Simulating gauge theories with cold atoms

Francesco Pepe

Centro Fermi & INFN Sezione di Bari

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Quantum simulation of gauge theories

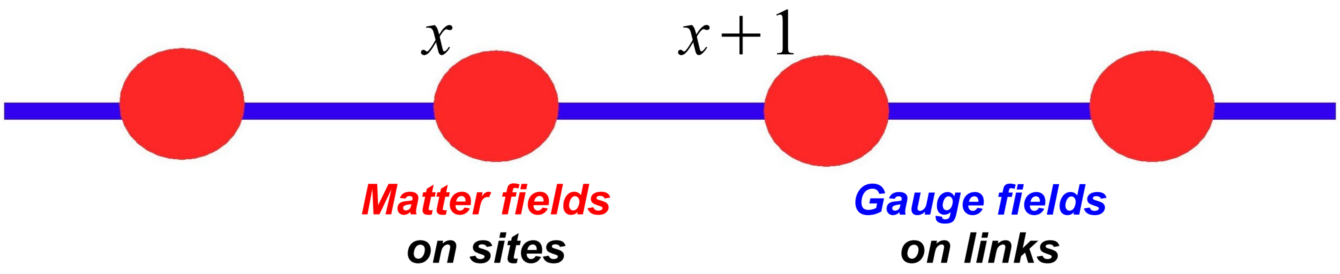
QUANTUM SIMULATOR: a special purpose Quantum Computer

- Exponential speedup with respect to classical computation
- Oriented at one specific (complex) physical system

Quantum simulation of lattice gauge theories

including

Quantum Electrodynamics on a one-dimensional lattice



Lewenstein *et al* 2013,
Cirac *et al* 2015

One-dimensional Lattice QED

$$H = -t\sum_x \left(\psi_x^\dagger e^{-iA_{x,x+1}}\psi_{x+1} + \text{H.c.}\right) + m\sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2}\sum_x (E_{x,x+1})^2$$

Parallel transporter
$A_{x,x+1}$ vector potential

Staggering

Wilson 1974
Kogut and Susskind 1975

Canonical gauge $\{\psi_x, \psi_{x'}\} = 0, \quad \{\psi_x^\dagger, \psi_{x'}\} = \delta_{x,x'}, \quad [E_{x,x+1}, A_{x',x'+1}] = i\,\delta_{x,x'}$

U(1) gauge invariance $$H = \prod_x e^{-i\alpha_x G_x} H \prod_y e^{i\alpha_y G_y}$$

GENERATOR $$G_x = \psi_x^\dagger \psi_x + \frac{(-1)^x - 1}{2} - (E_{x,x+1} - E_{x-1,x})$$

Charge (particle-hole picture) **Divergence of *E***

Gauge-invariant states
satisfy the discretized **Gauss law**

$$G_x |\phi\rangle = 0$$

Finite link spaces: algebra vs group

Encoding in condensed-matter systems:

- The number of degrees of freedom of links is *finite*
- The microscopic dynamics is *not* gauge-invariant

QUANTUM LINK models:

the gauge fields are represented by spin-*s* operators on links

$$\dim \mathcal{H}_{\text{link}} = 2s+1$$

$$E \to S^3, \qquad e^{\mp i A} \to S^{\pm} = S^1 \pm i S^2$$

$$[(S^3)_{x,x+1}, (S^+)_{x',x'+1}] = \delta_{x,x'} (S^+)_{x,x+1}$$

Our approach: preserving the *general* Weyl group relation

$$e^{-i\eta A} e^{-i\xi E} = e^{i\xi\eta} e^{-i\xi E} e^{-i\eta A} \quad \text{with } \xi, \eta \in \mathbb{R}$$

$$\Downarrow$$

Generalization to *unitary* operators on an n-dimensional space

Correspondence $\quad e^{-iE} \to V, \qquad e^{-iA} \to U, \qquad U^l V^k = e^{\frac{2\pi i}{n} k l} V^k U^l$

A Schwinger-Weyl QED model

$$e^{-iE} \to V, \quad e^{-iA} \to U$$

$V|v_k\rangle = v_k|v_k\rangle$ electric field basis

$U|u_k\rangle = u_k|u_k\rangle$ vector potential basis

$$H = -t\sum_x \left(\psi_x^\dagger U_{x,x+1}\psi_{x+1} + \text{H.c.}\right) + m\sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g_n^2}{2}\sum_x f(V_{x,x+1})$$

Cyclic permutation of the electric field basis
Correlated with fermion hopping

Electric field energy
$\sim E^2$ for large n

Gauge transformation $O \to \prod_x (T_x)^{-\nu_x} O \prod_y (T_y)^{\nu_y}$ with $\nu_x \in \mathbb{Z}$

$$T_x = \exp\left[\frac{2\pi i}{n}\left(\psi_x^\dagger\psi_x + \frac{(-1)^x - 1}{2}\right)\right] V_{x,x+1} V_{x-1,x}^+, \quad (T_x)^n = \mathbb{1}$$

$\mathbb{Z}_n$ GAUGE GROUP

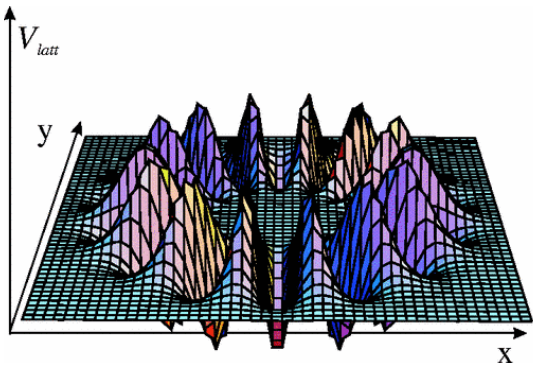
Physical states $T_x|\phi\rangle = |\phi\rangle$

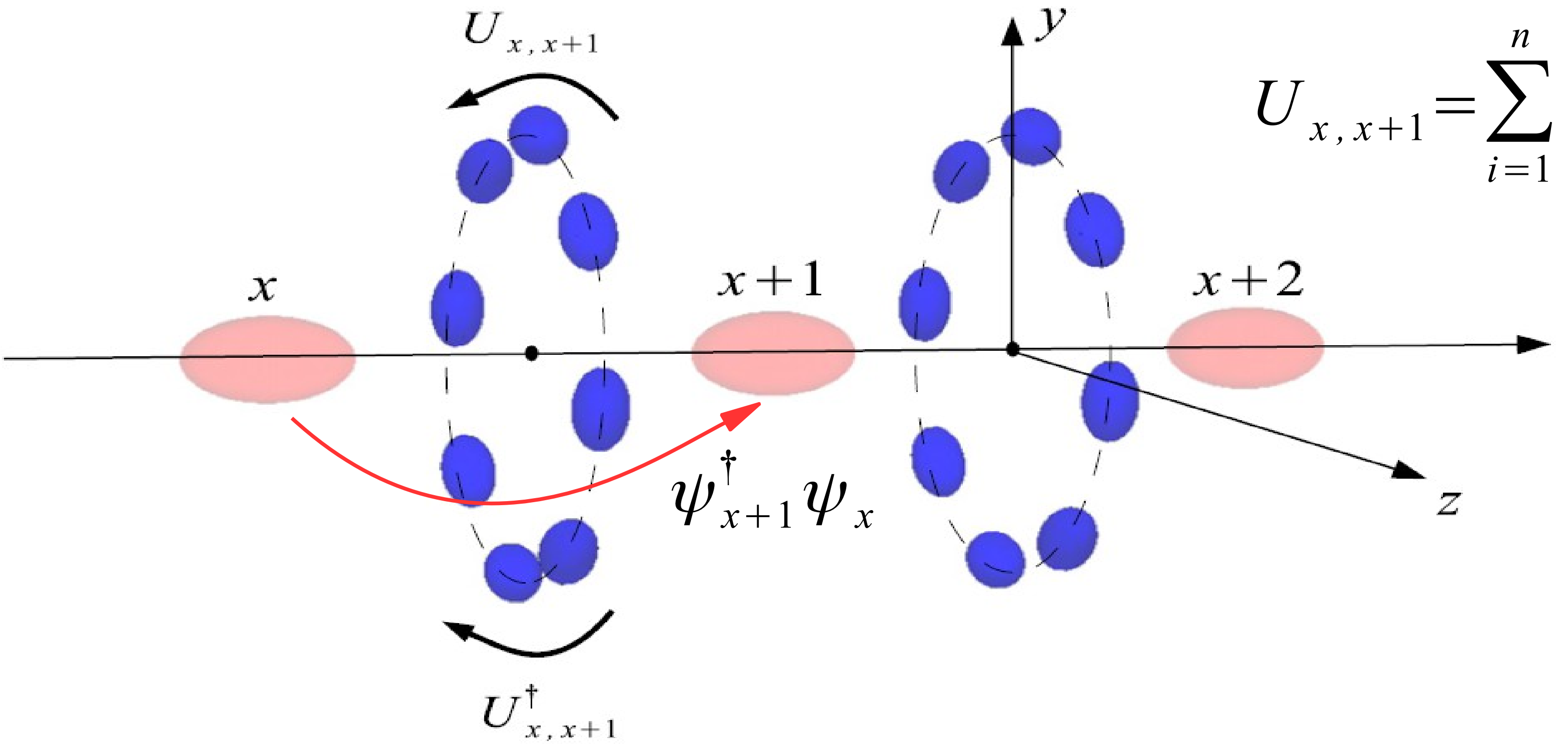
Implementation of links

Ring-shaped optical lattices can be obtained from the interference of a plane wave with a Laguerre-Gauss mode (Amico, Osterloh and Cataliotti 2005)

⇩

A possible tool to implement link dynamics





$$U_{x,x+1} = \sum_{i=1}^{n} c^\dagger_{(i+1)x,x+1} c_{(i)x,x+1}$$

How to implement the Gauss's law: OPEN PROBLEM

Thank you
for your attention

… e BUONE (MERITATE) VACANZE!