LOCAL QUANTUM THERMAL SUSCEPTIBILITY

arXiv: 1504.07787

SCUOLA NORMALE SUPERIORE

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TherMiQ

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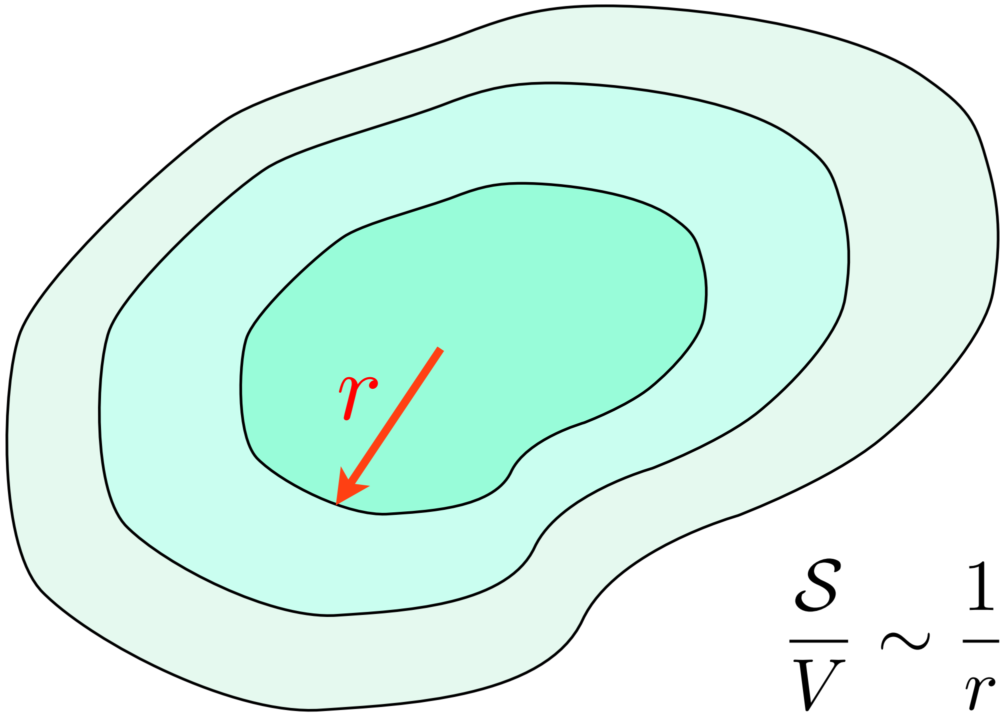
Outline

MAIN IDEA: quantum **estimation** approach to quantum **thermodynamics**

- Local quantum thermometry: LOCAL QUANTUM THERMAL SUSCEPTIBILITY
- Low temperature regime: a measure of STATE DISTINGUISHABILITY
- MANY-BODY SYSTEMS and quantum phase transitions

Thermodynamical description



$$\frac{\mathcal{S}}{V} \sim \frac{1}{r}$$

large systems at equilibrium are characterized
by only very few quantities: "typical behavior" of microstates

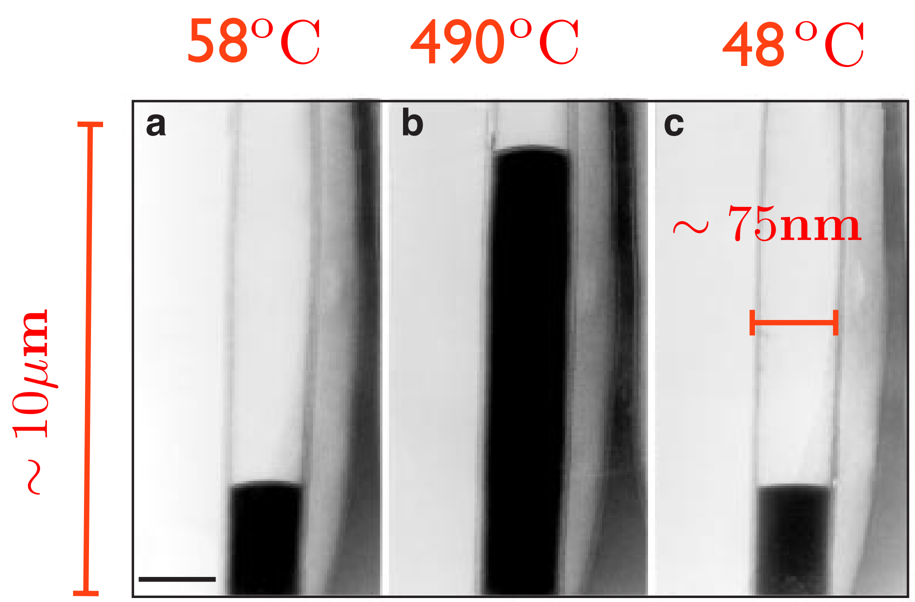
↕ ***Thermodynamic Limit***

at thermal equilibrium the interactions between
limiting regions, provided they are short ranged,
become negligible as far as the spatial extension of the
system is increased

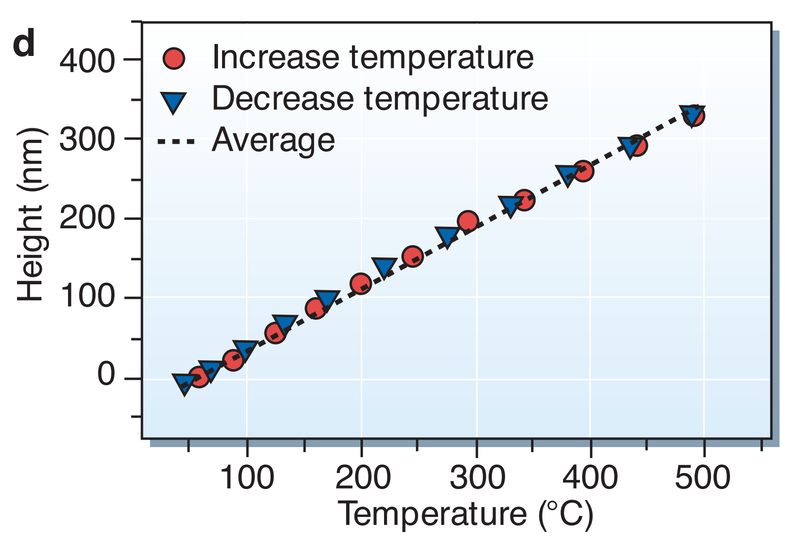
- ACCURACY LEVEL?
- MICROSCOPIC SYSTEM?

PRECISION NANOTHERMOMETRY

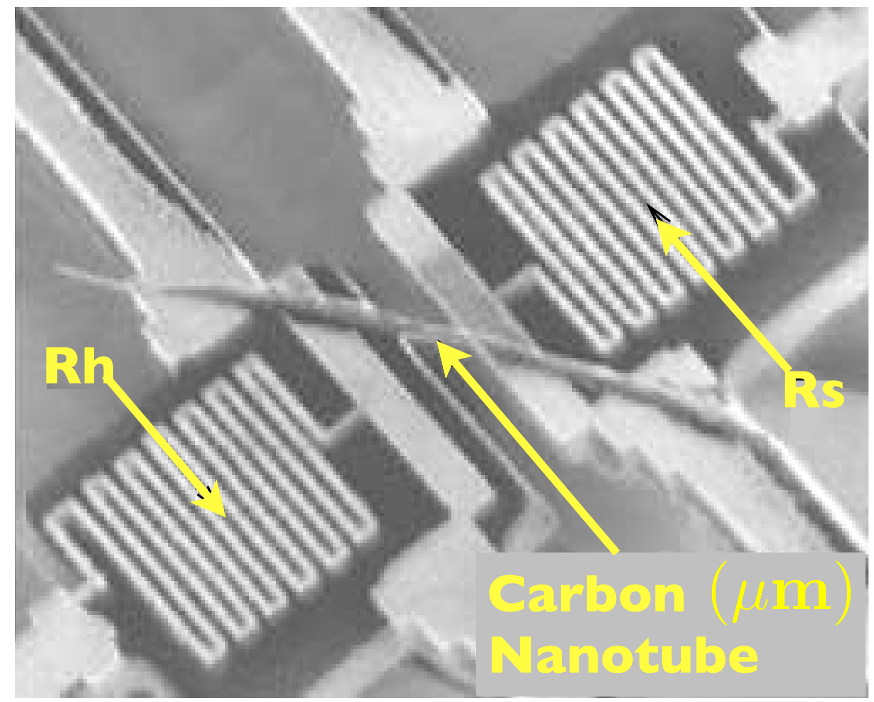
Microscopic limit of the thermodynamics



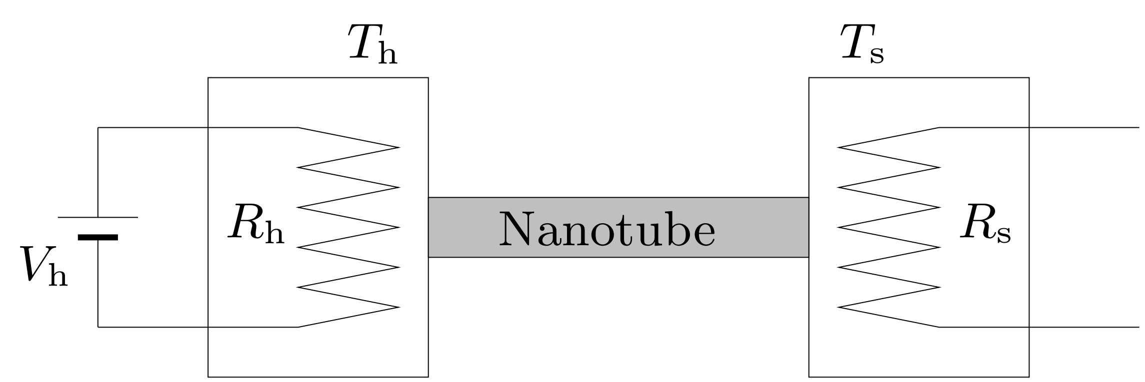
Expansion of gallium inside a carbon nanotube



Y. Gao and Y. Bando, NATURE 415, 599 (2002)



Heat condution across a carbon nanotube



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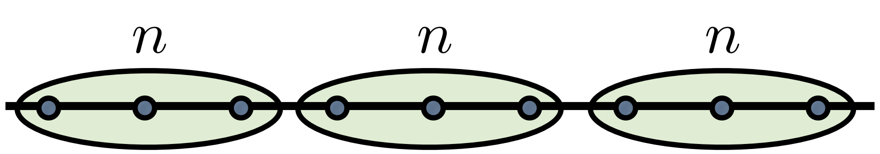
“Locality” of temperature

DEFINITION:
(Hartmann, Mahler ’04)

LOCAL TEMPERATURE exists if the considered part of the system is in a CANONICAL state

$e^{-\beta E}$
exp. decay function dependent on only one parameter

Hartmann, Mahler, Hess, PRL 93, 8 (2004); EPL 70, 579 (2005)



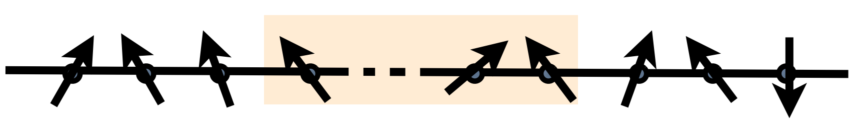
$$\Omega(\beta) = e^{-\beta H}/\mathcal{Z} \longrightarrow \begin{cases} \Omega_n(\beta) \sim e^{-\beta H_n}/\mathcal{Z}_n \\ n \geq n_{min}(\beta, H) \end{cases}$$

conditions on the interaction strength between subgroups

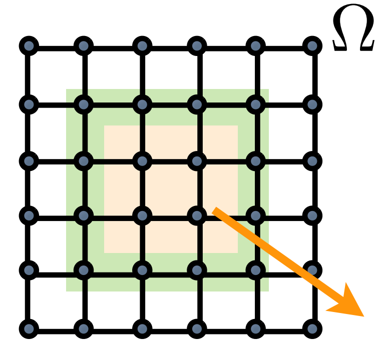
E.g. harmonic chain (iron, carbon, silicon)

Saez, Ferraro, Acin, PRA 79, 052340(2009); EPL 98, 10009 (2012)

$\rho_A(\beta)$

$\Omega(\beta)$ 

Ising chain



$\Omega(\beta)$

harmonic systems

$\rho_A(\beta)$

$$F\left[\rho_A(\beta), \Omega_A(\beta)\right] \longrightarrow F\left[\rho_A(\beta), \Omega_A(\beta')\right] \text{ or } F\left[\rho_A(\beta), \Omega'_A(\beta)\right]$$

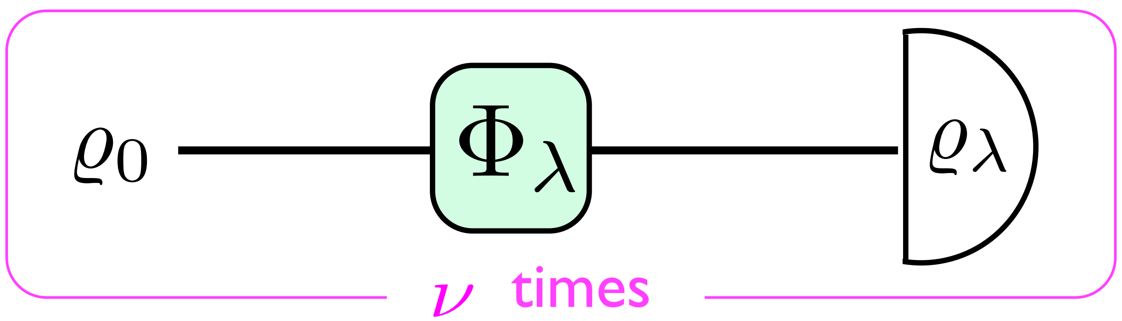
Fidelity
(state distinguishability)

Our approach

Temperature is always measured INDIRECTLY via quantum observables

QUANTUM ESTIMATION THEORY

$\lambda???$



+

data processing

quantum Cramer-Rao bound

$$\sqrt{\mathbb{E}[(\lambda^{est}-\lambda)^2]} \geq \frac{1}{\sqrt{\nu \mathcal{Q}}}$$

$$F(\rho,\sigma) = \mathrm{Tr}\left[\sqrt{\sqrt{\rho}\,\sigma\sqrt{\rho}}\right]$$

Fidelity
(state distinguishability)

Quantum **F**isher **I**nformation

- optimal quantum measurements
- depends on the geometrical structure of the statistical model

BURES DISTANCE

$$\mathcal{Q} = 8 \lim_{\varepsilon\to 0} \frac{1 - F(\rho_\lambda, \rho_{\lambda+\varepsilon})}{\varepsilon^2}$$

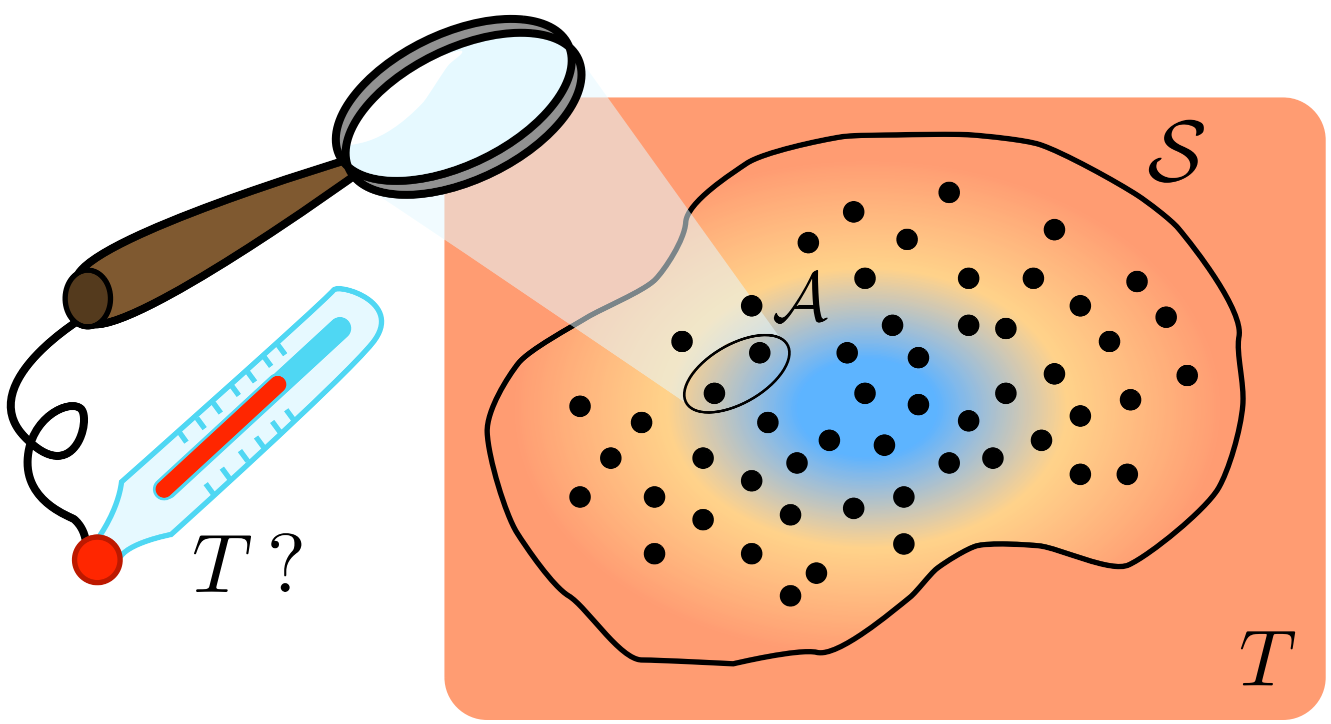
Local quantum thermometry

Temperature is always measured INDIRECTLY via quantum observables

QUANTUM ESTIMATION THEORY

how is the ACTUAL TEMPERATURE perceived LOCALLY?

only hp: $\rho_{\mathcal{S}} = \rho_\beta = \dfrac{e^{-\beta H}}{\mathcal{Z}}$



NO HYPOTHESIS ON THE REDUCED STATE OF $\mathcal{A}$

$\downarrow$

NO NOTION OF EFFECTIVE LOCAL TEMPERATURE

Local quantum thermometry

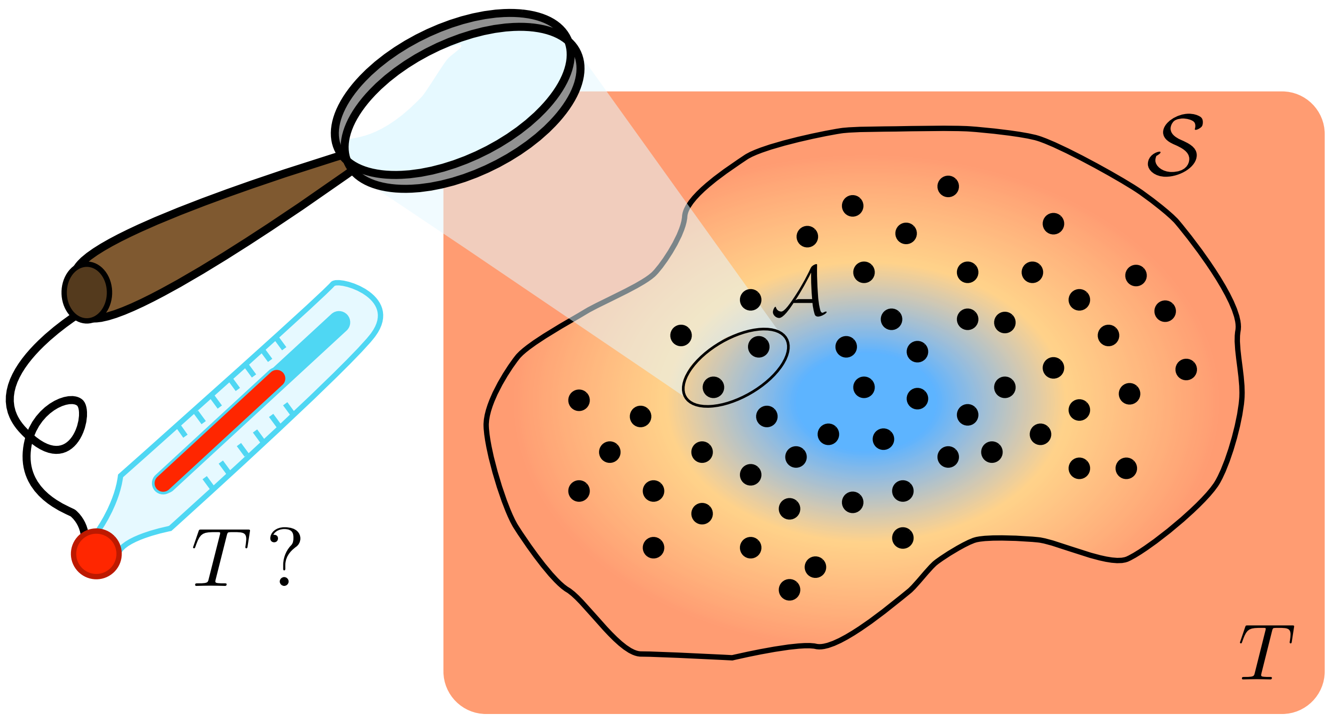
quantum
Cramer-Rao bound on T
for ν LOCAL
measurements on $\mathcal{A}$

$$\sqrt{\mathbb{E}[(T^{\text{est}} - T)^2]} \geq k_B T^2 / \sqrt{\nu \mathfrak{S}_{\mathcal{A}}[\rho_\beta]}$$

only hp: $\rho_{\mathcal{S}} = \rho_\beta = \dfrac{e^{-\beta H}}{\mathcal{Z}}$

$$\mathfrak{S}_{\mathcal{A}}[\rho_\beta] := 8 \lim_{\varepsilon \to 0} \frac{1 - \mathcal{F}\left(\rho_\beta^{\mathcal{A}}, \rho_{\beta+\varepsilon}^{\mathcal{A}}\right)}{\varepsilon^2}$$

Local Quantum
Thermal Susceptibility
(LQTS)



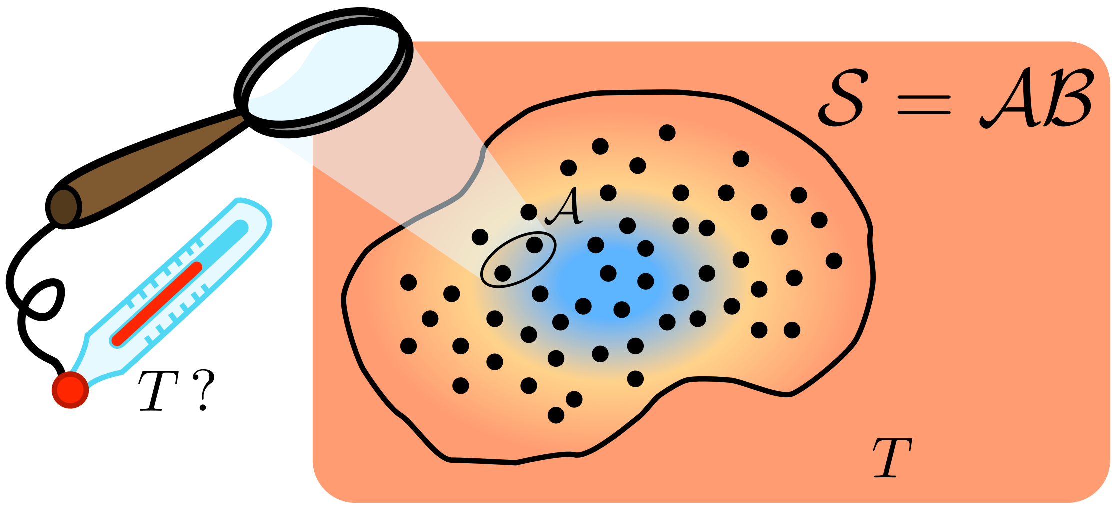
NB: if $\mathcal{A} \equiv \mathcal{S}$

$$\mathfrak{S}_{\mathcal{S}}[\rho_\beta] = \mathrm{Tr}[\rho_\beta H^2] - \mathrm{Tr}[\rho_\beta H]^2$$

heat capacity of global system

Zanardi, PRL 99, 100603 (2007)

Local Quantum Thermal Susceptibility



$$\mathfrak{S}_{\mathcal{A}}[\rho_\beta] := 8 \lim_{\varepsilon \to 0} \frac{1 - \mathcal{F}\left(\rho_\beta^{\mathcal{A}}, \rho_{\beta+\varepsilon}^{\mathcal{A}}\right)}{\varepsilon^2}$$

We introduce the ancillary system $a = \mathcal{B}\mathcal{S}'$ and apply the Uhlmann's theorem for the fidelity

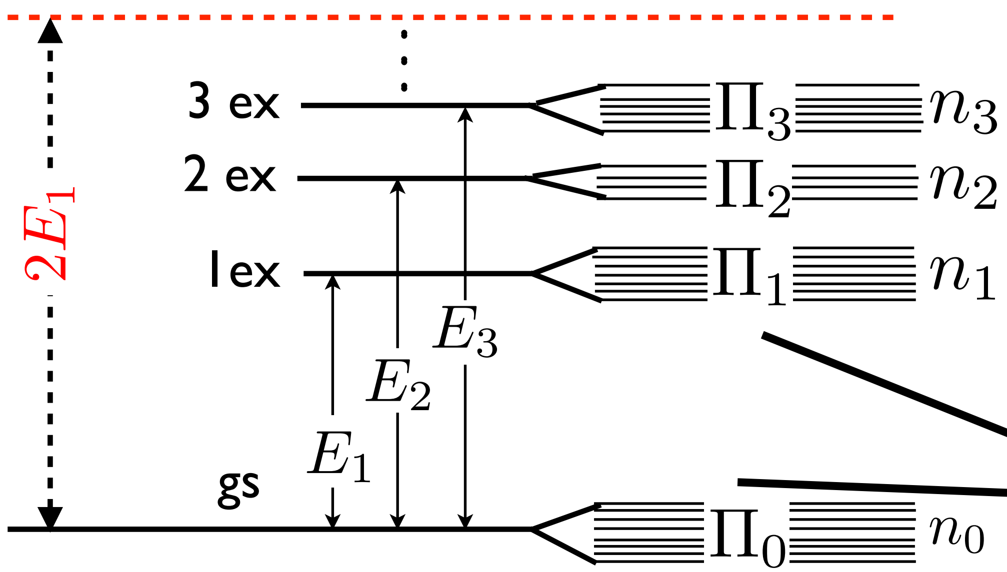
$$\boxed{\mathfrak{S}_{\mathcal{A}}[\rho_\beta] = \mathfrak{S}_{\mathcal{S}}[\rho_\beta] - 4\mathrm{Tr}[\rho_\beta^a\, \Omega^2]}$$

$$\Omega = \sum_{n=0}^{+\infty} (-1)^n \left(\rho_\beta^a\right)^n Q \left(\rho_\beta^a\right)^{-(n-1)}$$

$$Q = -\frac{i}{2}\mathrm{Tr}_{\mathcal{A}}\left\{\left[H, |\rho_\beta^{\mathcal{A}}\rangle\langle\rho_\beta^{\mathcal{A}}|\right]\right\}$$

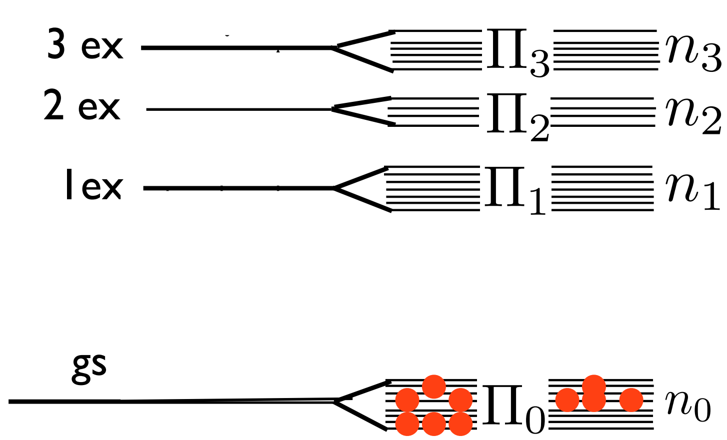
- $\mathfrak{S}_{\mathcal{A}}[\rho_\beta] \leq \mathfrak{S}_{\mathcal{S}}[\rho_\beta]$
- if $H = H_{\mathcal{A}} + H_{\mathcal{B}} \implies \mathfrak{S}_{\mathcal{A}}[\rho_\beta] \equiv$ heat capacity of $\mathcal{A}$

Low T regime: a measure of state distinguishability

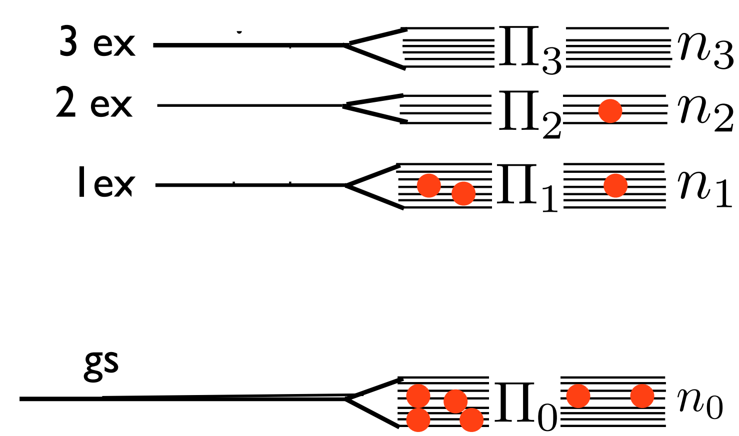


LQTS: how much does the ground state of the GLOBAL system $\mathcal{S}$ differ from the first excited subspaces, when observing it LOCALLY on $\mathcal{A}$?

Π_i = normalized projector on E_i subspace



$$\rho(T = 0)$$



$$\rho(T \sim 0)$$

Examples with 2 qubits

$$H_0 = \vec{\sigma}_{\mathcal{A}} \cdot \vec{\sigma}_{\mathcal{B}}$$

singlet subspace $\Pi_0 = |\psi_-\rangle\langle\psi_-|$

triplet subspace $\Pi_1 = \frac{1}{3}\left(|\psi_+\rangle\langle\psi_+| + |\uparrow\uparrow\rangle\langle\uparrow\uparrow| + |\downarrow\downarrow\rangle\langle\downarrow\downarrow|\right)$

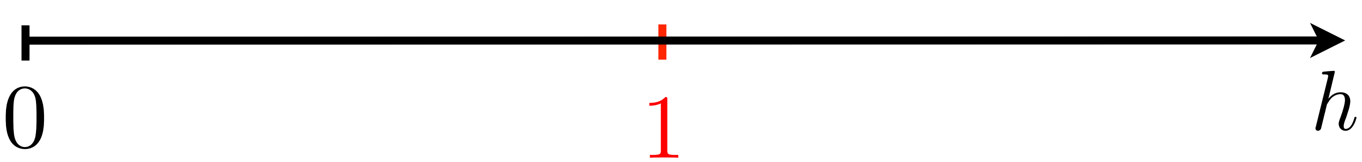
$$\rho_{\mathcal{A}}(\beta) = \mathrm{Tr}_{\mathcal{B}}[\rho_\beta] = \frac{\mathbb{I}_{\mathcal{A}}}{2}$$

$$\Longrightarrow \quad \mathfrak{S}_{\mathcal{A}}[\rho_\beta] = 0$$

$\boxed{\text{Indeed: } \Pi_0^{\mathcal{A}} = \Pi_1^{\mathcal{A}} \subseteq P_0^{\mathcal{A}}}$

no local distinguishability

$$H_{XX} = \sigma^x_{\mathcal{A}}\sigma^x_{\mathcal{B}} + \sigma^y_{\mathcal{A}}\sigma^y_{\mathcal{B}} + h(\sigma^z_{\mathcal{A}} + \sigma^z_{\mathcal{B}})$$



$h < 1$	$h > 1$
$\Pi_0 = \|\psi_-\rangle\langle\psi_-\|$	$\Pi_0 = \|\downarrow\downarrow\rangle\langle\downarrow\downarrow\|$
$\Pi_1 = \|\downarrow\downarrow\rangle\langle\downarrow\downarrow\|$	$\Pi_1 = \|\psi_-\rangle\langle\psi_-\|$

$$\mathfrak{S}_{\mathcal{A}}[\rho_\beta] \neq 0$$

$\boxed{\text{Indeed: } \Pi_0^{\mathcal{A}} \neq \Pi_1^{\mathcal{A}} \nsubseteq P_0^{\mathcal{A}}}$

local distinguishability

Local quantum thermometry in many-body systems

$$H_{\text{Ising}} = -\sum_i \left[\sigma_i^x \sigma_{i+1}^x + h\sigma_i^z\right]$$

FM PM

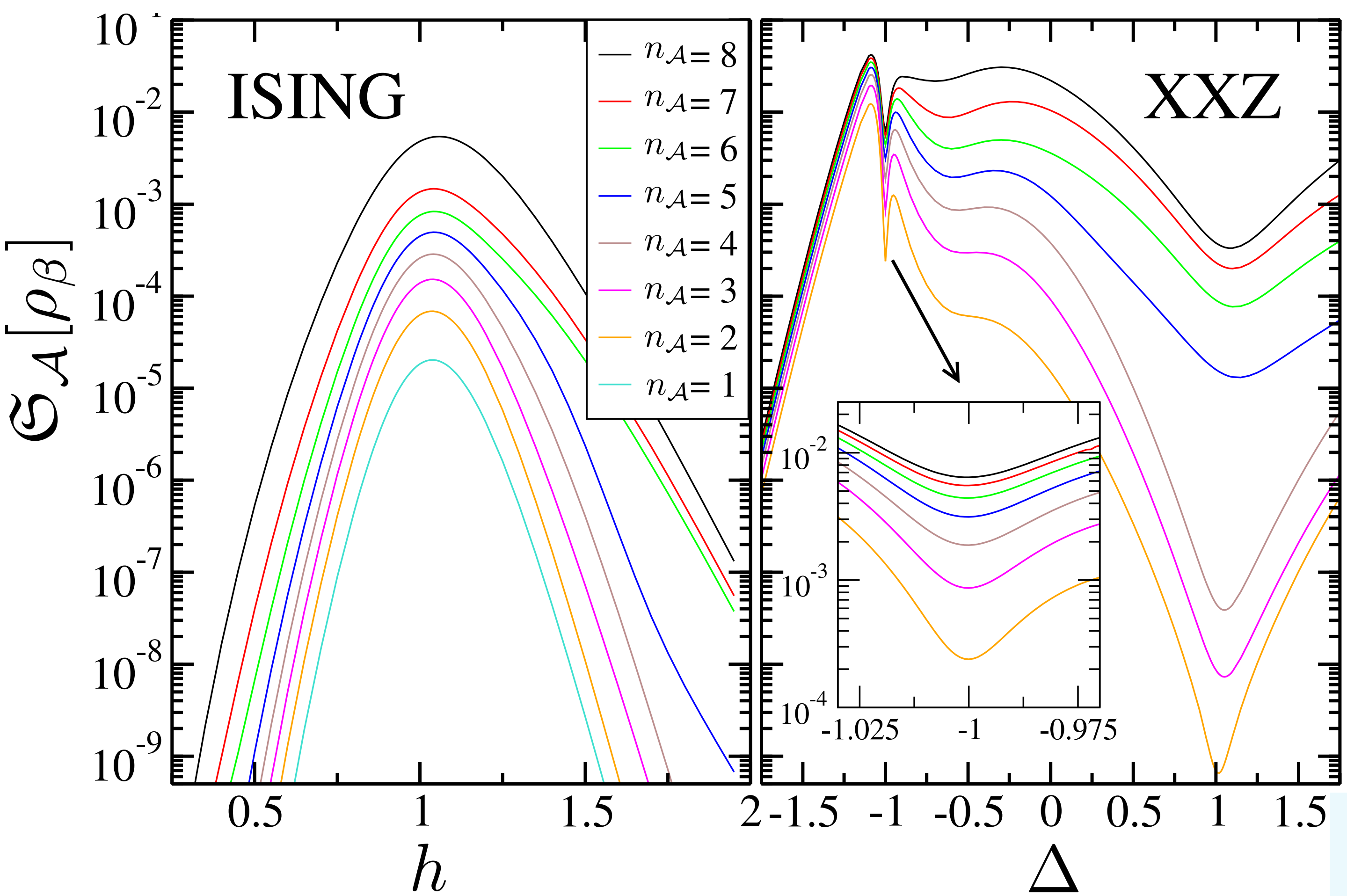
$0 \quad 1 \quad h$

$$H_{\text{XXZ}} = \sum_i \left[(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y) + \Delta\, \sigma_i^z \sigma_{i+1}^z\right]$$

FM LL AFM

$-1 \quad 1 \quad \Delta$

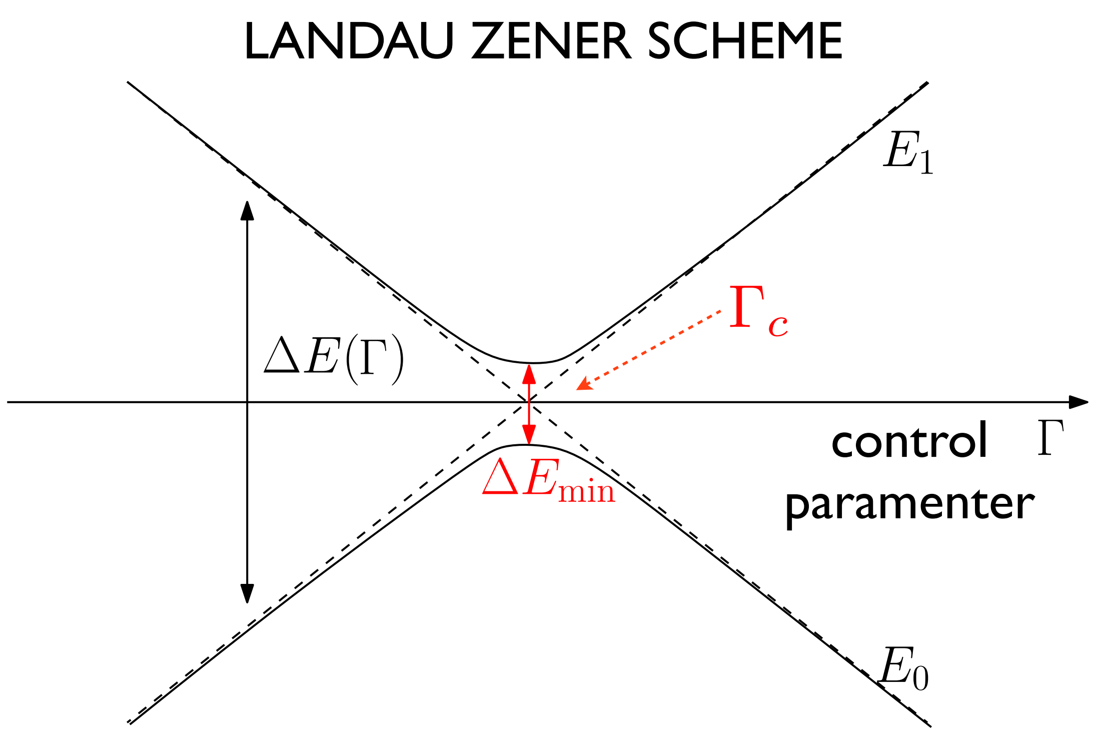
$\beta = 9 \qquad L = 8 \qquad \beta = 5$



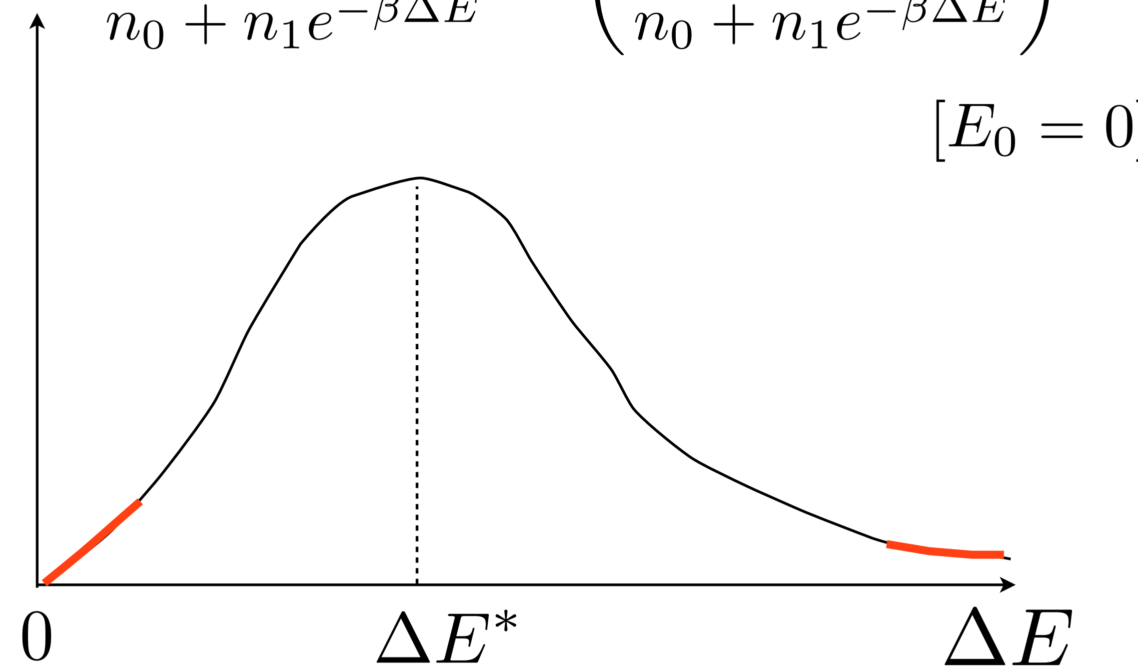
- LQTS monotonically decreases with decreasing the number $n_{\mathcal{A}}$ of contiguous spins in $\mathcal{A}$
- $n_{\mathcal{A}} = 2, 3$: similar thermal behavior of the global system
- LQTS is sensitive to the presence of critical regions

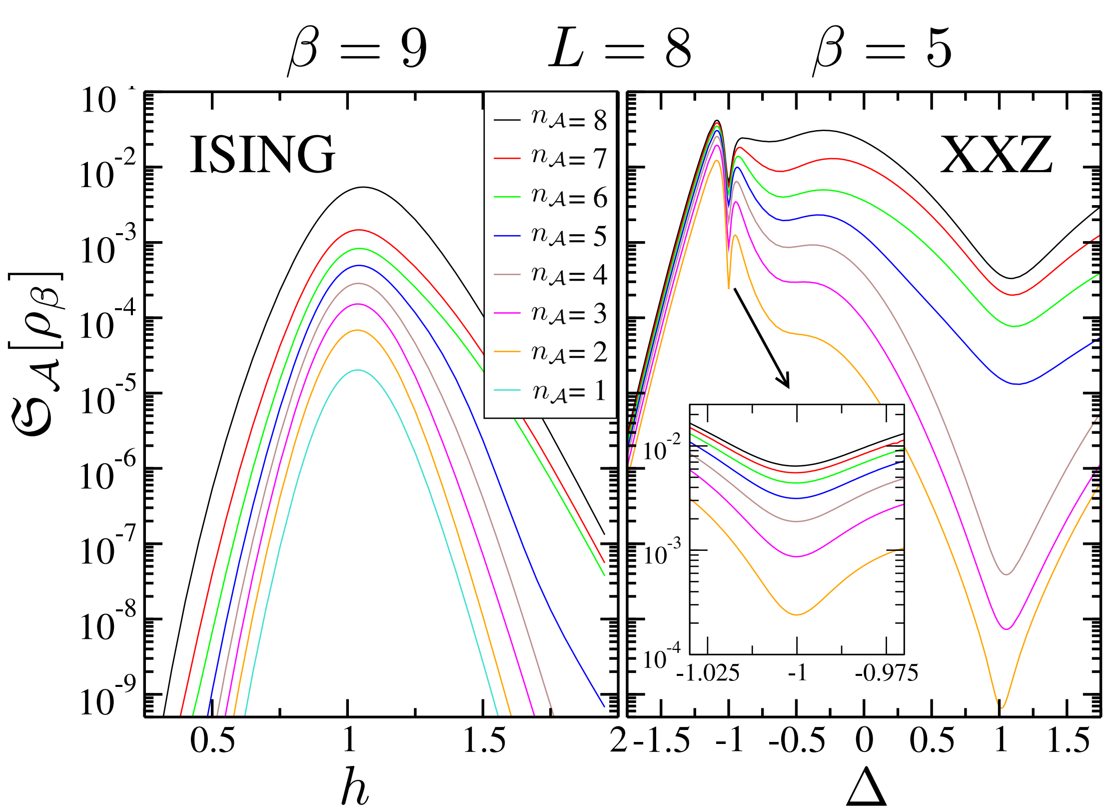
if $n_{\mathcal{A}} = L$ see also P. Zanardi, *et al.*, PRA 76, 062318 (2007); M. Mehboudi, *et al*, arXiv:1501.03095v1 (2015)

Local quantum thermometry in many-body systems



$$\mathfrak{S}_{\mathcal{S}}[\rho_\beta] = \frac{n_1(\Delta E)^2 e^{-\beta\Delta E}}{n_0+n_1 e^{-\beta\Delta E}} - \left(\frac{n_1\Delta E e^{-\beta\Delta E}}{n_0+n_1 e^{-\beta\Delta E}}\right)^2$$



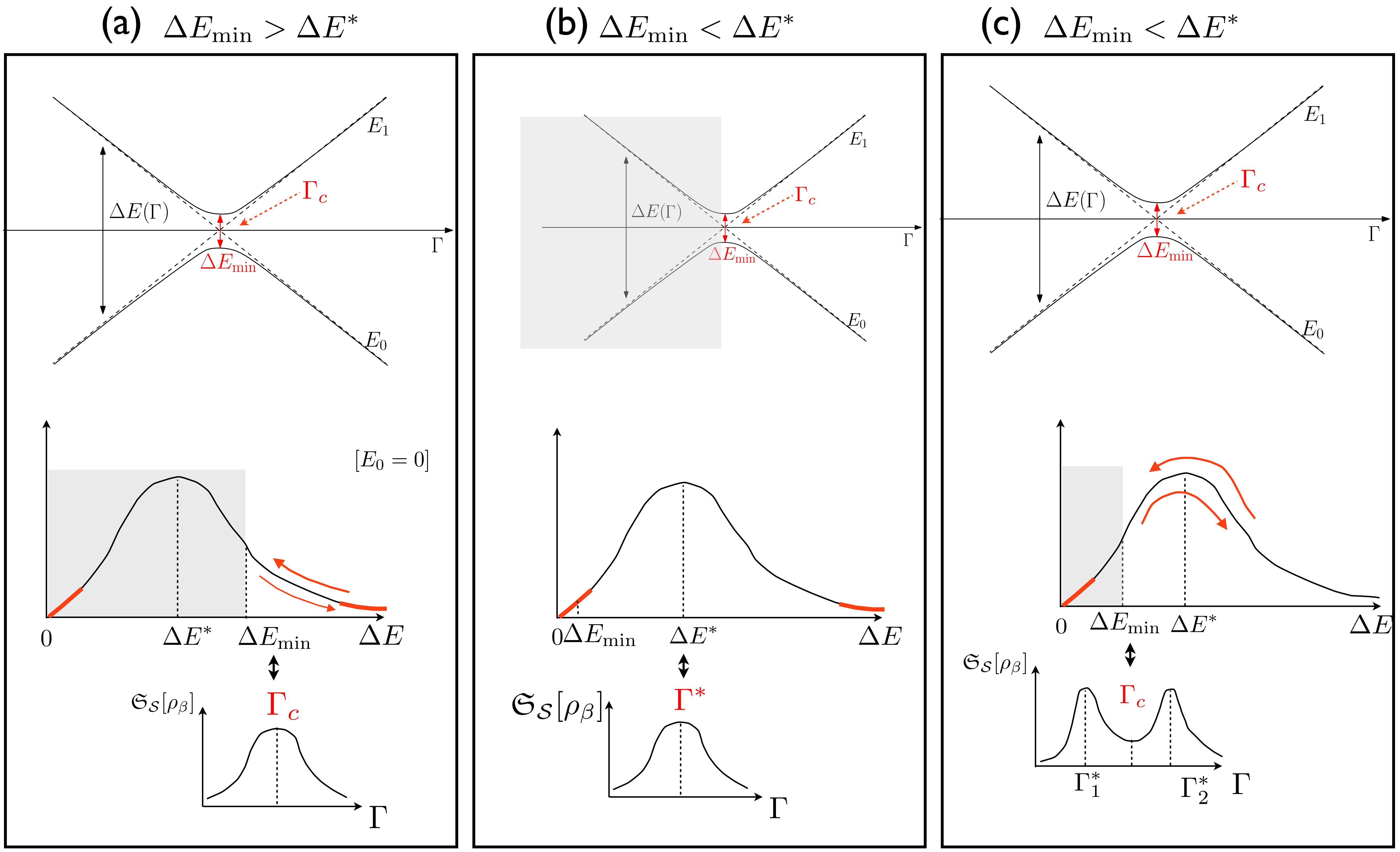


$$\mathfrak{S}_{\mathcal{S}}[\rho_\beta] \longrightarrow 0$$

$$\Delta E \to 0$$

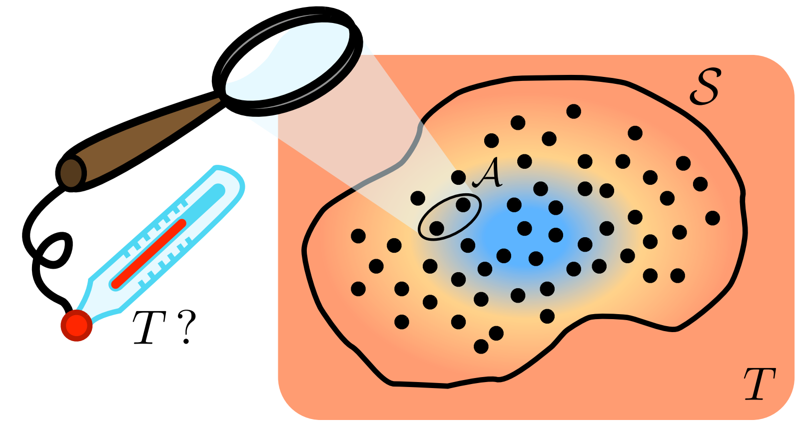
$$\Delta E \to \infty$$

Local quantum thermometry in many-body systems



Conclusions

- We have introduced a quantum estimation approach to quantum thermodynamics



- By removing the theoretical testbed of conserving thermal-like structure by parts of the total system in thermal state, we have introduced a functional (LQTS) with a clear operative meaning

- In the low temperature limit, our approach can be translated in terms of a receipt able to quantify the local distinguishability of the ground state from the first exicited levels

- Many-body systems: the thermal behavior of the system heat capacity can be predicted by optimal measurements on local subsystems, with reminiscences of critical regions