Signatures of glassiness in interacting fermions in one dimensions

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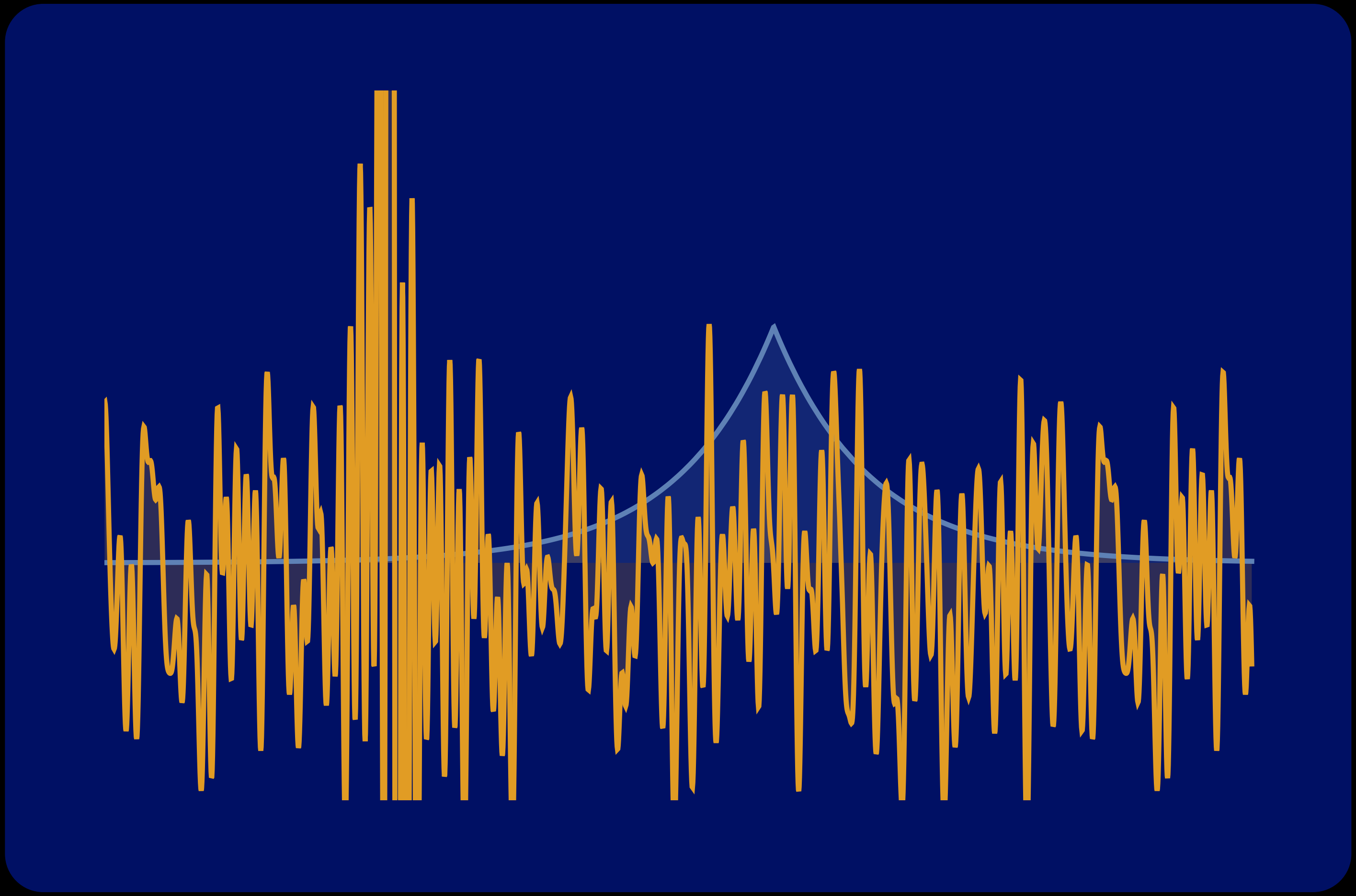
Overview

- Model and its dynamical phases
- Energy diffusion
- Particle sub-diffusion

Fermions with disorder

$$H = -t\sum_i (c^\dagger_{i+1}c_i + c^\dagger_i c_{i+1}) + \sum_i \epsilon_i c^\dagger_i c_i$$

Anderson problem, hopping on impurities



$$\rho(x,\infty) \propto e^{-|x-x_0|/\xi}$$

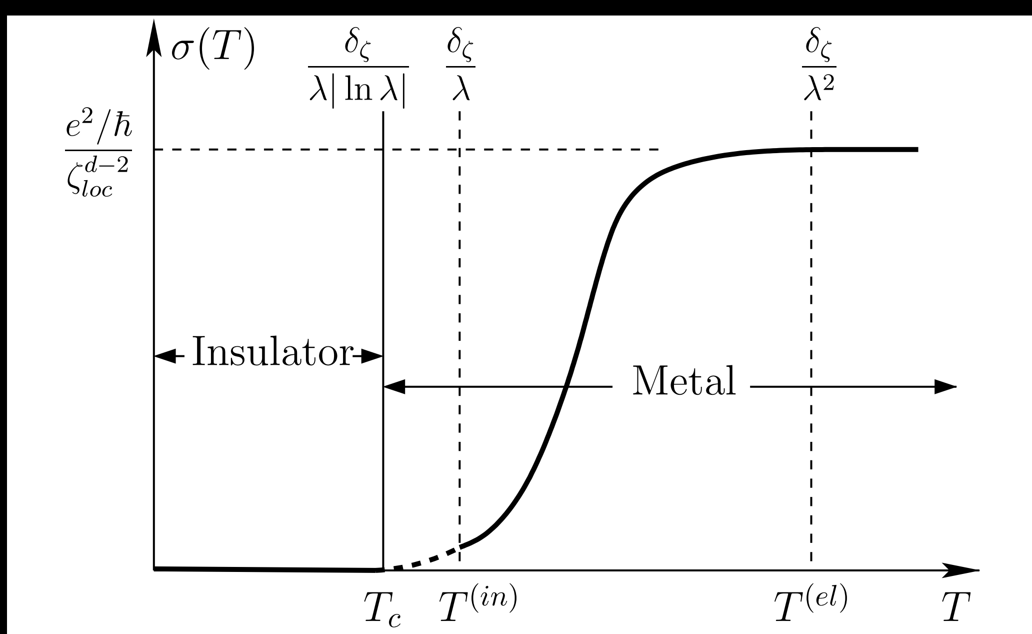
$$\sigma = 0$$

The response is dielectric

ξ is called localization length

Many-body localization

$$H = -t\sum_i c_{i+1}^\dagger c_i + c_i^\dagger c_{i+1} + \sum_i \epsilon_i n_i + \lambda \sum_{i,j} v(|i-j|)n_i n_j$$



Basko, Aleiner, Altshuler 2006

T here is the temperature associated to a given energy density: the system is not coupled to a bath

$$\lambda \lesssim \frac{\delta_\zeta}{T\ln(T/\delta_\zeta)} \quad \text{Insulator}$$

$$\lambda > \frac{\delta_\zeta}{T\ln(T/\delta_\zeta)} \quad \text{Conductor}$$

MBL in spin chains

Soon after BAA some authors have proposed that the same phenomenon should be observed in spin chains, even at **infinite** temperature

$$H = J\sum_i \vec{s}_i \cdot \vec{s}_{i+1} - \sum_i h_i s_i^z$$

$$H = J\sum_i c_i^\dagger c_{i+1} + h.c. + J\sum_i c_{i+1}^\dagger c_{i+1} c_i^\dagger c_i + \sum_i h_i c_i^\dagger c_i$$

long wavelength

$$M = \sum_{j=1}^{N} S_j^z e^{2\pi i j/L}$$

initial state

$$\rho = 1 + \epsilon M^\dagger$$

MBL in spin chains

a) Time evolution

$$M(t) = \mathrm{Tr}(\rho(t)M)$$

$$M(\infty) = \lim_{T\to\infty} \frac{1}{T}\int_0^T dt M(t)$$

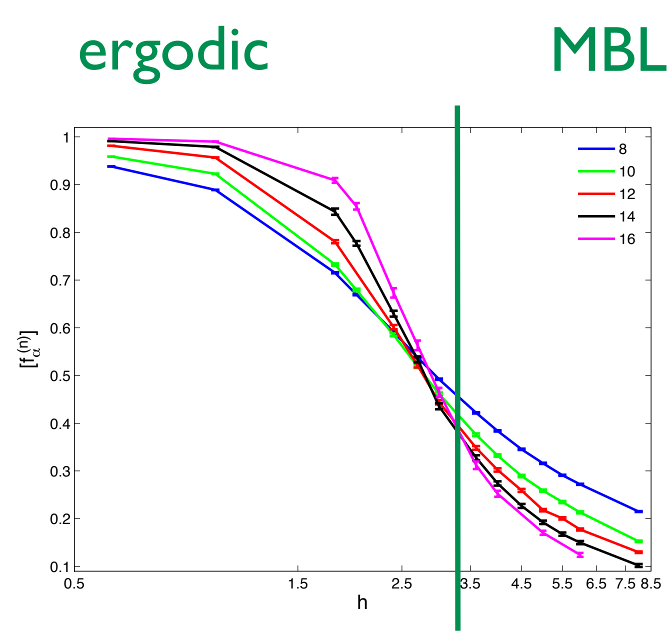


FIG. 2: (Color online) The fraction of the initial spin polarization that is dynamic (see text). The sample size L is indicated in the legend. In the ergodic phase (small h) the polarization decays substantially under the dynamics, while in the localized phase (large h) the decay is small, and this distinction gets sharper as L increases.

$$M(\infty) = 0$$ if ergodic

$$f = 1 - \frac{M(\infty)}{M(0)}$$ "ergodicity measure"

MBL in spin chains

"Ergodicity" here is intended in the sense:

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T dt \langle\psi(t)|O|\psi(t)\rangle = \mathrm{Tr}(O\rho_{th})$$

$$\rho_{th} \propto e^{-\beta H}$$

Where the temperature is that which correspond to the energy density of $\psi(0)$

All states with the same energy density give rise to the same values: memory of the initial condition is lost

Also: ETH (eigenstate thermalization hypothesis) holds

MBL in spin chains

b) Spectral signatures

WD

↓

Poisson

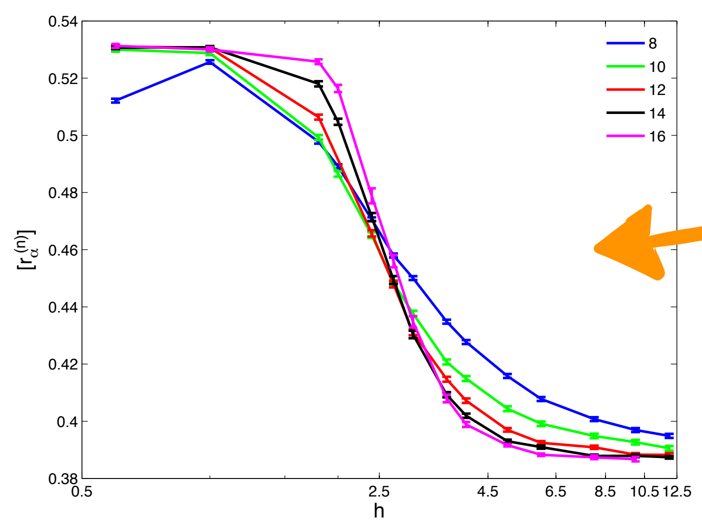
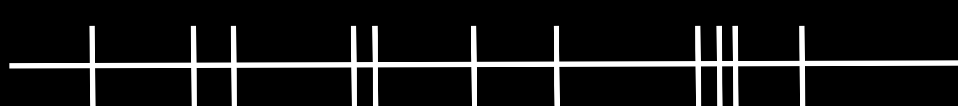


FIG. 3: (Color online) The ratio of adjacent energy gaps (defined in the text). The sample size L is indicated in the legend. In the ergodic phase, the system has GOE level statistics, while in the localized phase the level statistics are Poisson.

$$r = \left\langle \frac{\min(dE_i, dE_{i+1})}{\max(dE_i, dE_{i+1})} \right\rangle_i$$



Increasing disorder the level statistics goes from Wigner-Dyson to Poisson, and level repulsion disappears

Nature of the ergodic phase

Conduction is associated with diffusion

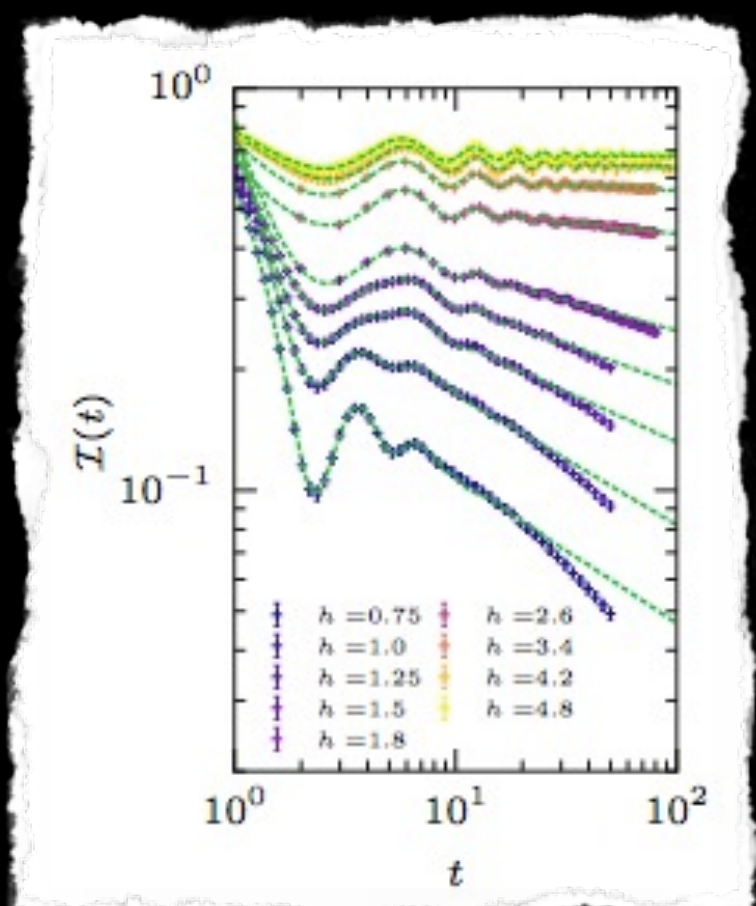
$\sigma \propto D$ Einstein's relation

Do we have diffusion in the ergodic phase?

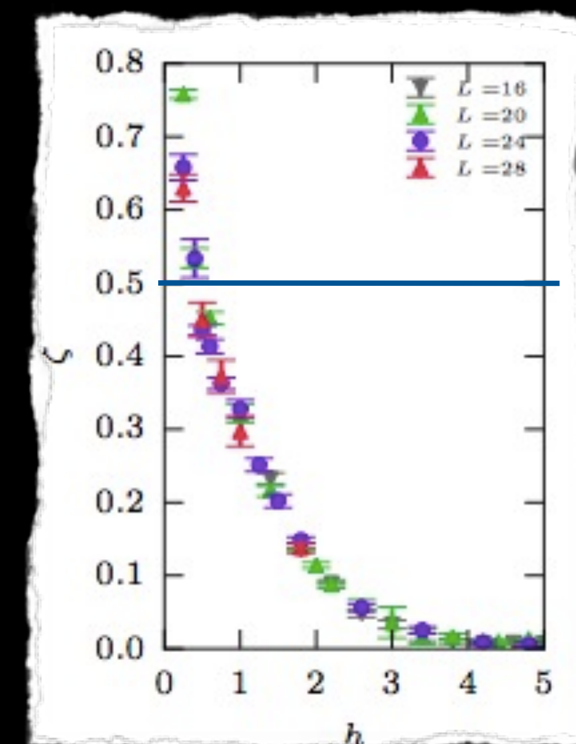
Answer: not for the spin/particle number, in one dimension

Nature of the ergodic phase

$$I(t) = 4 < \langle \psi_0 | s_i^z(t) s_i^z(0) | \psi_0 \rangle >_i$$



$$I(t) \sim t^{-\zeta}$$



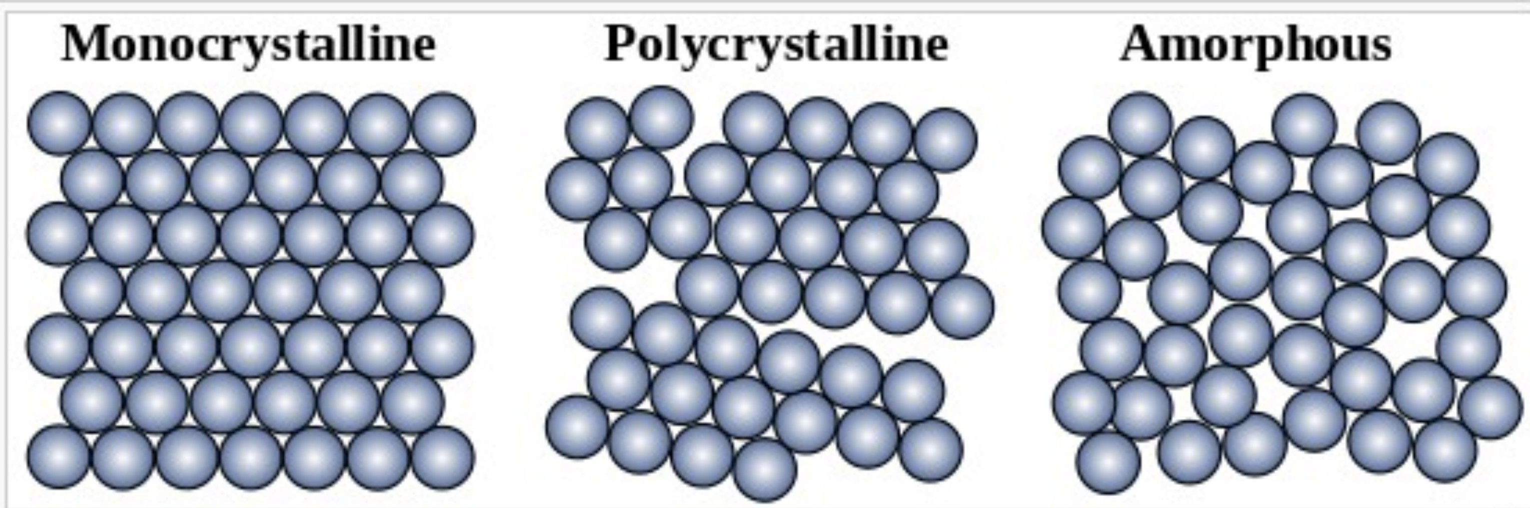
$$\rho(r,t) = \frac{1}{\sqrt{2\pi D t}} e^{-r^2/Dt}$$

Alet et al 2015

So this is a signal that particles do not diffuse

Nature of the ergodic phase

But is this the end of the story?

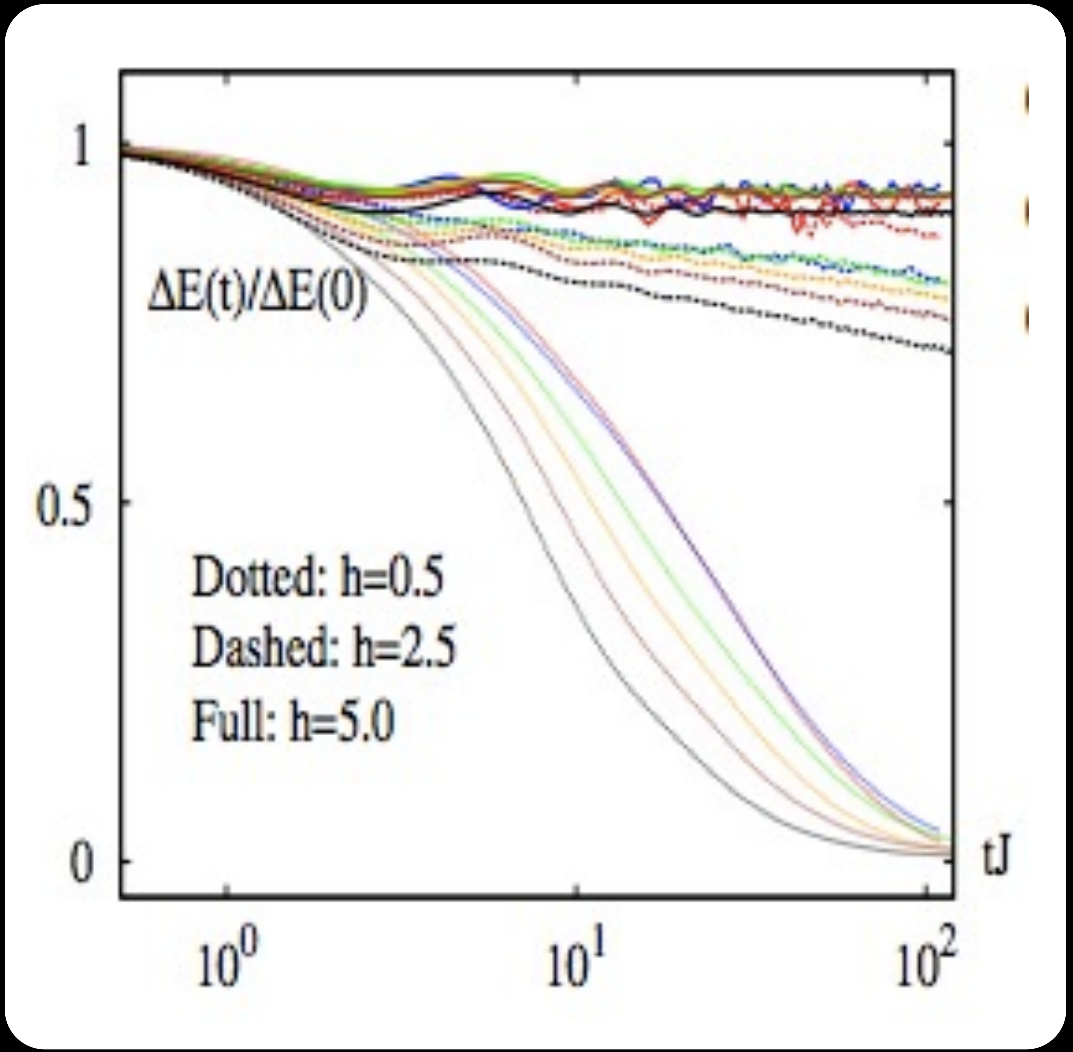


Schematic of allotropic forms of silicon: monocrystalline, polycrystalline, and amorphous silicon

Particles in amorphous materials sub-diffuse
But energy gets a conductivity!

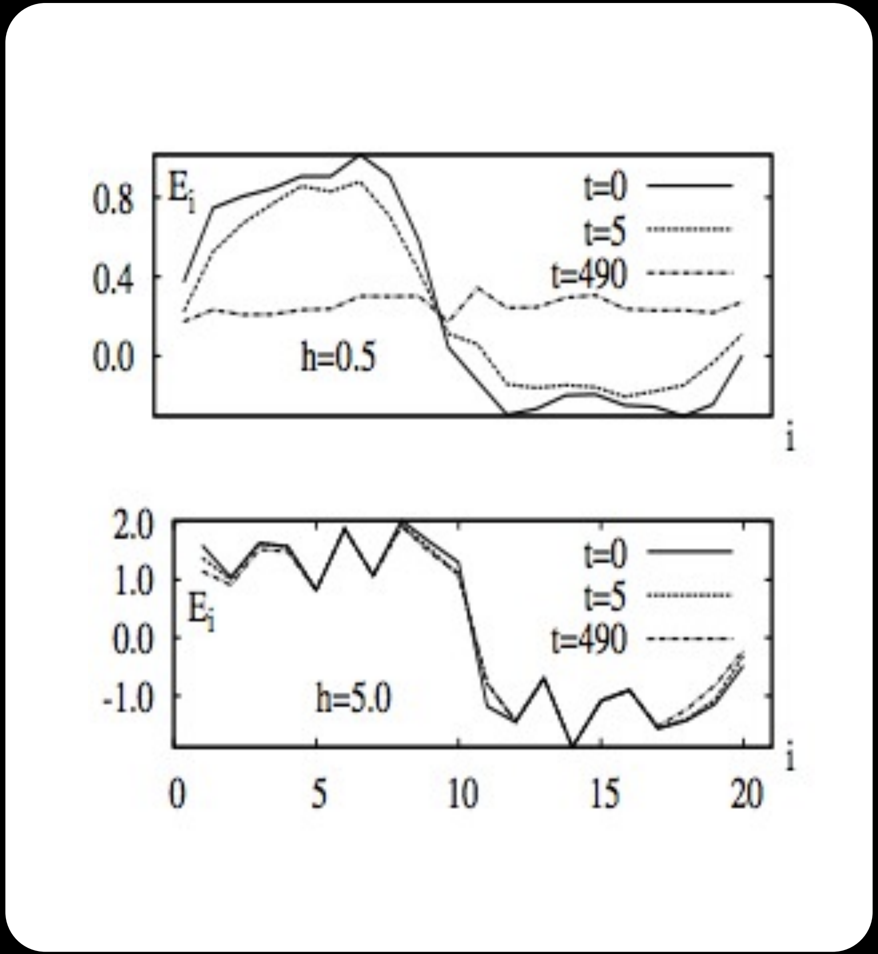
Nature of the ergodic phase

So we looked at energy diffusion



erg

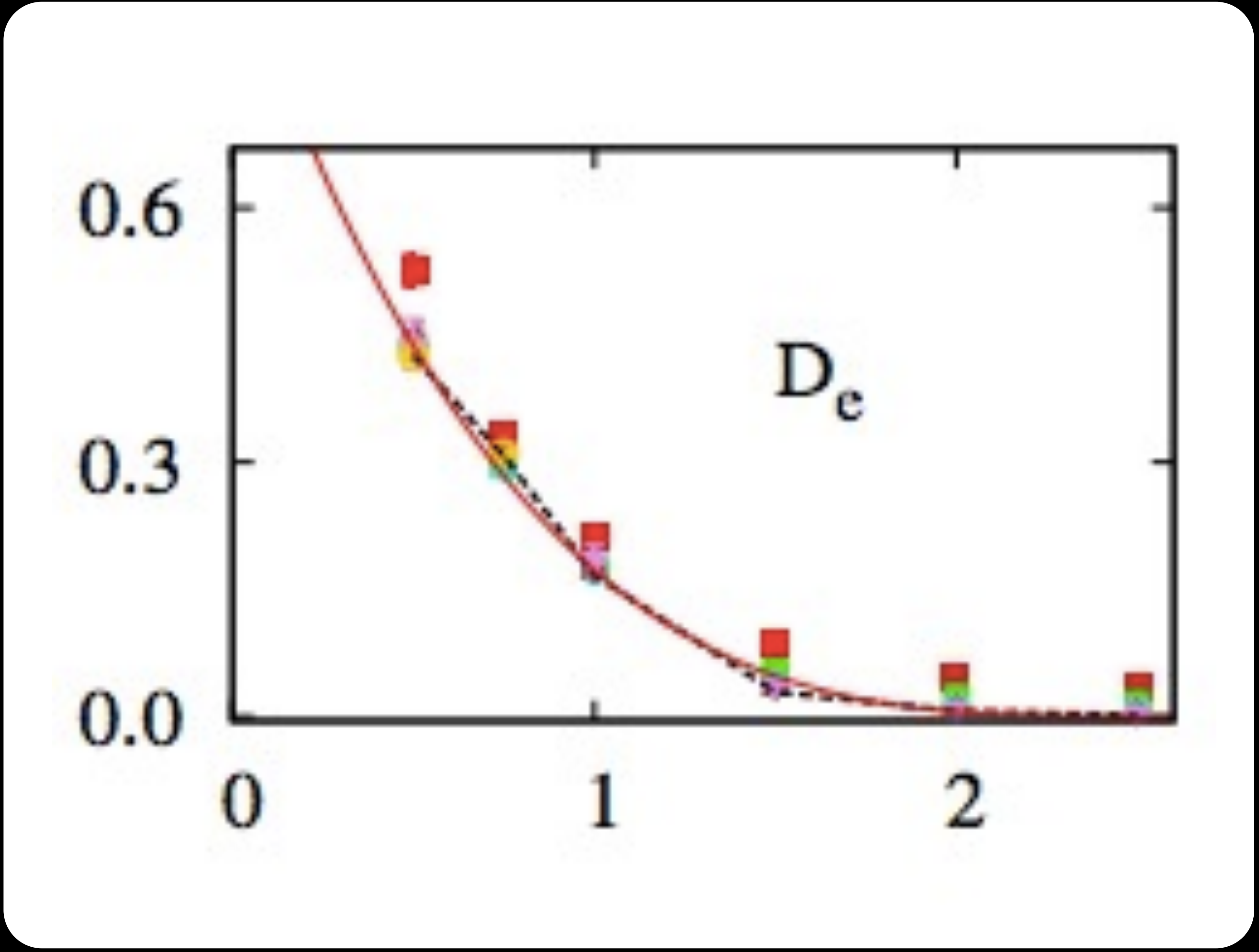
mbl



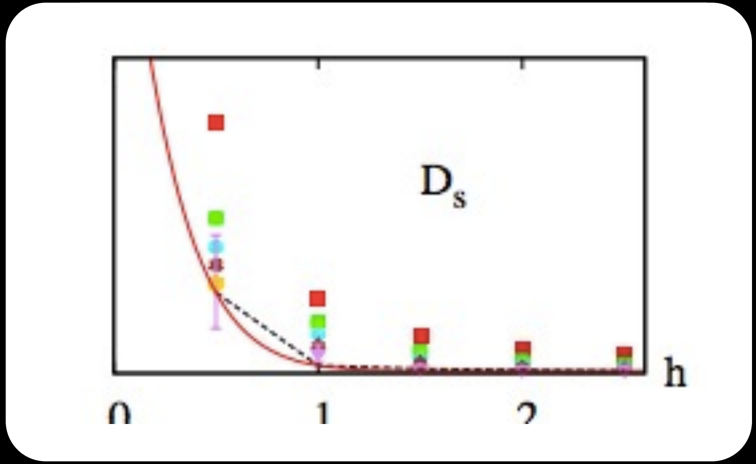
Nature of the ergodic phase

And we observe that there's diffusion!

Energy diffusion constant



Check: number diffusion constant is zero



Nature of the ergodic phase

- So we get that energy diffuses while particles do not
- How to reconcile this with theory?