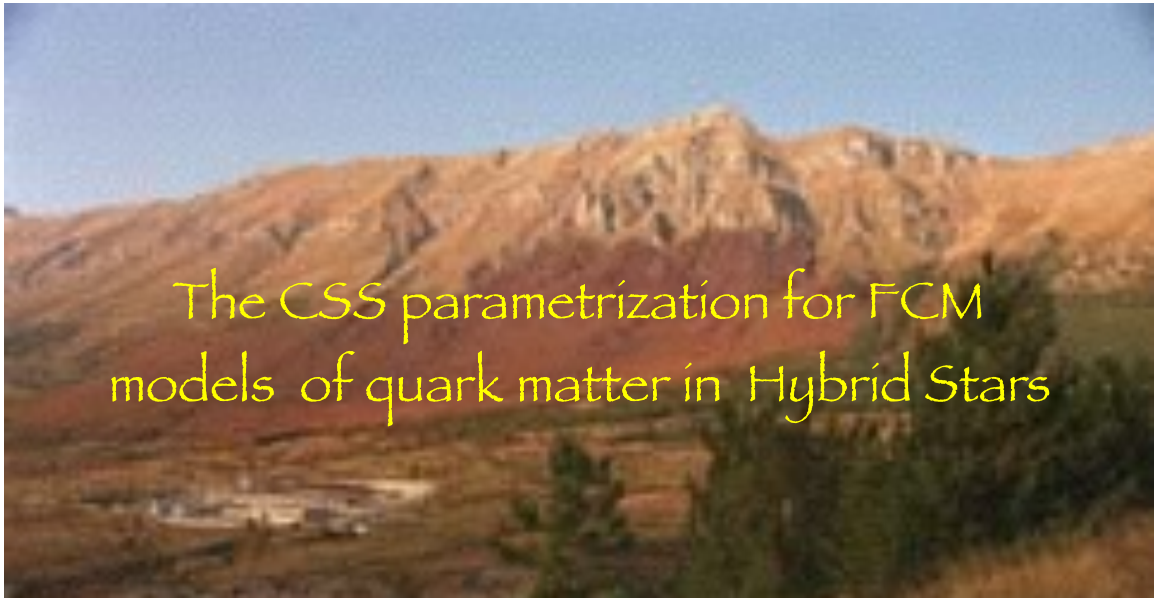
Compact Stars in QCD Phase Diagram V,
GSSI &LNGS, 23-27 May, 2016

Working Group 2 Meeting,
COST Action MP1304



The CSS parametrization for FCM models of quark matter in Hybrid Stars

Fiorella Burgio

INFN Sezione di Catania, Italy

with : M. Alford, S. Han, G. Taranto and D. Zappalà

based on : Phys. Rev. D**92**, 083002 (2015); EPJA **52**, 60 (2016)

Observing M(R) ⟷ Constraining QM EoS

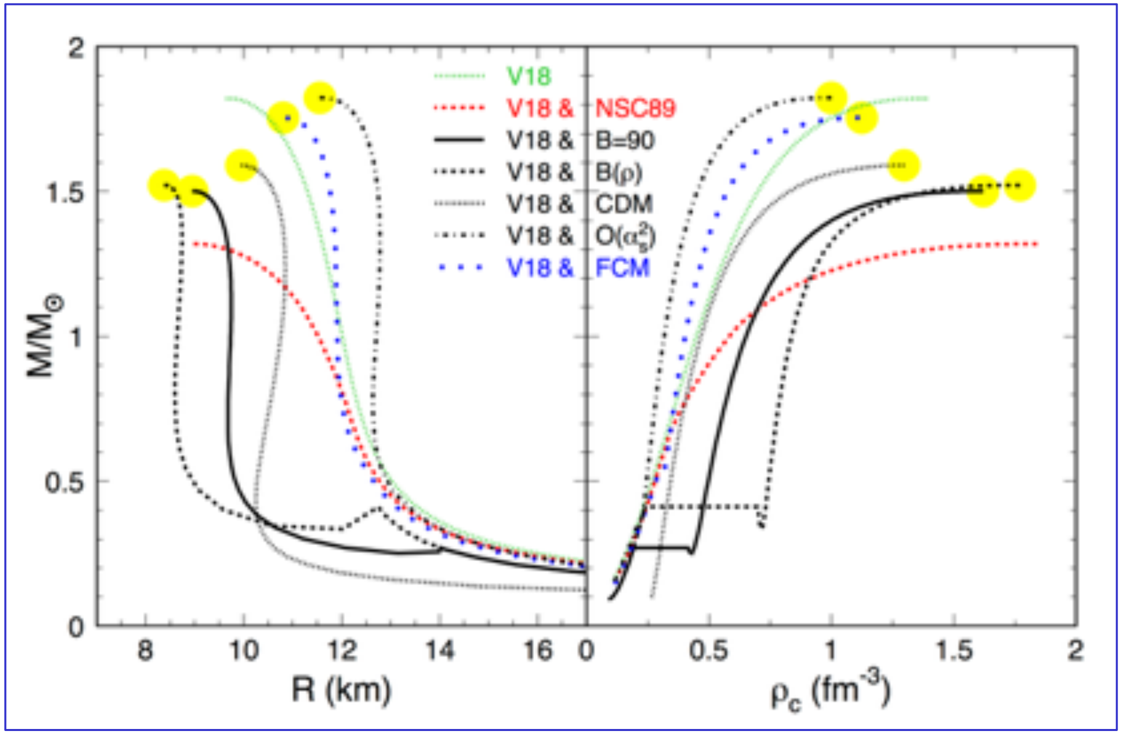
Does a NS contain quark matter ?

- MIT Bag Model (Chodos (1974), Fahri (1984), Baym (1985), Glendenning (1990))
- MIT Bag Model wt B(ρ); (Burgio et al., PLB**526** (2002) 19)
- MIT Bag Model wt. phen. corr.; (Alford et al., ApJ**629** (2005) 969)
- NJL Models; (Buballa et al., Nucl. Phys. A**703** (2002) 770)
- PNJL Models; (Blaschke et al., arXiv:1302.6275)
- 2-loop perturbation theory of the QCD EoS; (Kurkela et al., PR**D81** (2010) 105021)
- Color Dielectric Model; (Drago et al., PLB**380**, (1996) 13)
- Dyson-Schwinger Model; (Chen et al., PRD**84** (2011) 105023)
- Field Correlator Method; (Baldo et al., PRD**78** (2008); Bombaci et al., MNRAS**433** (2013))

They all give different hybrid star structure and mass limits.

Burgio, Chen, Schulze, Taranto, arXiv : 1301.4060

BHF (Av18 + UVIX) for the hadronic phase (**N**, ***NY***)
Different QM EoS : bag models, CDM, FCM
(hyperons prevent phase transition if NJL, or Dyson-Schwinger are used)



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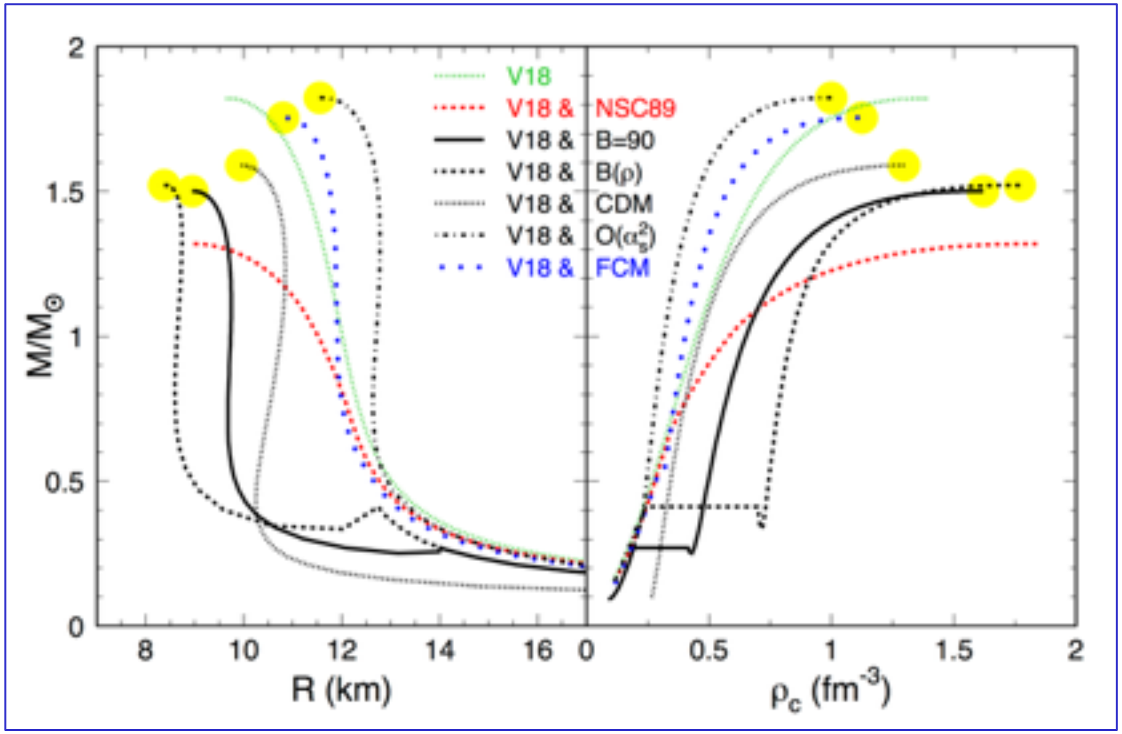
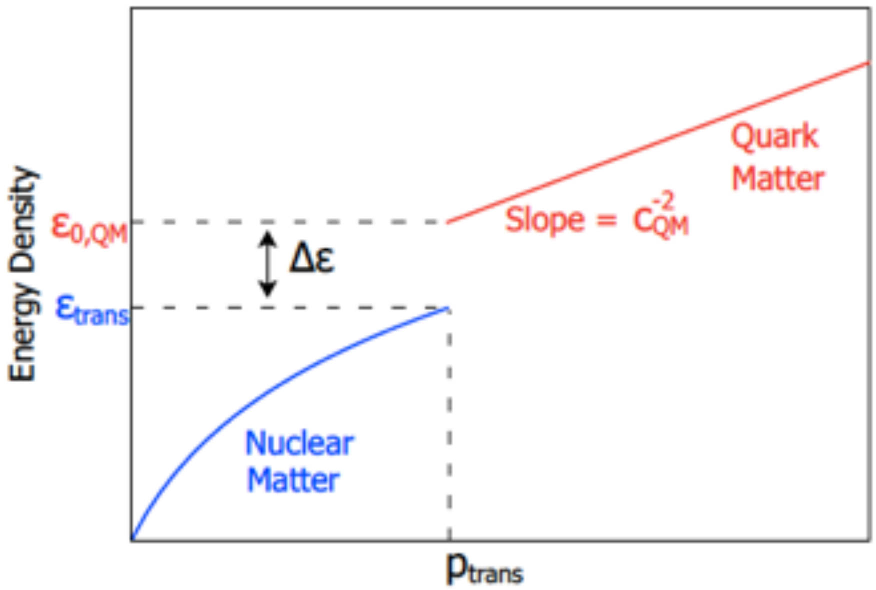


☛ Proposal : a generic framework for relating the different models to each other

Parametrization of a generic quark matter EOS

If a) sharp phase transition and b) c_{QM} is pressure independent :

$$\varepsilon(p) = \varepsilon_{\text{trans}} + \Delta\varepsilon + c_{\text{QM}}^{-2}(p - p_{\text{trans}})$$



QM EoS parameters

$p_{trans}/\varepsilon_{trans}$

$\Delta\varepsilon/\varepsilon_{trans}$

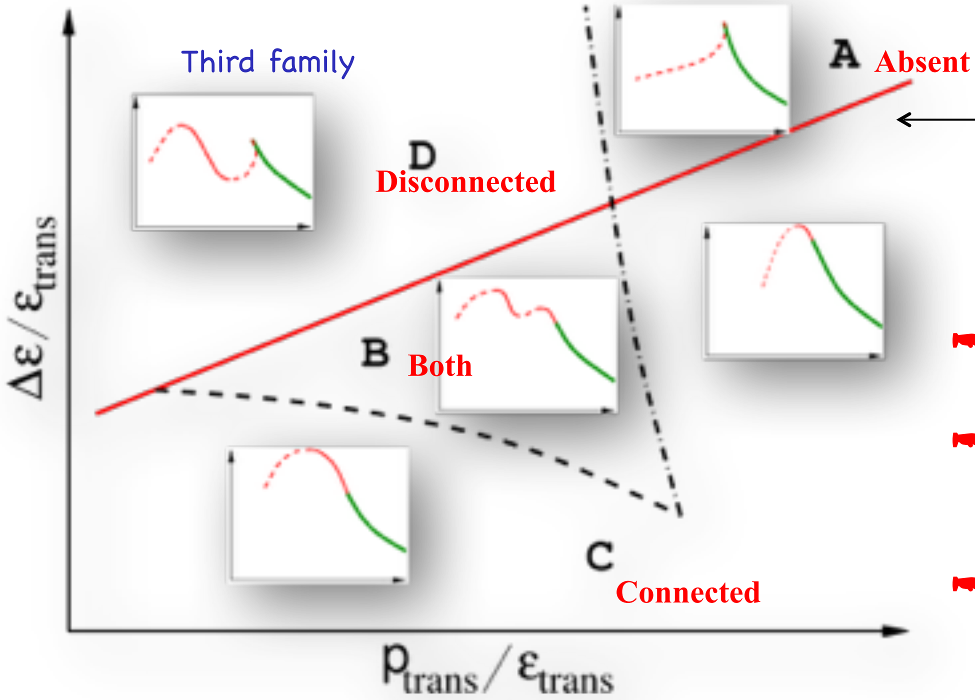
c^2_{QM}

"CSS" parametrization ⟶ M.G. Alford, S. Han and M. Prakash, PRD88 (2013)083013

The phase diagram of the M(R) curve

Green : hadronic branch

(Dashed) Solid Red : (un)stable hybrid stars



$$\frac{\Delta\varepsilon_{crit}}{\varepsilon_{trans}} = \frac{1}{2} + \frac{3}{2}\frac{p_{trans}}{\varepsilon_{trans}}$$

- Analytic result, independent of c^2_{QM} and NM EoS
- Small energy density jump at phase transition -> Connected branches of stable hybrid stars in C and B.
- Large energy density jump at phase transition -> Disconnected branches of stable hybrid stars in D and B.
- D region important : characteristic signature in M(R) observations.
- The size of the phase boundary depends on c_{QM} and (slightly) on the NM EoS

Seidov, Sov. Astron. (1971); Schaeffer, Zdunik, Haensel, A&A (1983); Lindblom Phys.Rev.D (1998)

➢ **BHF** with Av18 as NN potential and UVIX for the nucleonic three-body force

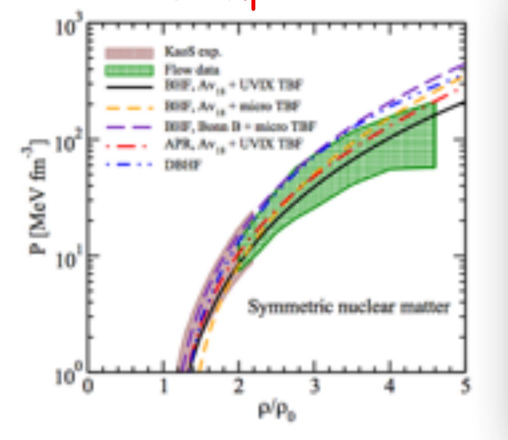
➢ **DBHF** with Bonn A as NN potential

Relevant properties :

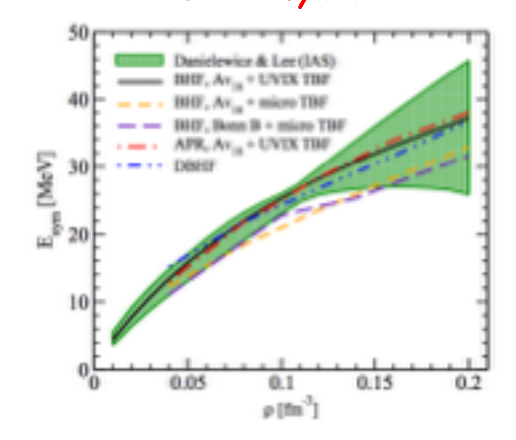
Property	BHF, Av_{18} + UVIX TBF	DBHF, Bonn A
Saturation baryon density n_0 (fm^{-3})	0.16	0.18
Binding energy/baryon E/A (MeV)	−15.98	−16.15
Compressibility K_0 (MeV)	212.4	230
Symmetry energy S_0 (MeV)	31.9	34.4
$L = 3n_0[dS_0/dn]_{n_0}$ (MeV)	52.9	69.4
Maximum mass of star ($M_\odot$)	2.03	2.31
Radius of the heaviest star (km)	9.92	11.26
Radius of $M = 1.4\ M_\odot$ star (km)	11.77	13.41

Good reproduction of the currently available data around the saturation density

HIC exp. data

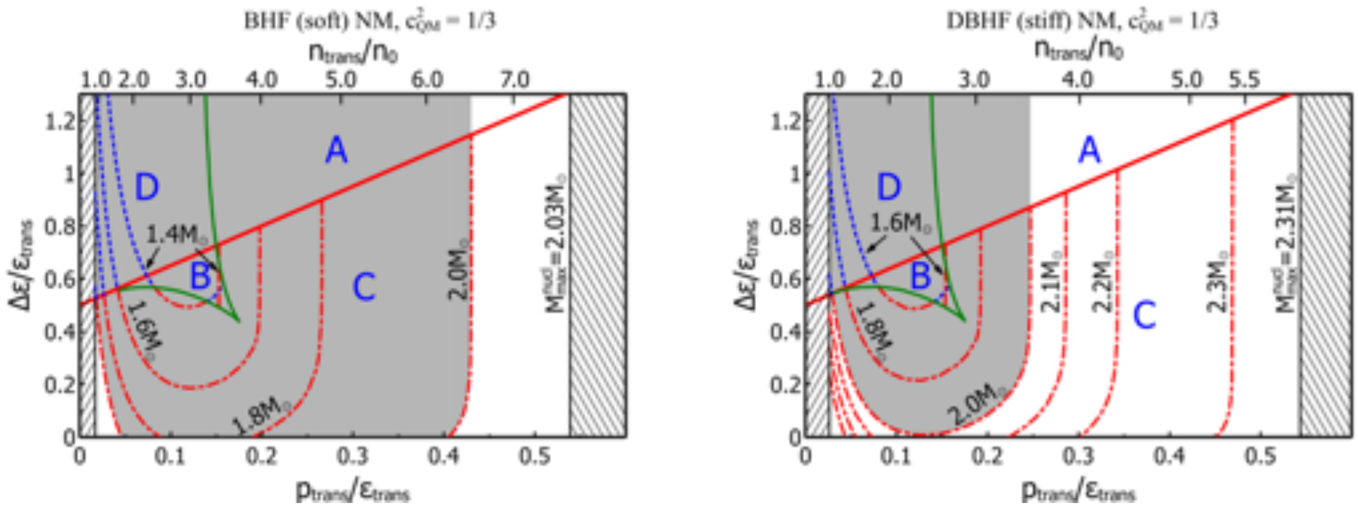


IAS analysis



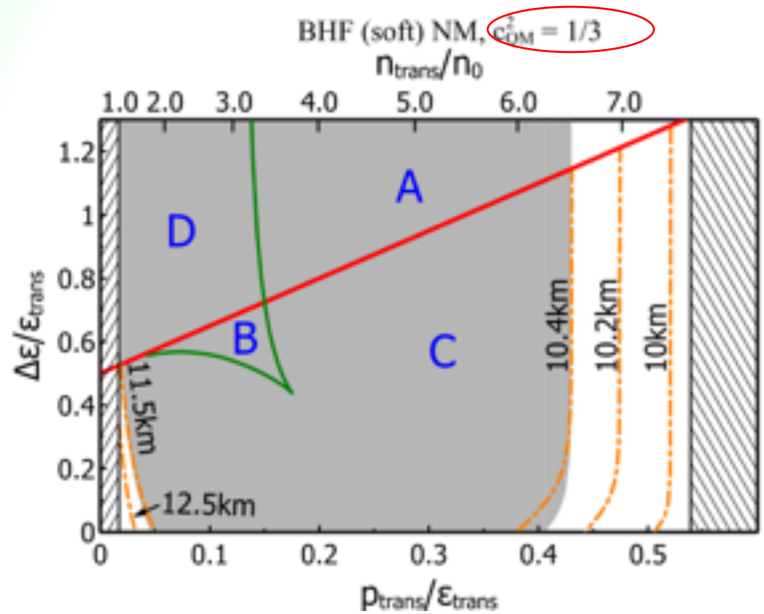
Can mass measurements constrain the CSS parameters ?

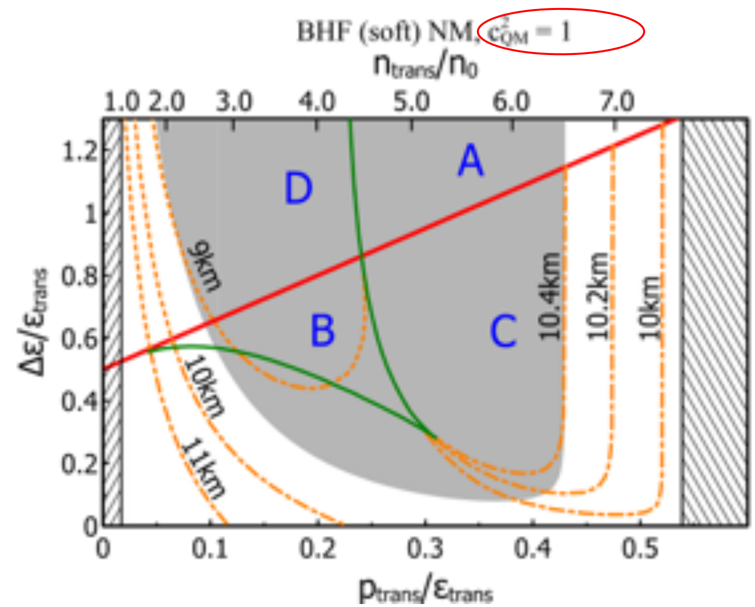
Contours of maximum hybrid star mass vs. the CSS parameters



- Several QM models characterized by $c^2_{QM} = 1/3$
- Grey shaded regions forbidden by the obs. of a $2M_o$ star.
- DBHF (stiffer) NM EoS gives heavier NS ⟹ wider range of CSS parameters compatible with $2M_o$ obs.
- Two regions of parameter space :
  - Low p_{trans} --- low density --- connected hybrid branch
  - High p_{trans} --- high density --- very small connected hybrid branch. Maximum hybrid stars mass close to the ones of purely hadronic stars with a tiny QM core. Small mass range < 10^(-3) Mo.
- No disconnected hybrid branches.

Contours of the radius of maximum-mass star vs. the CSS parameters





- Very small low-p_{trans} region separated from the high-p_{trans} region.
- Radius contours closely track the border of the allowed region -> R > 11.5 km.
- Similar result with DBHF.

- Disconnected branch stars with low P_{trans} and large $\Delta\varepsilon$.
- Small stars (R≈9km) occur.
- Observation of a smaller radius would imply that $c^2_{QM} > 1/3$

Application of the CSS parametrization to the Field Correlator Method for describing Quark Matter.

- The FCM model is able to cover the full T- μ plane
- Confinement is introduced ab initio through the QCD field correlators
- EoS expressed in terms of the $q\bar{q}$ potential V_1 and gluon condensate G_2

$$\boxed{P_{qg} = P_g + \sum_{j=u,d,s} P_q^j + \Delta\epsilon_{\mathrm{vac}}}$$

☛ Quark pressure :

$$P_q/T^4 = \frac{1}{\pi^2}\left[\phi_\nu\left(\frac{\mu_q - V_1/2}{T}\right) + \phi_\nu\left(-\frac{\mu_q + V_1/2}{T}\right)\right]$$

$$\phi_\nu(a) = \int_0^\infty du \frac{u^4}{\sqrt{u^2+\nu^2}} \frac{1}{(\exp[\sqrt{u^2+\nu^2} - a] + 1)} \qquad \nu = m_q/T$$

☛ Gluon pressure :

$$P_g/T^4 = \frac{8}{3\pi^2}\int_0^\infty d\chi \chi^3 \frac{1}{\exp(\chi + \frac{9V_1}{8T}) - 1}$$

☛ Eff. Bag constant

$$\Delta\epsilon_{\mathrm{vac}} \approx -\frac{(11 - \frac{2}{3}N_f)}{32}\frac{G_2}{2}$$

Di Giacomo (2002), Simonov and Trusov, (2007).

What about V_1 (μ_B,T) and G_2 (μ_B,T)?

- A few indications at $\mu_B = 0$ and $T=T_c$:

  From lattice $G_2(T_c) \approx G_2(T=0)/2$;

  $G_2(T=0)=0.012 \pm 0.006$ GeV^4 from QCD sum rules

Bombaci and Logoteta, MNRAS (2013)

$$T_c = \frac{a_0 G_2^{1/4}}{2}\left(1+\sqrt{1+\frac{V_1(T_c)}{2a_0 G_2^{1/4}}}\right) \longrightarrow \boxed{V_1(T_c) \leq 0.15\ \mathrm{GeV}}$$

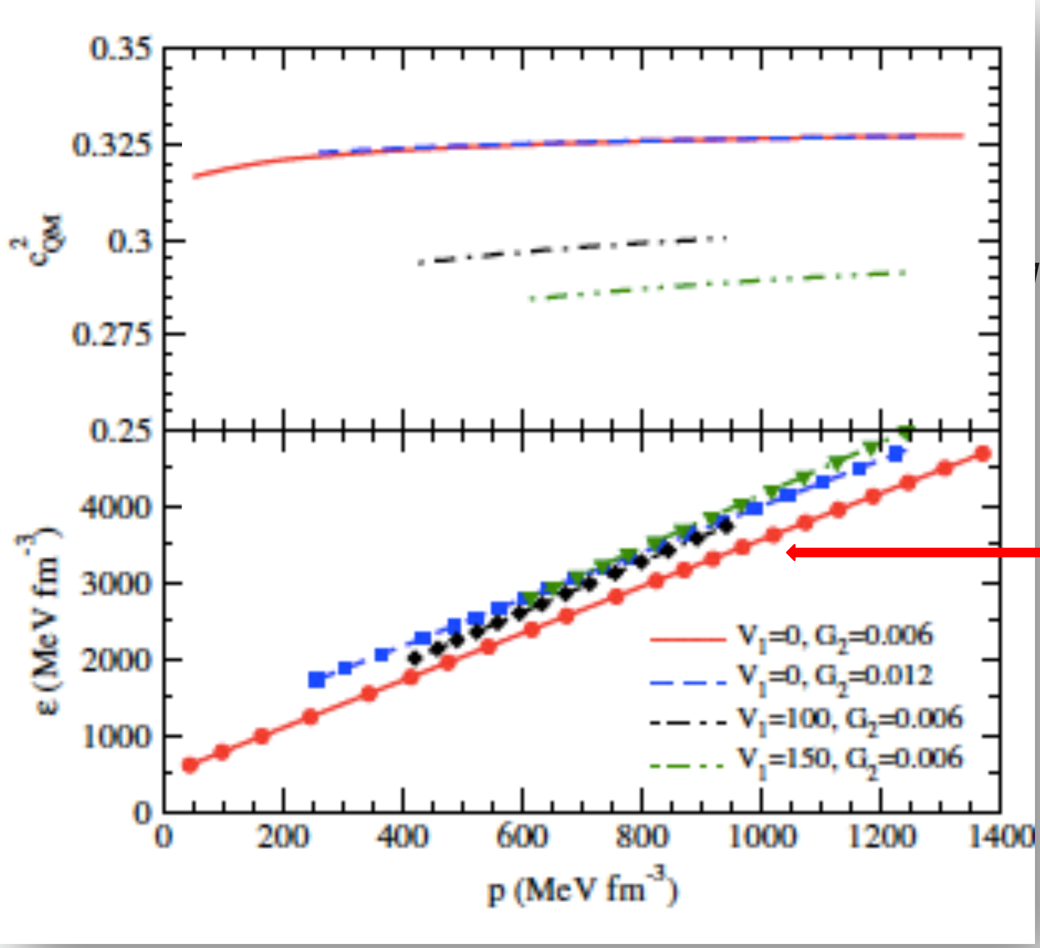
Plumari, Burgio, Greco and Zappalà, PRD(2013)

Comparing with lattice data $\longrightarrow \boxed{V_1(T_c) \approx 0.5 - 0.6\ \mathrm{GeV}}$

- No lattice simulations at finite μ_B and **T = 0**, no definite indications on **V_1 (μ_B,T=0)** and **G_2 (μ_B,T=0);**

  we keep it as free parameters !

CSS Parametrization suitable for the FCM EoS !



c^2_{QM} is almost pressure independent and $1/4 < c^2_{QM} < 1/3$ for the relevant values of (V_1, G_2)

Dots : FCM EoS

Solid lines : CSS parametrization

Alford, Han, Burgio, Taranto, Zappalà, PRD (2015)

The Hadron-quark phase transition

NS matter properties require
β-equilibrium; charge neutrality; N_B conservation

The transition between the hadronic and quark phase is determined by a crossing of the granpotential $\Omega=-PV$ in the two phases.

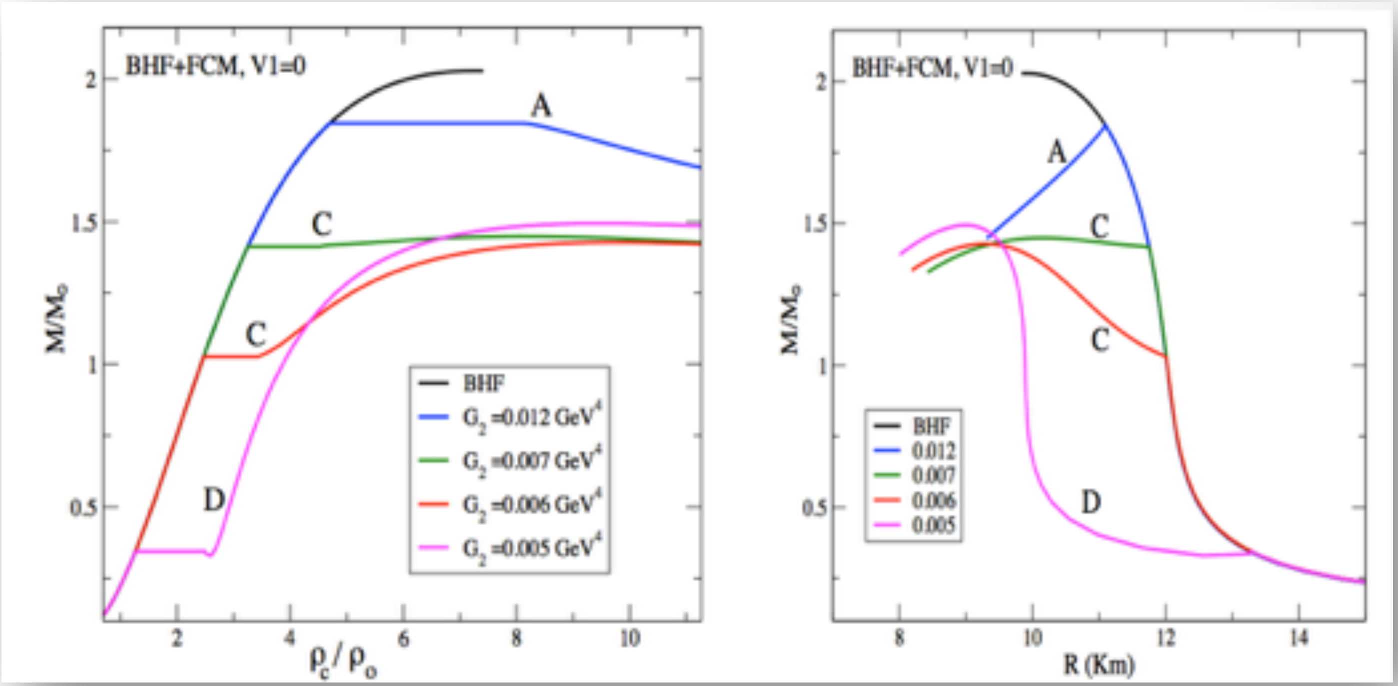
<u>Maxwell construction is implemented</u>

M(R) and M(ρ) obtained by solving the TOV eqs.

$$\frac{dP(r)}{dr} = -\frac{Gm(r)\varepsilon(r)}{r^2} \frac{\left[1+\frac{P(r)}{\varepsilon(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{m(r)}\right]}{\left[1-\frac{2GM(r)}{rc^2}\right]} \qquad \frac{dm(r)}{dr} = 4\pi r^2 \varepsilon(r)$$

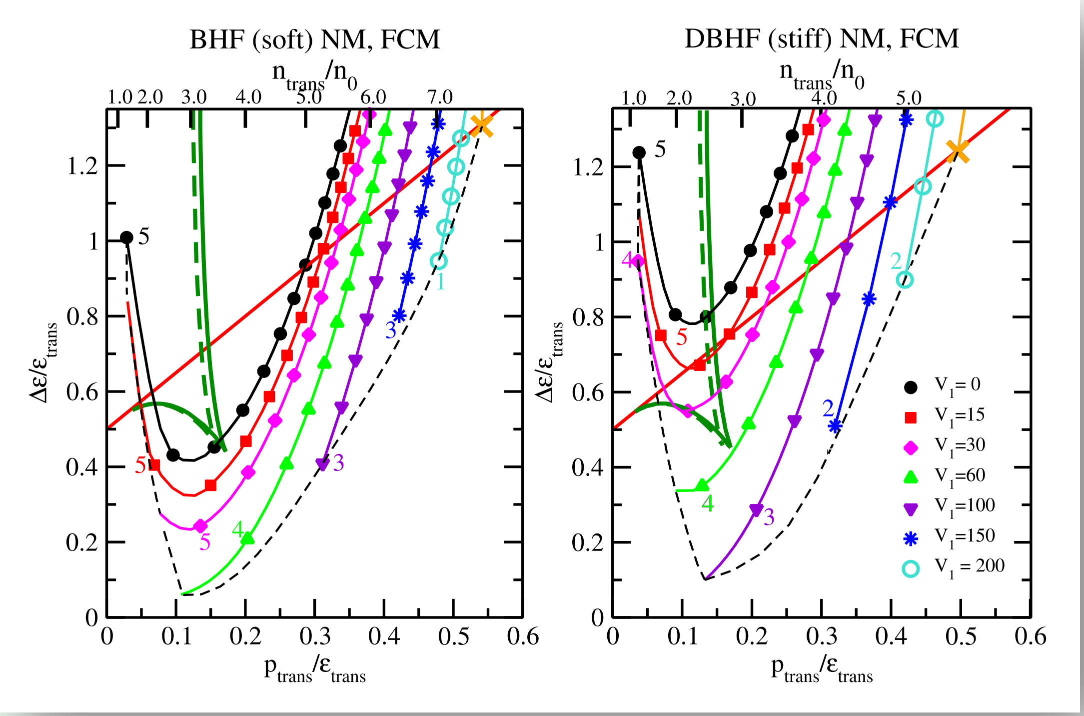
Some examples

- ✓ All kinds of topology are observed.
- ✓ High accuracy required in solving TOV eqs.



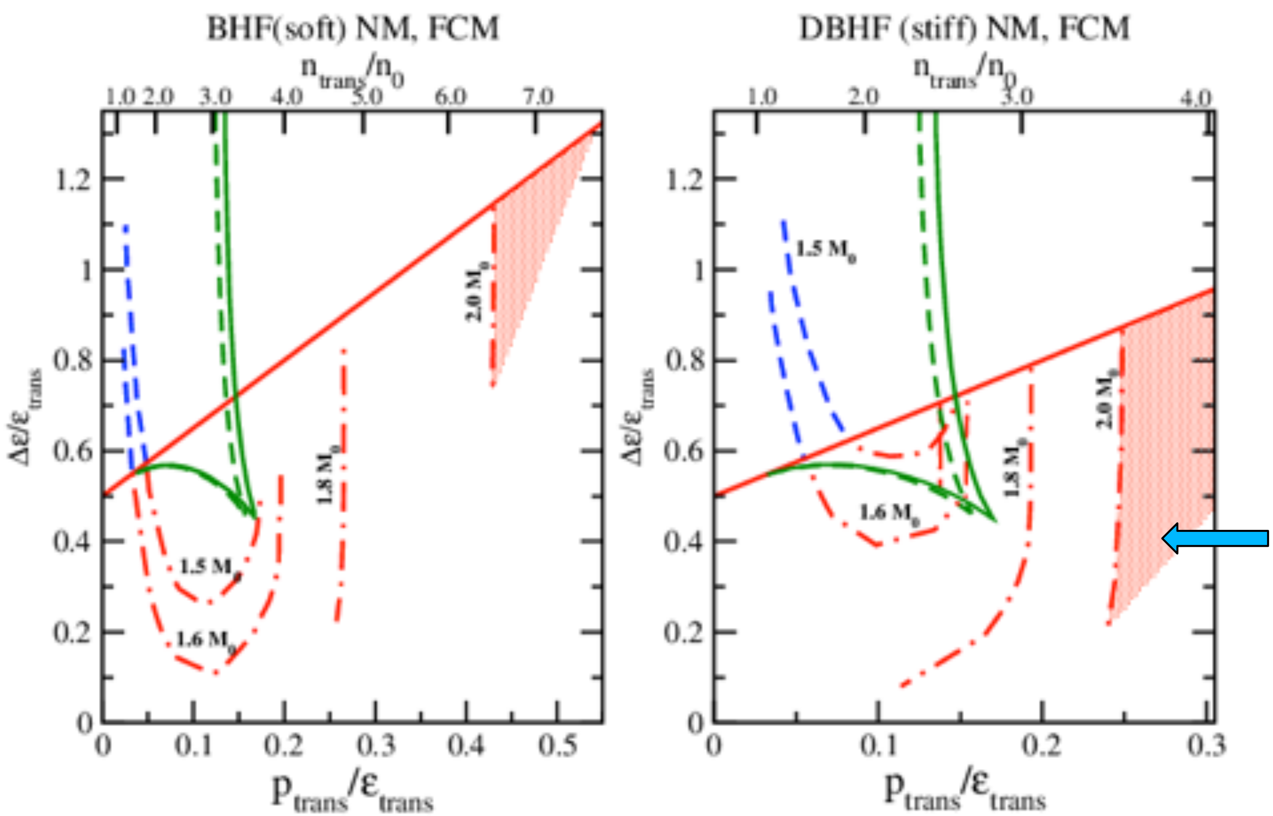
Mapping FCM onto the CSS parametrization

(V_1, G_2) vs. $(p_{trans}/\varepsilon_{trans}, \Delta\varepsilon/\varepsilon_{trans})$



- Green lines : phase boundaries for connected and disconnected hybrid branches.
- (Black curves) : Region yielded by the FCM EoS. $0 < V_1 < \max V_1$ at which hybrid stars occur (orange cross).
- Along each line, V_1 is kept constant while G_2 is changing, up to the last stable hybrid star mass configuration.
- BHF : mainly C, A regions are populated. D is scarcely populated.
- DBHF : D region more populated.

Contour plots of maximum mass hybrid star



Region accessible by FCM and compatible with the $2M_{\odot}$ obs.

- ✳ Hybrid stars in the shaded red area possess a small quark core and M,R similar to those of the heaviest purely nucleonic stars.
- ✳ V_1 lies in the range 200-250 MeV.
- ✳ No heavy hybrid stars at low p_{trans}

Paired quark matter

- Extension of the FCM model to include color superconductivity through the CFL mechanism ⟶ the mapping onto CSS param. still holds true !

- Dependence on (V_1, G_2 and Δ)

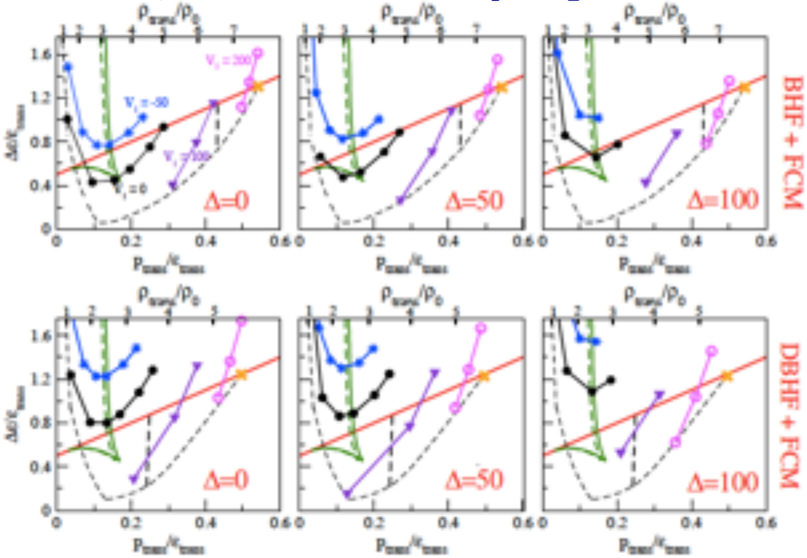


Table 2. The total radius R, the radius of the quark core R_Q, the radius of the hadronic layer R_H and the crust radius R_{crust} are given for a hybrid star mass $M = 2M_{\odot}$, for different choices of the hadronic EoS and Δ. All radii are given in km.

EoS	Δ (MeV)	R	R_Q	R_H	R_{crust}
BHF	0.	10.37	0.055	9.97	0.345
	100.	10.44	0.215	9.87	0.355
DBHF	0.	12.78	1.27	10.87	0.640
	100.	12.72	2.42	9.665	0.635

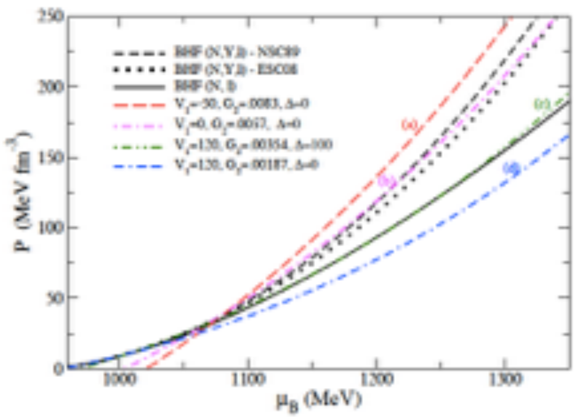
- Largest quark core for the stiffest EoS.
- Hadronic layer occupies the largest portion of the star.

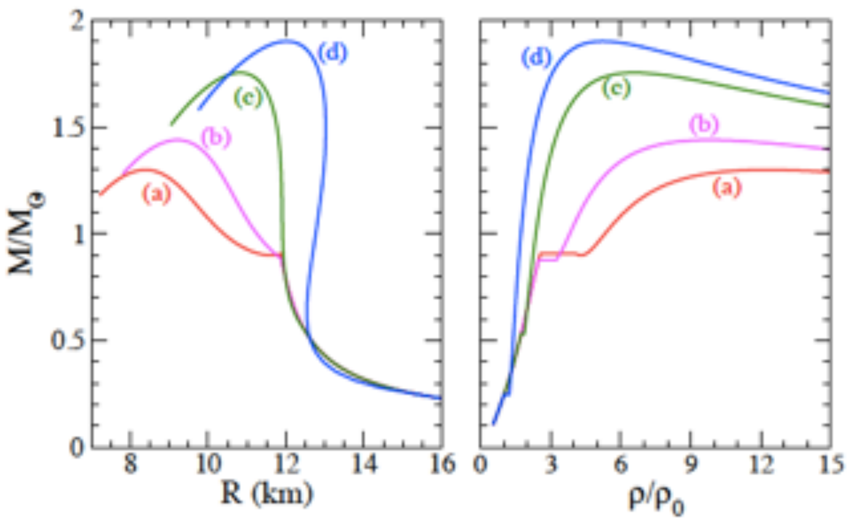
Burgio and Zappalà, EPJA (2016)

Effects of hyperons on the phase transition

Extension of BHF to hypernucler matter : Baldo, Burgio, Schulze, PRC(1998, 2000)

Nucleon-hyperon potentials (NSC89 and ESC08) : Schulze, Rjiken, PRC(2011)



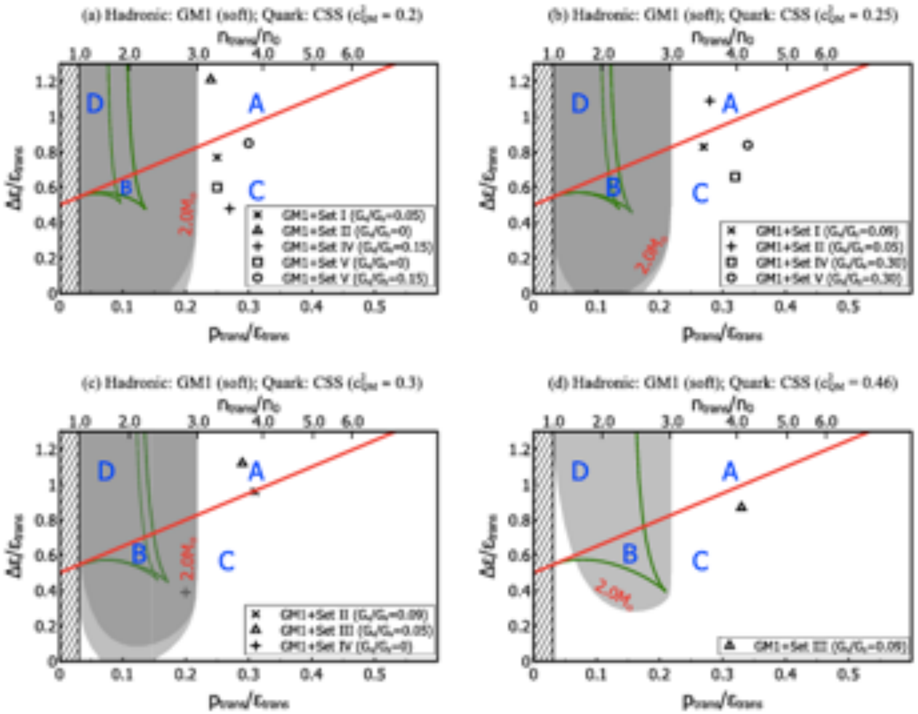


- Strong competition between QM and Hyperon Onset
- Maximum mass values below 2Mo.
- Similar result as obtained in 2-loop perturbation theory by Kurkela et al., PRD(2010)

CSS parametrization and NJL models

Ranea-Sandoval, Han, Orsaria, Contrera, Weber, Alford, PRC (2016)

- Local and nonlocal NJL models with repulsive vector interaction and different sets of parameters (m_s, G_V, ...)
- RMF EoS for hadronic matter : GM1 (soft), NL3 (stiff)



- "A" region : no hybrid stars because its phase transition is so strongly first order.
- "C" region : the connected branch is short, covering a range of no more than 0. $05M_0$. Very difficult to detect in mass-radius obs..
- "D" and "B" regions : ruled out because their heaviest star is too light.
- Similar results also for the stiff NL3 EoS.

Conclusions

- CSS (Constant Speed of Sound) as a generic language to connect different quark matter EoS.
- FCM QM model can be mapped onto the CSS parametrization, with and without CFL.
- Existence of a $2M_o$ star puts severe constraints on FCM and CSS parameters.

Conclusions

- CSS (Constant Speed of Sound) as a generic language to connect different quark matter EoS.
- FCM QM model can be mapped onto the CSS parametrization, with and without CFL.
- Existence of a $2M_{o}$ star puts severe constraints on FCM and CSS parameters.

- The appearance of the quark-matter core either destabilizes the star or leads to a very short hybrid star branch in the mass-radius relation, the reason being the transition pressure and energy density fairly high both in FCM and NJL models.

- A very small fraction of observed neutron stars expected to be in the hybrid branch, difficult to identify via mass and radius measurements, but they might have distinctive observable properties (cooling, GW emission)….

Thank you !