PULSAR DIFFERENTIAL ROTATION WITH SUPERFLUID ENTRAINMENT

ArXiv:

M. Antonelli, P. Pizzochero
“Axially symmetric equations for differential pulsar rotation with superfluid entrainment”

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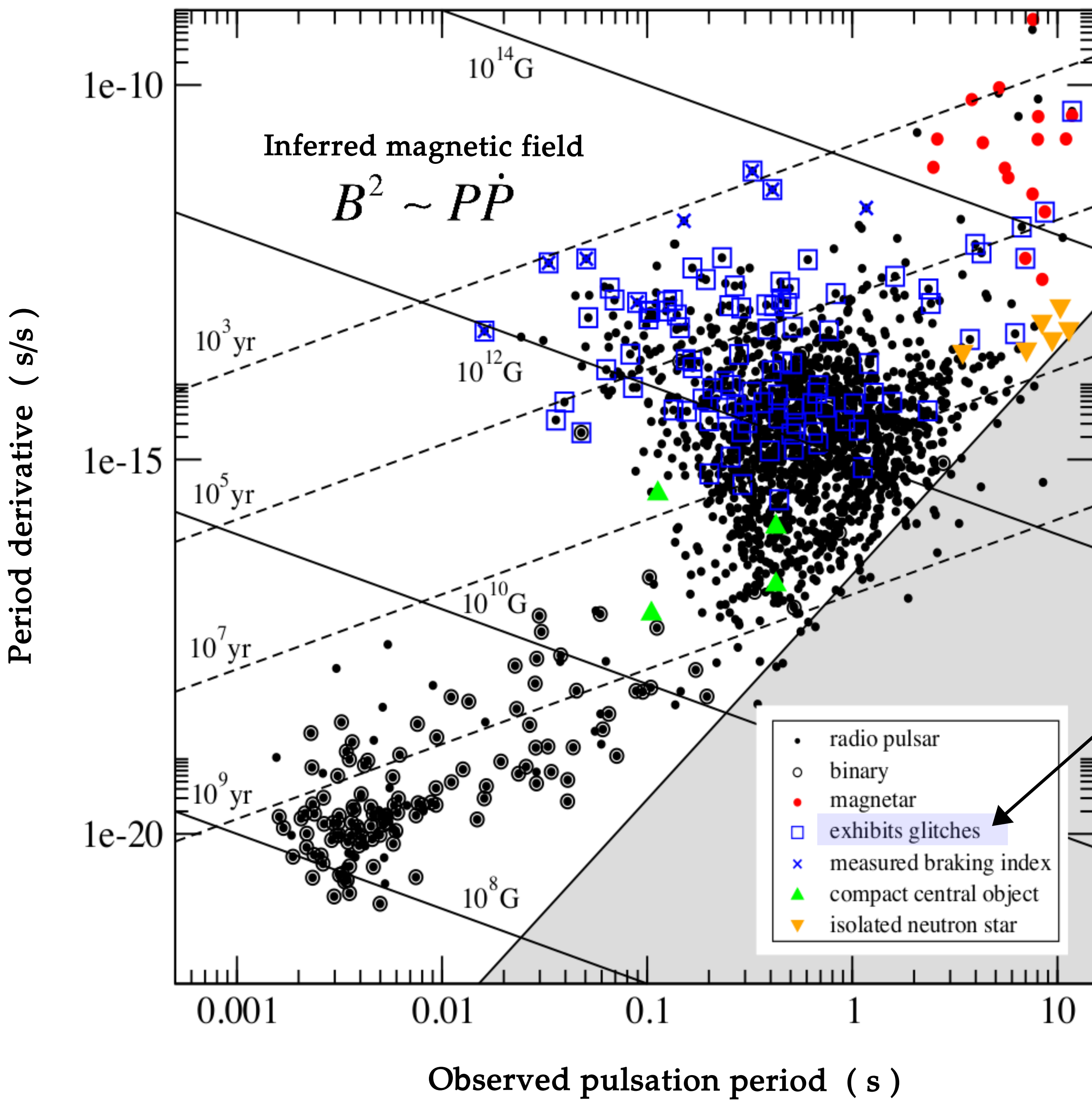
Pulsar timing



- Period and spin-down rate (period derivative) are precisely determined.

- Different classes populate different regions (inferred age and magnetic field).

- Stable clocks with predictable spin-down... except for random timing irregularities

→ it's interesting to study the rotational dynamics:

- evidence of superfluidity

- alternative to cooling

PULSAR GLITCHES (basic facts)

- **Lack of radiative/pulse profile changes**

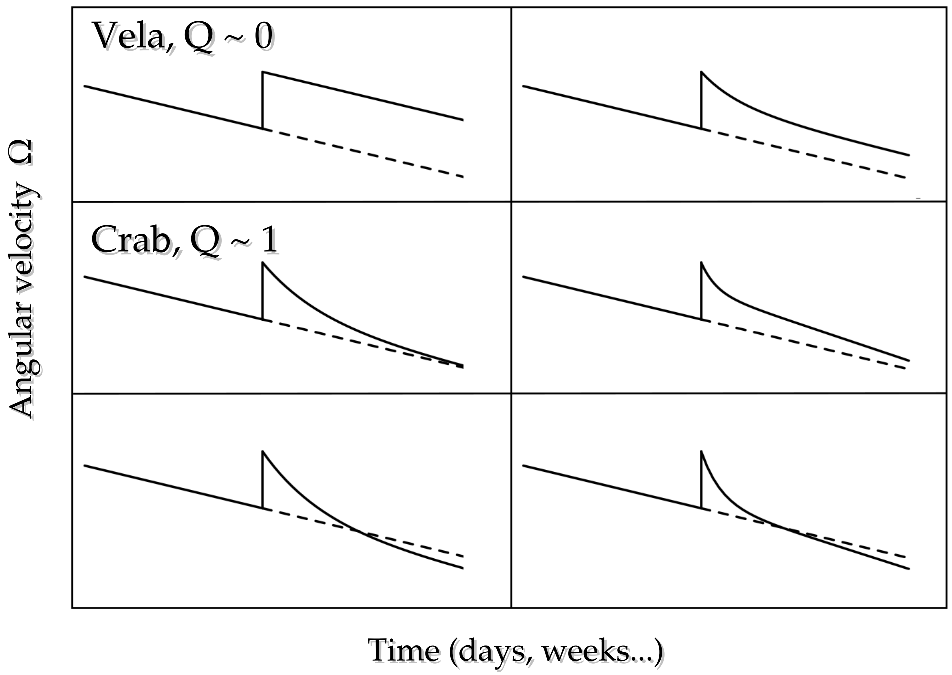
  → Evidence for internal origin

- **Long recoveries**

  → Thought to be due to superfluid component in the star

- **Diverse phenomenology**

  → different ages/mass/rotational parameters...



Key point: to describe glitches we need that a NS is comprised of (at least) two components that exchange angular momentum.

Can we identify the (two?) components ?

Which part of the NS provides the angular momentum to spin-up the "observable component" ?

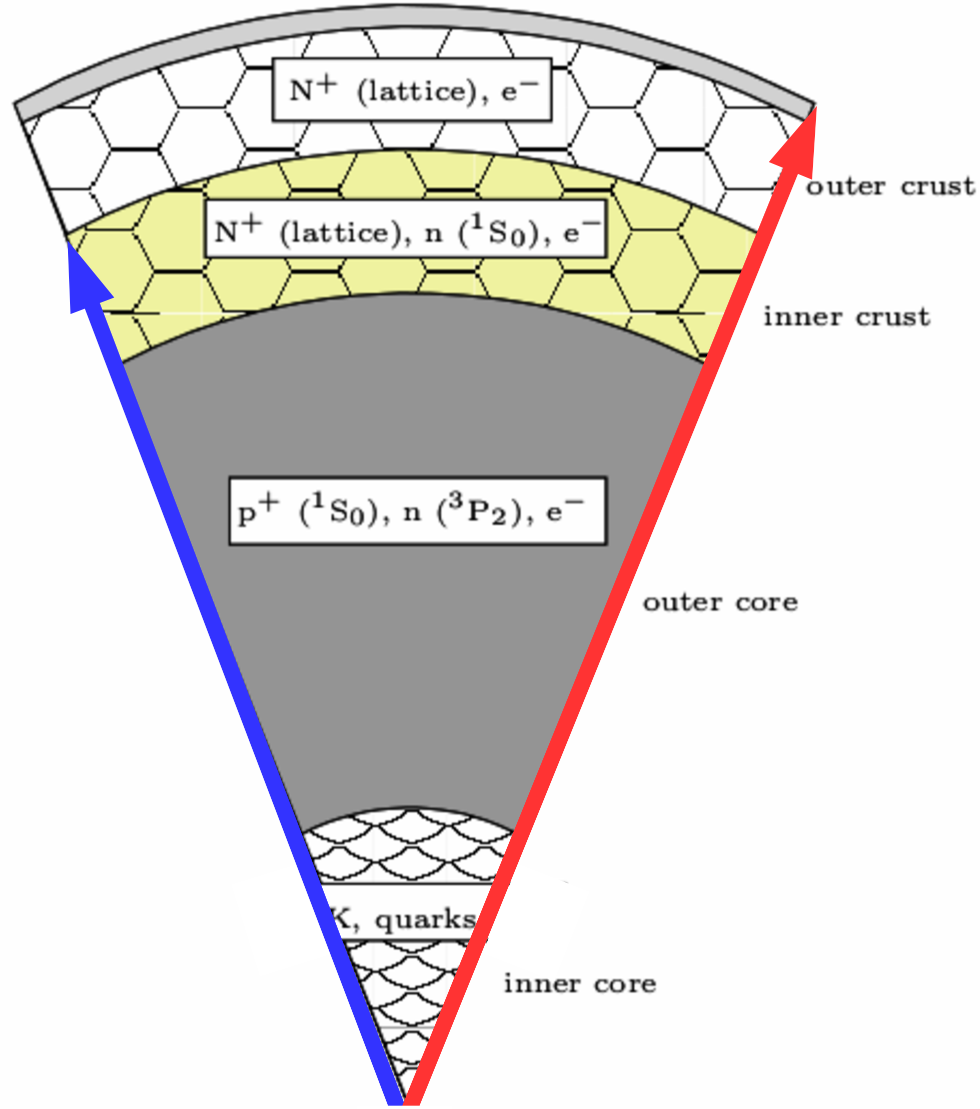
The “minimal” model

The inner crust & core contain a neutron superfluid (superfluid n-component). Everything else (proton superconductor, electron gas) is locked with the solid crust into the magnetic field (rigid p-component).

“Starquake model” of Baym, Pethick, Pines, Ruderman (1969):
Two rigid components differentially rotating

“crust” $$I_c \dot{\Omega}_c = \underbrace{-\alpha}_{\text{braking torque}} - \frac{I_c}{\tau_c}(\Omega_c - \Omega_n)$$

“neutrons” $$I_n \dot{\Omega}_n = \underbrace{\frac{I_c}{\tau_c}(\Omega_c - \Omega_n)}_{\text{Mutual friction}}$$



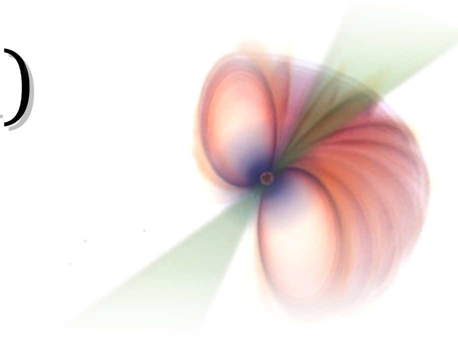
Spin-up is given by the settling of the crust under gravitational stresses

Phenomenological coupling timescale → postglitch relaxation

“Linear” mutual friction → exponential relaxation

IMPOSSIBLE TO EXPLAIN LARGE VELA GLITCHES!

Glitch mechanism (vortex-mediated)

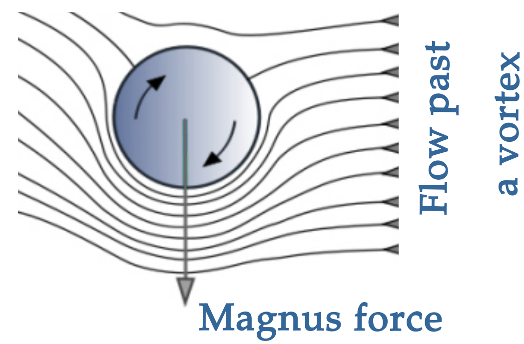


- The "p" follows the observed spin down of the pulsar

- Where vortex lines are pinned, the n-component cannot follow p...

  ...a velocity lag builds up between "n" and "p"

  ...motion of a neutron vortex is affected by fluid flow past it



- Hydrodynamical effect: when the Magnus force ≃ pinning force the vortex line unpins and is expelled → n looses angular momentum, p gains the same amount (dissipative MF)

$$\boldsymbol{f}_D = -\eta(\boldsymbol{x})(\boldsymbol{v}_L(\boldsymbol{x}) - \boldsymbol{v}_p(\boldsymbol{x}))$$ Drag force

$$\boldsymbol{f}_M = \rho_n(\boldsymbol{x})\boldsymbol{\kappa} \times (\boldsymbol{v}_L(\boldsymbol{x}) - \boldsymbol{v}_n(\boldsymbol{x}))$$ Magnus force

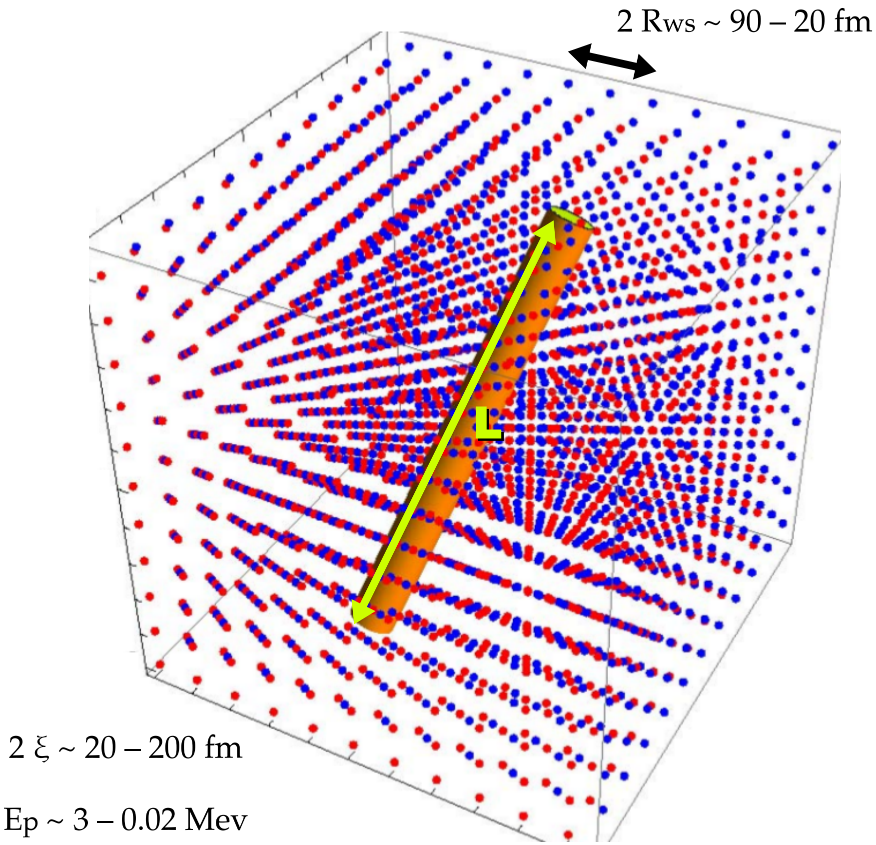
} Expulsion of vortex lines from bulk superfluid } Vortex mediated mutual friction between n and p

UNPINNING → (thermally activated) (soc, "external" trigger..?)

Local: vortex creep **Global: avalanche**

Mesoscopic pinning forces

(S. Seveso et al., MNRAS, 2016)



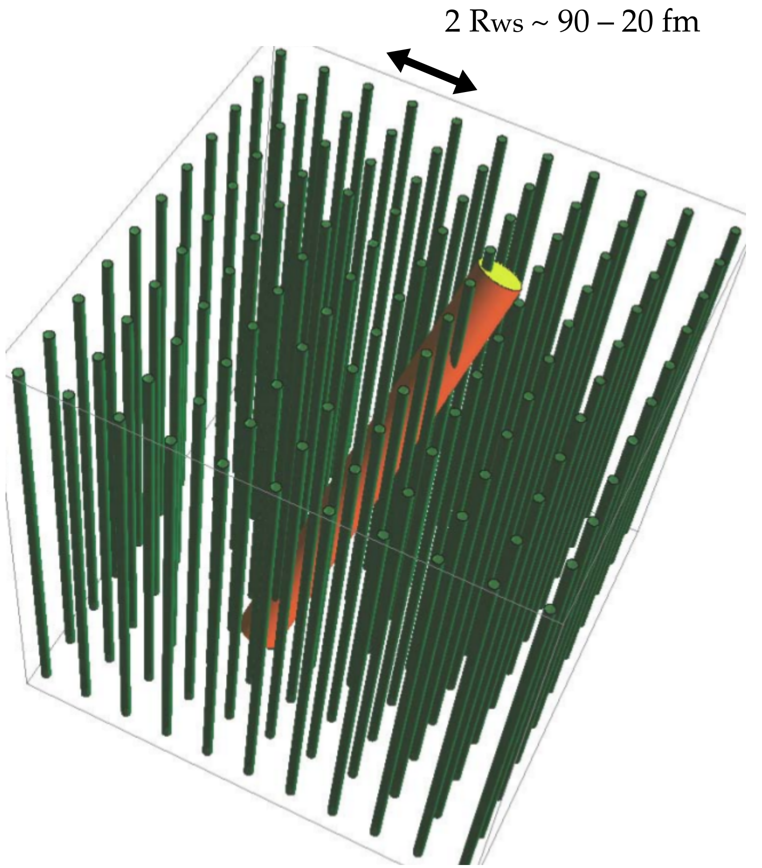
Inner crust:

Vortex-nucleus interaction → Vortex-lattice interaction per unit length of vortex line

IDEA: consider a segment of vortex line

(the length L is given by the tension)

Average over traslations and rotations of the total pinning force divided by L



Core:

Vortex-flux tube interaction → Vortex-array interaction

Pinning to flux-tubes negligible for normal pulsars

$$\xi_p \approx 16\, x_p^{1/3} \rho_{14}^{1/3} \Delta_p (\mathrm{MeV})^{-1} \mathrm{fm}$$

$$\xi_n \approx 16\, x_n^{1/3} \rho_{14}^{1/3} \Delta_n (\mathrm{MeV})^{-1} \mathrm{fm}$$

Entrainment effect:

- **Andreev-Baskin (1975): Three-velocity Hydrodynamics of Superfluid solutions**

  Viscosity → different velocity fields cannot coexist inside the same fluid

  Superfluid mixture → each superfluid can flow with its own velocity

  Interactions between particles → non-dissipative entrainment

Superfluid momenta in the frame of the normal fluid

$$\begin{cases} \mathbf{p}_1 = \rho_{11}^{(s)}(\mathbf{v}_1 - \mathbf{v}_n) + \rho_{12}^{(s)}(\mathbf{v}_2 - \mathbf{v}_n) \\ \mathbf{p}_2 = \rho_{21}^{(s)}(\mathbf{v}_1 - \mathbf{v}_n) + \rho_{22}^{(s)}(\mathbf{v}_2 - \mathbf{v}_n) \end{cases}$$

1: velocity of the superfluid "1"

2: velocity of the superfluid "2"

n: velocity of the normal component

...the momentum of one fluid is a linear combination of the velocities of all fluids!

- **N. Chamel & collaborators:**

  Entrainment also arise when a fluid is flowing through a solid...

  ...like electrons in metals or "free" neutrons in NS crust!

  → **non dissipative coupling** between "n" and "p"

  → NOT to be confused with the mutual friction

$$\frac{\bar{p}_i^{\mathrm{n}}}{m_{\mathrm{n}}} = v_i^{\mathrm{n}} + \varepsilon_{\mathrm{n}}(v_i^{\mathrm{p}} - v_i^{\mathrm{n}})$$

$$\frac{\bar{p}_i^{\mathrm{p}}}{m_{\mathrm{p}}} = v_i^{\mathrm{p}} + \varepsilon_{\mathrm{p}}(v_i^{\mathrm{n}} - v_i^{\mathrm{p}})$$

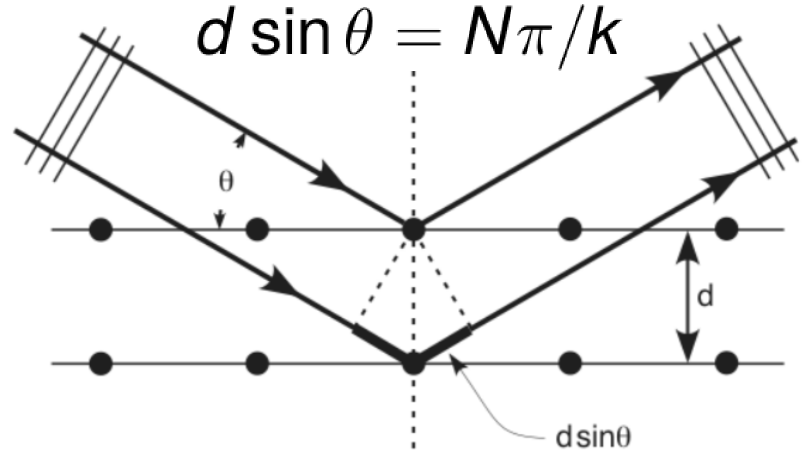
Entrainment effects in neutron stars

- In the crust:

- The entrainment parameters can be expressed in terms of effective masses:

$$\frac{\bar{p}_i^{\mathrm{n}}}{m_{\mathrm{n}}} = v_i^{\mathrm{n}} + \varepsilon_{\mathrm{n}}(\cancel{v_i^{\mathrm{p}}} - v_i^{\mathrm{n}})$$

$$\frac{\bar{p}_i^{\mathrm{p}}}{m_{\mathrm{p}}} = v_i^{\mathrm{p}} + \varepsilon_{\mathrm{p}}(v_i^{\mathrm{n}} - v_i^{\mathrm{p}})$$



- In the core:

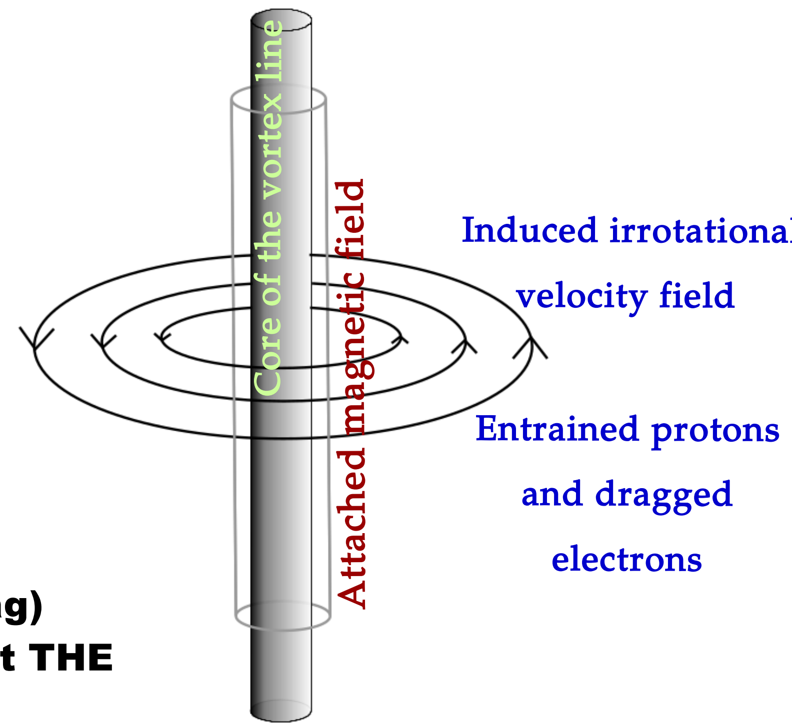
Entrainment is due to:

- Interaction between protons and neutrons

Effects: vortex lines are magnetized!

Scattering of electrons → vortex is dragged

Dipole-Dipole interaction with fluxtubes (core pinning?)



→ the conclusion is that the core should be coupled to the crust on the timescale of a second (el. scattering drag)

→ glitches originate in the crust... but thanks to entrainment THE CRUST IS NOT ENOUGH!

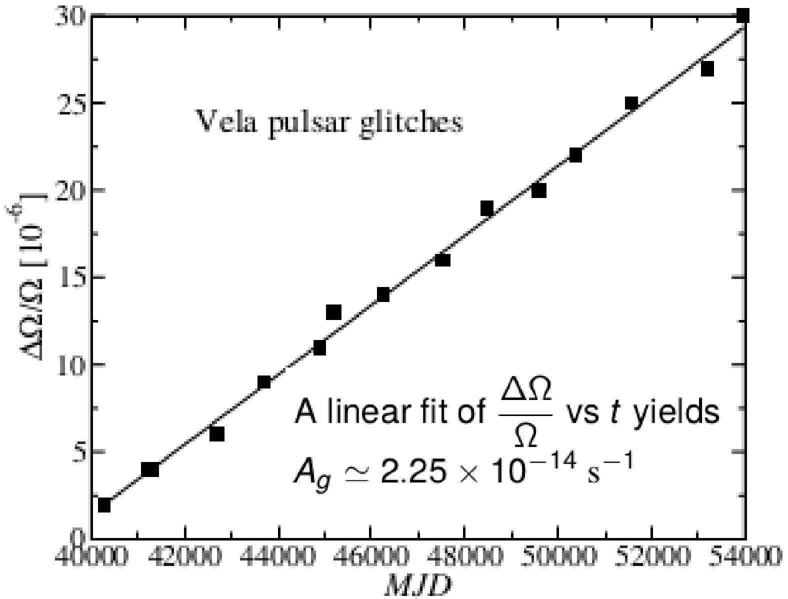
"The crust is not enough" (Andersson et al., 2012)

For systems that glitch regularly, one can estimate the moment of inertia of the superfluid component.

Need to involve up to 2% of the total moment of inertia.

The **crust superfluid** would be sufficient to explain the observations; as long as we do not worry about the entrainment.

Cumulated glitch amplitude



However, the large effective neutron mass in the crust (due to Bragg scattering of neutrons by the nuclear lattice) lowers the effective superfluid moment of inertia by a factor of 5 or so.

Entrainment correction on the moment of inertia of the superfluid

$$J_s = I_{ss}\Omega_s + (I_s - I_{ss})\Omega_c$$

Activity parameter

$$A_g = \frac{1}{t}\sum_i \frac{\Delta\Omega_i}{\Omega}$$

A simple but robust model assures that:

$$\frac{(I_s)^2}{I_{ss} I} \geq A_g \frac{\Omega}{|\dot{\Omega}|}$$

$$\frac{(I_s)^2}{I_{ss} I} \geq 1.6\%$$

For the Vela pulsar

Two fluids with entrainment

Motivation:

- Exchange of angular momentum → 2 components
- Long timescales → one component is superfluid
- Non realtivistic fluids (vel. equator < 20% c)

Conservation of mass and momenta

$$\frac{\bar{p}_i^{\rm n}}{m_{\rm n}} = v_i^{\rm n} + \varepsilon_{\rm n}(v_i^{\rm p} - v_i^{\rm n})$$

$$\frac{\bar{p}_i^{\rm p}}{m_{\rm p}} = v_i^{\rm p} + \varepsilon_{\rm p}(v_i^{\rm n} - v_i^{\rm p})$$

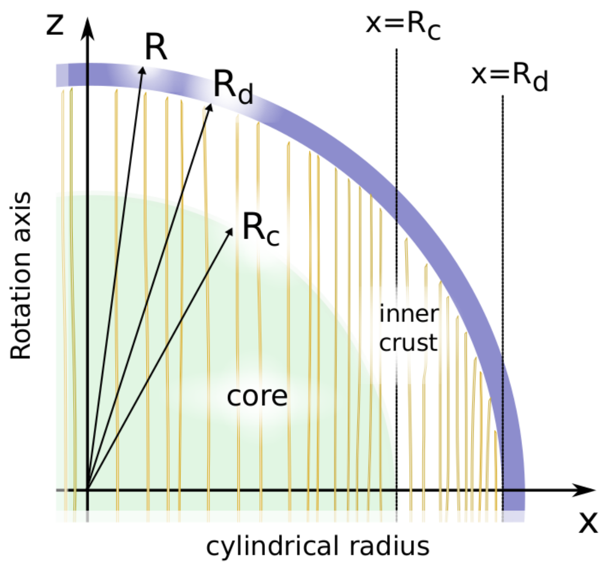
2 fluids hydro with entrainment

Chemical index:

X, Y = **"n"**, **"p"**

$$\begin{cases} \partial_t \rho_{\rm x} + \nabla_i(\rho_{\rm x} v_{\rm x}^i) = 0 \\ (\partial_t + v_{\rm x}^j \nabla_j)(v_i^{\rm x} + \varepsilon_{\rm x} w_i^{\rm yx}) + \nabla_i(\tilde{\mu}_{\rm x} + \Phi) + \varepsilon_{\rm x} w_{\rm yx}^j \nabla_i v_j^{\rm x} = f_i^{\rm x}/\rho_{\rm x} + \nabla_j D_i^j \\ f_i^{\rm x} = 2\rho_{\rm n}\mathcal{B}'\epsilon_{ijk}\Omega^j w_{\rm xy}^k + 2\rho_{\rm n}\mathcal{B}\epsilon_{ijk}\hat{\Omega}^j\epsilon^{klm}\Omega_l w_m^{\rm xy} \end{cases}$$

(Vortex mediated mutual friction)



"Cylindrical reduction"

By assuming columnar flow we can project the complicated 3D problem into a simpler one (1D radial) → this means that inside the star the vorticity of the superfluid is assumed to be parallel to the rotation axis of the star

1D **cylindrical** model:

(Antonelli & Pizzochero, 2015)

- Embeds the most important physical ingredients in a simple (but consistent) way
- Non uniform star, realistic EOS & layered structure: cylindrical projection "micro(r)" → "hydro(x)"
- Quantitative test without "tuning"

Assumptions:

- Azimuthal component of the "n" momentum → "x" only $\qquad \boldsymbol{p}_n(\boldsymbol{x}) \cdot \hat{\boldsymbol{e}}_\theta = p_n(x)$
- Charged component rotates as a rigid body (due to short Alf. timescale & electron screening length) $\qquad \boldsymbol{v}_p = x \Omega_p \hat{\boldsymbol{e}}_\theta$

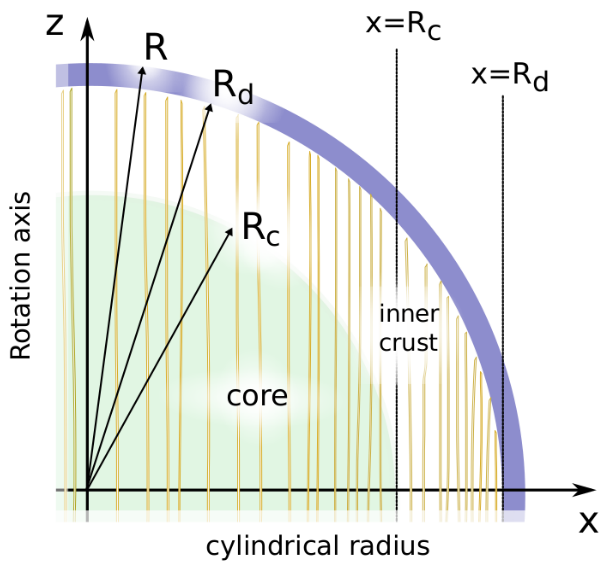
Strategy:

Define a fictitious "v" component

$$\Omega_v(x) = \frac{p_n(x)}{m_n\, x}$$

→ Fey. relation

$$\Omega_v(x) = \frac{\kappa\, N(x)}{2\pi\, x^2}$$



To encode entrainment consistently into a cylindrical model we have to consider a "v" (fictitious) component and the usual (rigid) "p" component

$$I_p + I_v = I$$

Macroscopic equations

$$I_c \dot{\Omega}_c = -\alpha - \frac{I_c}{\tau_c}(\Omega_c - \Omega_n)$$

$$I_n \dot{\Omega}_n = \frac{I_c}{\tau_c}(\Omega_c - \Omega_n),$$

Formal analogy with the early model of Baym

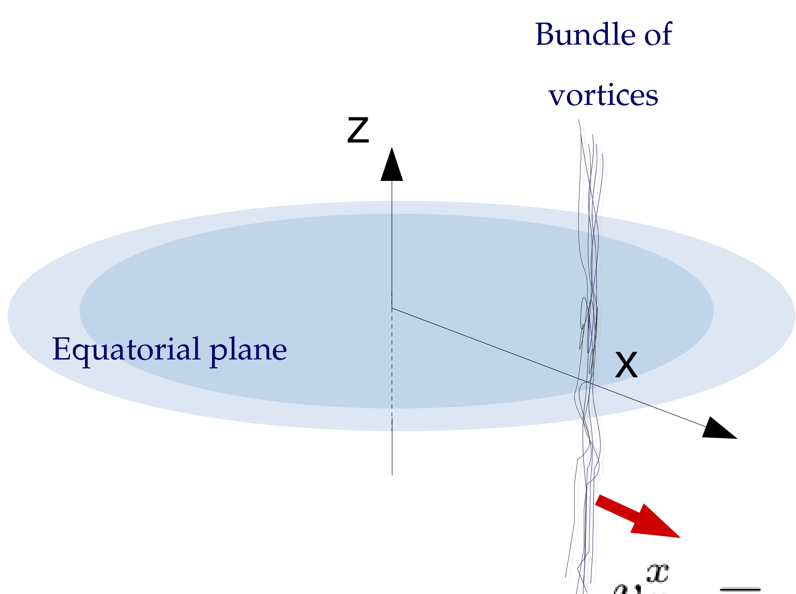
- Total angular momentum "conservation"
- Vortex lines must have a radial velocity is "v" is changing
- Just carry out the integrations on the z axis, from 0 to: $z(x) = \sqrt{R^2 - x^2}$
- Evolution slowly driven by external torque
- Non linear mutual friction (Drag functional) & no phenomenological timescales

$$\partial_t \Omega_v(x,t) = -\mathcal{B}[\Omega_v, \Omega_p, x]\,(2\Omega_v + x\partial_x\Omega_v)(\Omega_v - \Omega_p)$$

$$\partial_t \Omega_p(t) = -(1+q)\dot{\Omega}_\infty - q\langle\, \partial_t \Omega_v(t)\,\rangle$$

$$q = I_v / I_p$$

$$\dot{\Omega}_\infty = T_{ext}/I$$



Equation of motion for (rigid) vortex lines:

$$\boldsymbol{F}_{tot} = \int_L dl(\boldsymbol{x})\,[\boldsymbol{f}_M(\boldsymbol{x}) + \boldsymbol{f}_D(\boldsymbol{x}) + \boldsymbol{f}_P(\boldsymbol{x})] = 0$$

We can solve the equation of motion to find the outward velocity of vortices

$$\mathcal{B}[\Omega_v, \Omega_p, x] \approx Y[\omega, x]\,\mathcal{B}^x(x)$$

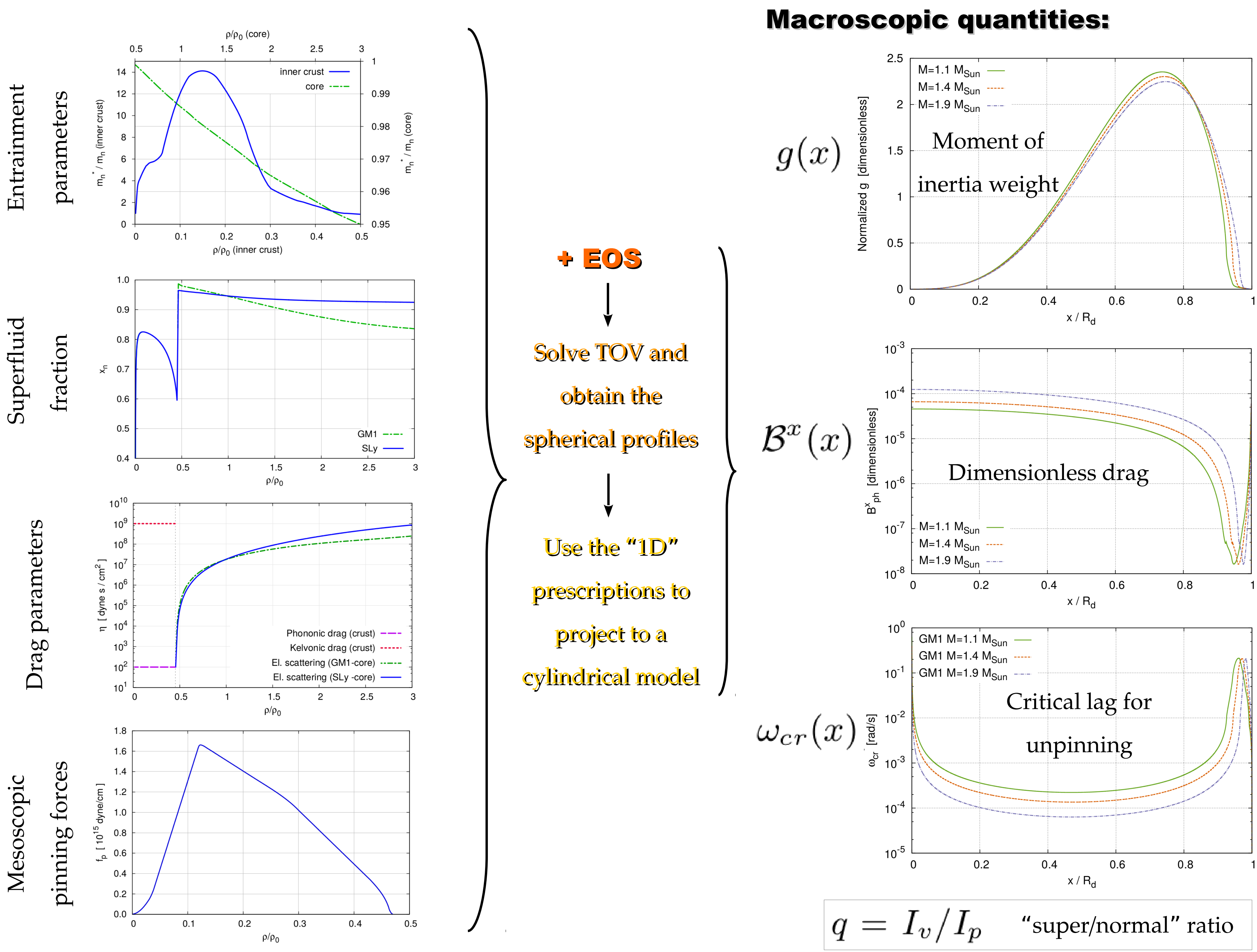
$$Y[\omega, x] = \theta(|\omega(x)| - \omega_{cr}(x))$$

$$v_L^x = x\,\mathcal{B}[\Omega_v, \Omega_p, x]\,(\Omega_v(x) - \Omega_p)$$

Radial (average) velocity of vortices

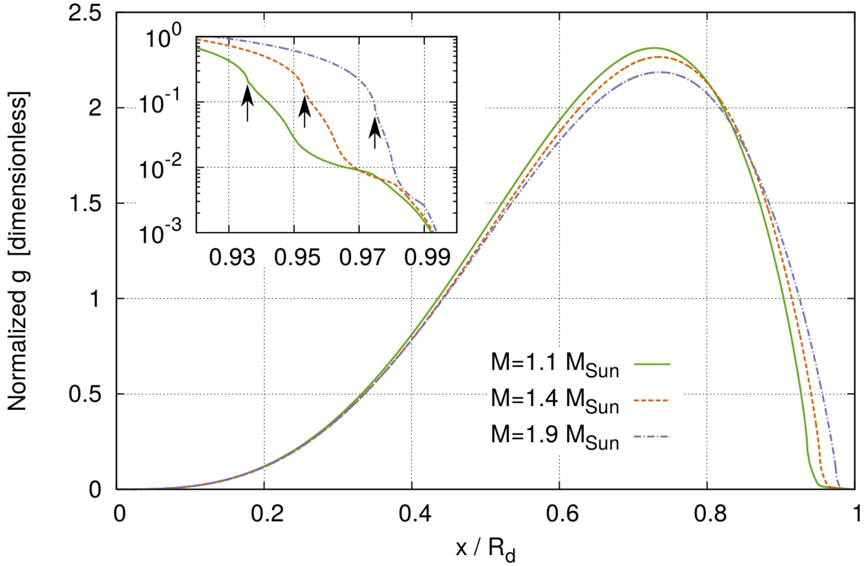
Dimensionless drag functional

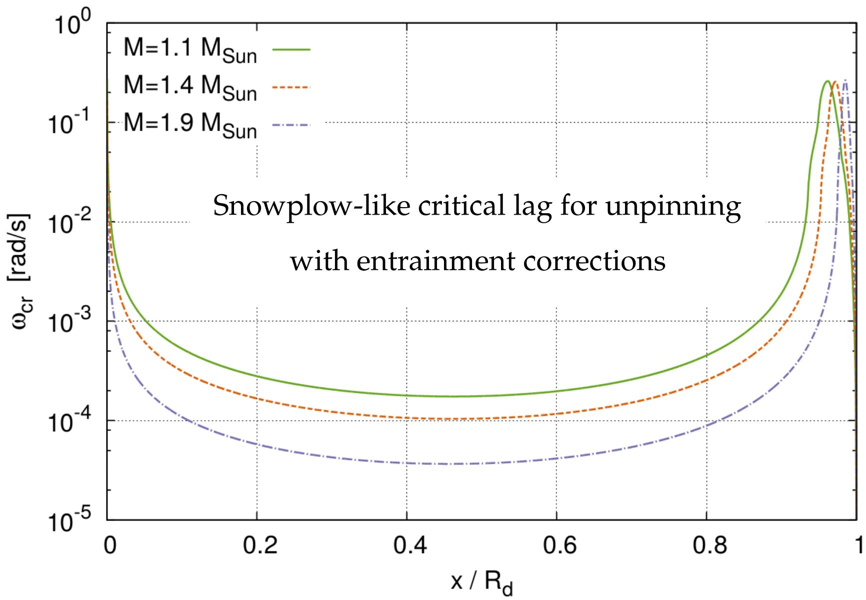
Angular velocity lag v-p


Macroscopic quantities:
Entrainment parameters
ρ/ρ0 (core)
inner crust
core
m*n / mn (inner crust)
m*n / mn (core)
ρ/ρ0 (inner crust)
Superfluid fraction
xn
GM1
SLy
ρ/ρ0
Drag parameters
η [dyne s / cm2]
Phononic drag (crust)
Kelvonic drag (crust)
El. scattering (GM1-core)
El. scattering (SLy -core)
Mesoscopic pinning forces
fp [10^15 dyne/cm]
+ EOS
Solve TOV and obtain the spherical profiles
Use the "1D" prescriptions to project to a cylindrical model
g(x)
Normalized g [dimensionless]
M=1.1 MSun
M=1.4 MSun
M=1.9 MSun
Moment of inertia weight
x / Rd
B^x(x)
B^x_ph [dimensionless]
Dimensionless drag
ω_cr(x)
ω_cr [rad/s]
GM1 M=1.1 MSun
GM1 M=1.4 MSun
GM1 M=1.9 MSun
Critical lag for unpinning
q = I_v / I_p "super/normal" ratio

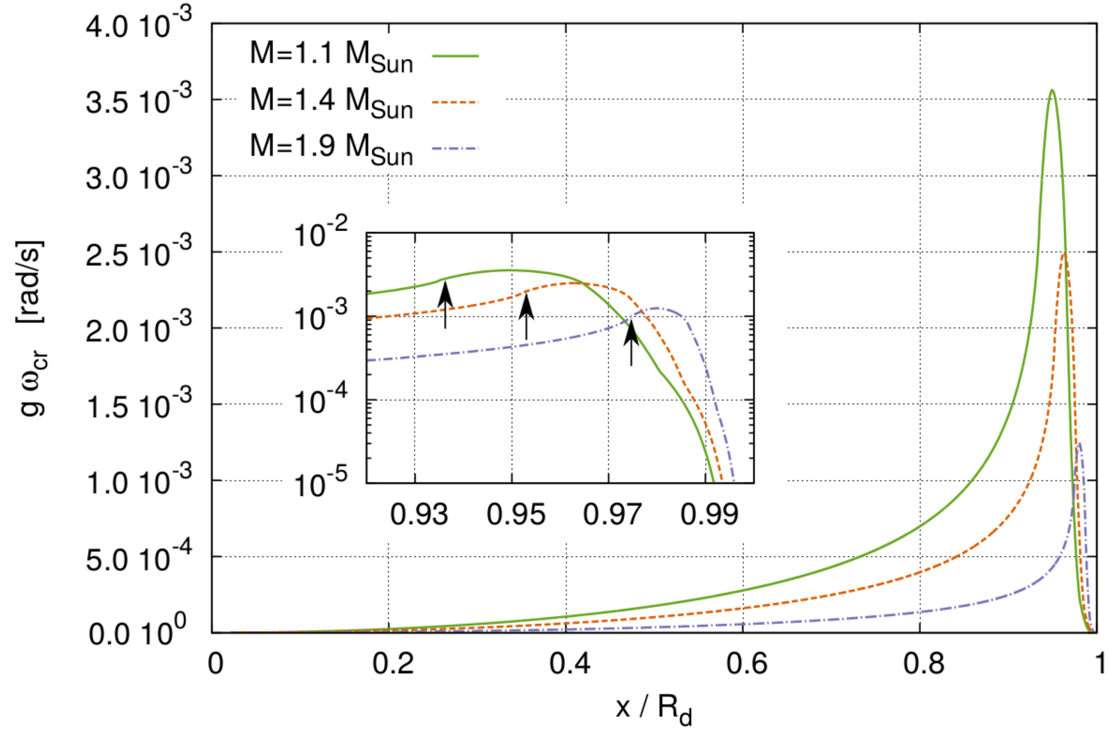
Angular momentum reservoir

(Figures: examples with the Sly EOS)





The arrows indicate the radius of the cylindrical region that is completely immersed into the crust



The product **Iv g(x) ω_{cr}(x)** is an uper bound to the angular momentum that can be stored in a cylindrical shell at radius x: If locally the lag **ω(x) > ω_{cr}(x)**, vortex lines **unpin** → expulsion of lines and the **lag decreases**

...simulation of a **GIANT** glitch

$$\begin{cases} \partial_t \Omega_v(x,t) = -\mathcal{B}[\Omega_v, \Omega_p, x]\,(2\Omega_v + x\partial_x \Omega_v)(\Omega_v - \Omega_p) \\ \partial_t \Omega_p(t) = -(1+q)\dot{\Omega}_\infty - q\langle\, \partial_t \Omega_v(t)\,\rangle \end{cases}$$

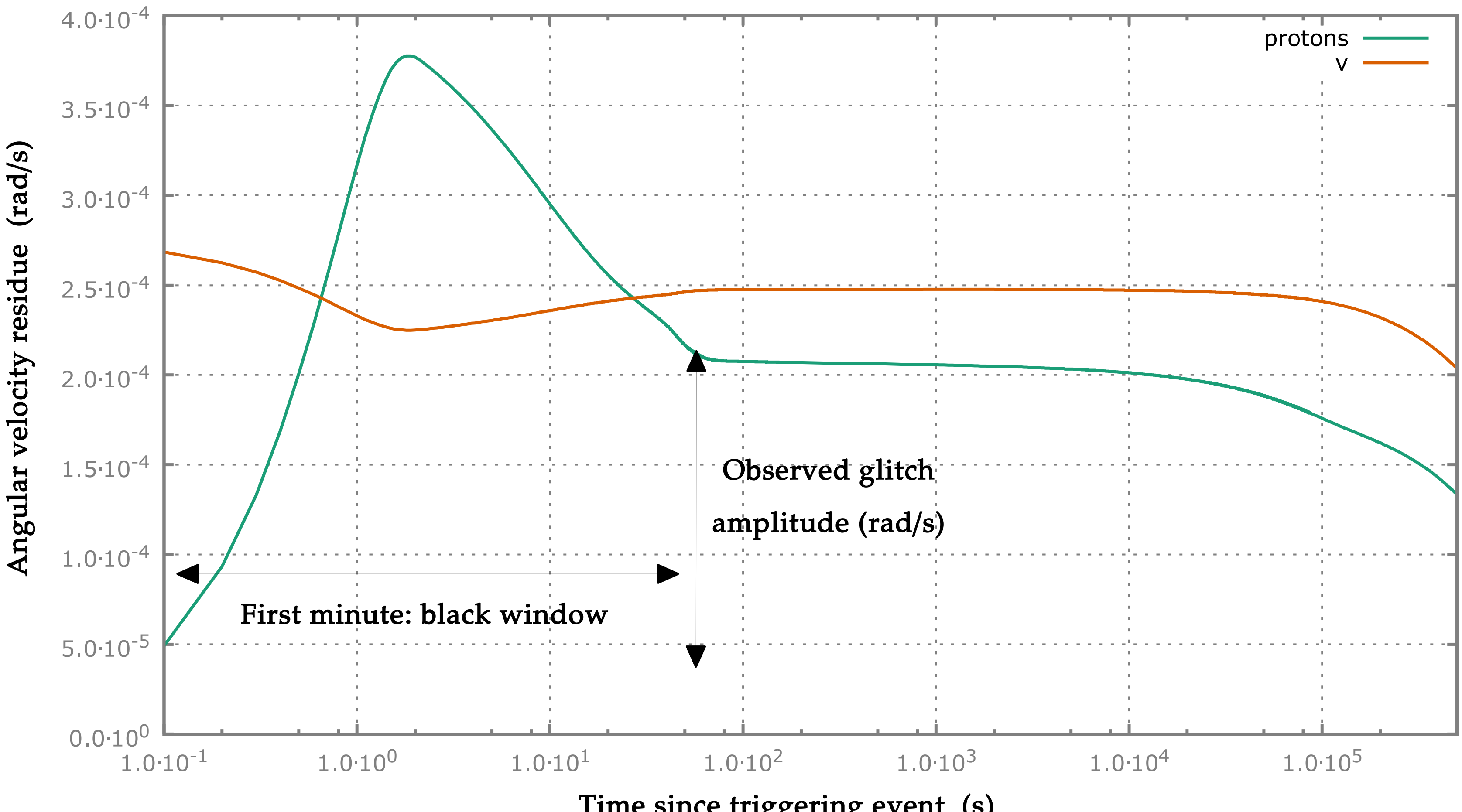
Here: rigid vortices approximation → Snowplow like drag functional

$$\mathcal{B}[\Omega_v, \Omega_p, x] \;\approx\; Y[\omega, x]\; \mathcal{B}^x(x)$$

$$Y[\omega, x] = \theta(|\omega(x)| - \omega_{cr}(x))$$

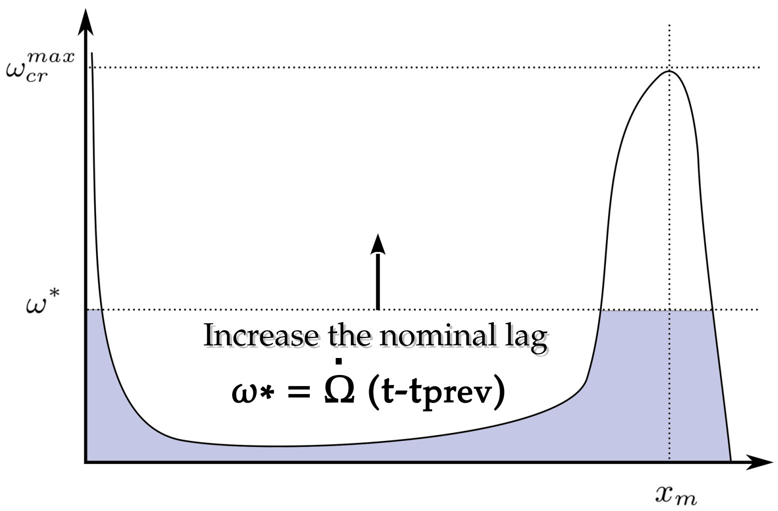
Glitch amplitude (Vela): $\Delta\nu / \nu \sim 10^{-6} \rightarrow \Delta\nu \sim 10^{-5}$ Hz

Differential rotation of the superfluid + non uniform structure of the star → different timescales in the relaxation



(Dynamical simulation: S. Seveso)

Mass upper bounds…

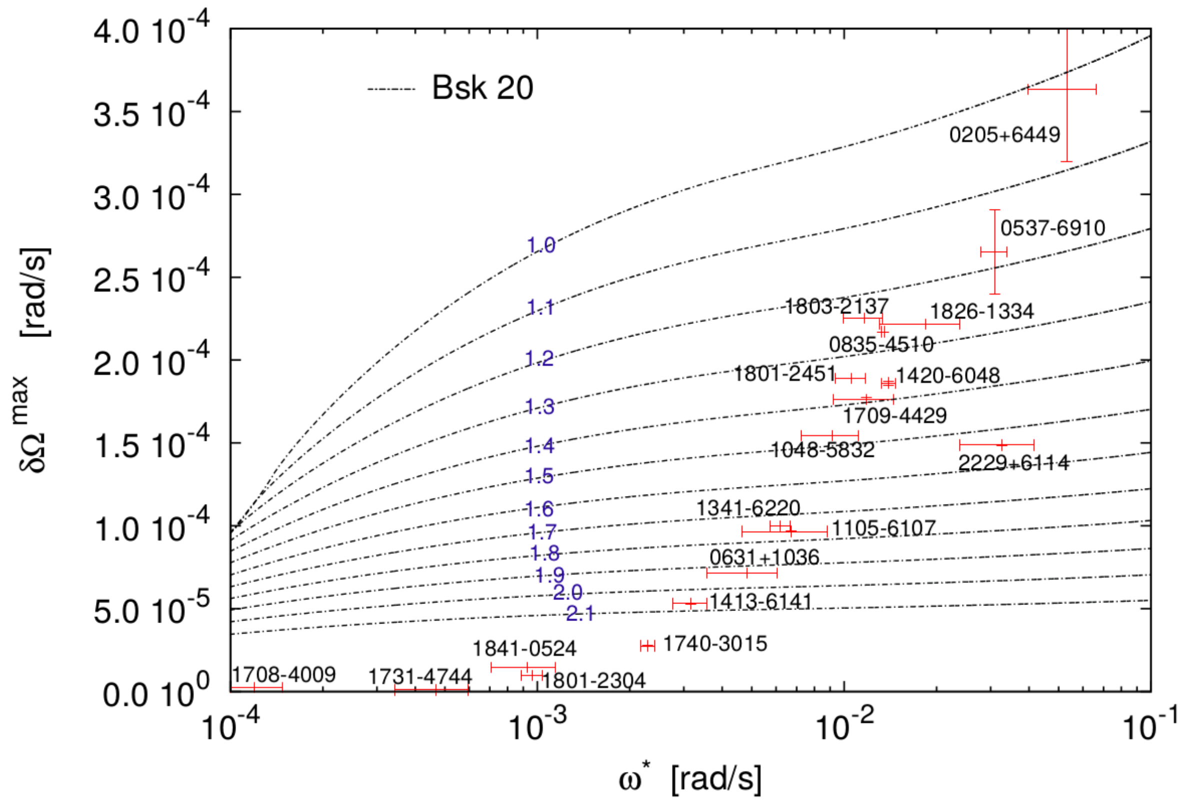


In the "mean time" between two large glitches the pulsar must be able to build a reservoir of angular momentum that is (at least) enough to produce the observed angular velocity jump .

For every pulsar, consider:

- The glitch amplitude of its maximal glitch **$\delta\Omega_{max}$**

- The waiting time mutiplied by the observed spin down rate → "nominal lag" **$\omega* = \dot{\Omega}$ (t-tprev)**



In the plane **$\delta\Omega_{max}$ – $\omega*$** we also plot the curves obtained by filling the critical lag as **$\omega*$** increases For different masses. We must have that:

$\delta\Omega_{max}$ ($\omega*$) > $\delta\Omega_{max}$ "observed"

This constrains the mass of the glitching pulsar!

NOTE: it is possible to do the same job by using thedirectly the dynamical equations instead of using this simple prescription

Final remarks

- Entrainment challenges our understanding of pulsar glitches…

- The crust is not enough (Andersson/Chamel…)
- New mesoscopic pinning forces (Seveso)
- Vortex lines that cross the superfluid core

Need to understand superfluid turbulence in NS… does it affects the amount of angular momentum that can be stored in the superfluid?

- MAIN WORKING ASSUMPTION:

- Axial symmetry → at the end the dynamical equations will depend only on "x"
- The charged component is a rigid body

- Results:

- General set of equations valid for any "cylindrical model"
- Consistent model with entrainment & stratification

Quantitative test of the widespread assumption of a "vortex array"

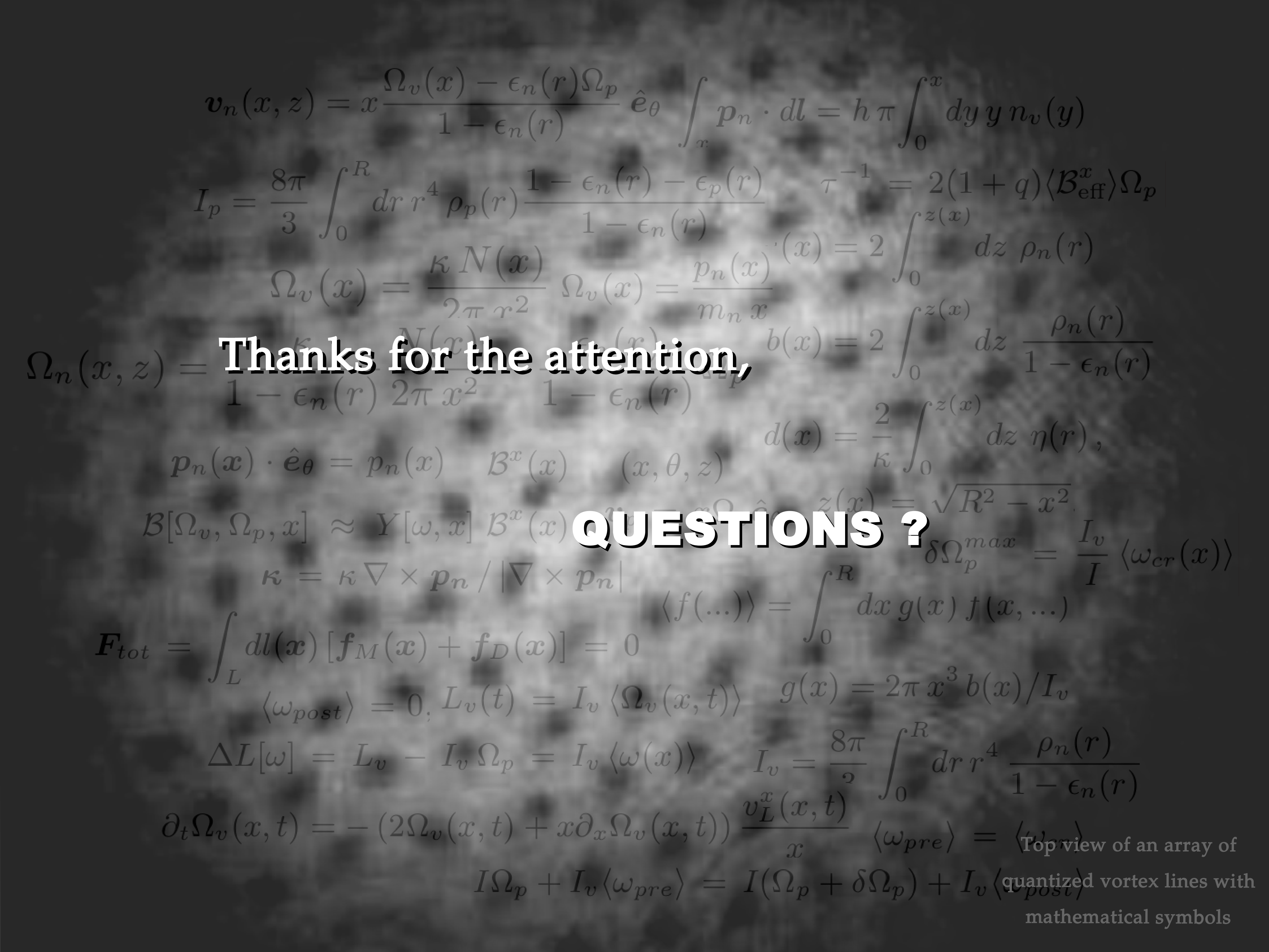
- Estimate of superfluid angular momentum reservoir → **upper limits on NS masses**

Details of the cylindrical model are on ArXiv (submitted to MNRAS):

M. Antonelli, P. Pizzochero
"Axially symmetric equations for differential pulsar rotation with superfluid entrainment"

Mass estimates: in preparation!

M. Antonelli, B. Haskell, P. Pizzochero, S. Seveso



Top view of an array of quantized vortex lines with mathematical symbols

Turbulence?

(Andersson, Sidery, Comer - 2007)

Study of mutual friction (force per unit volume) based on superfluid He analogy

$$f_i^{\mathrm{mf}} = \underbrace{\mathcal{B}' \rho_{\mathrm{n}} n_v \epsilon_{ijk} \kappa^j w_{\mathrm{np}}^k + \mathcal{B} \rho_{\mathrm{n}} n_v \epsilon_{ijk} \epsilon^{klm} \hat{\kappa}^j \kappa_l w_m^{\mathrm{np}}}^{\text{Mutual friction for straight vortices...}} - \rho_{\mathrm{n}} \tilde{\nu} n_v \left[\mathcal{B}' \hat{\kappa}^j \nabla_j \kappa_i + \mathcal{B} \epsilon_{ijk} \kappa^j \hat{\kappa}^l \nabla_l \hat{\kappa}^k \right]$$

...and with curvature

$$f_i^{\mathrm{mf}} = \frac{8\pi^2 \rho_{\mathrm{n}}}{3\kappa} \left(\frac{\chi_1}{\chi_2} \right)^2 \mathcal{B}^3 w_{\mathrm{pn}}^2 w_i^{\mathrm{pn}}$$

Gorter-Mellink

$$f_{\mathrm{HV}} \approx \mathcal{B} \rho_{\mathrm{n}} \kappa n_v w_{\mathrm{pn}}$$

Straight vortex array: "Hall-Vinen type" force

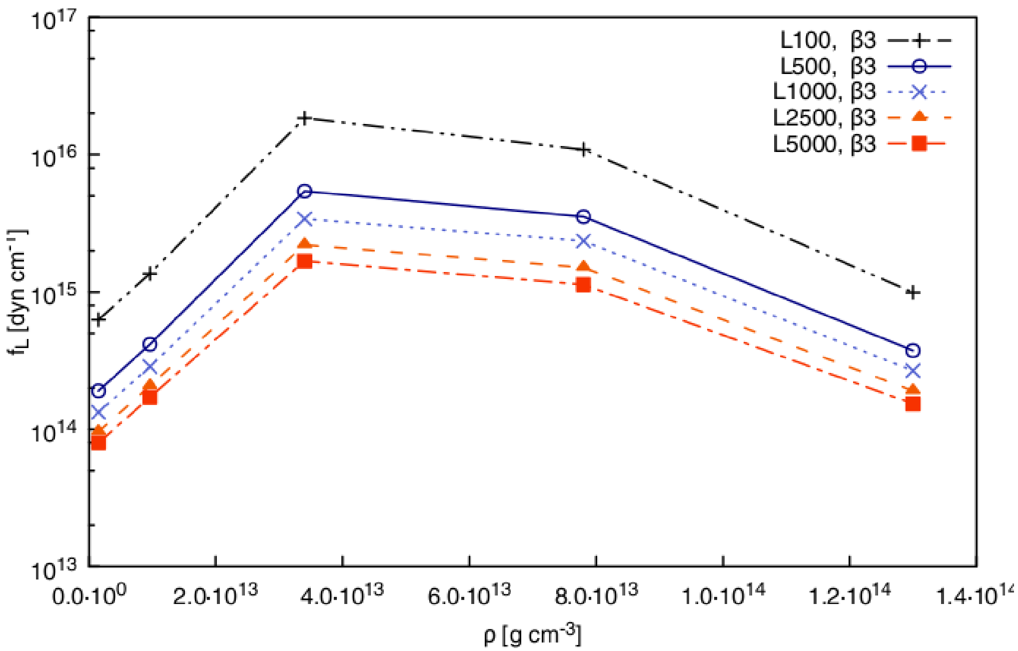
$$\chi_1/\chi_2 \sim 1 \qquad n_v \kappa \approx 2\Omega_{\mathrm{n}}, \quad \kappa = h/2m_n$$

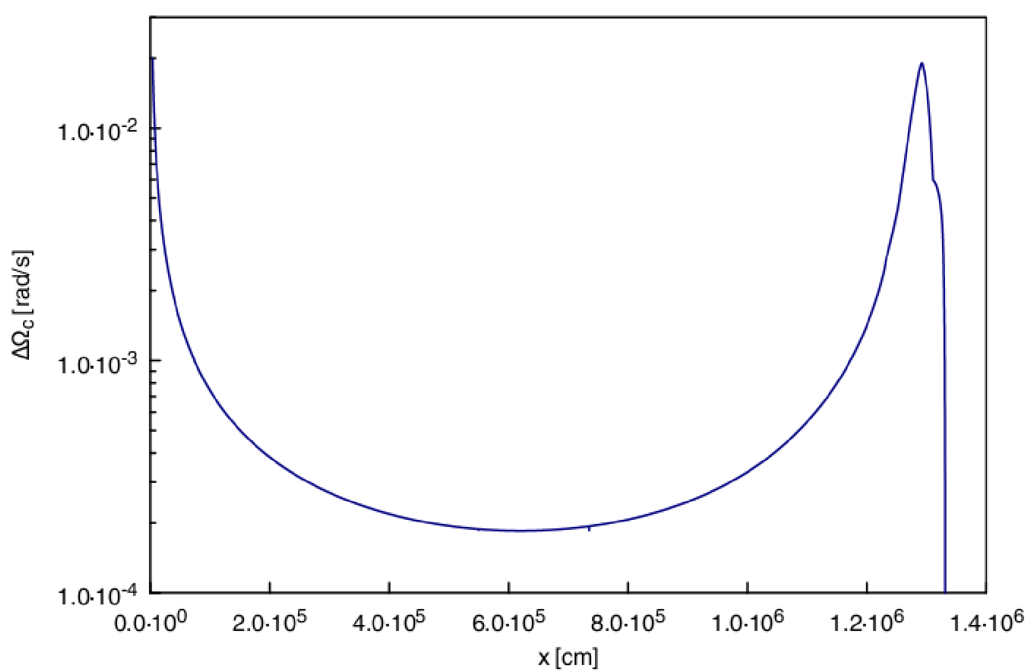
$$w_{\mathrm{pn}} = r\Delta\Omega = r(\Omega_{\mathrm{p}} - \Omega_{\mathrm{n}})$$

$$\frac{f_{\mathrm{GM}}}{f_{\mathrm{HV}}} \approx 250 \left(\frac{r}{10^6 \mathrm{cm}} \right)^2 \left(\frac{\Delta\Omega/\Omega_{\mathrm{n}}}{5 \times 10^{-4}} \right)^2 \left(\frac{P}{1\mathrm{s}} \right)^{-1}$$

Here r is the cylindrical radius, GM more important wrt HV at great r → where vortices are "more pinned"

Mesoscopic pinning forces





Mesoscopic pinning forces

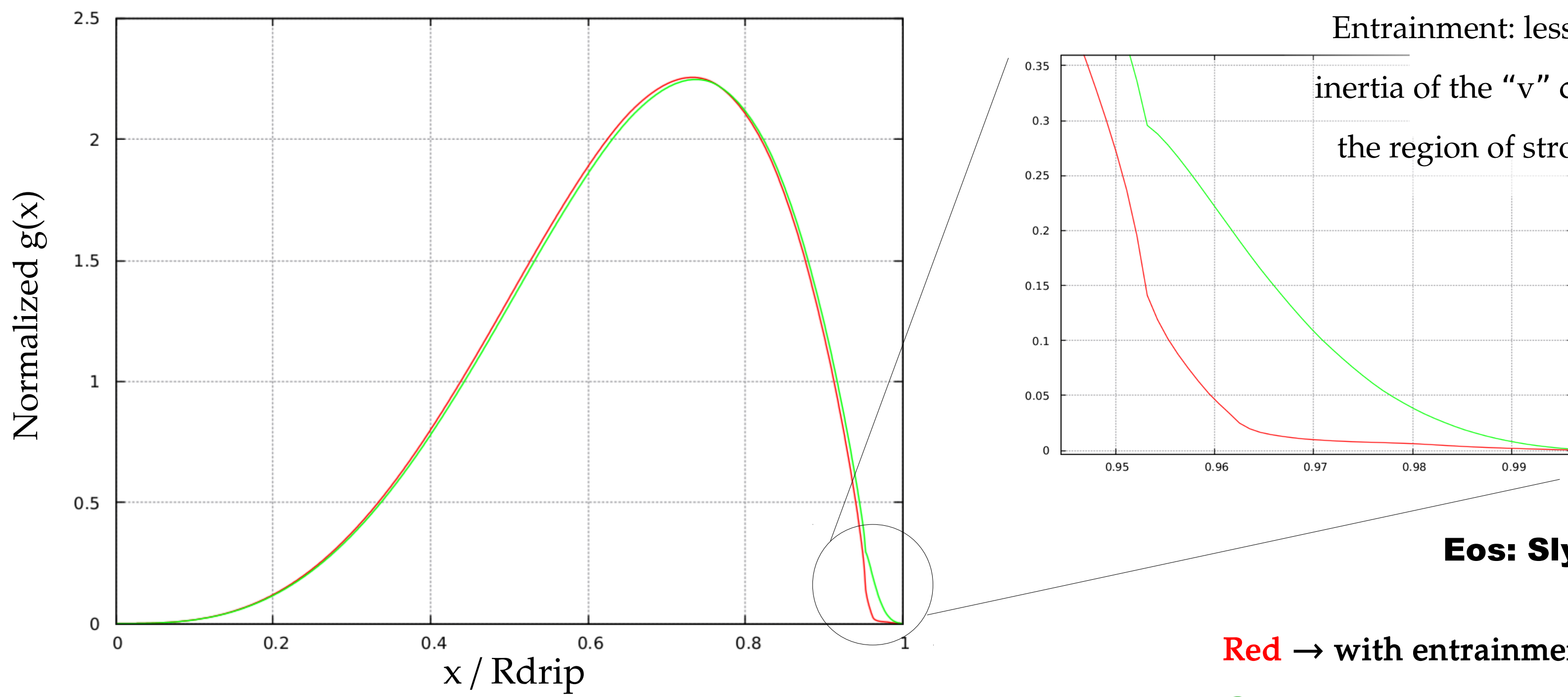
(vortex-lattice interaction per unit length of vortex line)

S. Seveso, F. Grill,
P. Pizzochero, B. Haskell

Resulting critical lag for unpinning (without entrainment)

$$Y[\omega, x] = \theta(|\omega(x)| - \omega_{cr}(x))$$

Example of entrainment corrections

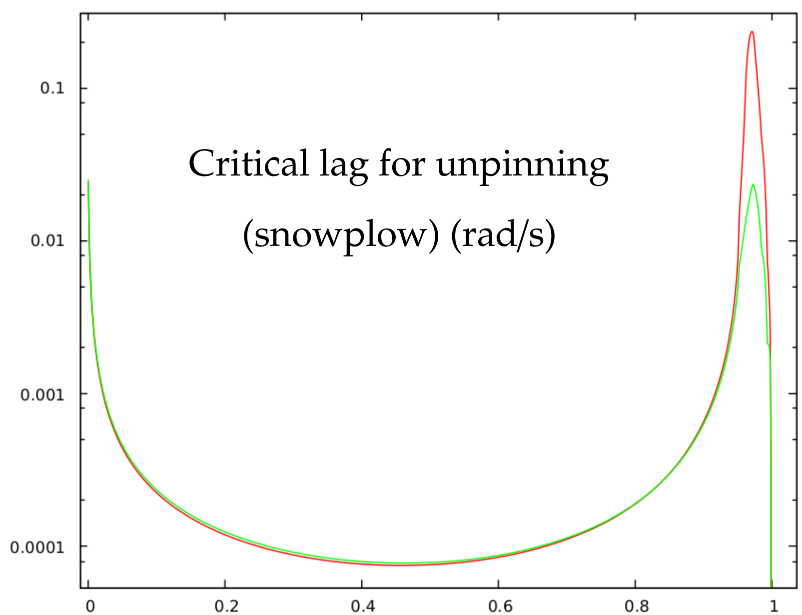


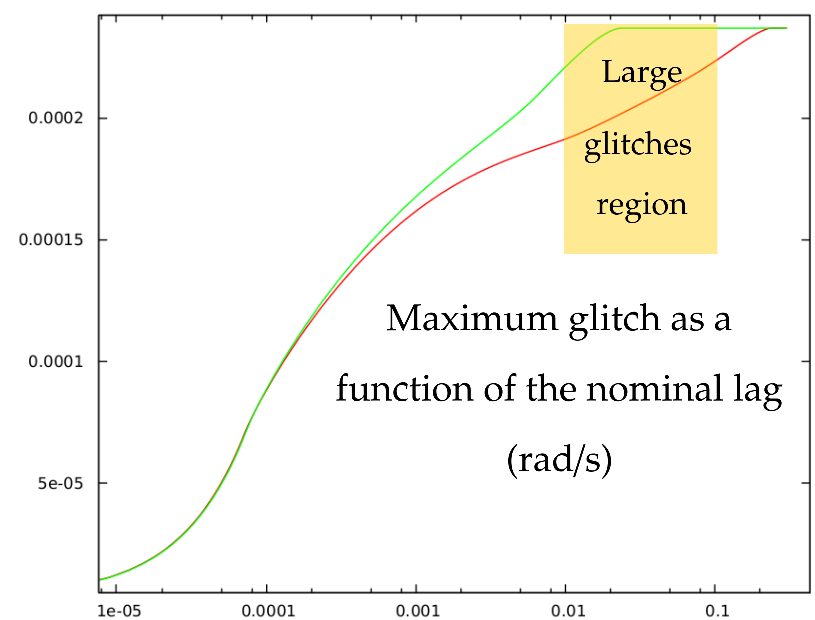
Entrainment: less moment of inertia of the “v” component in the region of strong pinning

Eos: Sly, M = 1.4 Msun

Red → with entrainment (Chamel)

Green → no entrainment





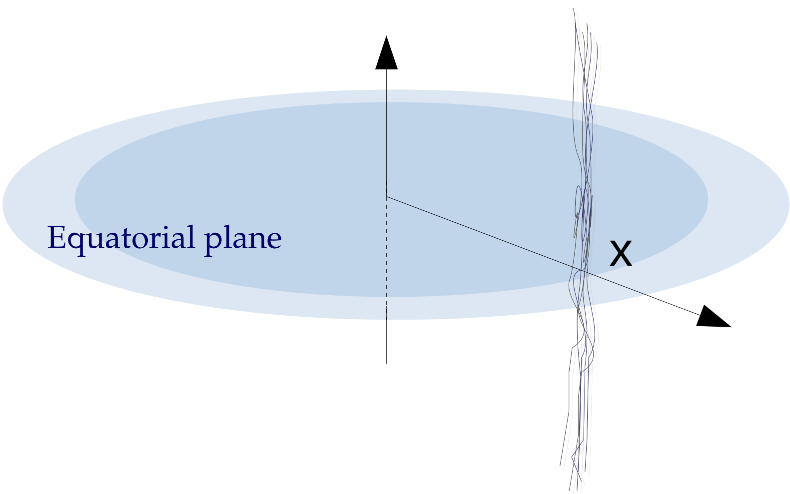
Macroscopic equations (similar eqs without entrainment → Alpar et al. 1984)

- The star spins-down under a constant external torque:

$$\boxed{\int d^3x \rho_p(\boldsymbol{r}) \left(\boldsymbol{x} \times \dot{\boldsymbol{v}}_p(x)\right) = -\boldsymbol{T}_{ext} - \int d^3x \rho_n(\boldsymbol{r}) \left(\boldsymbol{x} \times \dot{\boldsymbol{v}}_n(\boldsymbol{x})\right)}$$

$$\boldsymbol{p}_n(\boldsymbol{x}) = m_n \left[(1-\epsilon_n(\boldsymbol{x}))\, \boldsymbol{v}_n(\boldsymbol{x}) + \epsilon_n(\boldsymbol{x}) \boldsymbol{v}_p(\boldsymbol{x}) \right]$$

- v-component: $\Omega_v(x) = \dfrac{p_n(x)}{m_n\, x}$ → Feynman relation + continuity



$$\boxed{\partial_t n_v + \frac{1}{x}\partial_x (x\, n_v\, v_L^x) = 0}$$

2D density of vortex lines

Radial vel. of vortices

$$n_v(x) = \frac{(\nabla \times \boldsymbol{p}_n)\cdot \hat{\boldsymbol{e}}_z}{\kappa\, m_n} = \frac{1}{\kappa}(2\Omega_v(x) + x\partial_x \Omega_v(x))$$

$$v_L^x = x\ \mathcal{B}[\Omega_v, \Omega_p, x]\ (\Omega_v(x) - \Omega_p)$$

Dimensionless drag functional Angular velocity lag v-p

Radial (avereage) velocity of vortices

Macroscopic equations

- Just carry out the integrations on the z axis, from 0 to: $z(x) = \sqrt{R^2 - x^2}$

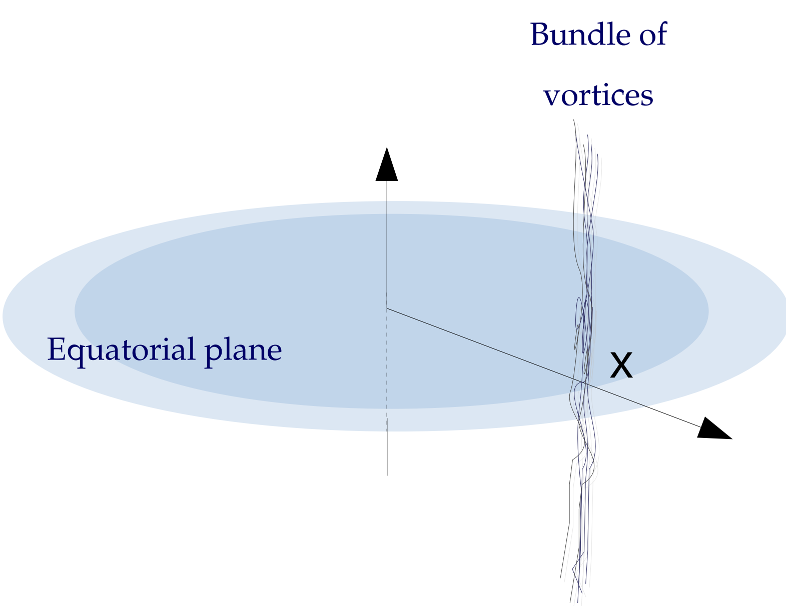
The general form of the equations → formally the same for the case without entrainment

$$\partial_t \Omega_v(x,t) = -\mathcal{B}[\Omega_v, \Omega_p, x]\,(2\Omega_v + x\partial_x \Omega_v)(\Omega_v - \Omega_p)$$

$$\partial_t \Omega_p(t) = -(1+q)\dot{\Omega}_\infty - q \underbrace{\langle\, \partial_t \Omega_v(t)\, \rangle}_{dI_v(x)/I_v \;=\; g(x)dx}$$

$$q = I_v / I_p$$

$$\dot{\Omega}_\infty = T_{ext}/I$$



Equation of motion for (rigid) vortex lines:

$$\boldsymbol{F}_{tot} = \int_L dl(\boldsymbol{x})\,[\boldsymbol{f}_M(\boldsymbol{x}) + \boldsymbol{f}_D(\boldsymbol{x}) + \boxed{\boldsymbol{f}_P(\boldsymbol{x})}] = 0$$

$$v_L^x = x\, \mathcal{B}^x(x)\, \omega(x,t)$$

Here: rigid vortices approximation → Snowplow like drag functional

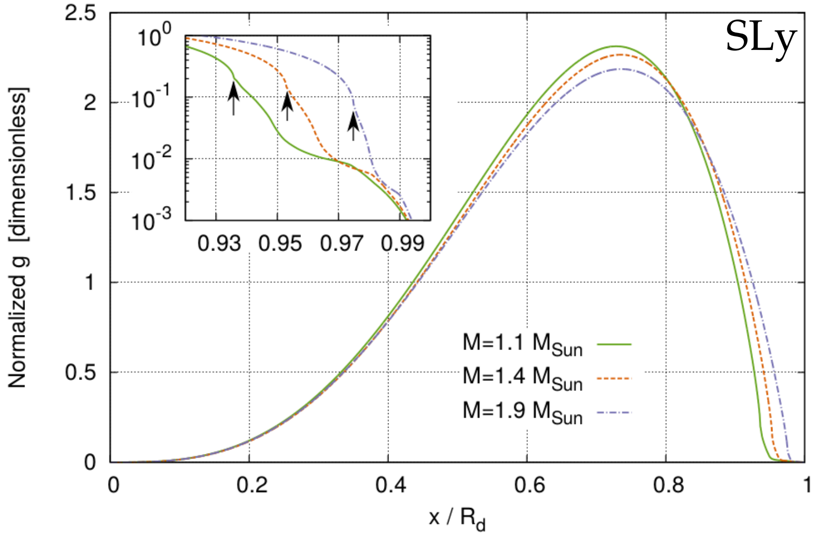
$$F_P(x) = |\boldsymbol{F}_M(x)|_{\boldsymbol{v}_L = \boldsymbol{v}_p}$$

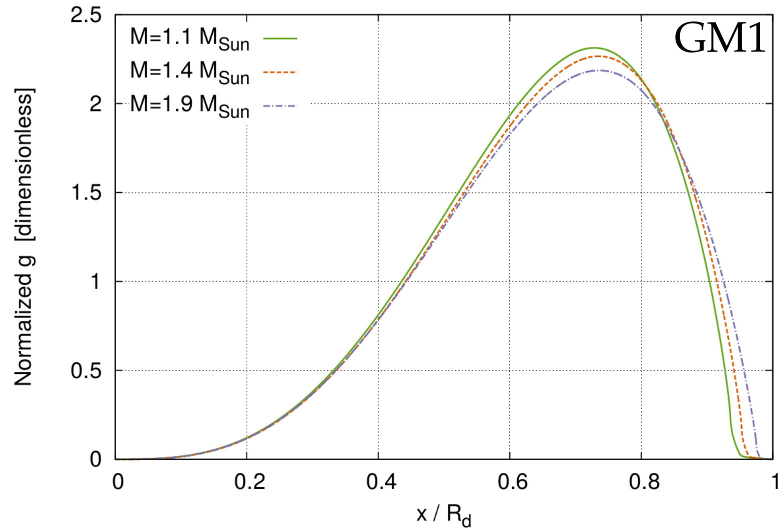
$$Y[\omega, x] = \theta(|\omega(x)| - \omega_{cr}(x))$$

$$\mathcal{B}[\Omega_v, \Omega_p, x] \approx Y[\omega, x]\, \mathcal{B}^x(x)$$

Some results for Sly & GM1 equations of state

Moments of inertia weights look the same (also because of normalization on the unit interval)





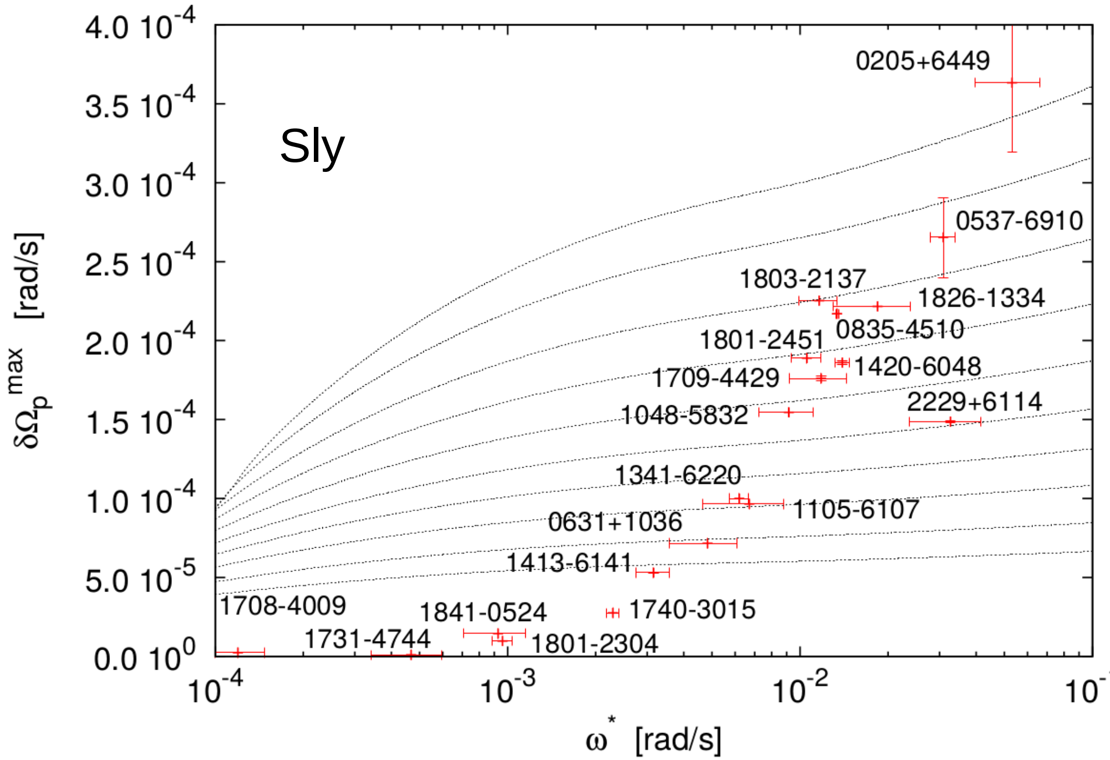
$$\tau^{-1} = 2(1+q)\langle \mathcal{B}^x_{\text{eff}} \rangle \Omega_p$$

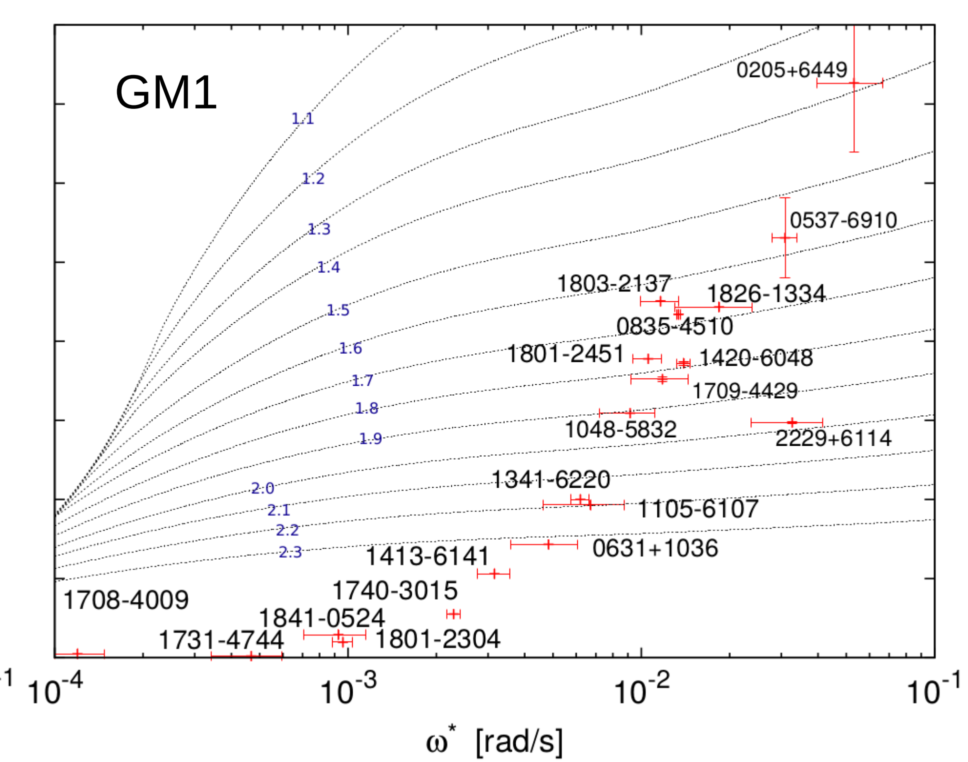
	M ($M_\odot$)	R_d (km)	R_c (R_d)	$I/10^{45}$ (g cm^2)	I_v (I)	q	τ_{ph} (P)	τ_{kv} (P)	$\langle\omega_{cr}\rangle$ (10^{-4}rad/s)	$\delta\Omega_p^{max}$ (10^{-4}rad/s)
SLy	1.1	11.41	0.936	0.86	0.928	12.9	62.1	13.2	4.17	3.87
	1.4	11.43	0.953	1.09	0.941	16.1	29.2	12.1	2.51	2.37
	1.9	10.92	0.974	1.35	0.958	22.9	9.0	7.2	0.92	0.89
GM1	1.1	13.08	0.926	1.14	0.901	9.2	445	13.9	6.99	6.30
	1.4	13.28	0.945	1.51	0.913	10.5	261	17.1	4.37	3.92
	1.9	13.24	0.966	2.04	0.916	10.9	136	26.3	2.04	1.87

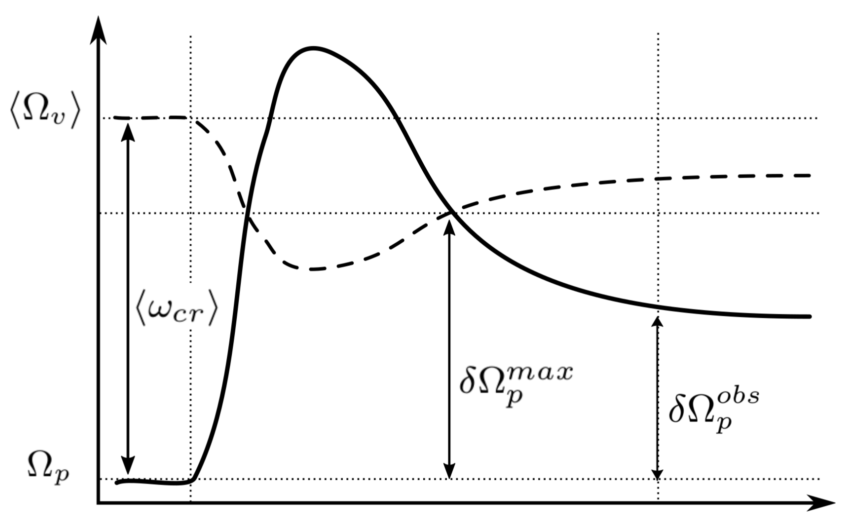
Estimates of the rise time

(all vortices unpinned)

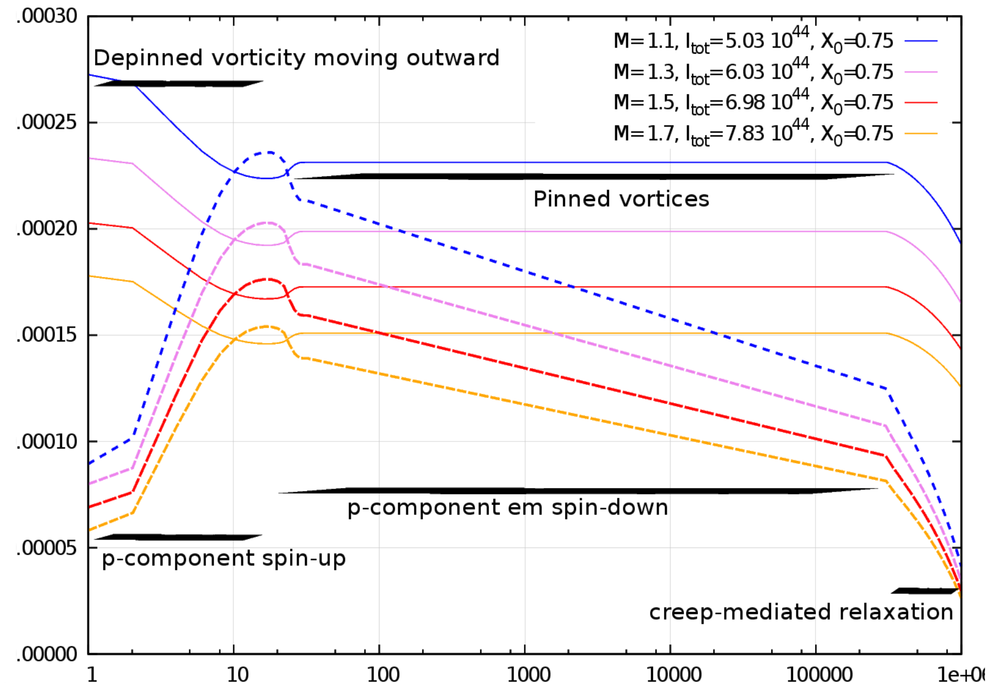
Maximal glitch amplitude





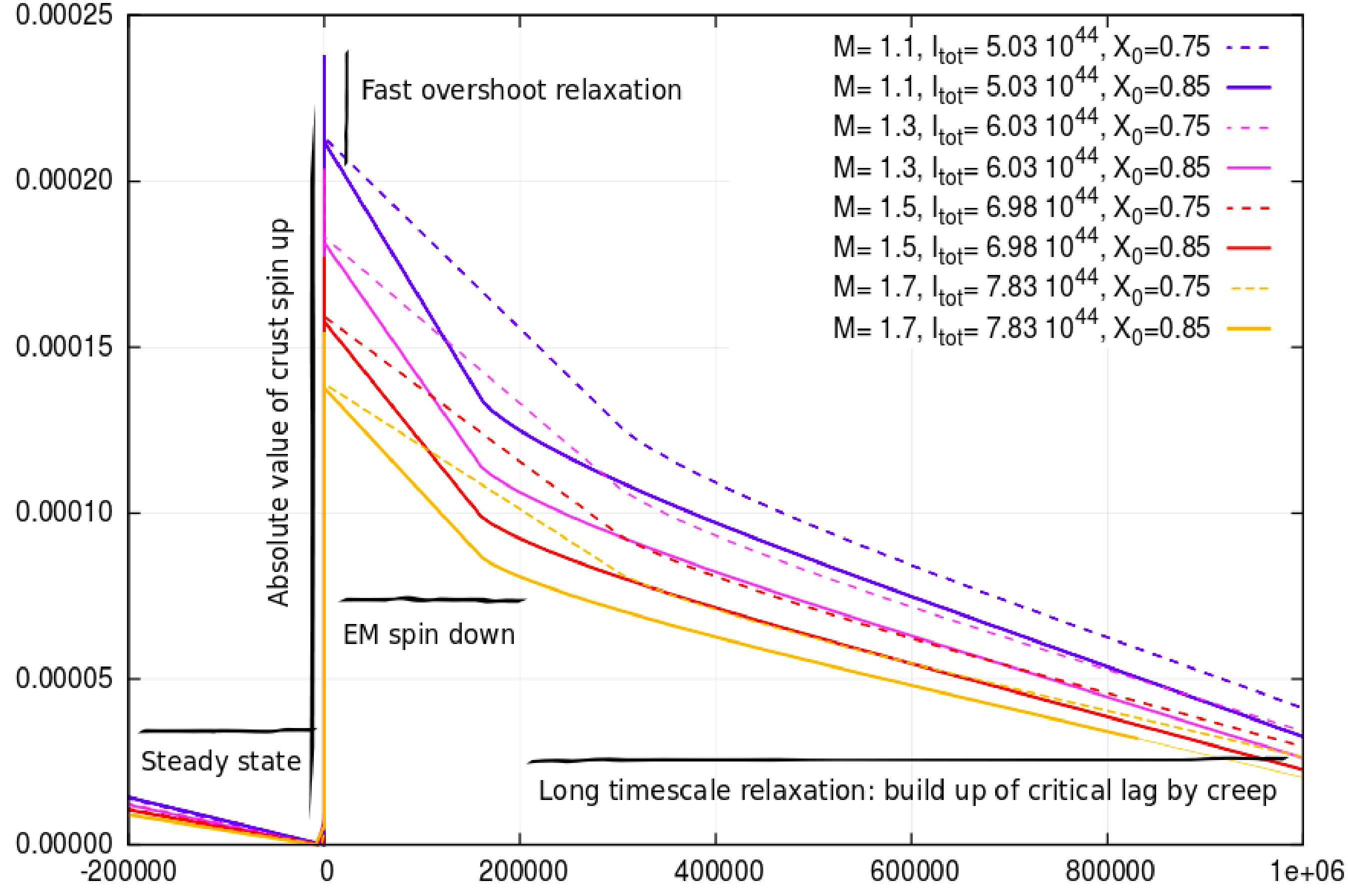


...why a **simple** but **consistent** model?



Dynamical simulation with:

- Polytropic EOS
- Proton fractions by hand
- Entrainment by hand
- Drag by hand (rise times are the same!)
- "Similar" pinning force profile
  (Pizzochero, "snowplow model")



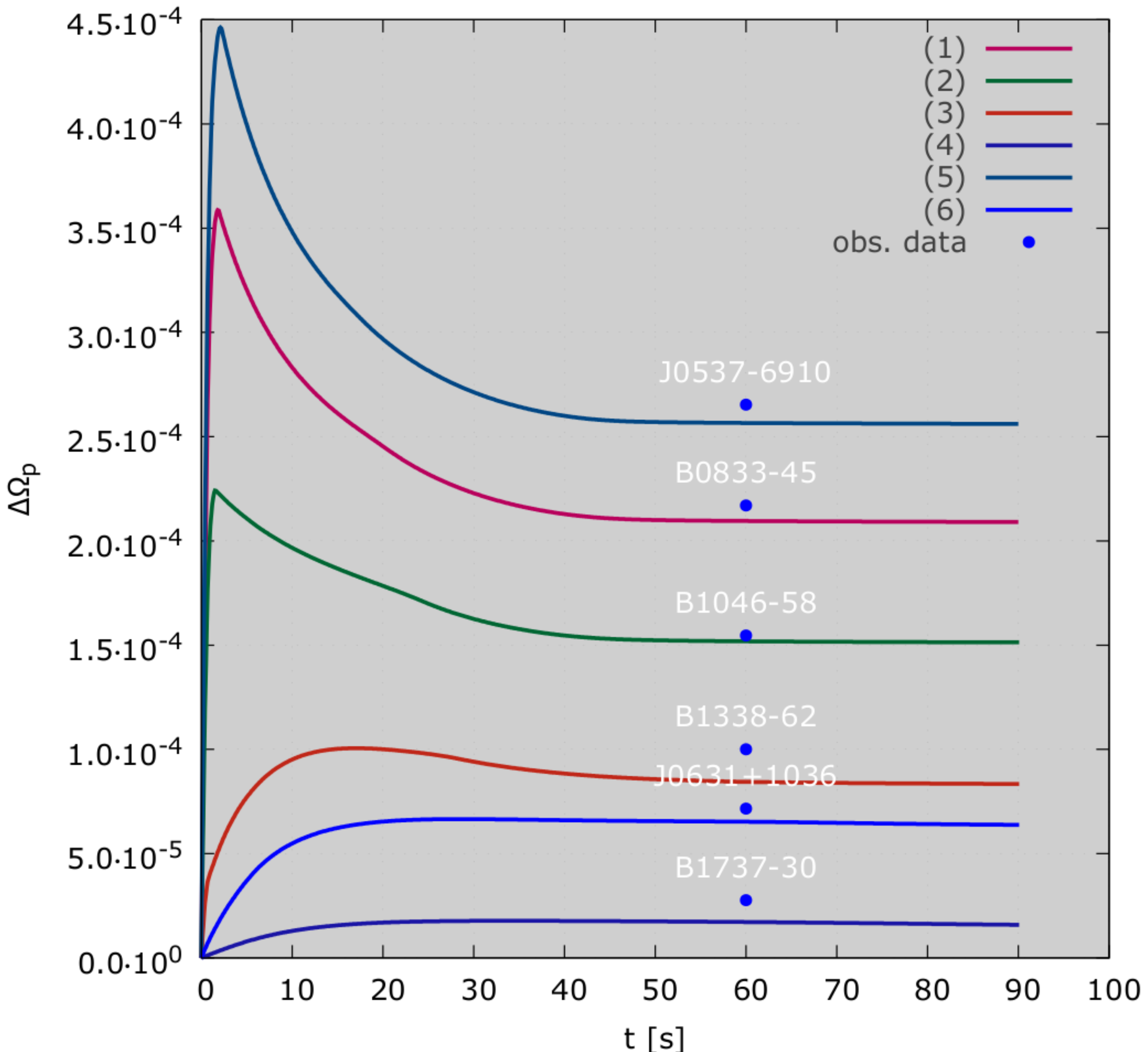
Important:
Large glitch → high mass

...why a **simple** but **consistent** model?

Dynamical simulations with:

- GM1 EOS
- EOS consistent proton fractions
- Entrainment in the core & crust (Chame
- Consistent drag in the core
  (el. scattering, Alpar, Andersson...)
- Drag in the crust (phonons, Jones 1991)
- New MESOSCOPIC pinning forces
  (Seveso, Haskell, Pzzochero 2016)

Large glitch → low mass
(true for all the EOSs that we used)



Dynamical simulation: S. Seveso

Vortex-mediated glitch theory in a nut shell

When $f_M = f_P$ vortices are unbound from the lattice

$\Downarrow$

Outward motion of vortices toward the crust

$\Downarrow$

Angular momentum is quantized in vortices

$\Downarrow$

Vortices give their angular momentum to the crust

$\Downarrow$

Glitch!