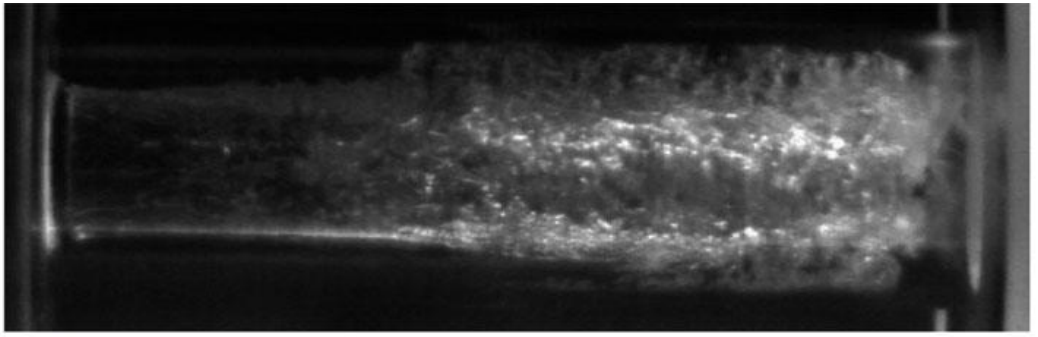
Bari Xmas Theory Workshop, 22nd December 2015

Cavitation inception of a van der Waals fluid at a sack-wall obstacle

Cavitation on ship propellers



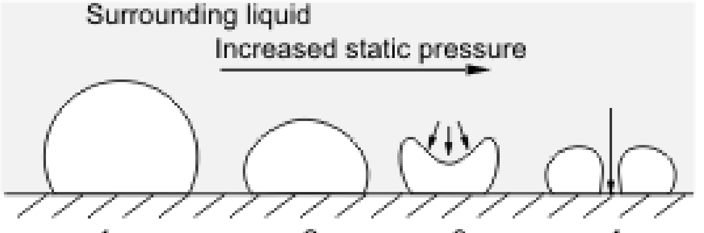
CCD image of cavitation inside a large-scale single-hole nozzle
E. Giannadakis et al. Journal of Fluid Mechanics 616, 153-193 (2008).

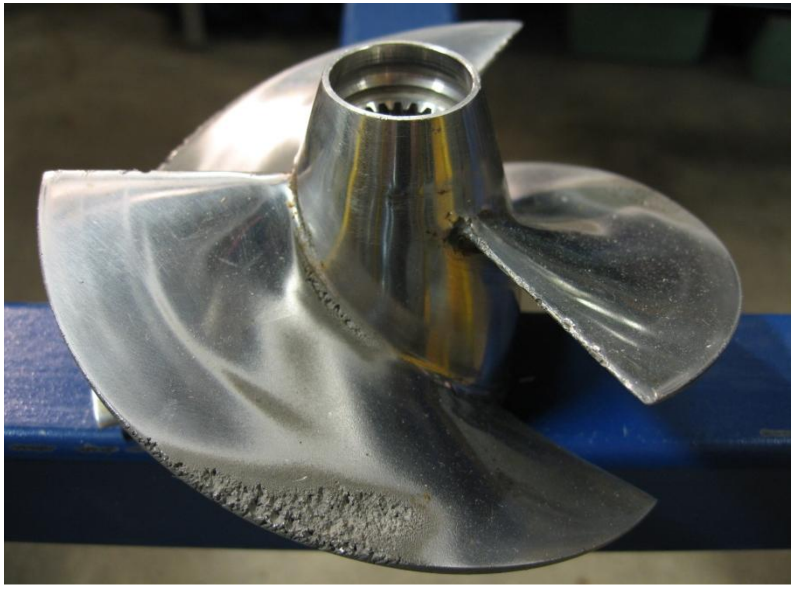
F. BONELLI

G. KÄHLER, G. GONNELLA, A. LAMURA

Cavitation

Cavitation behind a NACA 0015 hydrofoil[2]



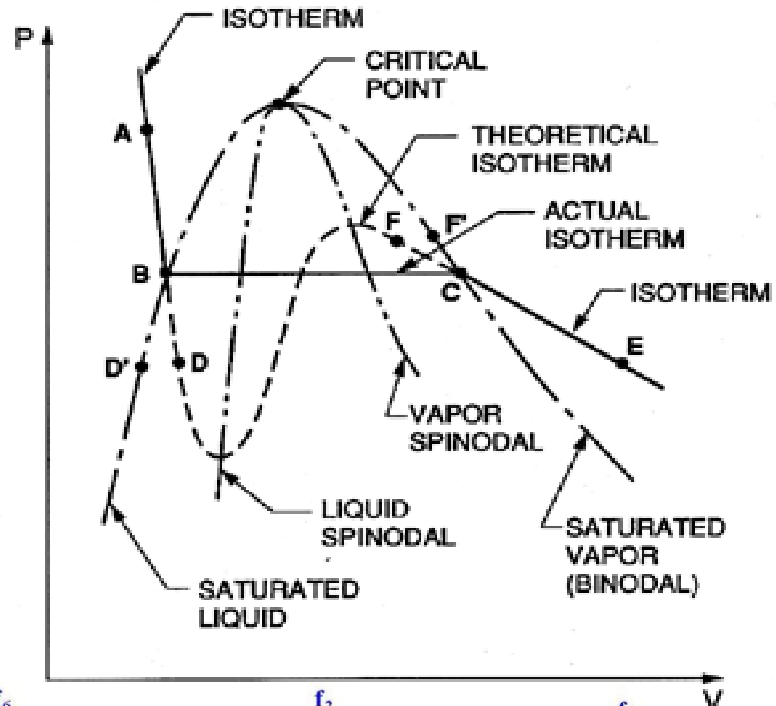


Cavitation propeller damage[3]

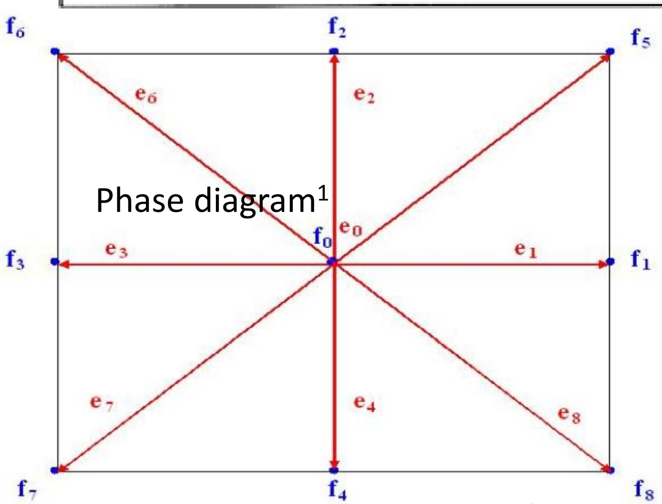
Cavitation is defined as the rupture of a liquid due to a pressure drop, which falls below a certain critical value, at approximately constant liquid temperature, with the subsequent formation of vapor bubbles[1]. This phenomenon causes noise, vibrations and above all damage and performance loss.

[1] C. E. Brennen, Cavitation and Bubble Dynamics (Oxford University Press, 1995).
[2] http://cav.safl.umn.edu/gallery.htm
[3] https://en.wikipedia.org/wiki/Cavitation

Van Der Waals EOS and Lattice Boltzmann Method



Phase diagram[1]



discrete velocities and distribution functions for the D2Q9

$$\Psi = \int d\mathbf{r}\left[\psi(n,T) + \frac{\kappa}{2}(\nabla n)^2\right]$$

For a van der Waals fluid the pressure can be derived from the free-energy functional

$$\psi = nT\ln\left(\frac{3n}{3-n}\right) - \frac{9}{8}n^2$$

bulk free-energy density

$$p^w = n\frac{\partial\psi}{\partial n} - \psi = \frac{3nT}{3-n} - \frac{9}{8}n^2$$

van der Waals equation of state with the critical point at n_c=1 and T_c=1

Lattice Boltzmann equation (LBE):

$$f_i(\mathbf{r}+\mathbf{e}_i\Delta t, t+\Delta t) - f_i(\mathbf{r},t) = -\frac{\Delta t}{\tau}\left[f_i(\mathbf{r},t) - f_i^{eq}(\mathbf{r},t)\right] + \Delta t\mathfrak{I}_i$$

collision $$f_i^{coll}(\mathbf{r},t) = f_i(\mathbf{r},t) - \frac{\Delta t}{\tau}[f_i(\mathbf{r},t) - f_i^{eq}(\mathbf{r},t)] + \Delta t\mathfrak{I}_i$$

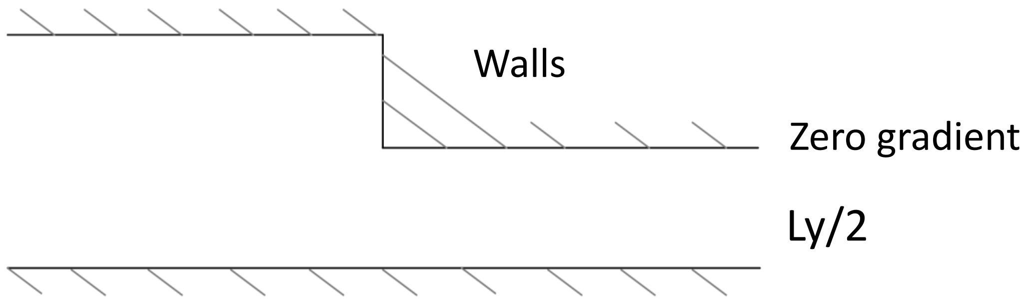
streaming $$f_i(\mathbf{r}+e_i\Delta t, t+\Delta t) = f_i^{coll}(\mathbf{r},t)$$

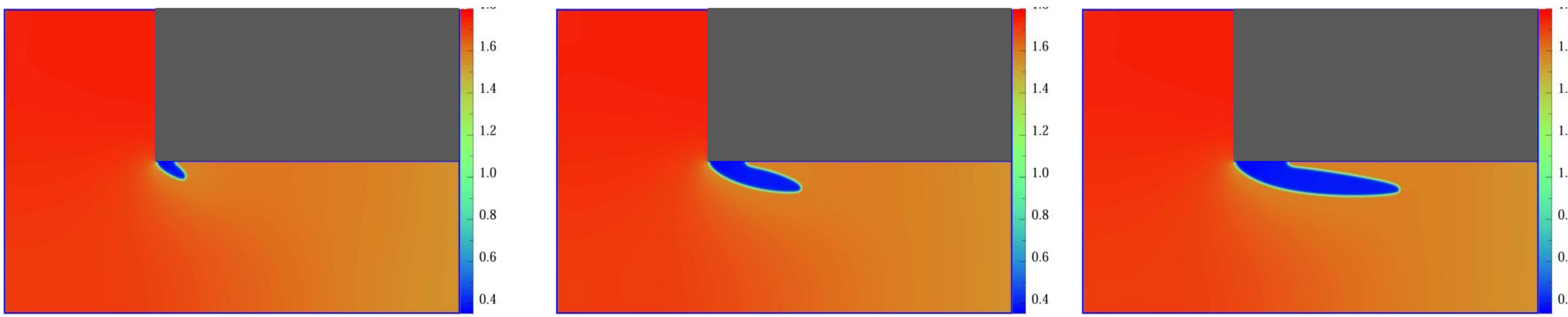
$$e_i = \begin{cases} \sqrt{3}, & if\ \ i = 1,\ 2,\ 3,\ 4 \\ \sqrt{6}, & if\ \ i = 5,\ 6,\ 7,\ 8 \\ 0, & if\ \ i = 0 \end{cases}$$

[4]Coclite, G. Gonnella, and A. Lamura,Physical Review E 89, 063303 (2014).
[5]Z. Guo, C. Zheng, and B. Shi, Physical Review E 65, 046308 (2002).
[6]X. Shan and H. ChenJ. Fluid Mech. 550, 413 (2006)

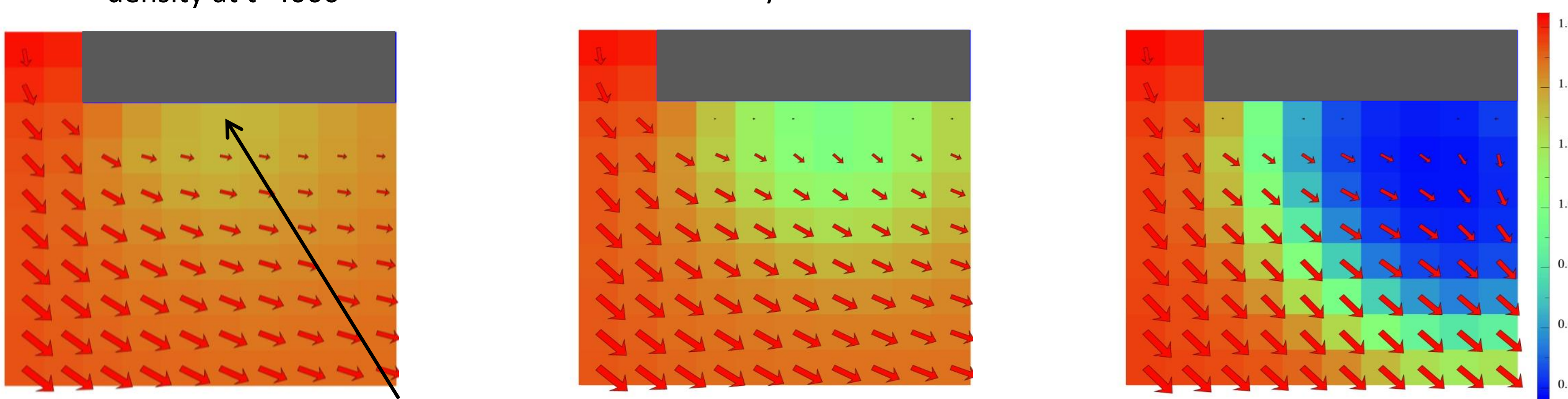
Geometry and Example of Cavitation

$T = 0.9$
$n_0 = 1.55$
$u_{x0} = 0.2$
Lx=601
Ly=401





density at t=4000 density at t=5000 density at t=6000



$t=t_{spinodal}$ $t=t_{spinodal}+200$ $t=t_{spinodal}+300$

Quantifying Cavitation : Stress Criterion

$\mathbf{T} = -p\mathbf{I} + \boldsymbol{\tau}$ total stress tensor

$\mathbf{T}_{\Pi} = -\boldsymbol{\Pi} + \boldsymbol{\tau}$ total stress tensor with the full pressure tensor that includes interfacial contributions

p bulk pressure

$\boldsymbol{\tau}$ viscous stress tensor

$\boldsymbol{\Pi}$ full pressure tensor

Joseph[7] suggested a local cavitation criterion based on the local total stress tensor

$T_{11} + p_c > 0$ [7] $\longrightarrow$ $-T_{11} < p_c$

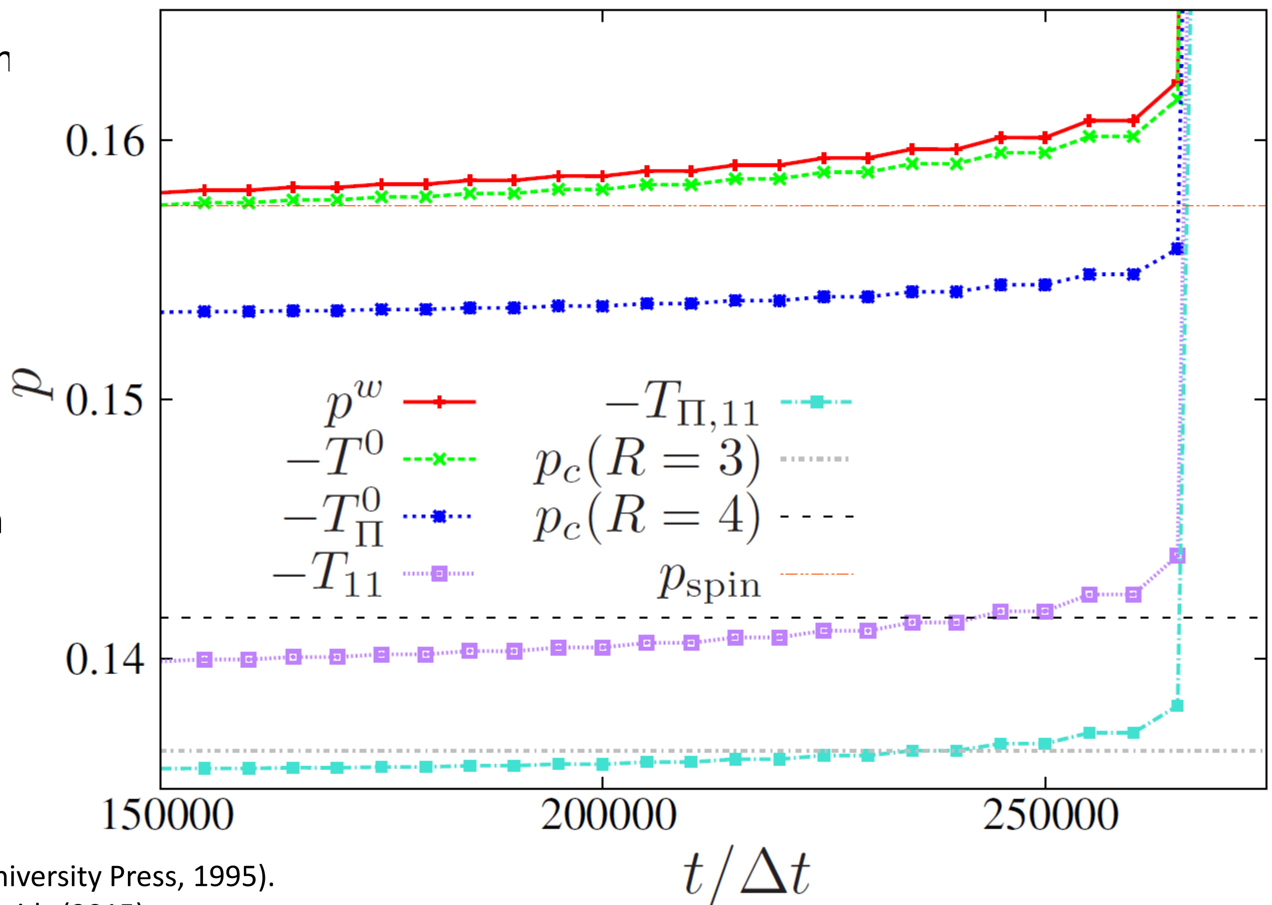
T_{11} maximum principal stress

$T^0 = \frac{1}{D} tr\mathbf{T}$ mean stress

$p_c = p_{spin} - \frac{\sigma}{R}$ [1] threshold corrected with the Laplace pressure

σ surface tension

R minimal radius of a vapor bubble



[7] D. D. Joseph, Journal of Fluid Mechanics 366, 367 (1998).

[1] C. E. Brennen, Cavitation and Bubble Dynamics (Oxford University Press, 1995).

[8] G. Kähler, F. Bonelli, G. Gonnella, A. Lamura, Physics of Fluids (2015).

Summary

- Reproduced cavitation formation in a bounded flow with a LBM based on well understood van der Waals EOS, no ad hoc requirements for phase separation or multi-phase behavior

- Local bulk pressure and mean stress are insufficient to explain cavitation inception. Results suggest that the viscous stress, interfacial contributions , and the Laplace pressure are relevant to the opening of a vapor cavity and can be described by an appropriate generalization of Joseph’s criterion