BEAM OPTICS AND MAGNET STUDIES FOR NEUTRALIZER STORAGE RINGS (NSR)

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The design of efficient storage rings with large acceptance so that a neutralizer gas cell can be inserted requires both linear matrix formalism and full field tracking calculation. Moreover large magnet aperture must be considered. First an unbiased search of suitable lattices is needed. Matrix formalism is simple enough to allow use of symbolic manipulation programs, with s the beam direction, x,y the transverse coordinates, and M_x, M_y the corresponding transport matrices: the conditions that $|\mathrm{trace}(M_x)| < 2$ and $|\mathrm{trace}(M_y)| < 2$ can be reduced (automatically) to simple inequalities for lattice side lengths. Numerical optimizations are also discussed. Differently from usual storage rings, primary beam consumes in few passage through neutralizer cell, so angle injection seems possible. Field tracking simulation needs a rapid method to calculate field from pole footprints and shape, which preferably avoids the use of differential formulas. The method proposed is compared with analytic result for flat poles. After determining suitable magnet poles, full 3D magnets can be designed, for verification. Analogies with Fixed-Field Alternating Gradient (FFAG) accelerators are being investigated.

I. INTRODUCTION: Storage rings and neutralizer

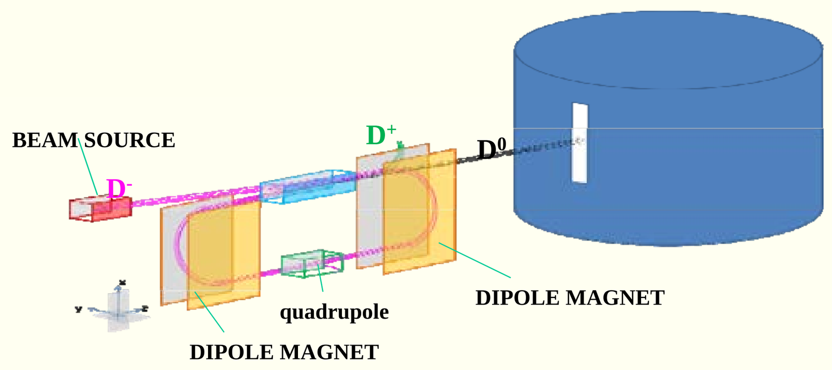


Figure 1: artistic view of ion source, storage ring and tokamak (see other poster); vertical mounting and recovery of D^+ beam will be understood in the following

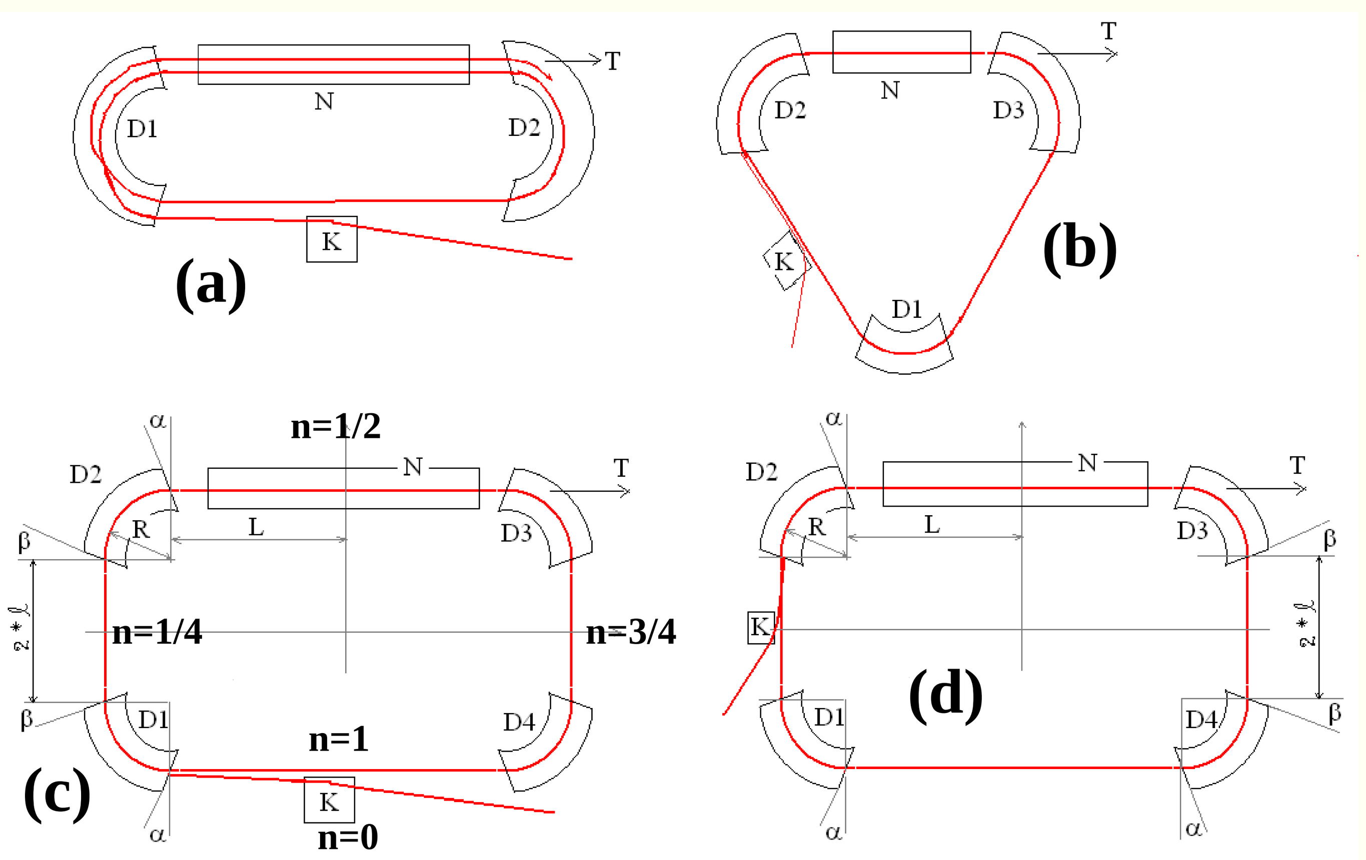


Figure 2: a) the lattice (magnet and beam path scheme) of fig. 1, with a different injection at K (inflector steerer); T is tokamak receiving neutral D^0 from neutralizer N; here number M of main dipoles is M=2. b) a different lattice, with triangular symmetry and M=3; c) the rectangular lattice (that is M=4), with some notation: R bending radius, D1, D2, D3 and D4 dipole magnets, ℓ and respectively L are half lengths of the short and long straight sections; α and β are entrance/exit angles (note that angles of size β are adjacent to short secion for rectangular symmetry); d) the rectangular lattice as before, but with injection on short section. Other polygonal lattice (with M>4 and/or different bending angles) may be also considered; the square lattice is as 'd', but with ℓ=L . Note the turn number n, and its origin n=0.

Design study can be divided into 3 phases, of progressively detailed specification:

1) **linear stability of x,y around the reference orbit x(z)=x'(z)=y(z)=y'(z)=0, where local system xyz is so defined: z tangent to reference orbit, x centrifugal force direction, y normal to both; moreover s is orbit arc length. Magnet field By is taken piecewise constant (no fringe field)**
2) **in the pole only approximation, the pole shape and gap are taken in account. Fringe field is usually inferred from empirical example, but here is computed by special boundary conditions applied to 3D simulation of the pole region only**
3) **After a lattice is known to work in previous two phases, coil and yoke 3D geometry can be specified, and verified by complete simulations**

The transfer matrix $M_x(s)$ for phase space (x,x') and $M_y(s)$ for phase space (x,x') are known as function of lengths L and ℓ and magnet parameter

$$t_a = \tan\alpha \quad , \quad t_b = \tan\beta \quad , \quad n = \frac{R}{B_y}\frac{\partial B_y}{\partial x}$$

at reference orbit; we consider only magnet with field index n=0 (that is flat poles). Let Mx(s) and My(s) be computed for half turn (s=2 L + 2 ℓ + π R), since lattice repeats each half a turn. Linear stability requires

$$-2 < \mathrm{Tr}\, M_x < 2 \quad , \quad -2 < \mathrm{Tr}\, M_y < 2 \qquad \text{(eq. 1)}$$

We set

$$\ell = 0.8R \quad , \quad L = 3.8R \qquad \text{(eq. 3)}$$

and compute where eq. 1 is satisfied (white area in Fig. 3). In this region we choose (to complete linear design and to obtain a phase advance per turn as in fig. 4 and 5)

$$t_a = t_b = 0.52 \qquad \text{(eq. 3)}$$

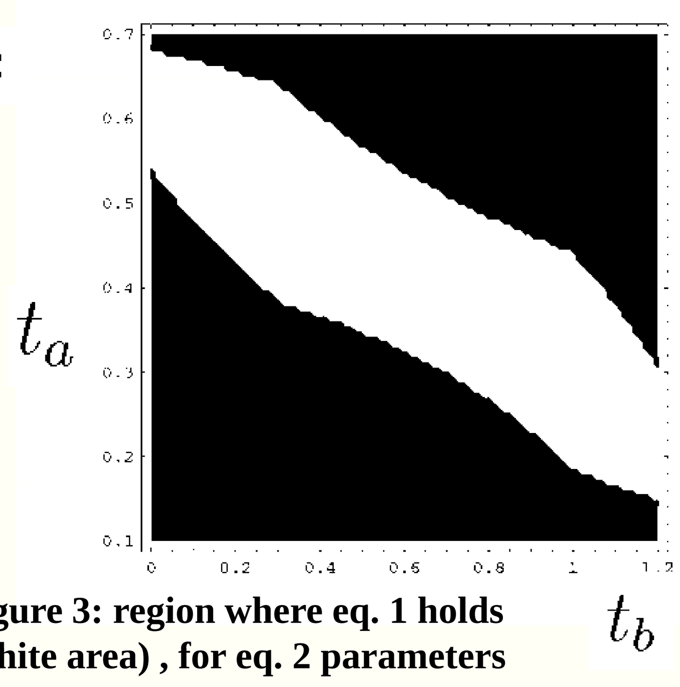


Figure 3: region where eq. 1 holds (white area) , for eq. 2 parameters

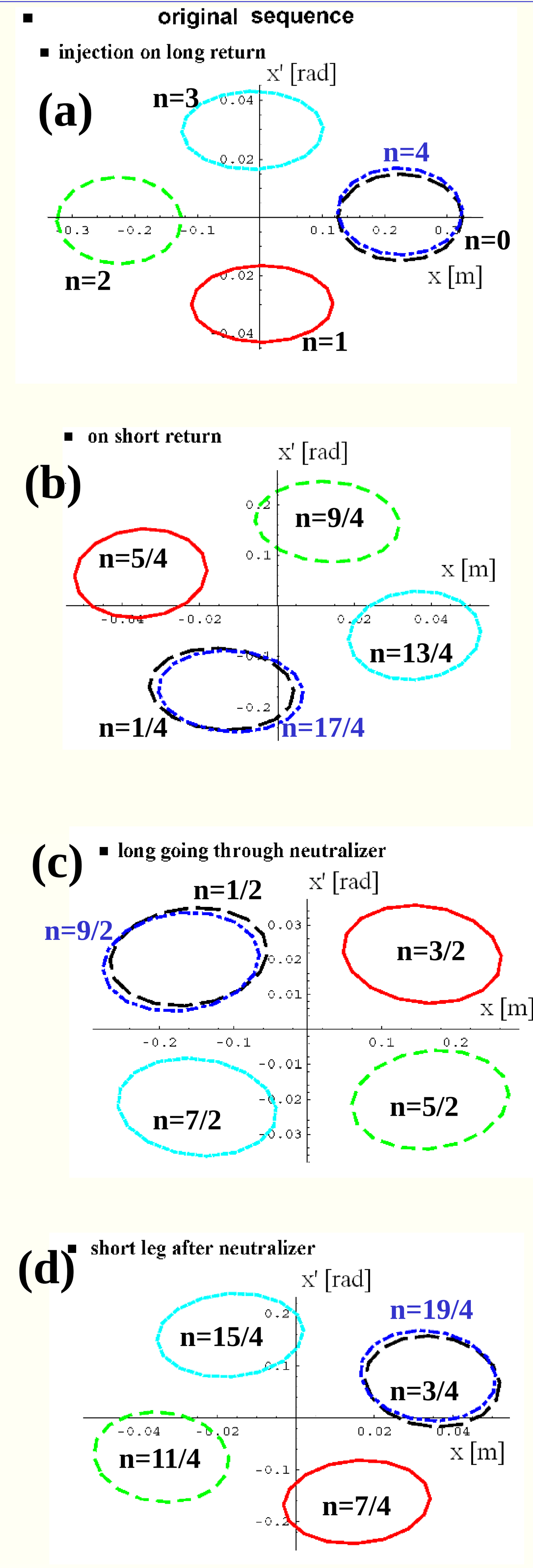


Figure 4: x,x' phase space nominal beam (colored ellipse), for the turn number n as labels. Fractional turn numbers, as defined in Fig. 2.c, refers to beam passage at symmetry planes. Remaining beam is ejected at n=4, by the action of the inflector K

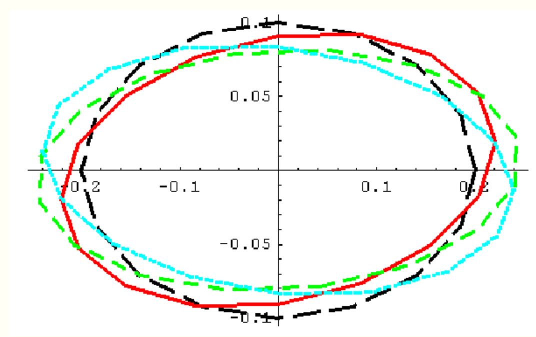
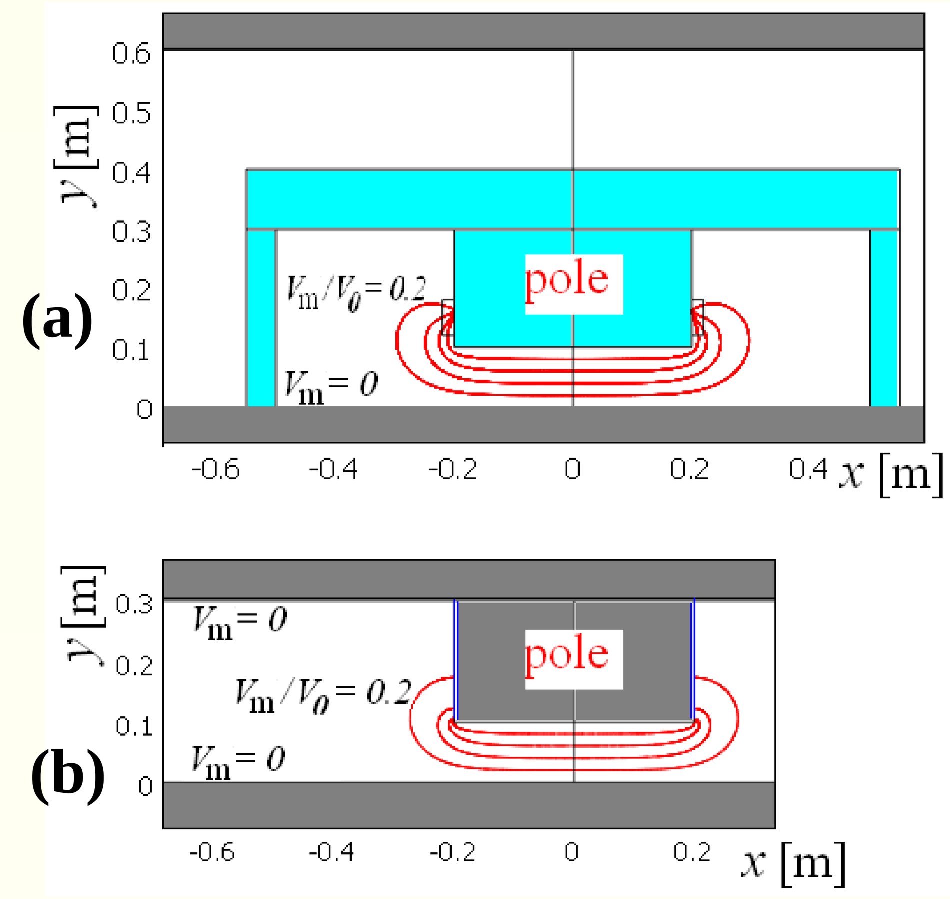


Figure 5: y, y' phase space nominal beam (colored ellipse), for the turn numbers n=0,1,2,3 (same colors as in fig 4.a). Remaining beam will be ejected at n=4, by the action of the inflector K

Phase 2: the pole only calculation

Figure 6: a) the magnetic potential Vm level lines, as computed in a realistic geometry including iron (azure region) and coil. b) The Vm level lines, computed by special boundary conditions as labelled. Gray region excluded from simulations.



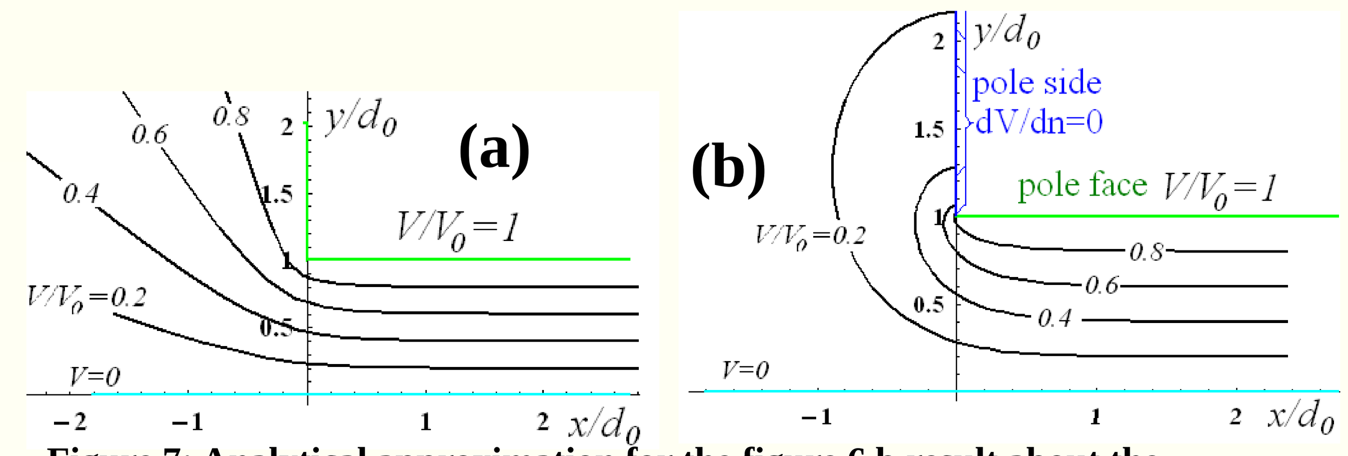


Figure 7: Analytical approximation for the figure 6.b result about the magnetic potential Vm level lines, with idealized geometry and boundary as labelled. In case (a), eq. 5 holds and green line is Vm=V0.
In case (b), eq, 6 holds, and double blue line is Neumann condition.

For figure 7 idealized geometry, a well known transform from z=x+iy to w plane

$$\frac{z}{d_0} = \frac{z_s(s)}{d_0} = \mathrm{i} + \frac{1}{\pi}\left(2s - \ln\frac{1+s}{1-s}\right) \qquad s = \mathrm{i}\sqrt{w-1} \qquad \text{(eq. 4)}$$

map the field region into the Im(w)>0 semiplane. With boundary condition of Fig. 7.a we have the solution

$$\frac{V}{V_0} = v_{w1}(w) = 1 - \frac{1}{\mathrm{i}\pi}\ln(w) \qquad v_{s1}(s) = 1 - \frac{\ln(1-s^2)}{\mathrm{i}\pi} \qquad \text{(eq. 5)}$$

With boundary condition of Fig. 7.b we find the solution

$$\frac{V}{V_0} = v_{s2}(s) = \frac{1}{\mathrm{i}\pi}\ln\frac{s-1}{s+1} \qquad \text{(eq. 6)}$$

Agreement of Fig 7 b and 6 b to true solution 6.a is reasonable and justifies use of 6.b assumptions, which allows the pole-only 3D calculation. Note an eigenfunction expansion of eq (6) as

$$H_x - \mathrm{i}H_y = -\frac{\partial V}{\partial z} = -\frac{\mathrm{i}V_0}{d_0}\left(1 + \sum_{n=1}^{\infty} a_n e^{-n\left(2 + \frac{\mathrm{i}\pi y}{d_0} + \frac{\pi x}{d_0}\right)}\right) \qquad \text{(eq. 7)}$$

$$a_1 = -4 \quad , \quad a_2 = 24 \quad , \quad a_3 = -156 \quad , \quad a_4 = 3152/3 \qquad \text{(eq. 8)}$$

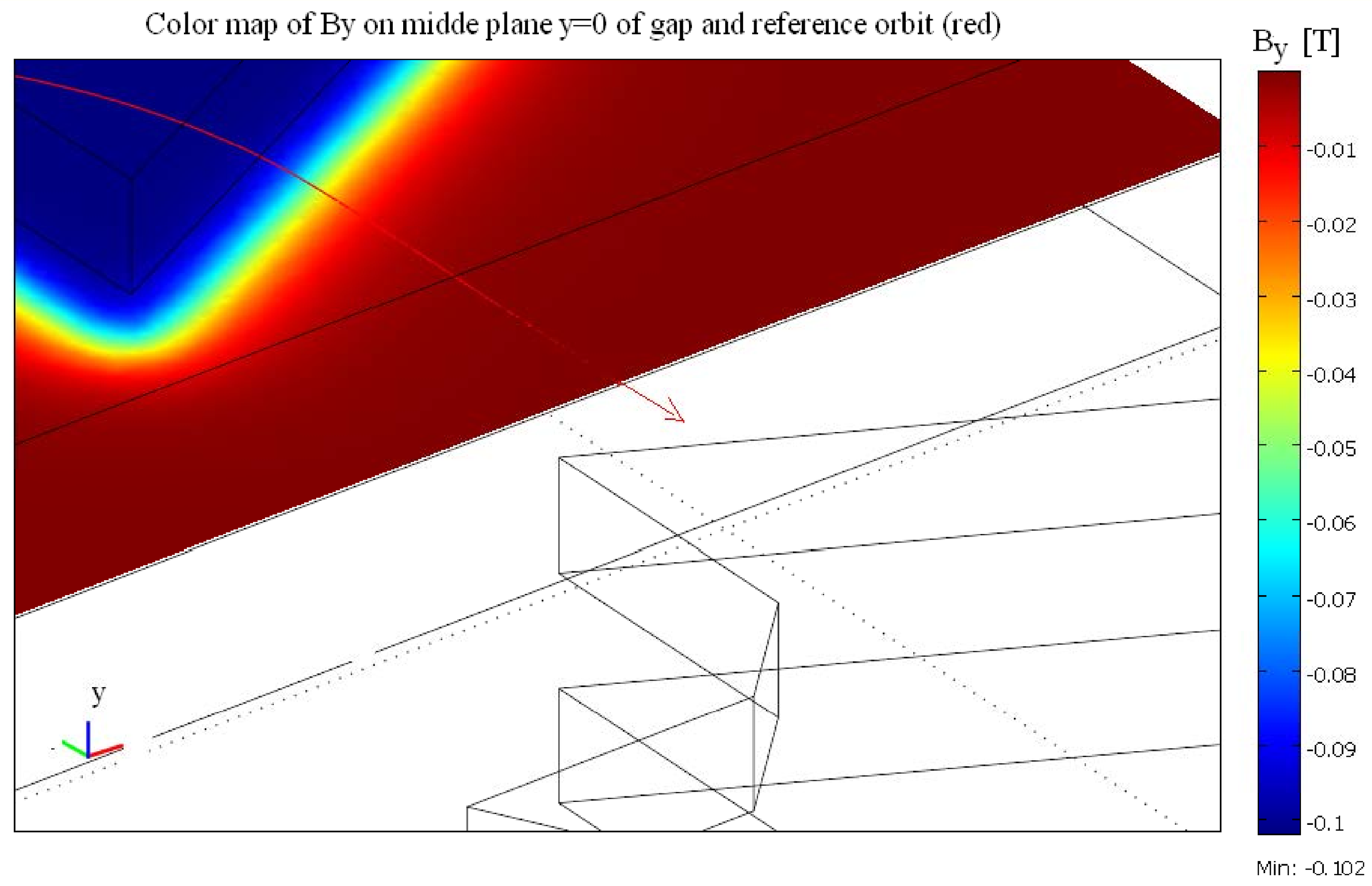


Figure 8: detail view of 3D simulation (pole only) where perpendicular crossing of reference orbit to a symmetry plane is visible; this was obtained by adjusting coil current.