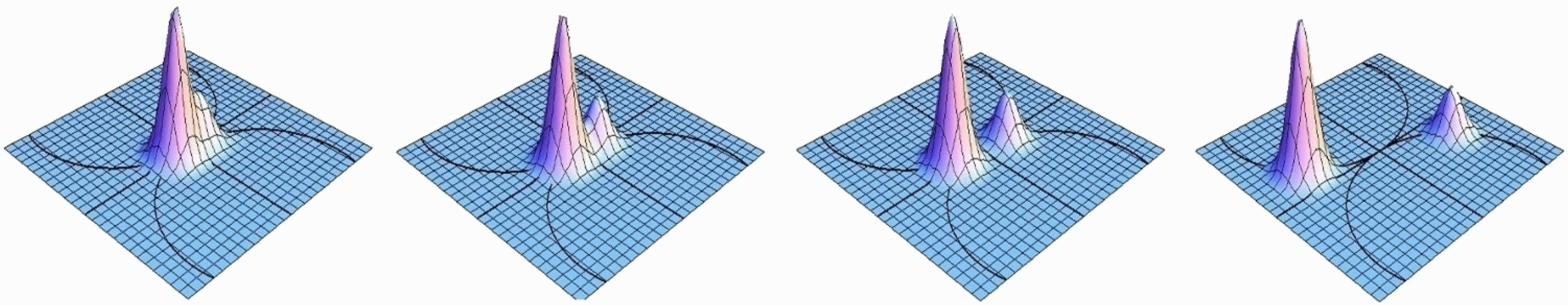
$\hat{H}_{qb} = \nu \hat{b}^{+}\hat{b} + g\sqrt{\hbar}\,\hat{\sigma}^z(\hat{b} + \hat{b}^{+})$; $[b, b^{+}] = \hbar$; $\mathcal{M}$ complex plane



$$\chi^2_t(\Omega) = \sum_\gamma |c_\gamma|^2 \, |\langle \Omega | R^\gamma_t \rangle|^2 = \sum_\gamma |c_\gamma|^2 \, h^\gamma_t(\Omega)$$

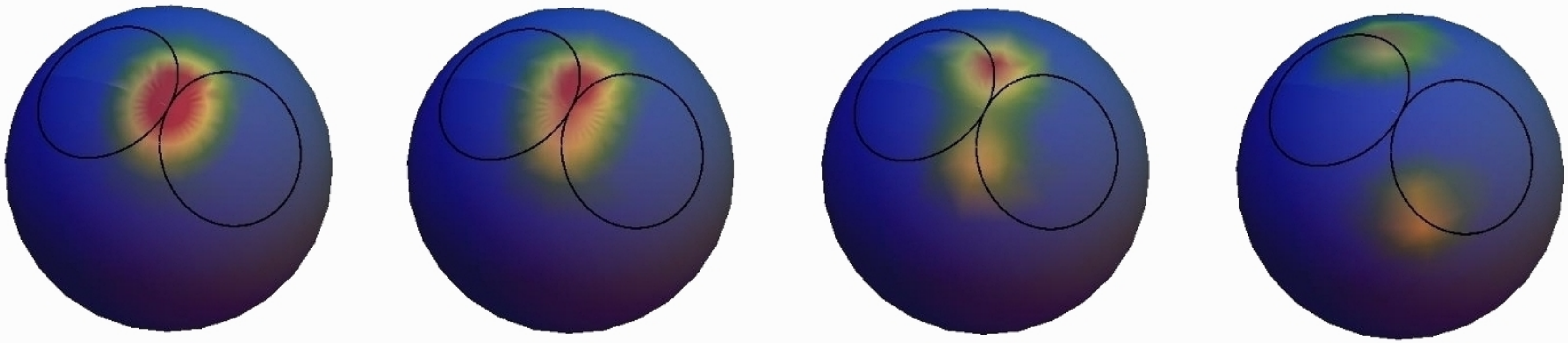


$\hat{H}_{qJ} = h\hat{J}_z + g\hat{\sigma}^z(\hat{J}_x)$; $[\hat{J}_\alpha, \hat{J}_\beta] = i\varepsilon^{\alpha\beta\delta}\hat{J}^\delta$; $\mathcal{M}$ unit 2-sphere



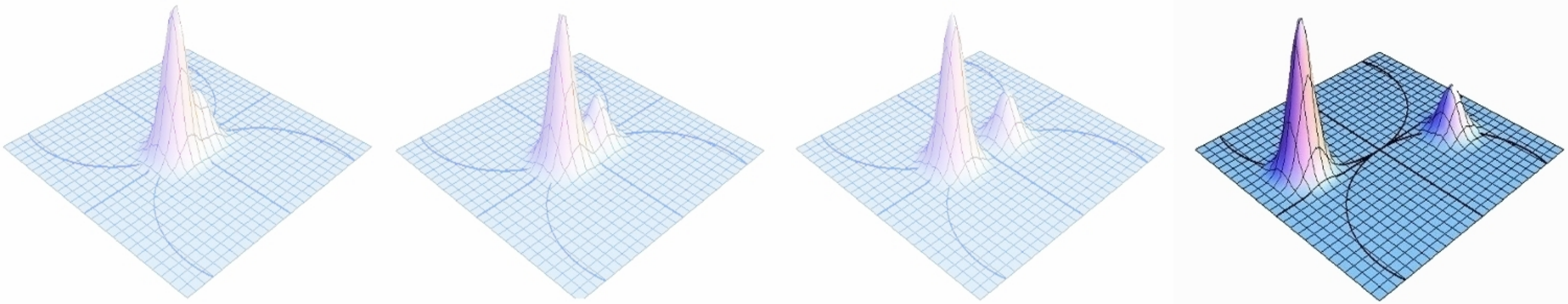
$x_t^2(\Omega) = \sum_\gamma |c_\gamma|^2 \, h_t^\gamma(\Omega)$ normalized distributions on $\mathcal{H}$ with ε-support

$\mathcal{S}_t^\gamma$: $h_t^\gamma(\Omega) > \varepsilon$ for $\Omega \in \mathcal{S}_t^\gamma$



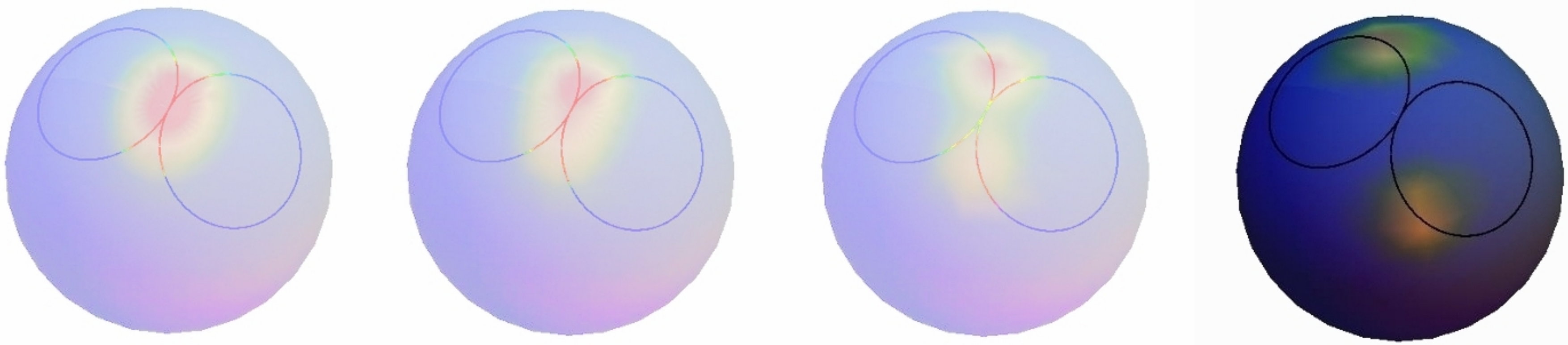
INFORMATIVE APPARATUS DECOHERENCE

IJTP (online 2015) arXiv: 1411.2797



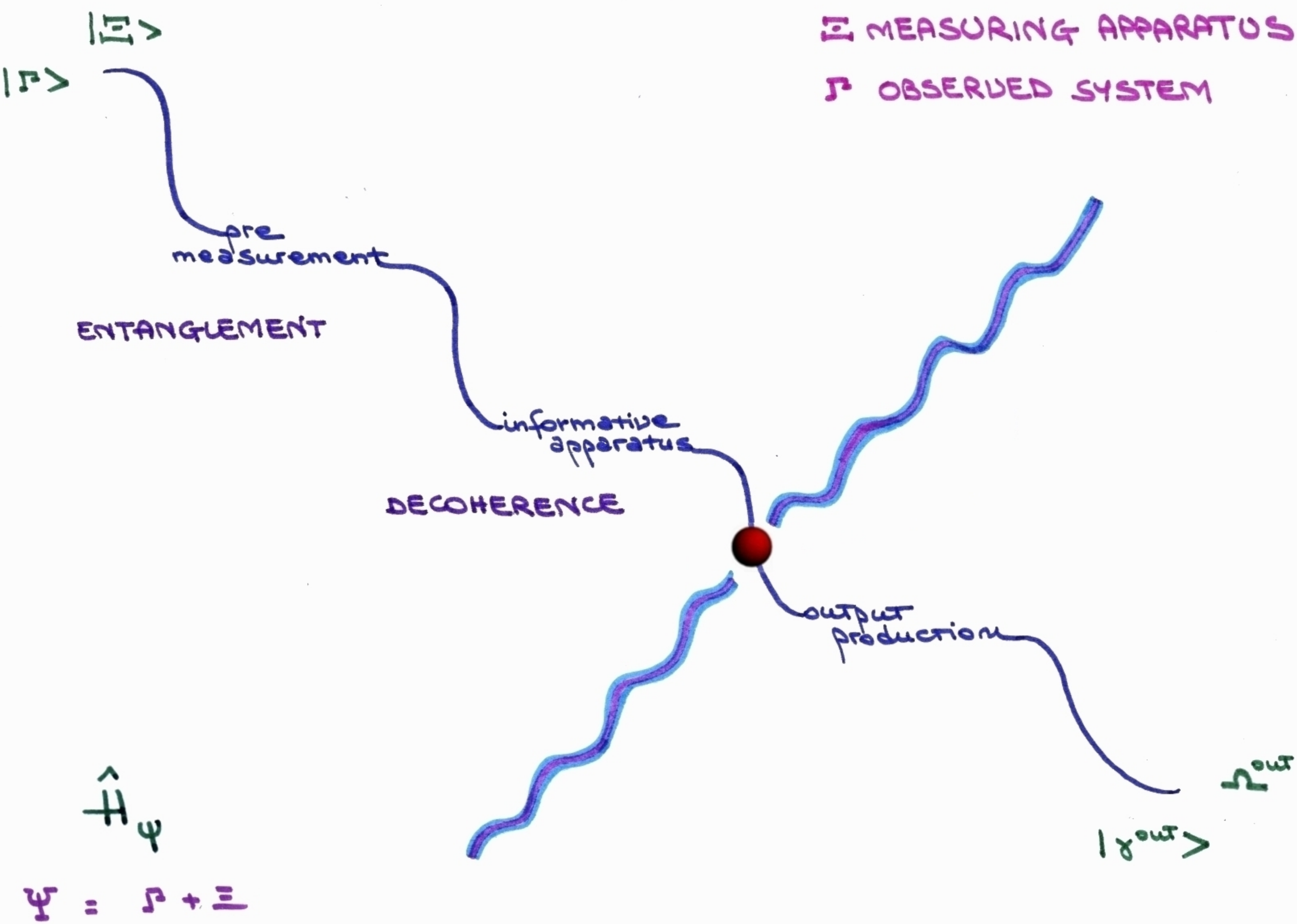
$x_t^2(\Omega) = \sum_\gamma |c_\gamma|^2 h_t^\gamma(\Omega)$ normalized distributions on $\mathcal{M}$ with ε-support

S_t^γ : $h_t^\gamma(\Omega) > \varepsilon$ for $\Omega \in S_t^\gamma$



$$S_t^\gamma \cap S_t^{\gamma'} = \emptyset$$

$$\rho_S(t) = \int d\mu(\Omega)\, x_t^2(\Omega) |\phi_t(\Omega)\rangle\langle\phi_t(\Omega)| = \sum_\gamma |c_\gamma|^2 |\gamma\rangle\langle\gamma|$$



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Ξ MEASURING APPARATUS
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"CAN ONE FIND A CLASSICAL SYSTEM WHOSE DYNAMICS IS EQUIVALENT TO SOME $N \to \infty$ LIMIT OF A GIVEN QUANTUM THEORY"

? L.G. YAFFE Rev. Mod. Phys. **54**, 407 ('82)

$|\Omega\rangle_k \, , \, \mathcal{H}_k$

G_k

Q_k

conditions ensuring

$$Q_k \xrightarrow[k \to 0]{} C$$

emerge in Terms of

COHERENT STATES $|\Omega\rangle_k$

$k \to 0$

C

classical phase-space Ω

" CAN ONE FIND A CLASSICAL SYSTEM WHOSE DYNAMICS IS EQUIVALENT TO SOME $N \to \infty$ LIMIT OF A GIVEN QUANTUM THEORY "

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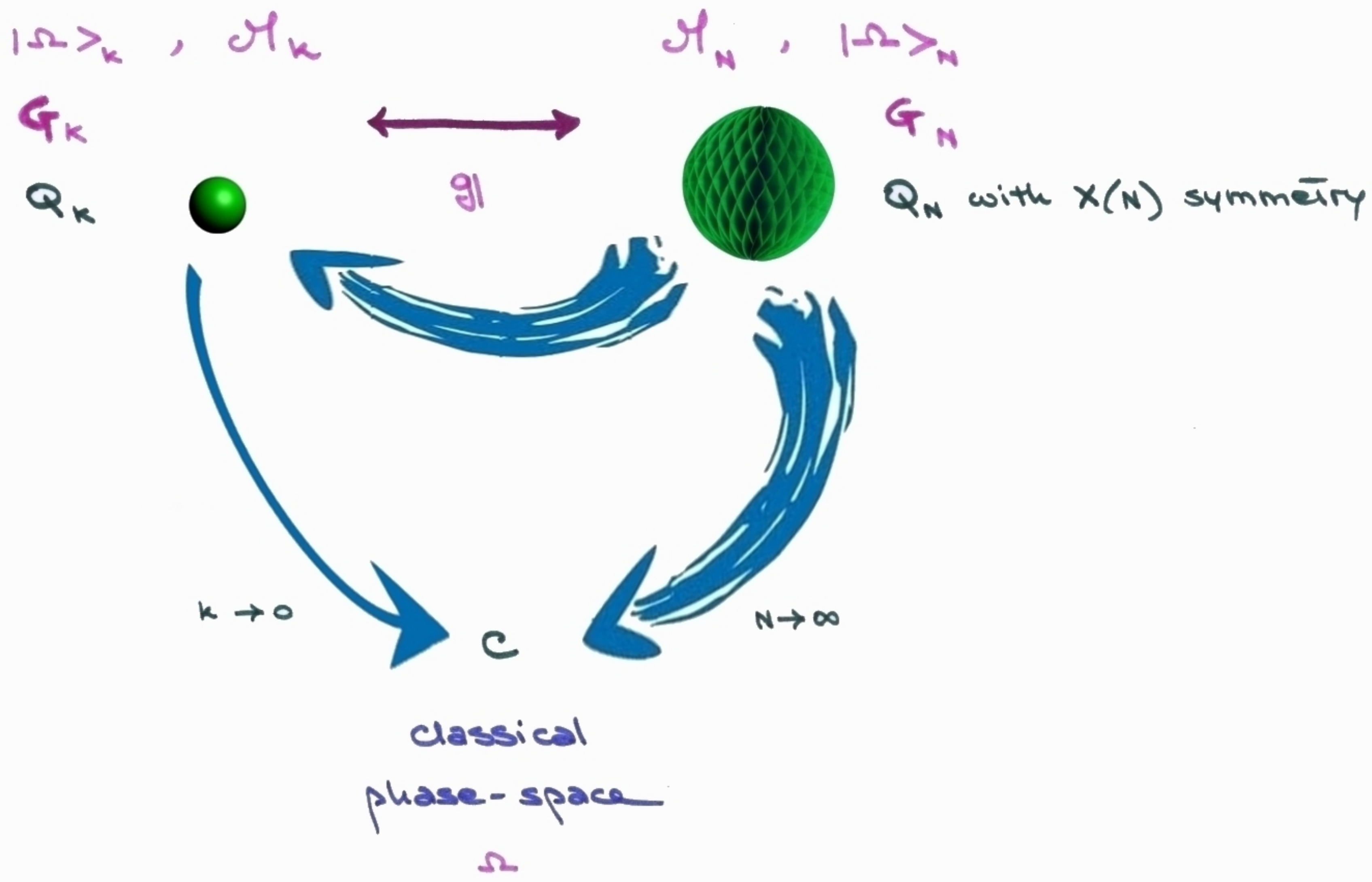
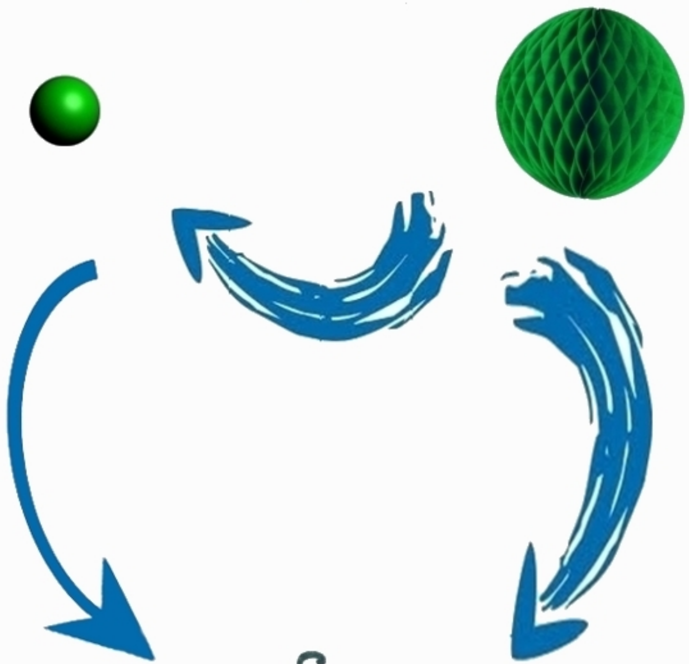


CLASSICALLY EQUIVALENT

$$\lim_{k\to 0} Q_k \quad = \quad \lim_{N\to\infty} Q_N$$



$|\Omega\rangle_k$

$\{|\Omega\rangle_N\}_\sim$

c

$\sim$

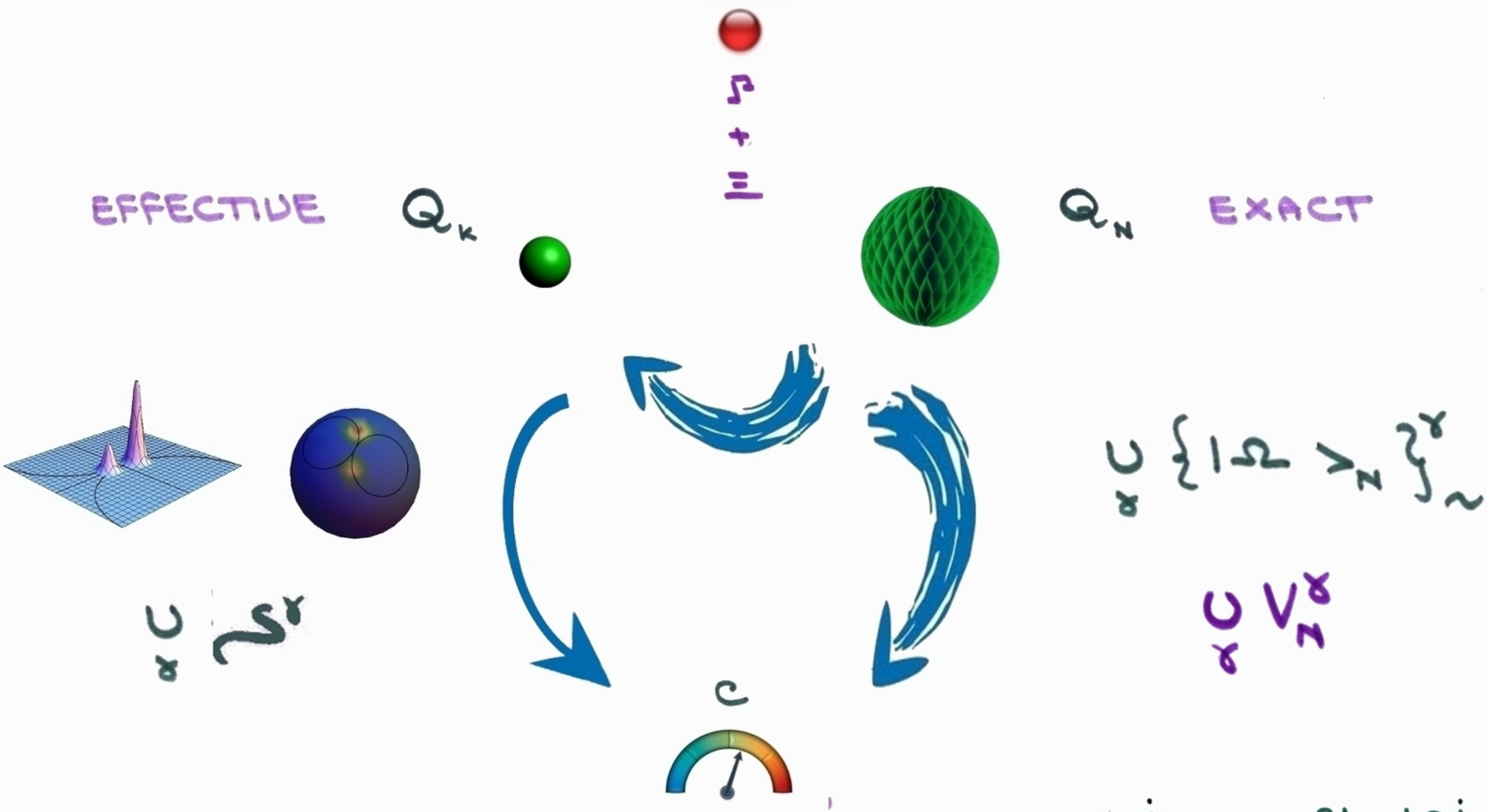
classically equivalent

$${}_k\langle\Omega|\hat{A}_k|\Omega\rangle_k \rightarrow A(\Omega) \leftarrow {}_N\langle\Omega^i|\hat{A}_N|\Omega^i\rangle_N$$

finite

! $\forall\, |\Omega^i\rangle_N \in \{|\Omega\rangle_N\}_\sim$

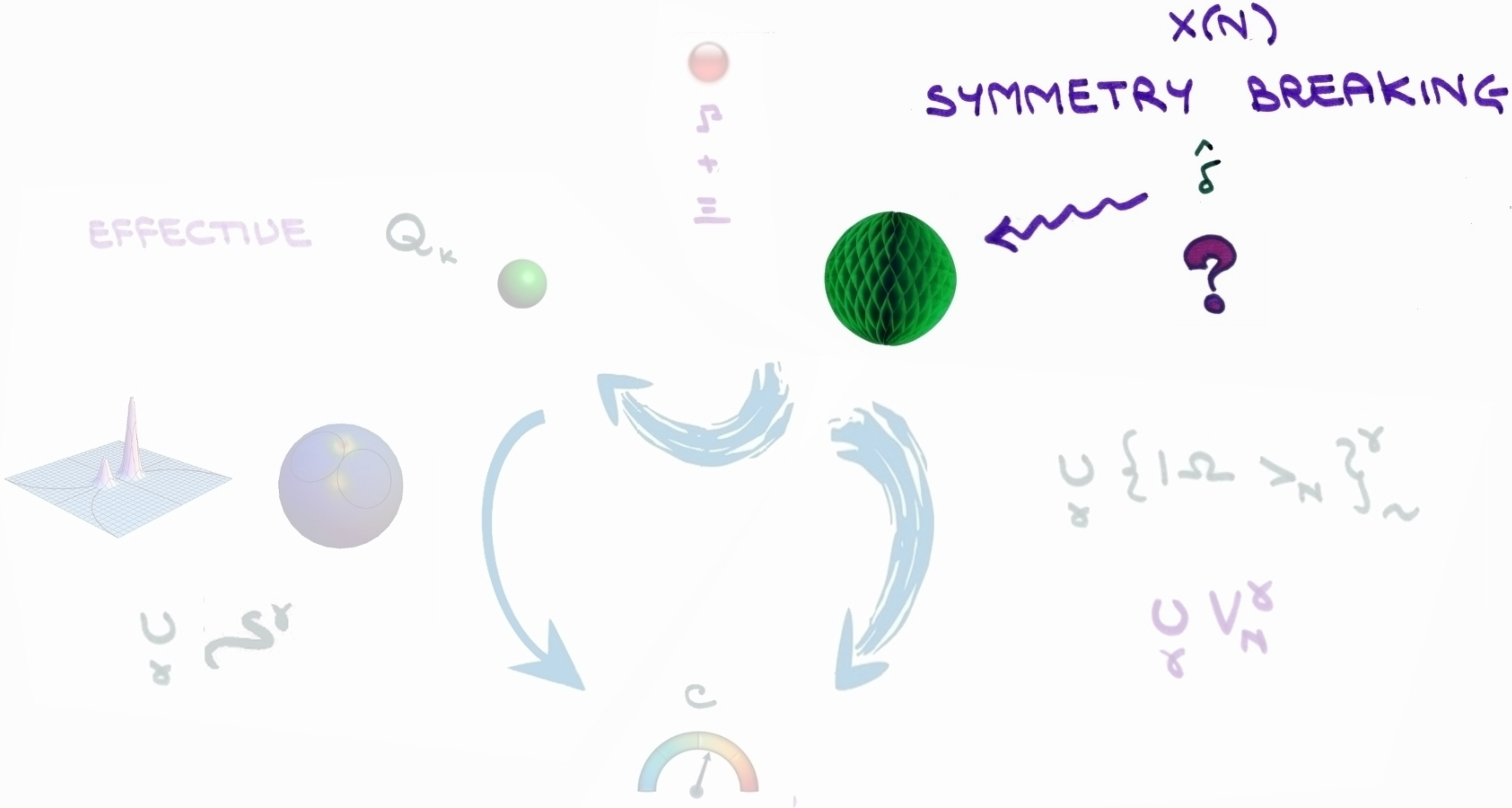
$|\Omega^i\rangle_N = U_{ij}\,|\Omega^j_N\rangle$



$|\Omega^i\rangle_N = \mathcal{U}_{ij}\, |\Omega^j_N\rangle$

they all have the same energy

they are not classically equivalent if $\gamma \neq \gamma'$


X(N)
SYMMETRY BREAKING
EFFECTIVE

X(N)

SYMMETRY BREAKING

$\Omega + \Xi$

$\hat{\delta}$

$$\bigcup_\gamma \{|\Omega\rangle_N\}^\gamma_\sim$$

$$\bigcup_\gamma V_N^\gamma$$

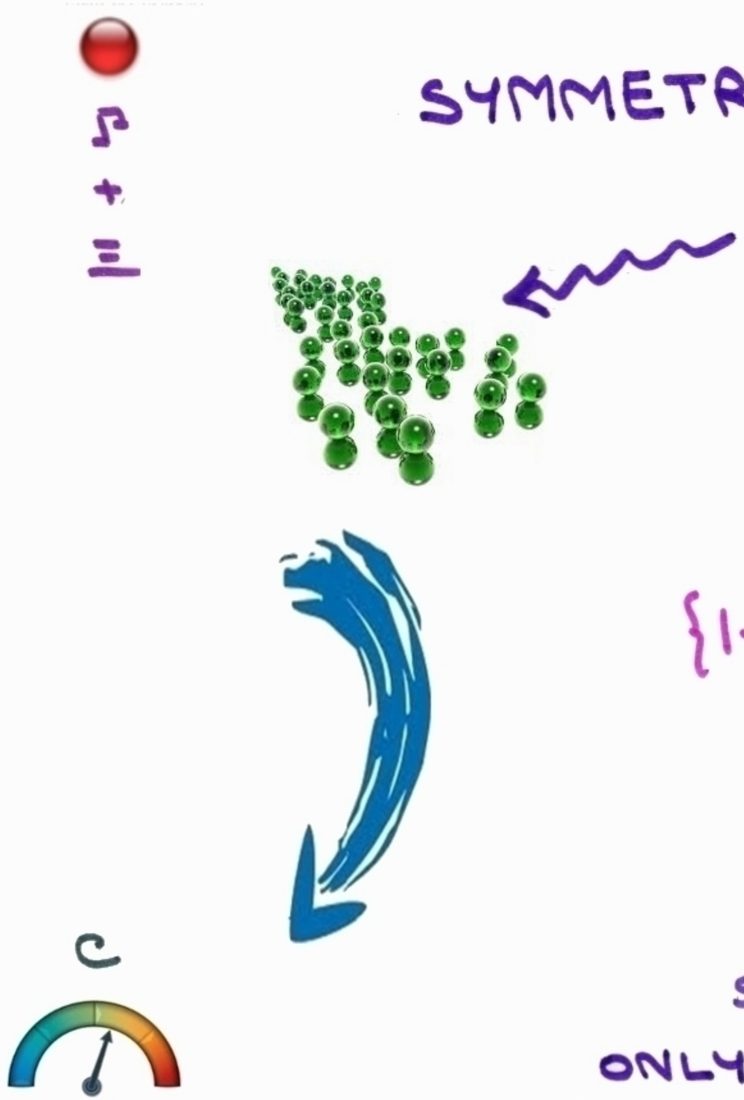
c

$\hat{\delta}$ cannot depend on $\{\omega_\gamma\}$

$$\hat{O}_\Omega = \sum_\gamma \omega_\gamma |\gamma\rangle\langle\gamma|$$

$$\hat{H}^\gamma \equiv u\omega_\gamma \hat{O}_\Xi + \hat{H}_{\Xi 0}$$

X(N)
SYMMETRY BREAKING



$\hat{\delta}$

!

$\{|\Omega\rangle_N\}_N^{\gamma_{out}}$

$V_N^{\gamma_{out}}$

SELECTS
ONLY ONE $|\Omega^{out}\rangle_N$

$\Omega^{out} \in V_N^{\gamma_{out}}$

$[\hat{\delta}, U_{ij}] \neq 0 \quad \forall i,j$

$\forall U_{ij} \in X(N)$

X(N)

SYMMETRY BREAKING

Ω

$+$

Ξ

$\hat{\delta}$

!

$\{|\Omega\rangle_N\}_N^{\gamma_{out}}$

$V_N^{\gamma_{out}}$

C

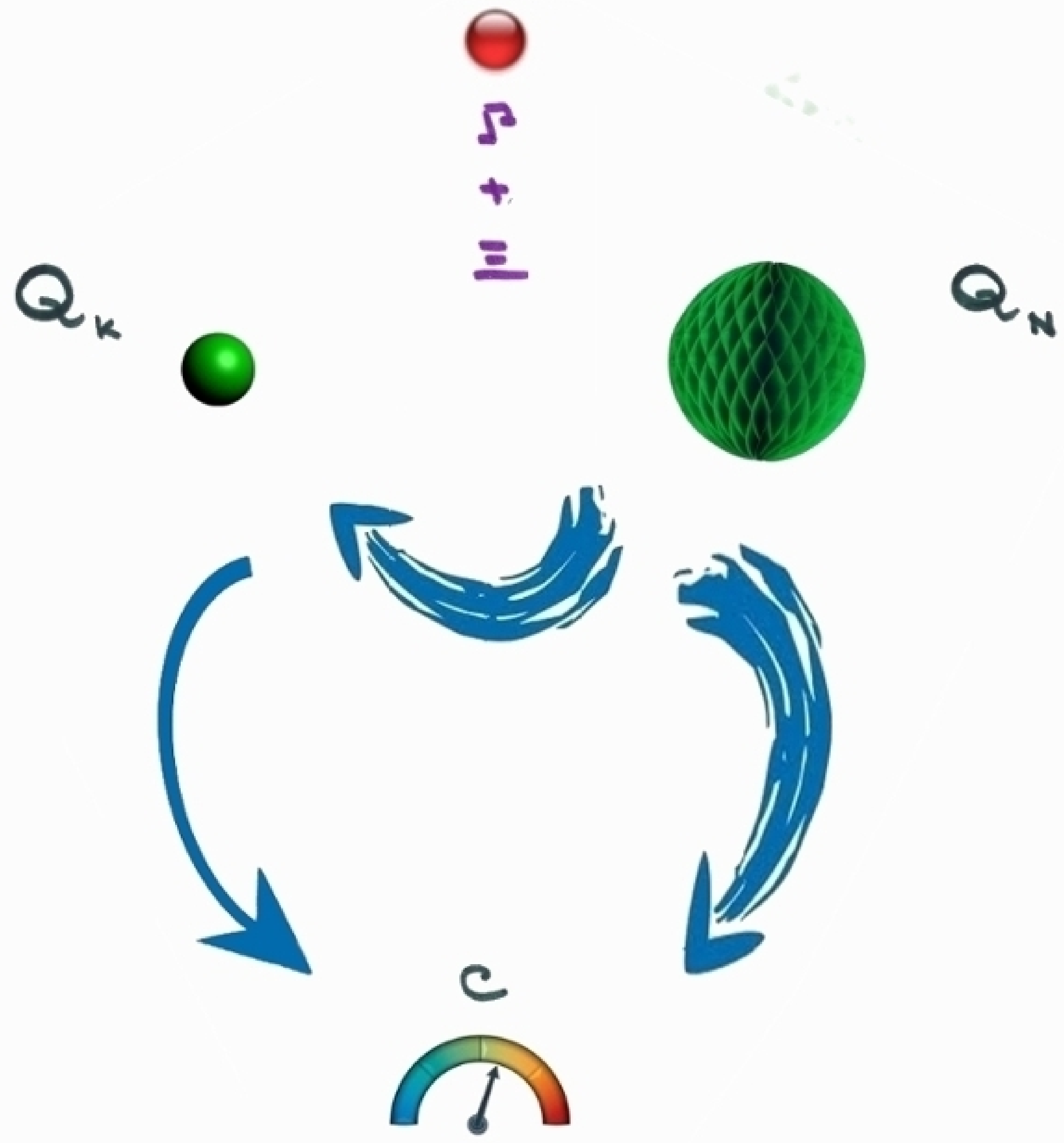
SELECTS ONLY ONE γ^{out}

$$A(\Omega^{\gamma_{out}}) \xleftarrow[\infty \leftarrow N]{} {}_N\langle \Omega^i | \hat{A}_N | \Omega^i \rangle_N$$

$p^{\gamma_{out}}$

ANY Ω^i_N in $V_N^{\gamma_{out}}$

$$V_N^\gamma$$



$$\langle \Psi | \mathbb{1}_\Omega \otimes \hat{A}_k | \Psi \rangle =$$

$$\lim_{k \to 0} \sum_\gamma \int_{S^\gamma} d\mu(\Omega)\, \chi^2(\Omega)\, {}_k\langle \Omega | \hat{A}_k | \Omega \rangle_k$$

$$= \lim_{N \to \infty} \sum_\gamma \int_{V_N^\gamma} d_N\mu(\Omega)\, {}_N\langle \Omega | \hat{A}_N | \Omega \rangle_N$$

at all times, for all $\{c_\gamma\}$, for all operators with a finite $k \to 0$ limit

$$V_N^\gamma \propto \lim_{k \to 0} \int_{S^\gamma} d\mu(\Omega)\, \chi^2(\Omega) = |c_\gamma|^2$$

BORN's RULE

ABOUT Ω

$$|\psi(t)\rangle = \int_{\mathcal{M}} d\mu(\Omega)\, \chi_t(\Omega)\, |\Omega\rangle |\phi_t(\Omega)\rangle$$

$$|\phi_t(\Omega)\rangle = \frac{1}{\chi_t(\Omega)} \sum_\gamma c_\gamma \langle \Omega | \xi \rangle |\gamma\rangle = \sum_\gamma \frac{e^{i\phi_t^\gamma}}{\left(1 + \sum_{\gamma' \neq \gamma} \frac{|c_{\gamma'}|^2 h_t^{\gamma'}(\Omega)}{|c_\gamma|^2 h_t^\gamma(\Omega)}\right)^{1/2}} |\gamma\rangle$$

if the selected $|\Omega^{out}\rangle_N$ is : $\Omega^{out} \in V_N^{\gamma_{out}}$ then

the only $h_t^\gamma(\Omega) \neq 0$ is $h_t^{\gamma_{out}}(\Omega)$

$$|\phi(\Omega)\rangle \longrightarrow |\gamma^{out}\rangle$$

STATE REDUCTION


|Ξ>
|Γ>
Γ OBSERVED SYSTEM
Ξ MEASURING APPARATUS
pre measurement
ENTANGLEMENT
informative apparatus
DECOHERENCE
LARGE-N
EXP
?
SYMMETRY BREAKING
output production
Γ + Ξ
k
g
σ
Γout
|γout>

THANK YOU

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