

Bari Theory Xmas Workshop 2015

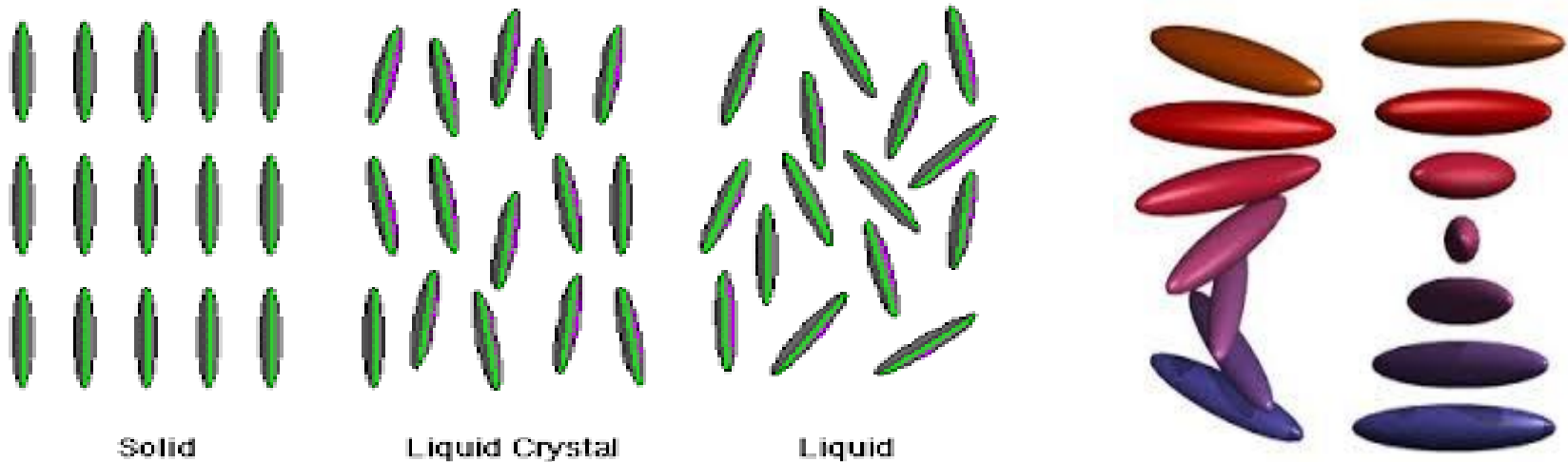
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Cholesteric liquid crystal droplets under electric fields



di
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What are cholesteric liquid crystals?



- Liquid crystals are an intermediate state of matter between solids and liquids -> they possess a long range orientational order and they also flow;
- The degree of order is quantified by the director, $|\mathbf{n}|=1$ and $\mathbf{n} \rightarrow -\mathbf{n}$ invariant!
- Tensorial order parameter, 2nd rank, symmetric, traceless with 5 independent components -> $Q_{\alpha\beta} = q \left(n_{\alpha} n_{\beta} - \frac{1}{3} \delta_{\alpha\beta} \right)$, $0 < q < 1$, $\alpha, \beta = x, y, z$
- Cholesteric liquid crystals or chiral nematic are an interesting mesophase of liquid crystals where molecules exhibit a helical arrangement with a certain pitch;

Dynamical equations

- Continuity equation:

$$\partial_t n + \partial_\alpha (n u_\alpha) = 0$$

- Navier-Stokes equation:

$$n(\partial_t + u_\beta \partial_\beta) u_\alpha = \partial_\beta (\Pi_{\alpha\beta}) + \eta \partial_\beta (\partial_\alpha u_\beta + \partial_\beta u_\alpha)$$

- Beris-Edwards equation:

$$(\partial_t + \vec{u} \cdot \nabla) \mathbf{Q} - \mathbf{S}(\mathbf{W}, \mathbf{Q}) = \Gamma \mathbf{H}$$

$$\mathbf{S}(\mathbf{W}, \mathbf{D}) = (\xi \mathbf{D} + \boldsymbol{\omega}) \left(\mathbf{Q} + \frac{\mathbf{I}}{3} \right) + \left(\mathbf{Q} + \frac{\mathbf{I}}{3} \right) (\xi \mathbf{D} - \boldsymbol{\omega}) + -2\xi \left(\mathbf{Q} + \frac{\mathbf{I}}{3} \right) \text{Tr}(\mathbf{Q}\mathbf{W}),$$

$$\mathbf{D} = \frac{(\mathbf{W} + \mathbf{W}^T)}{2}, \boldsymbol{\omega} = \frac{(\mathbf{W} - \mathbf{W}^T)}{2}, W_{\alpha\beta} = \partial_\alpha u_\beta$$

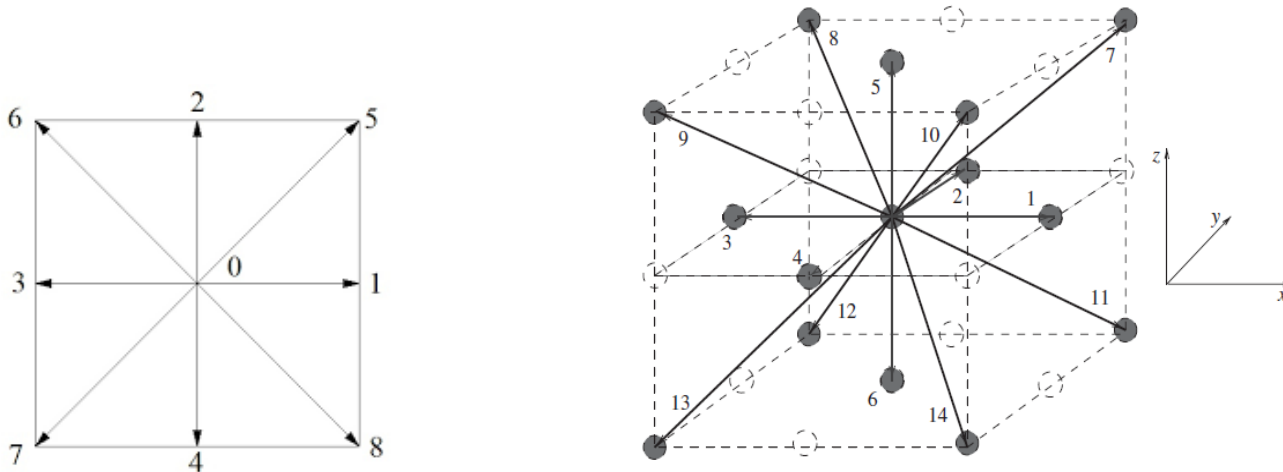
$$\mathbf{H} = -\frac{\partial \mathfrak{F}}{\partial \mathbf{Q}} + \frac{\mathbf{I}}{3} \text{Tr} \left(\frac{\partial \mathfrak{F}}{\partial \mathbf{Q}} \right)$$

- Cahn-Hilliard equation:

$$\partial_t \phi + \partial_\alpha (u_\alpha \phi) = D \nabla^2 \mu$$

Hybrid Lattice Boltzmann method

- Discretization of the space with a lattice \rightarrow **D2Q9** and **D3Q15**



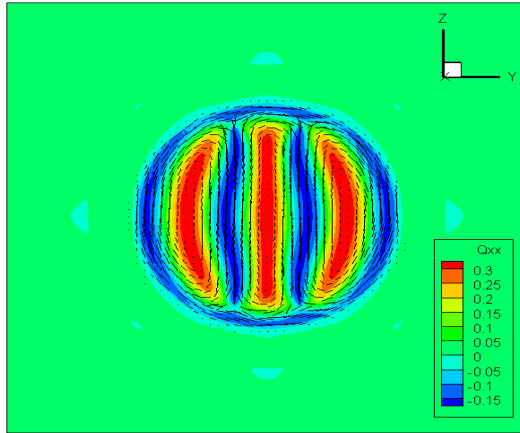
- In the continuum limit *the continuity* and *Navier Stokes equations* are obtained!!!
- Why hybrid??? \rightarrow This method solves the *continuity* and *Navier-Stokes equations* in the standard way and the *Beris-Edward* and the *Cahn-Hilliard equations* with a finite-difference method.

Influence of the electric fields

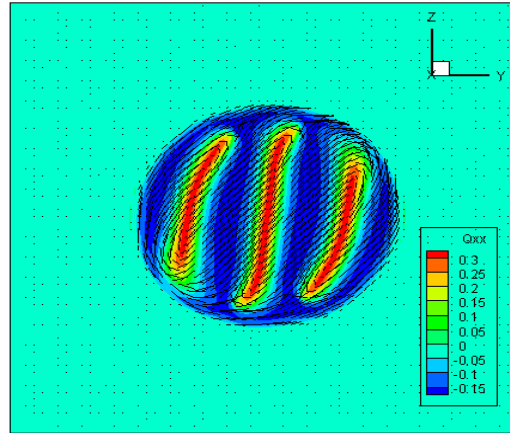
- If a DC electric field is switched on the director moves parallel to the field if $\epsilon > 0$ and orthogonal if $\epsilon < 0$. (*P.G. de Gennes and J. Prost, The Physics of Liquid Crystals, Clarendon Press, 2° edn, 1993; S. Chandrasekhar, Liquid Crystals, Cambridge University Press, (1980), A.N. Beris, B. J. Edwards, Thermodynamics of Flowing System, Oxford University Press, Oxford (1994)*)
- *Lehmann rotation (1901)* when the cholesteric droplet is under a temperature gradient -> recent works (*Physical Review E* **68**, 031708 (2003), *Physical Review E* **89**, 022509 (2014), *Optics Express* 6877 2008, *Phil. Trans. R. Soc. A* (2006) **364**, 2789-2805).

Simulations and results

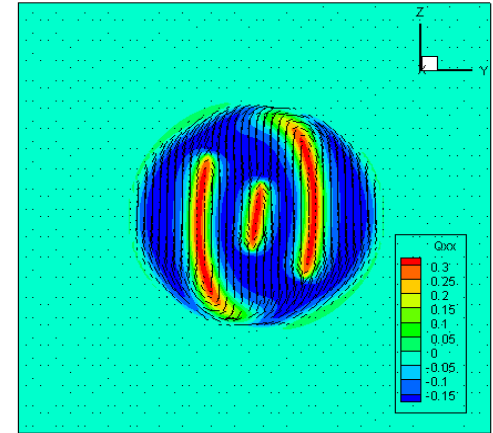
$R=32$
 $\rho_0=32$
 $\omega=0.001$



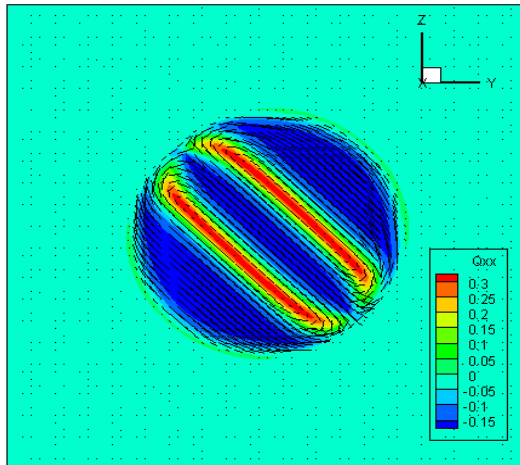
$t=400000$



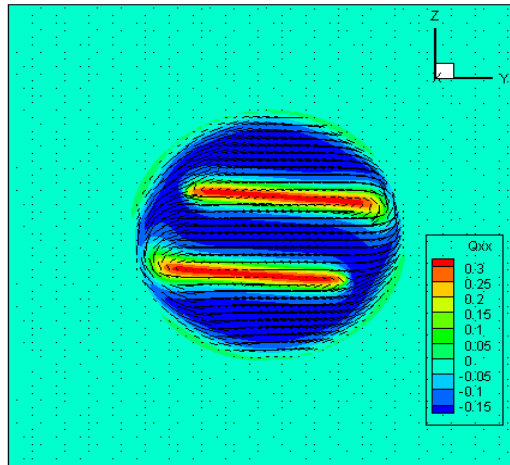
$t=401000$



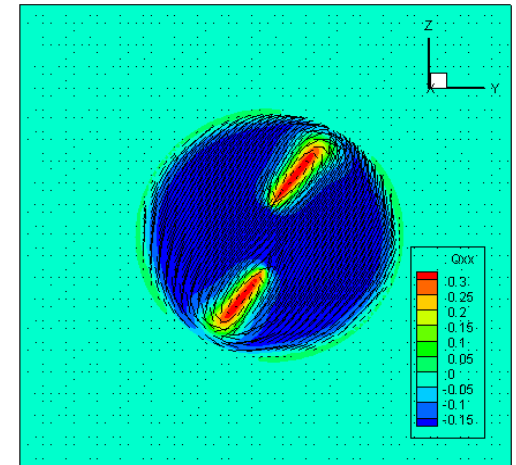
$t=450000$



$t=500000$



$t=550000$



$t=600000$