Chiral Transport Phenomena

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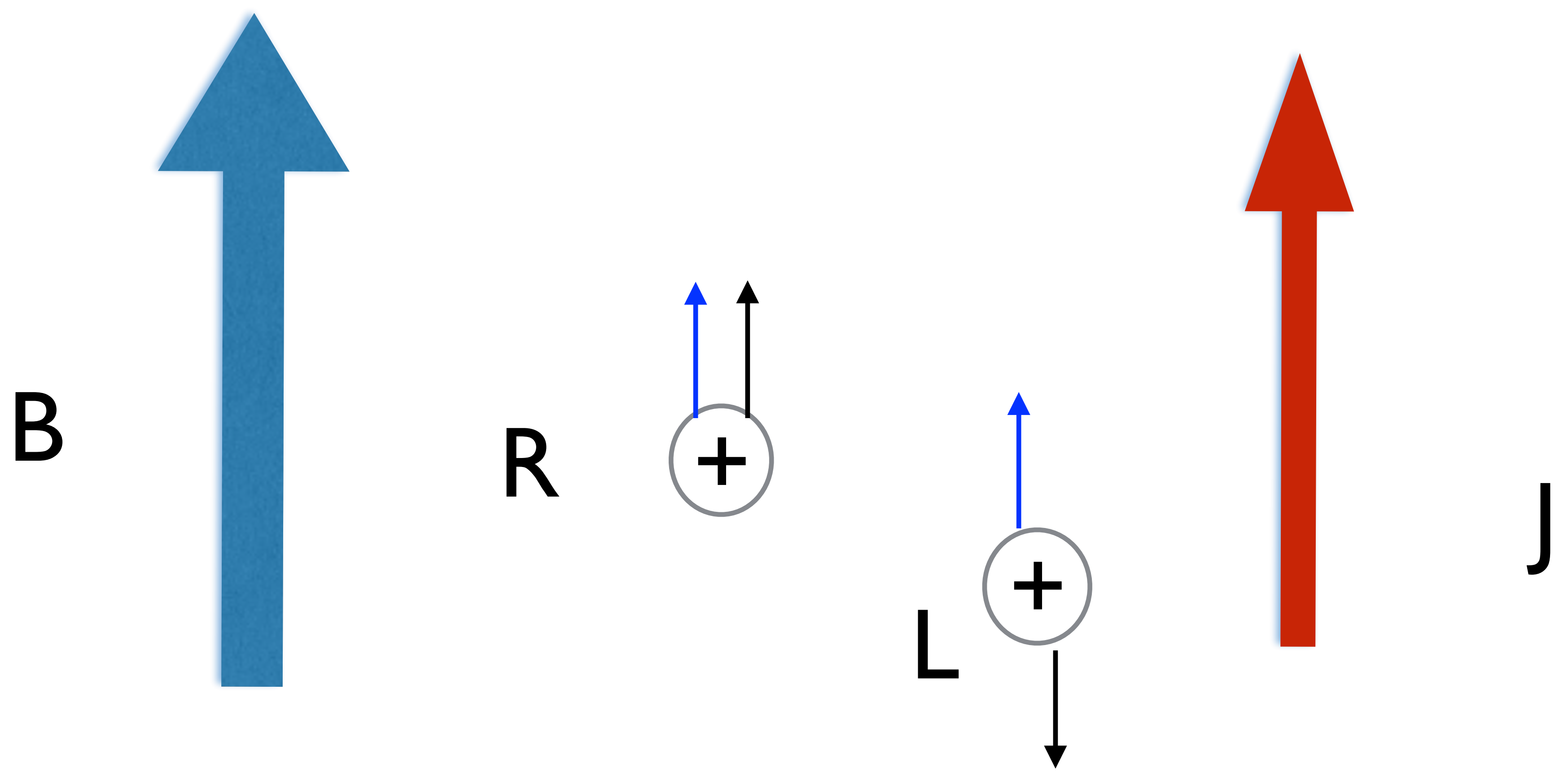
Outline

- Chiral Magnetic Effect
- Chiral Transport Equation
- Anomalous Maxwell Equations
- Applications to astrophysics

Chiral Magnetic Effect



$$\mathbf{J} = \frac{e^2 \mu_5}{4\pi^2} \mathbf{B}$$

In a B a misbalance in the population of L/R handed fermions leads to an e.m. current I I to B

Similarly, in the presence of fluid vorticity

$$\mathbf{J} = \frac{\mu \mu_5}{2\pi^2} \omega \qquad \omega = \frac{1}{2} (\nabla \times \mathbf{v})$$

Chiral Magnetic Effect

- First seen in EW (Vilenkin, 80)
- Discussed in the framework of HIC Kharzeev, McLerran, Fukushima, Warringa, '08
- Discussed in AdS/CFT Yee, Landsteiner et a,l, etc
- Studied in the lattice Buividovich et al; M Abramczyk et al
- Derived in hydrodynamics Son and Surowka
- Derived in kinetic theory (Son and Yamamoto, Stephanov and Yin, CM and Torres-Rincon)
- Observed in Dirac semimetals Kharzeev et al, '15

All these ideas here discussed are relevant for condensed matter physics

Discovery of Weyl semimetals and Weyl fermions as quasiparticles

Hasan et al, '15; Weng et al,

Chiral Transport Equation

Son and Yamamoto, '12; Stephanov and Yin, '12,
CM and J. Torreses-Rincón, '14)

In a collisionless case

$$\frac{\partial f_p}{\partial t} + (1 + e\hbar \mathbf{B}\cdot\mathbf{\Omega})^{-1}\left\{\left[\tilde{\mathbf{v}} + e\hbar\,\tilde{\mathbf{E}}\times\mathbf{\Omega} + e\hbar\,\mathbf{B}(\tilde{\mathbf{v}}\cdot\mathbf{\Omega})\right]\cdot\frac{\partial f_p}{\partial \mathbf{r}} + e\left[\tilde{\mathbf{E}} + \tilde{\mathbf{v}}\times\mathbf{B} + e\hbar\mathbf{\Omega}\,(\tilde{\mathbf{E}}\cdot\mathbf{B})\right]\cdot\frac{\partial f_p}{\partial \mathbf{p}}\right\} = 0$$

where

$$\tilde{\mathbf{E}} = \mathbf{E} - \frac{1}{e}\frac{\partial \epsilon^+_{\mathbf{p}}}{\partial \mathbf{r}}$$

$$\tilde{\mathbf{v}} = \frac{\partial \epsilon^+_{\mathbf{p}}}{\partial \mathbf{p}}$$

$$\mathbf{\Omega} = \frac{\mathbf{p}}{2p^3}$$

One can reproduce the chiral anomaly equation

$$\frac{\partial n}{\partial t} + \nabla \cdot \mathbf{j} = -e^2 \hbar \int \frac{d^3 p}{(2\pi\hbar)^3} \left(\mathbf{\Omega} \cdot \frac{\partial f_p}{\partial \mathbf{p}} \right) \mathbf{E} \cdot \mathbf{B}$$

In a thermal plasma: take into account both particles/antiparticles to correctly reproduce the chiral anomaly

$$f_p^{R,L} = \frac{1}{\exp\left[\frac{1}{T}\left(p - \mu_{R,L}\right)\right] + 1}$$

$$\bar{f}_p^{L,R} = \frac{1}{\exp\left[\frac{1}{T}\left(p + \mu_{R,L}\right)\right] + 1}$$

$$\partial_\mu j_A^\mu = \frac{e^2}{2\pi^2 \hbar^2} \mathbf{E} \cdot \mathbf{B} \qquad \partial_\mu j_V^\mu = 0$$

The chiral transport equation can be deduced simply by computing (for m=0) the first quantum corrections to the classical eqs. of motion

FW diagonalization

EFT methods

Semiclassical chiral transport equation

Foldy-Wouthuysen Diagonalization

- The Dirac eq. for a free fermion mixes particles and antiparticles d.o.f.
- FW found a representation where these can be separated, through a canonical transformation

exact for the free theory

approx. for an interacting theory

$$H\psi = i\frac{\partial\psi}{\partial t} \qquad H' = UHU^\dagger \qquad \psi' = U\psi$$

$$\mathrm{H}_0 = \alpha\cdot(\mathbf{P} - e\mathbf{A}(\mathbf{R})) + \beta m + eA_0(\mathbf{R})$$

$$\alpha_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},$$

At order $\mathcal{O}(\hbar^0)$

$$U = \frac{E + m + \beta\alpha\cdot(\mathbf{P} - e\mathbf{A}(\mathbf{R}))}{\sqrt{2E(E+m)}}$$

$$H_D = UH_0U^\dagger = \beta E + eA_0(\mathbf{R})$$

$$E \equiv \sqrt{(\mathbf{P} - e\mathbf{A}(\mathbf{R}))^2 + m^2}.$$

At order $\mathcal{O}(\hbar)$ $[R_i, P_j] = i\hbar\delta_{ij}$

Gosselin, Berard and Mohrbach 2007

Give a prescription to deal with products of R, P

Keep unitarity; project over the diagonal

Rotate all operators

$$\begin{aligned} \mathbf{r} &= \mathcal{P}[U(\mathbf{P},\mathbf{R})\,\mathbf{R}\;U^\dagger(\mathbf{P},\mathbf{R})] = \mathbf{R} + \mathcal{P}(\mathcal{A}_R)\ , \\ \mathbf{p} &= \mathcal{P}[U(\mathbf{P},\mathbf{R})\,\mathbf{P}\;U^\dagger(\mathbf{P},\mathbf{R})] = \mathbf{P} + \mathcal{P}(\mathcal{A}_P) \end{aligned}$$

$$\mathcal{P}(\mathcal{A}_{R_i}) = -\hbar\frac{E[\mathbf{\Sigma}\times(\mathbf{P}-e\mathbf{A})]_i}{2E^2(E+m)}\ , \qquad \mathcal{A}_{P^i} = e\,\nabla_{R^i}A_k(\mathbf{R})\mathcal{A}_{R^k}$$

$$\Sigma_k = \begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix}$$

In terms of the rotated variables

$$\boxed{H_D = \beta\left(E - \frac{e\hbar\boldsymbol{\Sigma}\cdot\mathbf{B}}{2E} - \frac{e\mathbf{L}\cdot\mathbf{B}}{E}\right) + eA_0(\mathbf{r})}$$

$$E = \sqrt{(\mathbf{p} - e\mathbf{A}(\mathbf{r}))^2 + m^2}$$

$$\mathbf{L} = \tilde{\mathbf{p}} \times \mathcal{P}(\mathcal{A}_{\mathbf{R}}) = \hbar\,\frac{\tilde{\mathbf{p}} \times (\tilde{\mathbf{p}} \times \boldsymbol{\Sigma})}{2E(E+m)} \qquad \tilde{\mathbf{p}} \equiv \mathbf{p} - e\mathbf{A}(\mathbf{r})$$

Gauge invariance kept at order of accuracy

The new variables are non canonical

$$\boxed{\begin{aligned}
[r_i, r_j] &= i\hbar^2 G_{ij} = -i\hbar^2 \epsilon_{ijk} G_k \\
[\tilde{p}_i, \tilde{p}_j] &= ie\hbar F_{ij} + ie^2\hbar^2 F_{ik} F_{jm} G_{km}\ , \\
[r_i, \tilde{p}_j] &= i\hbar\delta_{ij} + ie\hbar^2 F_{jk} G_{ik}
\end{aligned}}$$

$$\mathbf{G}(\tilde{\mathbf{p}}) = \frac{1}{2E^3}\left(m\boldsymbol{\Sigma} + \frac{(\boldsymbol{\Sigma}\cdot\tilde{\mathbf{p}})\tilde{\mathbf{p}}}{E+m}\right)$$

Massless fermions

$$\tilde{\mathbf{p}} \to \mathbf{p}$$

$$\mathbf{G} = \lambda \boldsymbol{\Omega} \,, \qquad \boldsymbol{\Omega} = \frac{\mathbf{p}}{2p^3} \qquad \lambda = \frac{\Sigma \cdot \mathbf{p}}{p}$$

Fermion dispersion law in an B field is modified

$$\boxed{\epsilon^{\pm}_{\mathbf{p}} = \pm p \left(1 - e\hbar\, \lambda \frac{\mathbf{B} \cdot \mathbf{p}}{2p^3} \right)}$$

Semiclassical equations of motion (e.g. right-handed)

$$\boxed{\begin{aligned} \dot{\mathbf{p}} &= -\frac{\partial \epsilon^{+}_{\mathbf{p}}}{\partial \mathbf{r}} + e(\mathbf{E} + \dot{\mathbf{r}} \times \mathbf{B}) \,, \\ \dot{\mathbf{r}} &= \frac{\partial \epsilon^{+}_{\mathbf{p}}}{\partial \mathbf{p}} - \hbar(\dot{\mathbf{p}} \times \boldsymbol{\Omega}) \end{aligned}}$$

EFT approach to the FW diagonalization - OSEFT

$\hbar = 1$

Separating fermion/antifermion d.o.f. within QFT (HQET, NRQED, LEET, HDET, ...)

Describing physics for an almost on-shell m=0 fermion

$$q^\mu = p v^\mu + k^\mu \qquad q^\mu = -p\tilde{v}^\mu + k^\mu$$

$$v^\mu = (1, \mathbf{v}) \qquad \tilde{v}^\mu = (1, -\mathbf{v}) \qquad v^2 = 0$$

$$\begin{aligned} \psi_v(x) &= e^{-ipv\cdot x}\left(P_{+v}\chi_v(x) + P_{-v}H_v^1(x)\right) \\ &\quad + e^{ip\tilde{v}\cdot x}\left(P_{-v}\xi(x) + P_{+v}H^2(x)\right) \end{aligned}$$

$P_{\pm v}$ particle/antiparticle projectors

Integrate out the off-shell modes H

$$\mathcal{L}_f = \sum_{\mathbf{v}} \left(\chi^\dagger_{+v}(x) i v \cdot D \chi_{+v}(x) - \chi^\dagger_{+v}(x) \frac{(\not{D}_\perp)^2}{2p} \chi_{+v}(x) \right)$$
$$+ \sum_{\mathbf{v}} \left(\xi^\dagger_{-v}(x) i \tilde{v} \cdot D \xi_{-v}(x) + \xi^\dagger_{-v}(x) \frac{(\not{D}_\perp)^2}{2p} \xi_{-v}(x) \right)$$

This produces the same FW Hamiltonian we obtained before for fermions!

Some advantages:

NLO corrections easier to obtain

It can be used for corrections to HTL diagrams

Transport theory provides a perfect framework to study the dynamical evolution of the system, where different anomalous effects can be taken into account

Including collisions, in the RTA

$$C^{\rm RTA}[f_p] = -\frac{1}{\tau}(f_p - f_p^{eq})$$

Solve the dynamics for time scales larger than the relaxation time

$$\mathbf{J} = \sigma \mathbf{E} + \sum_{s=1}^{N_s} \frac{e_s^2\, \mu_5}{4\pi^2}\, \mathbf{B}$$

Anomalous Maxwell Equations

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\sigma}\nabla^2 \mathbf{B} + \frac{C\alpha\mu_5}{\pi\sigma}\nabla \times \mathbf{B} - \frac{1}{\sigma}\frac{\partial^2 \mathbf{B}}{\partial t^2}$$

$$\frac{d(n_R - n_L)}{dt} = \frac{2C\alpha}{\pi}\frac{1}{V}\int d^3x\, \mathbf{E}\cdot\mathbf{B} = -\frac{C\alpha}{\pi}\frac{d\mathcal{H}}{dt}$$

$$C \equiv \sum_{s=1}^{N_s} \frac{e_s^2}{e^2}$$

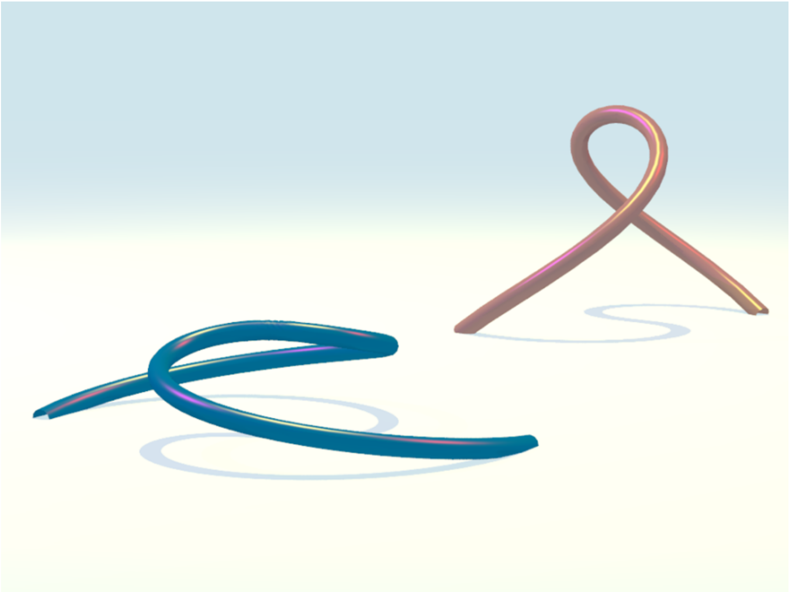
Linked dynamical evolution of magnetic fields and chiral fermion imbalance

Magnetic Helicity Density

(or Chern-Simons number)

$$\mathcal{H}(t) = \frac{1}{V}\int_V d^3x\, \mathbf{A}\cdot\mathbf{B}$$

gives a measure of a non-trivial topology of the B lines



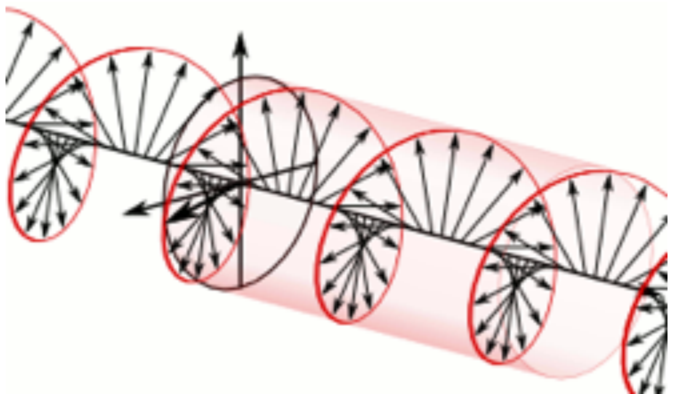
gauge invariant if B=0 on ∂V (or B n=0)

In Fourier modes, using vector polarization vectors describing circular polarized waves $(\mathbf{e}_+, \mathbf{e}_-, \hat{\mathbf{k}})$

$$\mathbf{B}_\mathbf{k} = B^+_\mathbf{k} \mathbf{e}_+ + B^-_\mathbf{k} \mathbf{e}_-$$

$$\mathcal{H} = \frac{1}{V} \int \frac{d\Omega_\mathbf{k}}{4\pi} \int dk \frac{k}{2\pi^2} \left(|B^+_\mathbf{k}|^2 - |B^-_\mathbf{k}|^2 \right)$$

it gives account of an asymmetry of L(+)/R(-) polarized fields



(Integrated) Anomaly Equation

Expresses a conservation law of total helicity

$$\frac{d}{dt}\left(n_5 + \frac{C\alpha}{\pi}\mathcal{H}\right) = 0$$

chiral fermion imbalance can be converted into magnetic helicity and vice versa

$$\frac{1}{\sigma}\frac{\partial^2 B^{\pm}_{\mathbf{k}}}{\partial t^2} + \frac{\partial B^{\pm}_{\mathbf{k}}}{\partial t} = -\left(\frac{1}{\sigma}k^2 \mp \frac{C\alpha\mu_5 k}{\pi\sigma}\right) B^{\pm}_{\mathbf{k}} \,,$$

$$\frac{dn_5}{dt} = -\frac{C\alpha}{V}\int\frac{d\Omega_{\mathbf{k}}}{4\pi}\int dk\,\frac{k}{2\pi^2}\frac{d}{dt}\left(|B^{+}_{\mathbf{k}}|^2 - |B^{-}_{\mathbf{k}}|^2\right) - \Gamma_f n_5 \,.$$

helicity flipping rate

Note: L/R handed polarized fields evolve differently with fermion chiral imbalance!

(circular dichroism)

We will solve the dynamics for both B and n5 assuming $t \ll 1/\Gamma_f$

Assume an initial monochromatic helicity

$$|B^+_{\mathbf{k},0}|^2 = |B^+_0|^2 \delta(k - k_0)$$

$$\mathcal{H}(t) = h_0 \exp\left[\frac{2k_0}{\sigma}\left(K_p - k_0\right) t\right]$$

Chiral magnetic instability $\quad k < \dfrac{C\alpha\mu_5}{\pi}$

B grows till reaching the tracking solution

$$\mu_{5,\mathrm{tr}} = \frac{\pi k_0}{C\alpha} \qquad t_{\mathrm{inst},h} \approx \frac{\pi\sigma}{2k_0 C\alpha(\mu_{5,0} - \mu_{5,tr})}$$

Several modes: inverse cascade phenomenon H transferred from the highest to lowest modes

used in cosmological scenarios

Chiral magnetic instability

possible mechanism to generate large B fields with magnetic helicity in

- cosmological scenarios — Shaposhnikov et al, ‘97
- neutron stars — Ohnishi Yamamoto et al, ‘14
- heavy ion collisions — CM and Torres-Rincon, ’15; Hirono, Kharzeev, Yin, ‘15

Proto-neutron stars

In the process of formation of a neutron star there is chiral imbalance: neutronization

$$p + e_L \to n + \nu_e^L$$

misbalance among R and L electrons might produce helical fields??

No, as electrons change helicity by scattering off with protons

Grabowska, Reddy and Kaplan, '14

Chirality of neutrinos?

Conclusions

- Chiral transport equation includes quantum corrections that allow us to study anomalous effects, such as the CME
- Anomalous Maxwell's equations: magnetic helicity and chiral fermion imbalance linked
- Applications of these ideas in different systems - could they explain the magnetic helicity of neutron stars?