# Moving Walls and Geometric Phases

Giancarlo Garnero

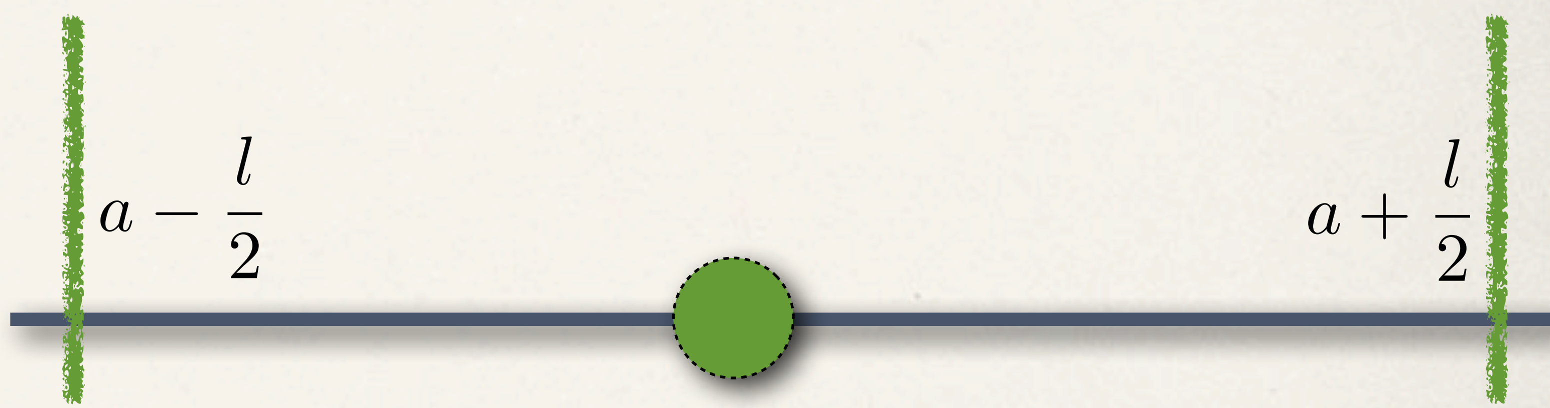
joint work with P.Facchi, G.Marmo, J.Samuel

# Moving Walls

- Free particle in a one dimensional box.
- Non mixing boundary conditions:

$$\psi\left(a-\frac{l}{2}\right)=\eta\psi\left(a+\frac{l}{2}\right),\ \bar{\eta}\psi'\left(a-\frac{l}{2}\right)=\psi'\left(a+\frac{l}{2}\right)$$

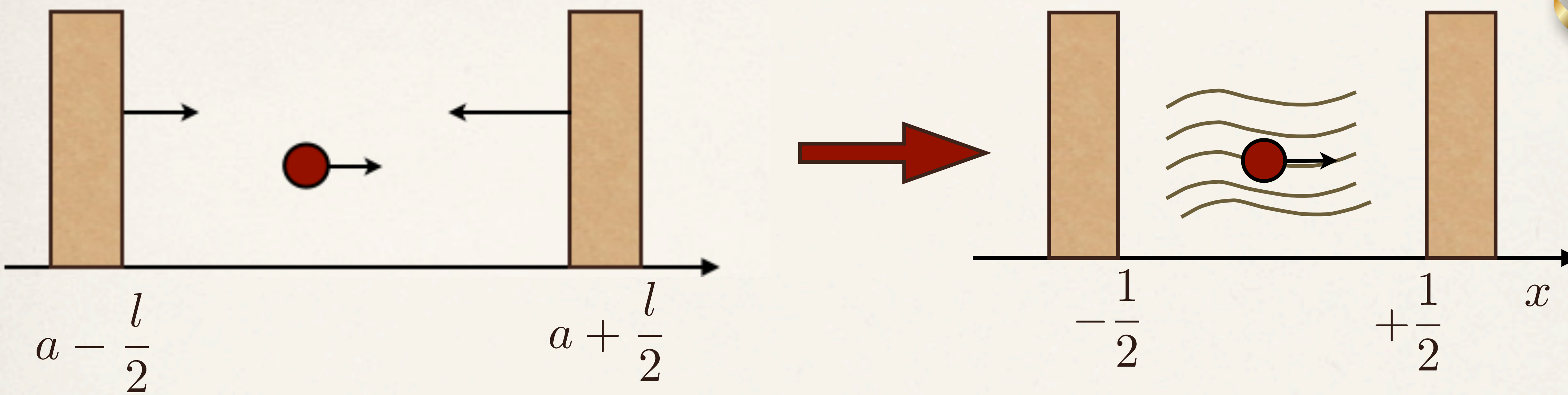
- "Moving boundaries": $t \to a(t)$, $t \to l(t)$.
- Cyclical evolution $\mathcal{C}$




$$\gamma_n(\mathcal{C}) = \oint_{\mathcal{C}} \mathcal{A}^{(n)} = i\oint_{\mathcal{C}} \langle \psi_n, \mathrm{d}\psi_n \rangle$$

We need to compute the spectral decomposition of this Hamiltonian!

# Moving Walls




$$I_{l,a} = \left[a - \frac{l}{2}, a + \frac{l}{2}\right] \longrightarrow I = \left[-\frac{1}{2}, \frac{1}{2}\right]$$

translation + dilation

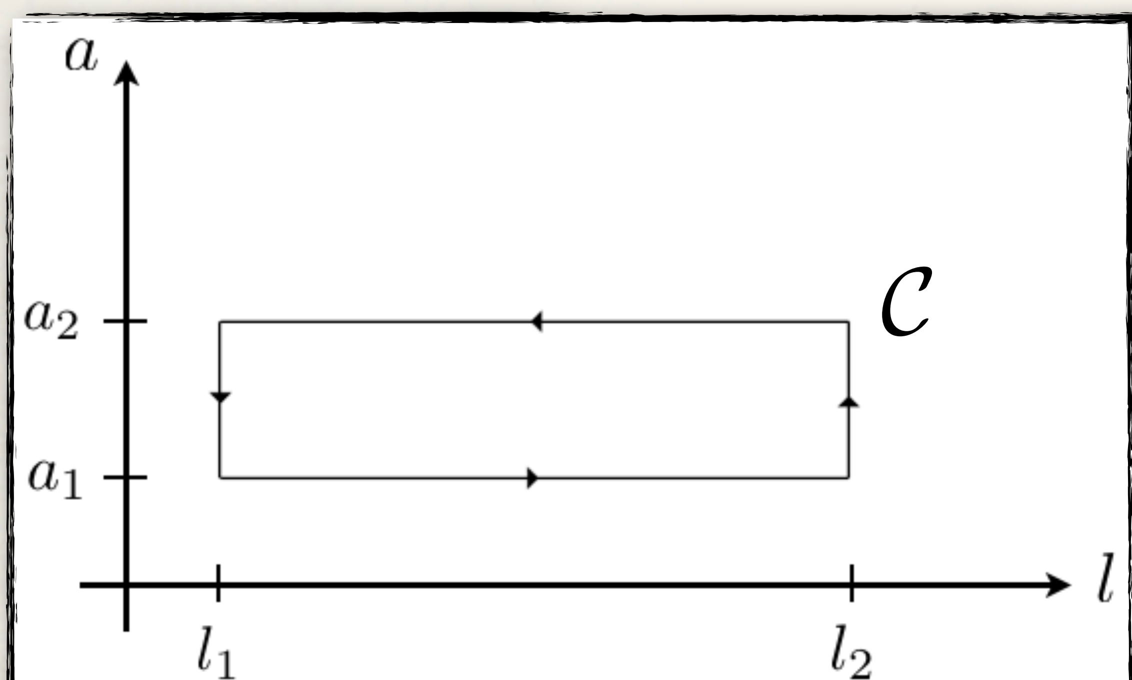
- They can be lifted to unitary transformations on $L^2(\mathbb{R})$
- Spectral decomposition on fixed domain.
- Get back to the initial problem with the inverse transformation!
- Computation of the Berry 1-form!

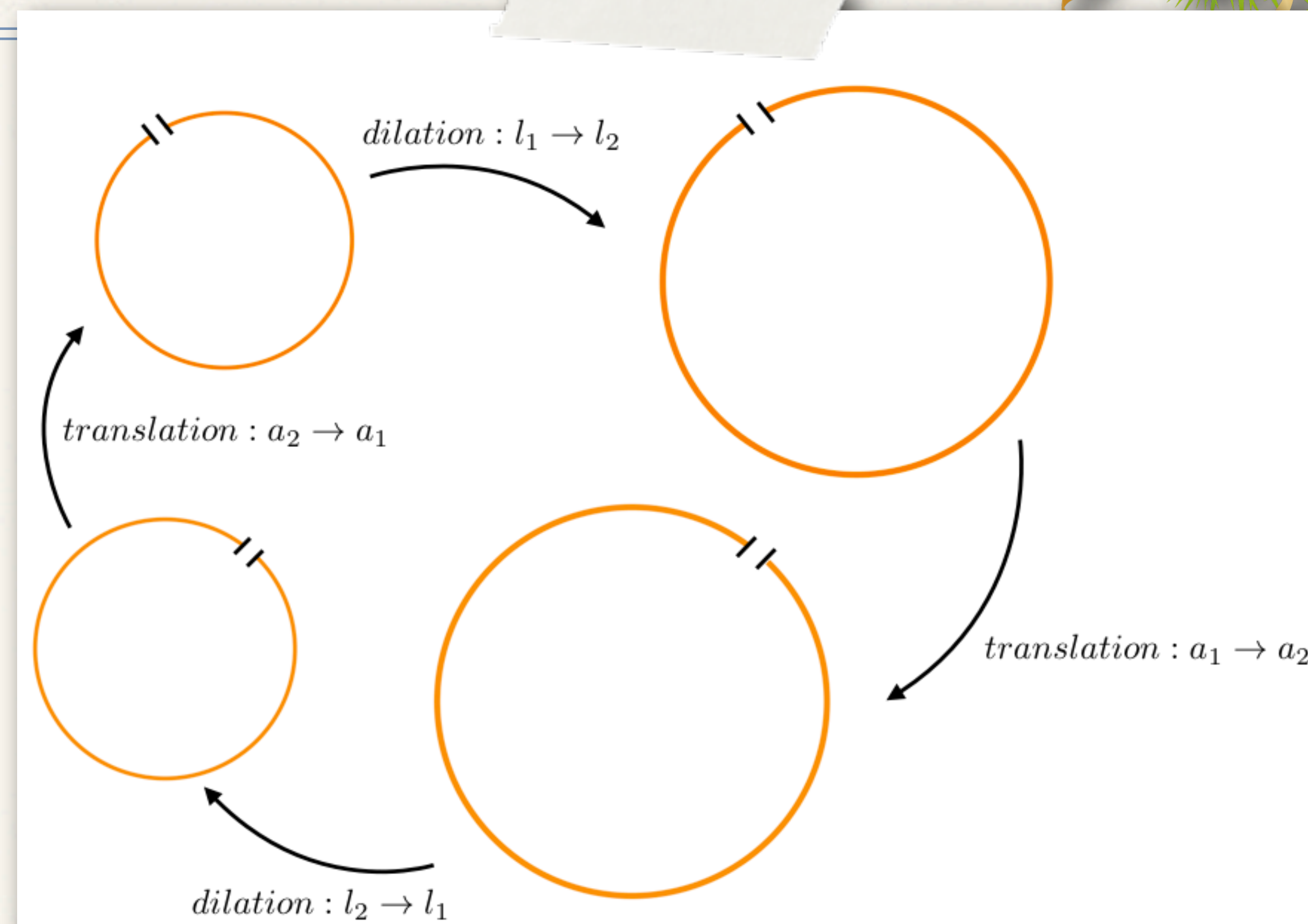
# Moving Walls and Geometric Phases

- Berry one-form

$$\langle \psi_n, d\psi_n \rangle = i\left(\frac{k_n}{l}\sin\alpha\right)\mathrm{d}a$$

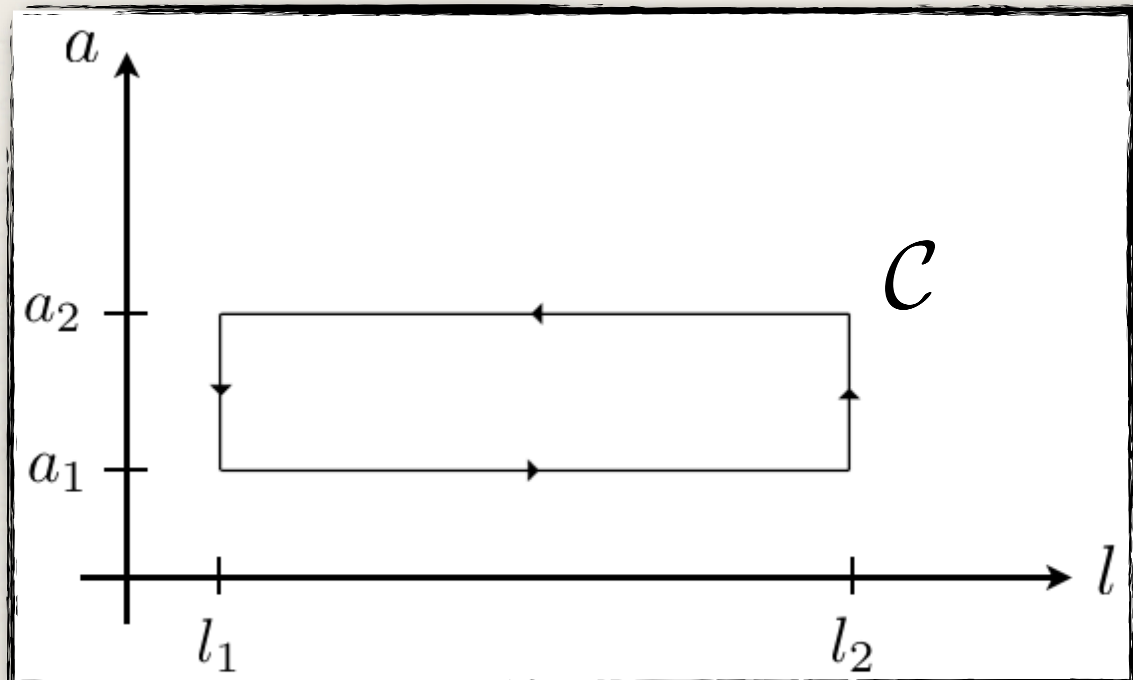
- Depends on: the n-th energy level
- Depends on: the boundary conditions $\alpha$ .
- $\alpha = \alpha\,(\eta)$.




- Interesting technical problem: renormalization of the 1-form coefficients.
- Distributional derivatives: divergences from the border
- Unbounded operator

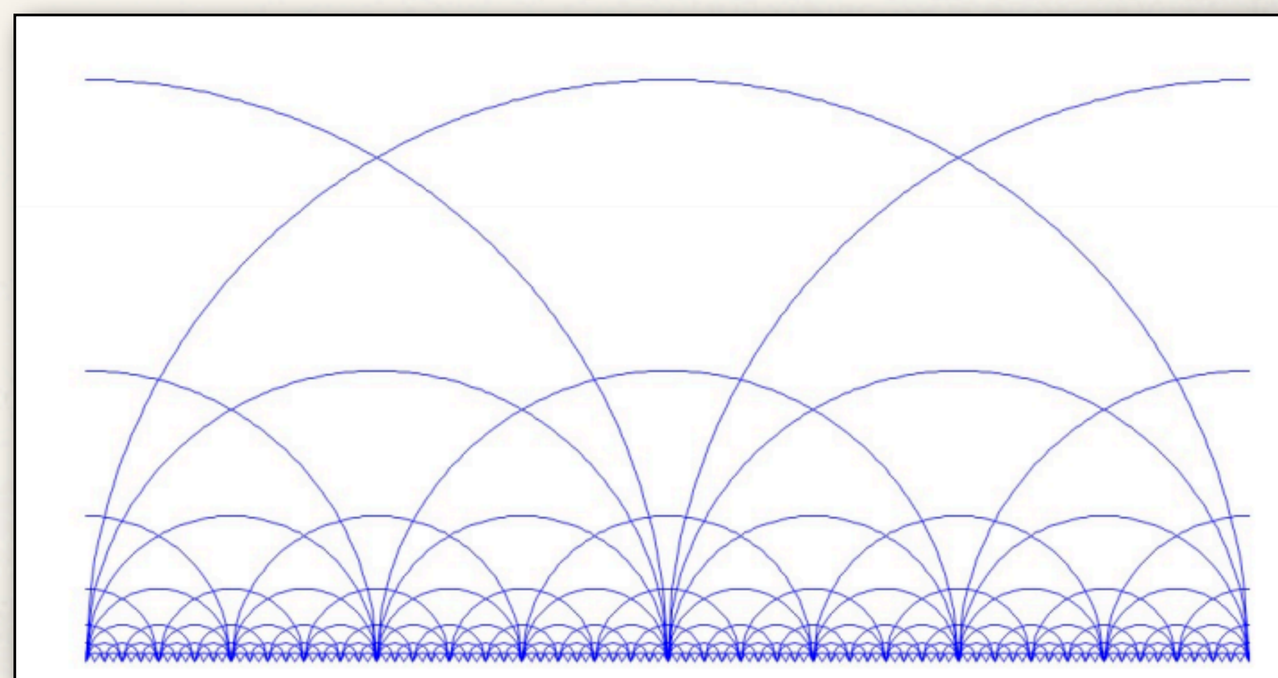
# Moving Walls and Geometric Phases




$$\gamma_n(\mathcal{C}) = \oint_{\mathcal{C}} \mathcal{A}^{(n)} = i \oint_{\mathcal{C}} \langle \psi_n, \mathrm{d}\psi_n \rangle$$

$$\gamma_n(\mathcal{C}) = k_n \left( \frac{1}{l_1} - \frac{1}{l_2} \right) (a_2 - a_1) \sin\alpha$$

$$\mathcal{F}^{(n)} = \mathrm{d}\,\mathcal{A}^{(n)} = \frac{k_n}{l^2} \sin\alpha \, \mathrm{d}l \wedge \mathrm{d}a$$



- Curvature of a hyperbolic manifold.
- Non-Euclidean geometry.
- Ricci tensor: *Poincarè half-plane* .

# Buon Natale!