Constraints on the isospin dependence of the lambda-nucleon force from hypernuclear binding energies





Francesco Pederiva



Physics Department - University of Trento

INFN - TIFPA, Trento Institute for Fundamental Physics and Applications

Collaborators

- Diego Lonardoni (FRIB/MSU-LANL)
- Alessandro Lovato (ANL)





Outline

- Why do we care about details in the YN interaction
- A non-relativistic model of the hyperon(Λ)-nucleon interaction (for "normal" baryonic matter).
- Connection to the existing (and future?) experimental data: computation of BE in hypernuclei by QMC.
- Conclusions

Open questions...

The *fine tuning* of the hyperon-nucleon interaction is essential to understand the behaviour of matter in extreme conditions.



Neutron star structure



Fortin, M., Zdunik, J. L., Haensel, P., & Bejger, M. (2015)

Astronomy & Astrophysics, 576, A68

Internal composition still largely unknown

EOS	$P(n_0)$ (10 ³³ dyn cm ⁻²)	$\rho(n_0)$ (10 ¹⁴ g cm ⁻³)	R ^(CL) (km)	<i>R</i> _{1.4} (km)	Ls (MeV)	R _{Mmax} (km)	$M_{\rm max}$ M_{\odot}
APR	3.05	2.72	15.01	11.34	59	9.93	2.19
BSk20	3.20	2.72	14.95	11.75	37	10.18	2.17
DH	3.60	2.72	15.03	11.73	46	9.99	2.05
BM165	6.45	2.74	15.46	13.59	74	10.68	2.03
DS08	7.58	2.74	15.52	13.91	88	12.02	2.05
GM1Z0	7.45	2.72	15.51	13.95	94	12.05	2.29
M.CQMCC	7.47	2.73	15.61	13.97	91	12.12	2.08
SA.BSR2	5.60	2.70	15.40	13.51	62	11.65	2.03
SA.TM1	9.58	2.82	16.35	14.86	110	12.52	2.10
G.TM1	8.78	2.75	15.91	14.51	110	12.51	2.06
M.TM1C	8.77	2.74	15.94	14.57	111	12.61	2.03
SA.NL3	8.91	2.72	16.14	15.02	118	12.83	2.32
M.NL3B	8.97	2.74	15.98	14.92	118	13.18	2.07
M.GM1C	7.45	2.72	15.61	14.06	94	12.28	2.14
SA.GM1	7.41	2.71	15.64	14.03	94	11.98	2.02
UU1	9.95	2.72	15.78	15.04	117	11.97	2.21
UU2	10.09	2.73	15.79	13.81	117	10.98	2.12

Table A.1. Parameters of the EOS and of NS models based on them.



Fortin, M., Zdunik, J. L., Haensel, P., & Bejger, M. (2015). *Astronomy & Astrophysics*, *576*, A68.

							\square
EOS	$P(n_0)$	$\rho(n_0)$	$R_{1.4}^{(CL)}$	<i>R</i> _{1.4}	Ls	R _{Mmax}	M _{max}
	$(10^{33} \text{ dyn cm}^{-2})$	$(10^{14} \text{ g cm}^{-3})$	(km)	(km)	(MeV)	(km)	M₀
APR	3.05	2.72	15.01	11.34	59	9.93	2.19
BSk20	3.20	2.72	14.95	11.75	37	10.1	2.17
DH	3.60	2.72	15.03	11.73	46	9.9	2.05
BM165	6.45	2.74	15.46	13.59	74	10.63	2.03
DS08	7.58	2.74	15.52	13.91	88	12.02	2.05
GM1Z0	7.45	2.72	15.51	13.95	94	12.(5	2.29
M.CQMCC	7.47	2.73	15.61	13.97	91	12.12	2.08
SA.BSR2	5.60	2.70	15.40	13.51	62	11.(5	2.03
SA.TM1	9.58	2.82	16.35	14.86	110	12.12	2.10
G.TM1	8.78	2.75	15.91	14.51	110	12.1	2.06
M.TM1C	8.77	2.74	15.94	14.57	111	12.61	2.03
SA.NL3	8.91	2.72	16.14	15.02	118	12.83	2.32
M.NL3B	8.97	2.74	15.98	14.92	118	13.13	2.07
M.GM1C	7.45	2.72	15.61	14.06	94	12.23	2.14
SA.GM1	7.41	2.71	15.64	14.03	94	11.9	2.02
UU1	9.95	2.72	15.78	15.04	117	11.91	2.21
UU2	10.09	2.73	15.79	13.81	117	10.98	2.12

Table A.1. Parameters of the EOS and of NS models based on them.

Fortin, M., Zdunik, J. L., Haensel, P., & Bejger, M. Astronomy & Astrophysics, 576, A68 (2015)



								1.6
EOS	$P(n_0)$	$\rho(n_0)$	$R_{1.4}^{(\text{CL})}$	<i>R</i> _{1.4}	Ls	R _{Mmax}	M nax	(Hyperons)
	$(10^{33} \text{ dyn cm}^{-2})$	$(10^{14} \text{ g cm}^{-3})$) (km)	(km)	(Me)) (km)	Λο	12 17 3.5
APR	3.05	2.72	15.01	11.34	59	9.93	2 19	- I.2 H Area Trace
BSk20	3.20	2.72	14.95	11.75	37	10.18	2,17	_ 1.0
DH	3.60	2.72	15.03	11.73	46	9.93	2.05	N (Nuclear) * APR
BM165	6.45	2.74	15.46	13.59	74	120	2.03	
DS08	7.58	2.74	15.52	13.91	88	12.02	2.05	
GM1Z0	7.45	2.72	15.51	13.95	94	12.05	1.29	0.6 BM165
M.CQMCC	7.47	2.73	15.61	13.97	91	12.12	2 08	0 4 2 D\$08
SA.BSR2	5.60	2.70	15.40	13.51	62	11.65	2 03	• GJTM1
SA.TM1	9.58	2.82	16.35	14.86	110	12.52	2,10	0.2 1.5
G.TM1	8.78	2.75	15.91	14.51	110	12.51	2,06	
M.TM1C	8.77	2.74	15.94	14.57	111	12.61	2, 03	2 4 6 8 10 12 14 16 18
SA.NL3	8.91	2.72	16.14	15.02	118	12.83	2.32	$P_c [10^{-2} \text{ erg cm}^{-2}]$
M.NL3B	8.97	2.74	15.98	14.92	118	13.18	2 07	
M.GM1C	7.45	2.72	15.61	14.06	94	12.28	2 14	
SA.GM1	7.41	2.71	15.64	14.03	94	11.98	2 02	
UU1	9.95	2.72	15.78	15.04	117	11.97	1.21	Radii roughly divided in two
UU2	10.09	2.73	15.79	13.81	117	10.98	2.12	riadin reaging arriada in the
Fortin, M. Astronom	, Zdunik, J. L., y & Astrophysi	Haensel, P <i>cs</i> , <i>576</i> , A6	., & Be 8.	jger,	M. (20	015).		groups.

Table A.1. Parameters of the EOS and of NS models based on them.

EOS	(10 ^{3*}	$P(n_0)$ dyn cin	$\rho(n_0)$ (10 ¹⁴ g cm ⁻³)	$R_{1.4}^{(\text{CL})}$ $R_{1.4}$ (km) (km)	Ls (MeV)	R _{Mmax} M (km) N	í _{max} M⊙											
APR		3.05	2.72	15.01 11.34	59	9.93 2	.19											
BSk20		3.20	2.72	14.95 11.75	37	10.18 2	.17	1	11									-
DH		3.60	2.72	15.03 11.73	46	9.99 2	05	1	10						A .U	102		I.
BM165		6.45	2.74	15.46 13.59	74	10.68 2	03											SA
DS08		7.58	2.74	15.52 13.91	88	12.02 2	05	23	9							G.TM		¢
GM1Z0		7.45	2.72	15.51 13.95	94	12.05 2), CII	8				i		DŞ		M.1MIQ	4
M.CQMCC		7.47	2.73	15.61 13.97	91	12.12 2	.08	erg	7						GM1Z0	SA GM	n l	
SA.BSR2		5.60	2.70	15.40 13.51	62	11.65 2	03	32	· [1		BM16	5		T
SA.TM1		9.58	2.82	16.35 14.86	110	12.52 2	.10	11(6						SA BSR	2	-+	+
G.TM1		8.78	2.75	15.91 14.51	110	12.51 2	.06	$\binom{n_0}{n}$	5					7		<u> </u>		4
M.TM1C		8.77	2.74	15.94 14.57	111	12.61 2	2.03	d									+	ł
SA.NL3		8.91	2.72	16.14 15.02	118	12.83 2	.32		*	*	DH							1
M.NL3B		8.97	2.74	15.98 14.92	118	13.18 2	07		3 1	APR	BSR20							+
M.GM1C		7.45	2.72	15.61 14.06	94	12.28 2	.14		11.0	11.5	12.0	12.5	13.0	13	5 1	4.0	14.5 1	5
SA.GM1		7.41	2.71	15.64 14.03	94	11.98 2	02		11.0	11.0	12.0	12.0	R ₁	4 [km]		1.0		
UU1		9.95	2.72	15.78 15.04	117	11.97 2												
UU2		10.09	2.73	15.79 13.81	117	10.98 2	.12											

Table A.1. Parameters of the EOS and of NS models based on them.

Fortin, M., Zdunik, J. L., Haensel, P., & Bejger, M. (2015). Astronomy & Astrophysics, 576, A68.

Many possible description of the YN interaction

NON RELATIVISTIC:

write an Hamiltonian including some potential and try to solve a many-body Schroedinger equation.

- The potential energy is **not an observable**: several different equivalent descriptions are possible.
- The interaction can be based on some more or less phenomenological scheme (fit the existing experimental data, rely on some systematic meson exchange model), or can be inferred from EFT systematic expansions.
- Only **accurate many-body calculations** can help distinguishing among different realisations of the potential.

RELATIVISTIC:

write a Lagrangian including relevant fields, and try to solve the field theoretical problem (usually RMF calculations are performed).

Some hints from LQCD.....



Fig. 10. Left: The central potential in the ${}^{1}S_{0}$ channel of the ΛN system in 2 + 1 flavor QCD as a function of r. Right: The central potential in the ${}^{1}S_{0}$ channel of the $\Sigma N(I = 3/2)$ system as a function of r.





Fig. 11. Left: The central potential (circle) and the tensor potential (triangle) in the ${}^{3}S_{1} - {}^{3}D_{1}$ channel of the ΛN system as a function of r. Right: The central potential (circle) and the tensor potential (triangle) in the ${}^{3}S_{1} - {}^{3}D_{1}$ channel of the $\Sigma N(I = 3/2)$ system as a function of r.

Model Hyperon-nucleon interaction

er Set of the set up some understanding, we need to set up some scheme.

 $_{j}) \mathcal{O}_{ij}^{p}$



Fig. 1. Total cross section for Ap scattering as a function of c.m. kinetic energy E(MeV). The

Model Hyperon-nucleon interaction

Model interaction (Bodmer, Usmani, Carlson):

A. Bodmer, Q. N. Usmani, and J. Carlson, Phys. Rev. C 29, 684 (1984).

from Kaon exchange terms (not considered explicitly in our calculations)



$$V_{\Lambda i}(r) = v_0(r) + v_0(r)\varepsilon(P_x - 1) + \frac{1}{4}v_\sigma T_\pi^2(m_\pi r)\boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_i$$

Two-body potential: accurately fitted on p- Λ scattering data

Q. N. Usmani and A. R. Bodmer, Phys. Rev. C 60, 055215 (1999).



$$V_{\Lambda ij} = V_{\Lambda ij}^{2\pi} + V_{\Lambda ij}^D$$

Parameters to be determined from calculations

Non trivial isospin dependence in the three-body sector?

In hypernuclei it is possible that the Λ NN interaction is not well constrained, especially in the isospin triplet channel:



On can try o do the exercise of re-projecting the interaction in the isospin singlet and triplet channels and try to explore the dependence of the hypernuclei binding energy on the relative strength.



Charge symmetry breaking

Obviously one also has to consider CSB interactions

$$\sum_{\lambda_i} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{i=1}^$$

This adds a further parameter in the interaction:

$$v_{\lambda i}^{CSB} = C_{\tau} T_{\pi}^2 (r_{\lambda i}) \tau_i^z$$

A. R. Bodmer, Q. N. Usmani, Phys.Rev.C 31, 1400 (1985)

Notice that no spin dependence has been added here. CSB interaction can be clearly seen in light nuclei. What about heavier hypernuclei?

$$v_{\lambda ij} = v_{\lambda ij}^{2\pi,P} + v_{\lambda ij}^{2\pi,S} + v_{\lambda ij}^{D}$$

$$\left(v_{\lambda}^{2\pi,P} = -\frac{C_P}{C_P} \left\{ X_i \lambda X_{\lambda i} \right\} \tau_i \cdot \tau_i$$

Input from experiment

We need to fit the three body interaction against some experimental data. There are available several measurements of the binding energy of Λ -hypernuclei, i.e. nuclei containing a Λ hyperon. The idea is to compute such binding energies. We can then compute the hyperon separation energy:



where B_{hyp} is the total binding energy of a hypernucleus with A nucleons and one Λ , and B_{nuc} is the tot du inding up ergy of the g_{40} MeV corresponding nucleus with A nucleons. This number can be used to gauge the coefficients in the nucleon- Λ interaction. $\frac{-1}{\Sigma^{-}} \frac{dd}{s} \frac{-1/2}{100} \frac{ud}{s} \frac{\Lambda}{\Sigma^{0}} \frac{1/2}{s} \frac{uu}{s} \frac{\Lambda}{\Sigma^{+}} \frac{1}{L} \frac{116}{1200} \frac{MeV}{MeV}$

-1

Hypernuclei data





J. Pochodzalla, Acta Phys. Polon. B 42, 833-842 (2011)



- The available data are very limited.
- There are several planned and ongoing systematic measurements.
- At present no proposals for gathering more Λ -nucleon scattering data
- Essentially no information on $\Lambda\Lambda$ interaction
- (Almost) nothing on Σ or Ξ hypernulcei



Many-Body theory: projection Monte Carlo

We compute ground state energies of nuclei by means of projection Monte Carlo methods. The ground state of a many-body system is computed by applying an "imaginary time propagator" to an arbitrary state that has to be non-orthogonal to the ground state (power method):

$$\langle R|\Psi(\tau)
angle = \langle R|e^{-(\hat{H}-E_0)\tau}|R'
angle\langle R'|\Psi(0)
angle$$

In the limit of "short" τ (let us call it " $\Delta \tau$ "), the propagator can be broken up as follows (Trotter-Suzuki formula): $W(R, R', \Delta \tau)$

$$\langle R | e^{-(\hat{H} - E_0)\Delta\tau} | R' \rangle \sim e^{-\frac{(R - R')^2}{2\frac{\hbar}{m}\Delta\tau}} e^{-\left(\frac{V(R) + V(R')}{2} - E_0\right)\Delta\tau}$$
Kinetic term Potential term ("weight")
Sample a new point from the
Gaussian kernel
$$|R_2\rangle \quad |R_1\rangle$$
Create a number of copies
proportional to the weight
$$M = (int)[W(R, R', \Delta\tau) + \frac{1}{16} and()] | R_4\rangle$$

Many-nucleon systems

PROBLEM

for realistic many-nucleon Hamiltonians, propagators must be evaluated on wave functions that have a number of components exponentially growing with A (spin/isospin singlet/triplet state for each pair of nucleons)

Very accurate results have been obtained in the years for the ground state and some excitation properties of nuclei with A≤12 by the Argonne based group (GFMC calculations by Pieper, Wiringa, Carlson, Schiavilla...). These calculations include twoand three-nucleon interactions.



Courtesy of R. Wiringa, ANL

Auxiliary Field Diffusion Monte Carlo (AFDMC)

Stefano Fantoni & Kevin Schmidt, 1999

The computational cost of GFMC can be reduced by introducing a way of sampling over the space of states, rather than summing explicitly over the full set.

For simplicity let us consider only one of the terms in the interaction. We start by observing that:

$$\sum_{i < j} v(r_{ij}) \vec{\sigma}_i \cdot \vec{\sigma}_j = \frac{1}{2} \sum_{i;\alpha,j;\beta} \sigma_{i;\alpha} A_{i;\alpha,j;\beta} \sigma_{j;\beta} = \sum_{n=1}^{3A} \lambda_n \hat{O}_n^2$$

Then, we can linearize the operatorial dependence in the propagator by means of an integral transform: auxiliary fields→Auxiliary Field Diffusion Monte Carlo

$$e^{-\frac{1}{2}\lambda\hat{O}_n^2\Delta\tau} = \frac{1}{\sqrt{2\tau}}$$

$$dxe^{-\frac{x^2}{2}}e^{-x\sqrt{\lambda\Delta\tau}}\hat{O}_n$$

K. E. Schmidt and S. Fantoni, Phys. Lett. B 446, 99 (1999). S. Gandolfi, F. Pederiva, S. Fantoni, and K. E. Schmidt, Phys. Rev. Lett. 99, 022507 (2007)

Hubbard-Stratonovich transformation

Auxiliary Field Diffusion Monte Carlo (AFDMC)

The operator dependence in the exponent has become linear.

In the Monte Carlo spirit, the integral can be performed by sampling values of x from the Gaussian $e^{-\frac{x^2}{2}}$ For a given x the action of the propagator will become:

$$e^{-x\sqrt{\lambda\Delta\tau}\hat{O}_n}|S\rangle = \prod_{k=1}^{3A} e^{-x\sqrt{\lambda\Delta\tau}\phi_n^k\sigma_k}|S\rangle$$

In a space of spinors, each factor corresponds to a rotation induced by the action of the Pauli matrices



Input from experiment





Assumption: use of a simplified NN interaction cancel in the difference and therefore the estimate of BA is accurate (verified!)



Only two parameters are relevant (one of them is essentially ineffective)

The *nucleon-nucleon* interaction that we use in our hypernuclear calculations is not the full realistic one, but the simpler AV4'+ the central (repulsive) term of the Urbana IX potential (UIX_c). Despite this simplification, the description of closed shell nuclei is not so bad. Here we report some results.

nucleus	AV4'	AV6′	AV7'	$AV4'+UIX_c$	\exp
${}^{4}\text{He}(0^{+})$	-32.83(5)	-27.09(3)	-25.7(2)	-26.63(2)	-28.295
$^{15}O\left(\frac{1}{2}^{-}\right)$	_	_	_	-99.43(2)	-111.955
$^{16}O(0^+)$	-180.1(4)	-115.6(3)	-90.6(4)	-119.9(2)	-127.619
39 K $\left(\frac{3}{2}^{+}\right)$	_	_	_	-360.8(2)	-333.724
40 Ca (0 ⁺)	-597(3)	-322(2)	-209(1)	-383.3(3)	-342.051
^{44}Ca (0 ⁺)	_	_		-397.8(5)	-380.960
${}^{47}{ m K}\left(\frac{1}{2}^{+}\right)$	_	_	preliminary —	-386.3(2)	-400.199
48 Ca (0 ⁺)	-645(3)	_	_	-413.2(3)	-416.001



Diego Lonardoni, Alessandro Lovato, FP, Stefano Gandolfi



D. L., F. Pederiva, S. Gandolfi, Phys. Rev. C 89, 014314 (2014) F. Pederiva, F. Catalano, D. L., A. Lovato, S. Gandolfi, arXiv:1506.04042 (2015)













preliminary

Charge symmetry preaking (improved¹⁺²)



Can we really constrain ANN interaction from hyper nuclear data?

In hypernuclei it is possible that the Λ NN interaction is not well constrained, especially in the isospin triplet channel:



We are doing the exercise of re-projecting the interaction in the isospin singlet and triplet channels and try to explore the dependence of the hypernuclei binding energy on the relative strength.



Can we really constrain the interaction from hyper nuclear data?



New experiments needed



 $^{A}Z(K^{-},\pi^{-})^{A}_{\Lambda}Z$

 $^{A}Z(\pi^{+},K^{+})^{A}_{\Lambda}Z$

 $^{A}Z(e, e'K^{+})^{A}_{\Lambda}[Z-1]$

Proposal presented at JLAB:

"A study of the Λ -N interaction through the high precision spectroscopy of Λ -hypernuclei with electron beam" (spokepersons: S. Nakamura, F. Garibaldi, P.E.C. Markowitz, J. Reinhold, L. Tang, G.M. Urciuoli)

Including measurements of ${}^{48}_{\Lambda}$ K and ${}^{40}_{\Lambda}$ K, but hopefully also light hypernuclei and hyper-Pb (EoS...)

A-neutron matter



Neutron star structure



Within this model the repulsion needed to correctly describe hypernuclear binding energy is so strong that hyperons would not be present in 2M_☉ stars!



Neutron star structure



Diego Lonardoni, A. Lovato, S. Gandolfi, FP, arXiv:1407.4448 [nucl-th], submitted to Phys. Rev. Lett.

Conclusions

- Our philosophy in attacking the problem of the hyperon-nucleon interaction: we do not want to add more information than the one that experiments can give us. Having too many parameters will result in a substantially arbitrary prediction of the EoS, and consequently adjustable predictions on the Neutron Star structures.
- AFDMC calculations are evolving. Better accuracy, better performance. This reflects on the work on hypernuclei Accessible systems: definitely A=90. For heavier systems one can possibly use alternative approaches.
- At this point there is real need of accurate experiments on hypernuclei in order to be able to gain more insight on NS interior at densities > $2\rho_0$.