Uniwersytet Wrocławski

simultaneous chiral symmetry restoration and deconfinement - consequences for the QCD phase diagram -

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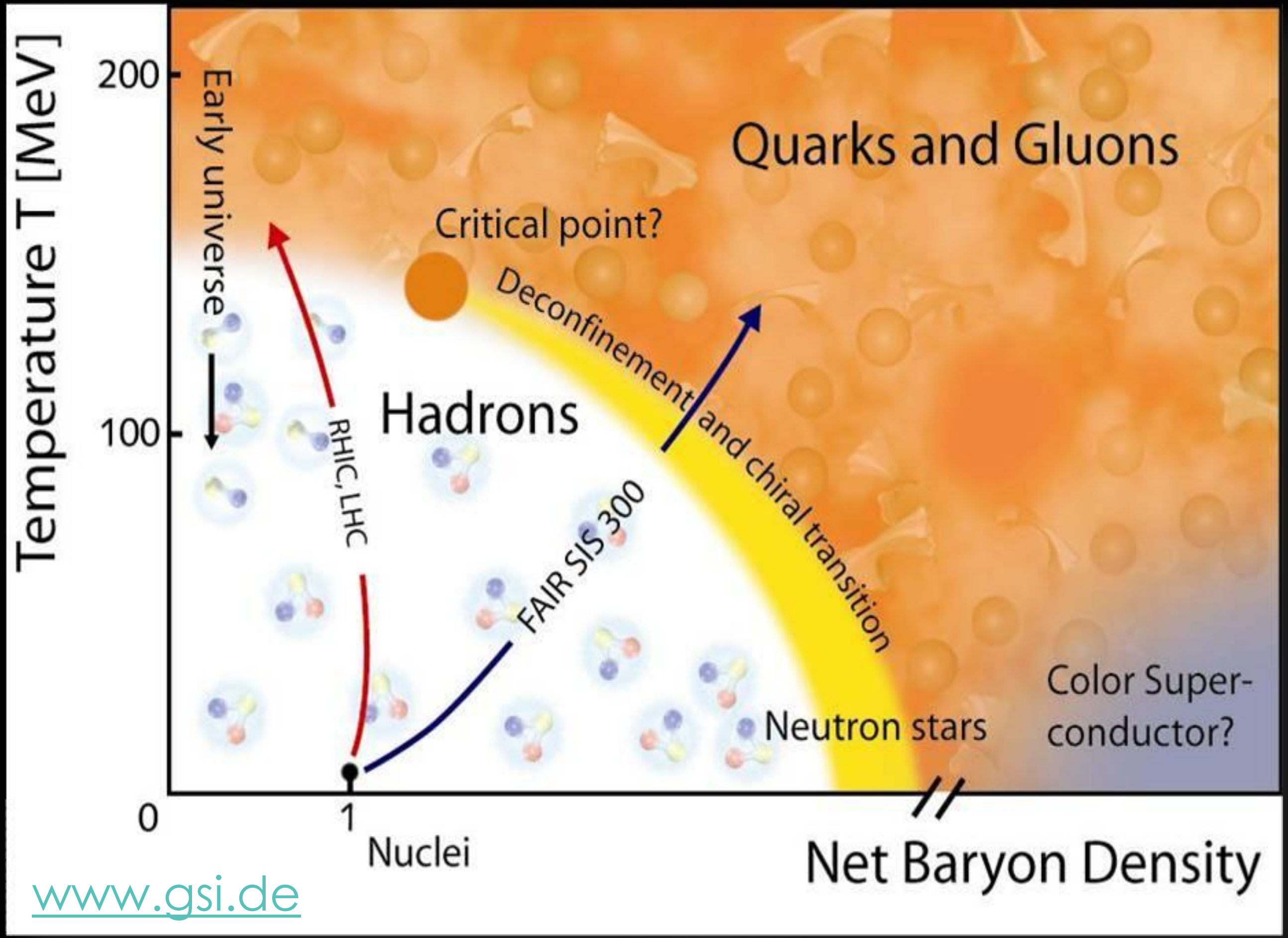
QCD Phase Diagram

- **dense hadronic matter**

<u>HIC in collider experiments</u>
Won't cover the whole diagram
Hot and 'rather' symmetric

<u>NS as a 2nd accessible option</u>
Cold and 'rather' asymmetric

Problem is more complex than
It looks at first gaze



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QCD Phase Diagram

- **dense hadronic matter**

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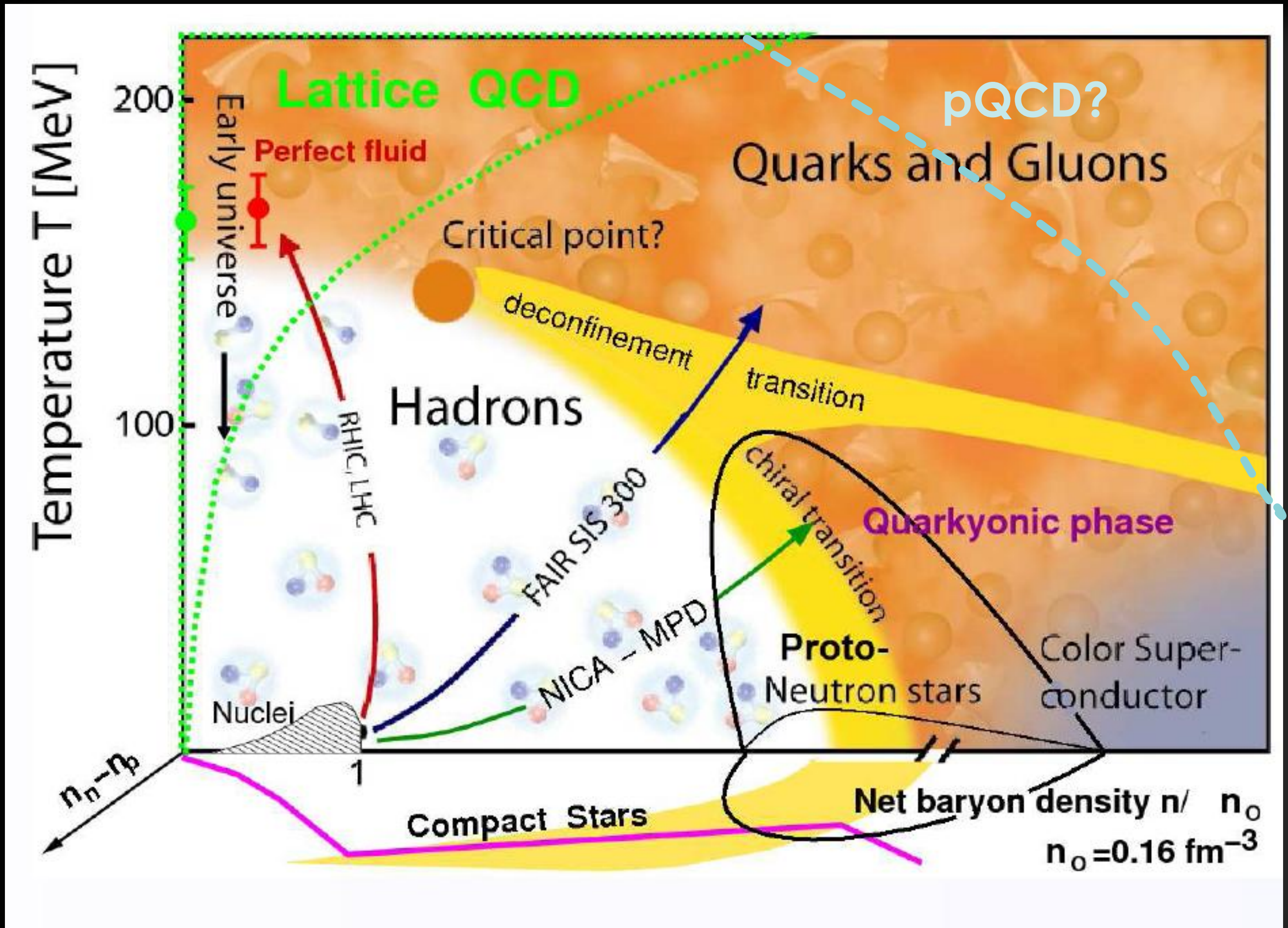


1st order phase transition observable in neutrino signal

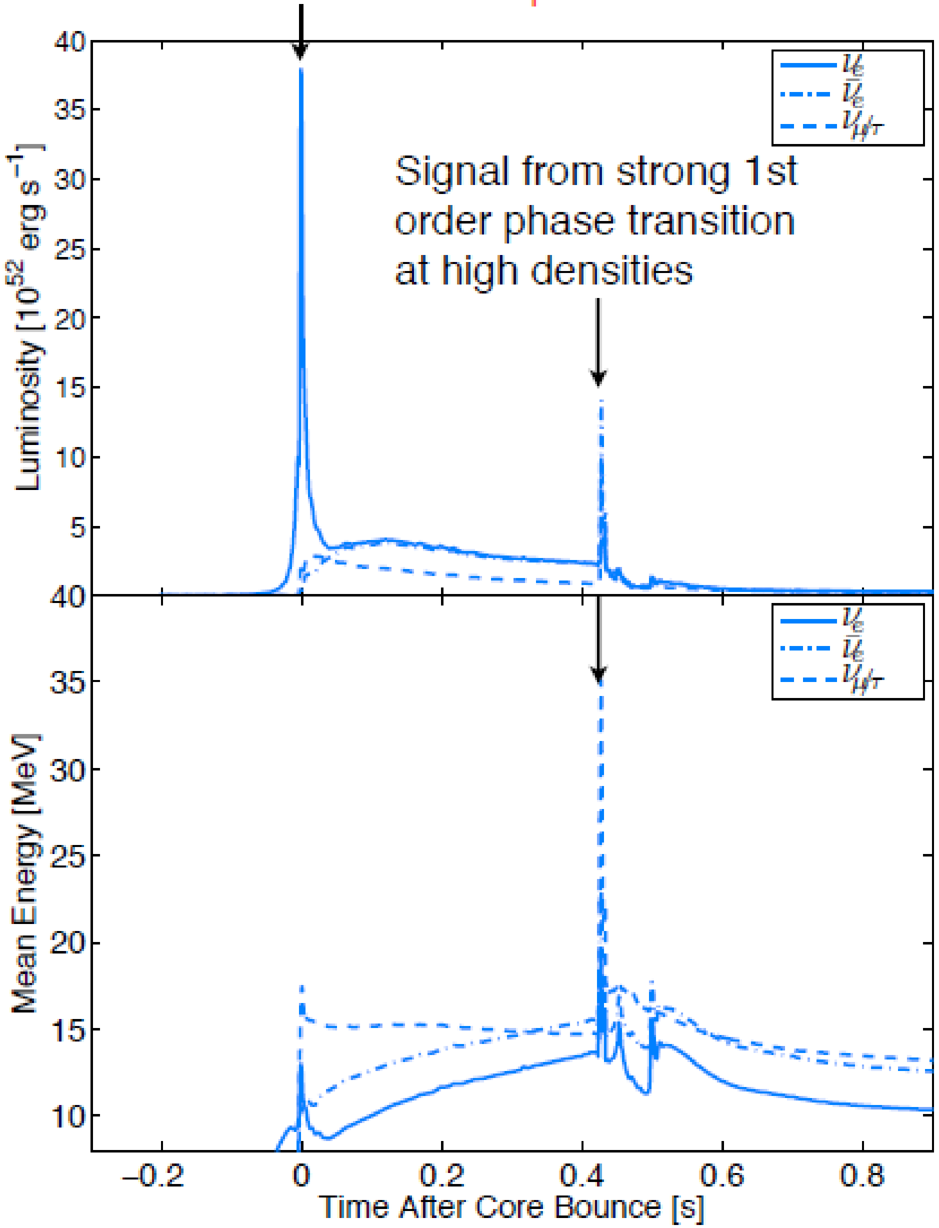
High mass NSs do not rule out QM cores

They are no evidence neither.

General problem:
Which observable would convince that QCD phase transition happens in nature?

Fischer et al. →

Standard feature: deleptonization burst form core bounce



1st neutrino burst shortly after core bounce, deleptonization burst, standard feature in all supernova models

Hard to detect because it comes in ν_e

2nd burst due to 2nd-shock propagation across neutrinospheres, dominated by:

$$\bar{\nu}_e \quad (\nu_{\mu/\tau}, \bar{\nu}_{\mu/\tau})$$

Neutrinos are emitted locally and come from low densities (hadronic phase)

2nd burst last only few milliseconds

Accompanied by significant rise of average neutrino energies

Observable for currently operating neutrino-detector facilities

Dasgupta & T.F. et al. (2010), PRD 81, 103005

Problem: Violation of current constraints from astrophysics

High mass NSs do not rule out QM cores

They are no evidence neither.

General problem:
Which observable would be convincing that QCD phase transition happens in nature?

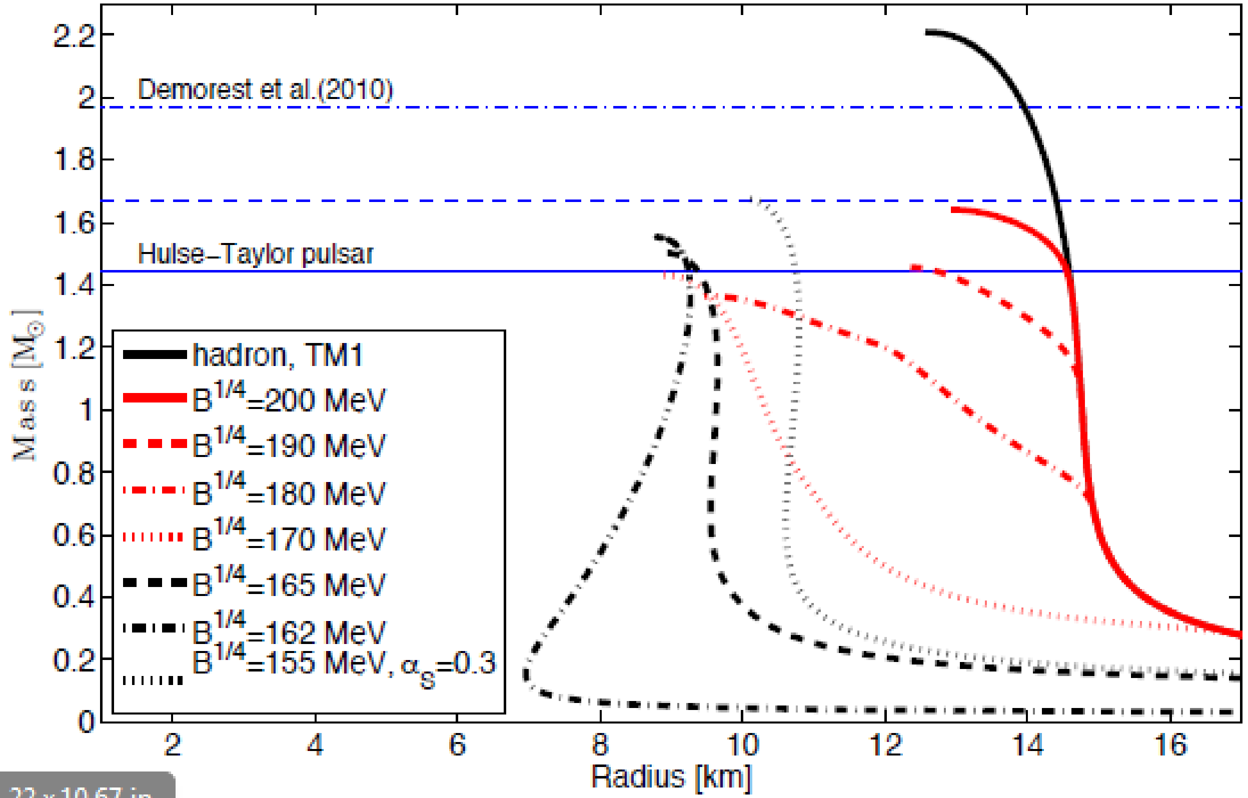
Fischer et al. →

Demorest et al. (2010), Nature 09466, **J1614-2230**

Antoniadis et al. (2013), Science 340, 448, **J1614-2230**

Steiner et al. (2010), ApJ 722 (Bayesian analysis of few selected low-mass X-ray binary systems)

$$M_{\rm max} = (1.97 - 2.01) \pm 0.04\ {\rm M}_\odot$$
$$R|_{M=1.4\ {\rm M}_\odot} = 12 \pm 1\ {\rm km}$$



14.22 x 10.67 in

All quark-bag hybrid EOS tested are ruled out !

Quark Matter

What is so special about quarks?

Confinement: No isolated quark has ever been observed
Quarks are confined in baryons and mesons

Dynamical Mass Generation:

Proton 940 MeV, 3 constituent quarks with each 5 MeV
→ 98.4% from .... somewhere?

and then this:
eff. quark mass in proton: 940 MeV/3 ≈ 313 MeV
eff. quark mass in pion : 140 MeV/2 = 70 MeV

quark masses generated by interactions only
‚out of nothing'
interaction in QCD through (self interacting) gluons
dynamical chiral symmetry breaking (DCSB)
is a distinct nonperturbative feature!

Confinement and DCSB are connected. Not trivially seen from QCD Lagrangian.
Investigating quark-hadron phase transition requires nonperturbative approach.

Quark Matter

Confinement and DCSB are features of QCD.
It would be too nice to account for these phenomena when describing QM in Compact Stars...

Current approaches mainly used to describe dense, deconfined QM:

Bag-Model :
While Bag-models certainly account for confinement (constructed to do exactly this) they do not exhibit DCSB (quark masses are fixed - bare quark masses).

Chodos, Jaffe et al: Baryon Structure (1974)
Farhi, Jaffe: Strange Matter (1984)

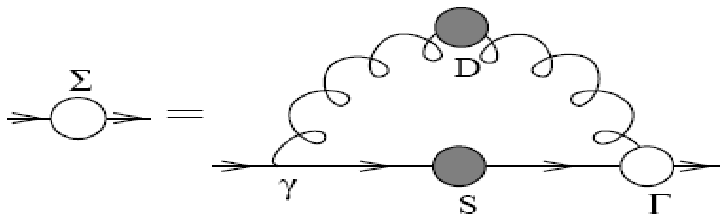
NJL-Model :
While NJL-type models certainly account for DCSB (applied, because they do) they do not (trivialy) exhibit confinement.

Nambu, Jona-Lasinio (1961)

Modifications to address confinement exist (e.g. PNJL) but are not entirelly satisfying
Both models: Inspired by, but not originally based on QCD.

Lattice QCD still fails at T=0 and finite μ

Dyson-Schwinger Approach
Derive gap equations from QCD-Action. Self consistent self energies.
Successfully applied to describe meson and baryon properties
Extension from vacuum to finite densities desirable
→ EoS within QCD framework



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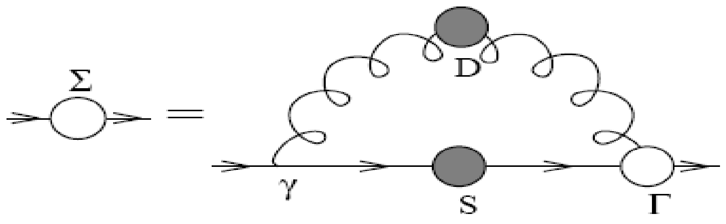
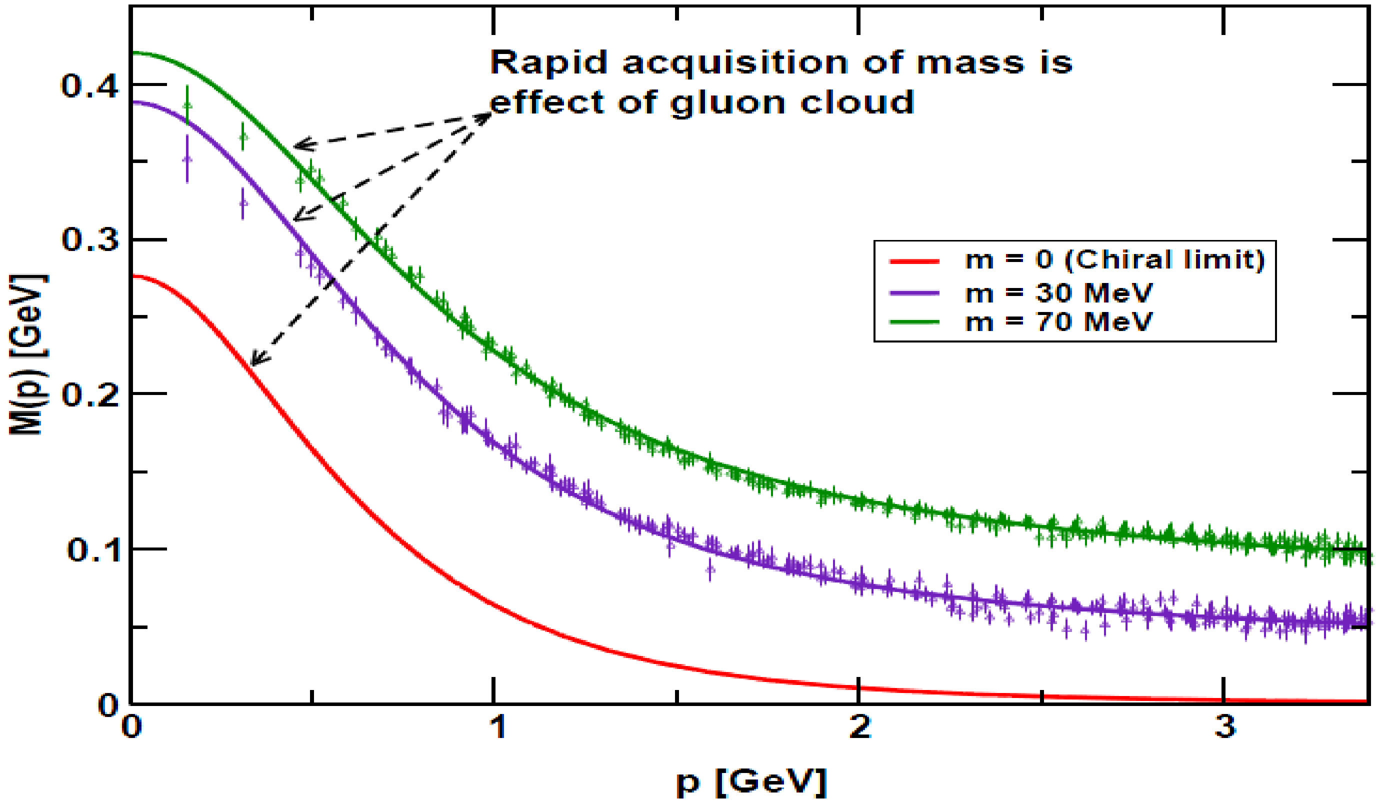
Derive gap equations from QCD-Action. Self consistent self energies.
Successfully applied to describe meson and baryon properties
Extension from vacuum to finite densities desirable
→ EoS within QCD framework
→ **THIS TALK: Bag and NJL model as simple limits within DS approach**



DSE : dynamical, momentum dependent mass generation



momentum dep. (here @ T=μ=0)
LQCD as benchmark

Neither NJL nor BAG have this

How do momentum dependent gap solutions affect
- EoS of deconfined quark matter?
- EoS of confined quark matter?
- transport properties in medium?

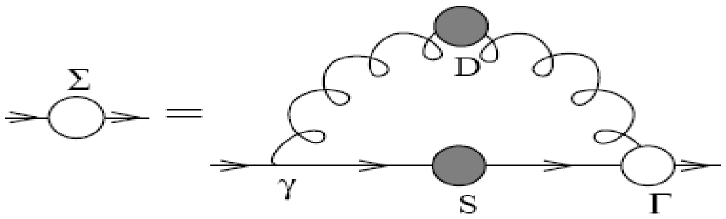
Roberts (2011)
Bhagwat et al. (2003,2006,2007)
P. O. Bowman et al. (2005)

Bag model: bare quark mass at all momenta and densities
NJL model: dressed quark mass at all momenta, changing dynamically with chemical potential

Dyson Schwinger Perspective

One particle gap equation(s)

$$S^{-1}(p;\mu) = i\vec{\gamma}\vec{p} + i\gamma_4(p_4 + i\mu) + m + \Sigma(p;\mu)$$



Self energy -> entry point for simplifications

$$\Sigma(p;\mu) = \int_\Lambda \frac{d^4q}{(2\pi)^4} g^2 D_{\rho\sigma}(p-q)\gamma_\rho \frac{\lambda^a}{2} S(q)\Gamma^a_\sigma(p;q)$$

General (in-medium) gap solutions

$$S^{-1}(p;\mu) = i\vec{\gamma}\vec{p}A(p;\mu) + i\gamma_4(p_4 + i\mu)C(p;\mu) + B(p;\mu)$$

Effective gluon propagator

$$S(p;\mu)^{-1} = Z_2(i\,\vec{\gamma}\,\vec{p} + i\,\gamma_4(p_4 + i\mu) + m_{\mathrm{bm}}) + \Sigma(p;\mu)$$

$$\Sigma(p;\mu) = Z_1 \int_q^{\Lambda} g^2(\mu) D_{\rho\sigma}(p-q;\mu) \frac{\lambda^a}{2} \gamma_\rho S(q;\mu) \Gamma_\sigma^a(q,p;\mu)$$

Ansatz for self energy (rainbow approximation, effective gluon propagator(s))

$$Z_1 \int_q^{\Lambda} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma_\nu^a(q,p) \to \int_q^{\Lambda} \mathcal{G}((p-q)^2)\, D_{\mu\nu}^{\mathrm{free}}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \frac{\lambda^a}{2} \gamma_\nu$$

Specify behaviour o $\mathcal{G}(k^2)$

$$\frac{\mathcal{G}(k^2)}{k^2} = 8\pi^4 D \delta^4(k) + \frac{4\pi^2}{\omega^6} D k^2 \mathrm{e}^{-k^2/\omega^2} + 4\pi \frac{\gamma_m \pi}{\frac{1}{2} \ln\left[\tau + \left(1 + k^2/\Lambda_{\mathrm{QCD}}^2\right)^2\right]} \mathcal{F}(k^2)$$

Infrared strength (zero width + finite width contribution)

running coupling for large k

EoS (finite densities):

1st term (Munczek/Nemirowsky (1983)) delta function in momentum space → Klähn et al. (2010)

2nd term → Chen et al.(2008,2011)

NJL model: $g^2 D_{\rho\sigma}(p-q) = \frac{1}{m_G^2} \delta_{\rho\sigma}$ delta function in configuration space = const. In mom. space

Munczek/Nemirowsky -> NJL's complement

Wigner Phase $\frac{\mathcal{G}(k^2)}{k^2} = 8\pi^4 D\delta^4(k) + \frac{4\pi^2}{\omega^6} D k^2 \mathrm{e}^{-k^2/\omega^2} + 4\pi \frac{\gamma_m \pi}{\frac{1}{2}\ln\left[\tau + \left(1 + k^2/\Lambda^2_{\mathrm{QCD}}\right)^2\right]} \mathcal{F}(k^2)$

$B_W = 0, A_W = C_W$:

$$C_W(p,\mu) = \frac{1}{2}\left(1 + \sqrt{1 + \frac{2\eta^2}{p_3^2 + (p_4 + i\mu)^2)}}\right)$$

Nambu Phase

$A_N = C_N$.

$\Re(\tilde{p}^2) < \frac{\eta^2}{4}$:

$$\begin{aligned} B_N(p,\mu) &= \sqrt{\eta^2 - 4(p_3^2 + (p_4 + i\mu)^2))} \\ C_N(p,\mu) &= 2 \end{aligned}$$

$\Re(\tilde{p}^2) > \frac{\eta^2}{4}$:

$$A_N = A_W, B_N = B_W, C_N = C_W.$$

<u>MN antithetic to NJL</u>

NJL:contact interaction in x

MN:contact interaction in p

(background field in x)

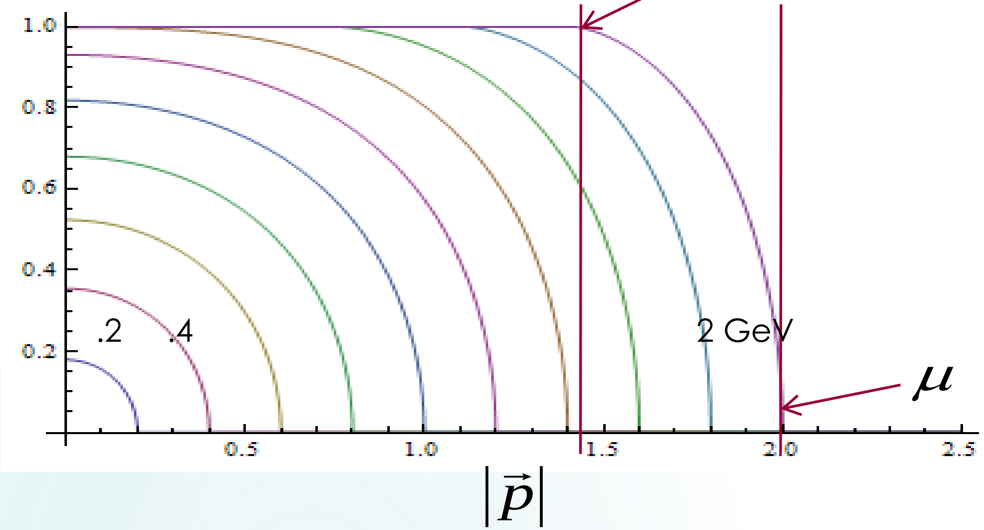
Munczek/Nemirowsky

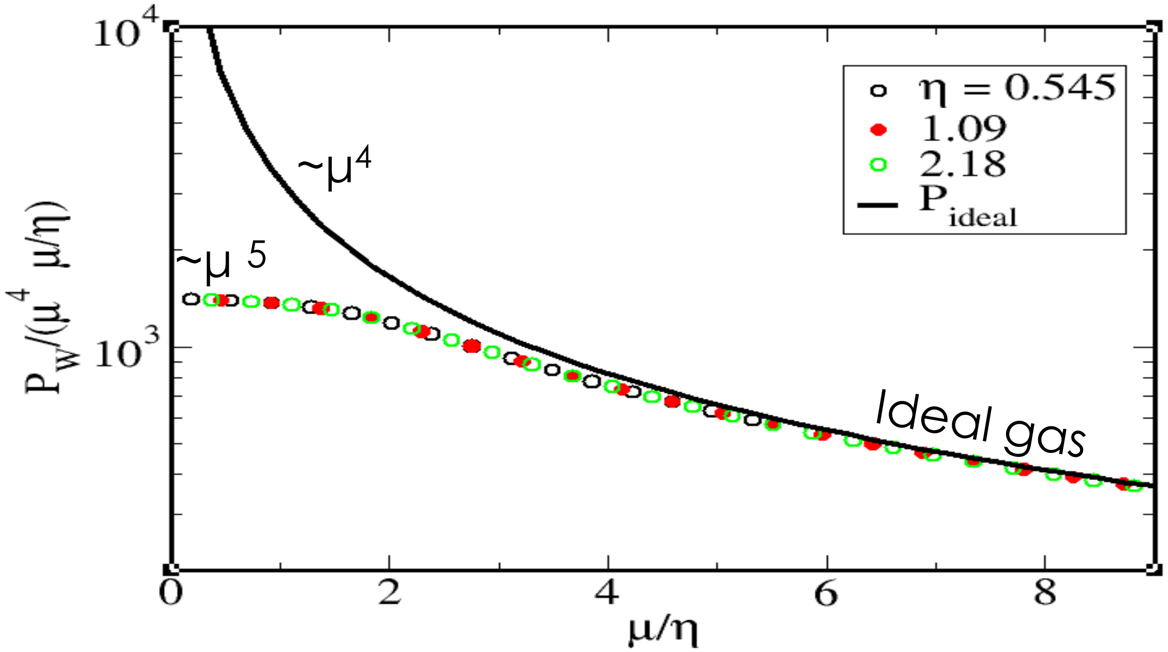
$$f_1(|\vec{p}|;\mu) = \frac{1}{4\pi}\int_{-\infty}^{\infty} dp_4\, \mathrm{tr_D}(-\gamma_4) S(p;\mu)$$

$$\boxed{P(\mu<\eta) = P_0 + \int_0^{\mu} d\mu'\, n(\mu') \propto P_0 + const \times \mu^5}$$

Wigner Phase

$\vec{p}^{\,2} = \mu^2 - 2\eta^2$





$\mu^2 \geq 2\eta^2$ to obtain $f_1(\vec{p}^{\,2}=0)=1$ model is scale invariant regarding μ/η

$P(\mu) \propto \mu^5$ well satisfied up to $\mu/\eta \approx 1$

$(\eta = 1.09\,\mathrm{GeV})$

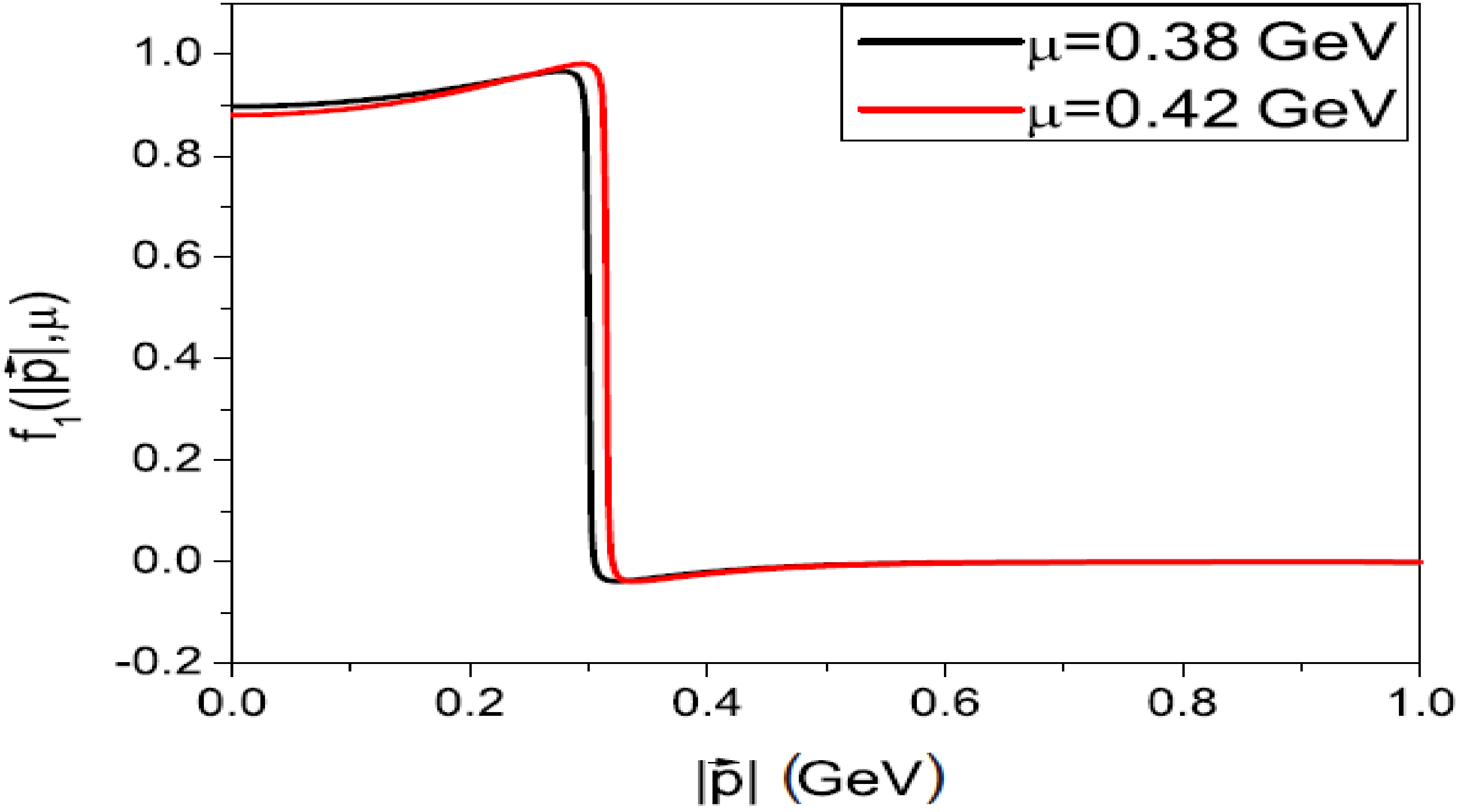
‚small‘ chem. Potential: $f_1(\vec{p}^{\,2}=0, \mu<\eta) \propto \mu \leftarrow$ $\boxed{n(\mu<\eta) = \frac{2N_c N_f}{2\pi^2}\int d^3\vec{p}\ f_1(|\vec{p}|) \propto \mu^4}$

T. Klahn, C.D. Roberts, L. Chang, H. Chen, Y.-X. Liu PRC 82, 035801 (2010)

DSE – simple effective gluon coupling

$$\frac{\mathcal{G}(k^2)}{k^2} = 8\pi^4 D\delta^4(k) + \frac{4\pi^2}{\omega^6} D k^2 \mathrm{e}^{-k^2/\omega^2} + 4\pi \frac{\gamma_m \pi}{\frac{1}{2}\ln\left[\tau + \left(1 + k^2/\Lambda_{\mathrm{QCD}}^2\right)^2\right]} \mathcal{F}(k^2)$$

Wigner Phase Less extreme, but again, 1particle number density distribution different from free Fermi gas (quasi particle) distribution



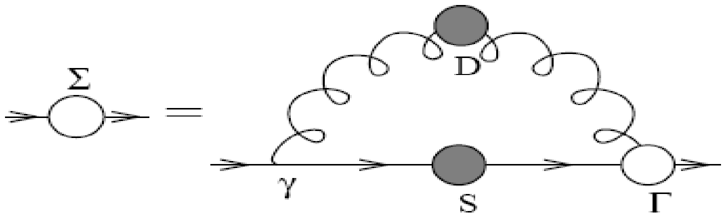
Chen et al. (TK) PRD 78 (2008)

DSE -> NJL model

$$g^2 D_{\rho\sigma}(p-q) = \frac{1}{m_G^2}\delta_{\rho\sigma},$$
$$\Gamma_\rho^a(p;q) = \frac{\lambda^a}{2}\gamma_\rho.$$

Gluon contact interaction in configuration space (other models exist)

Rainbow approximation



$$\vec{p}^2 A_p = \vec{p}^2 + \frac{8N_c}{9m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{\vec{p}\vec{q}A_q}{\vec{q}^2A_q^2+\widetilde{q}_4^2C_q^2+B_q^2},$$

$$A = 1$$

$$B_p = m + \frac{16N_c}{9m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{B_q}{\vec{q}^2A_q^2+\widetilde{q}_4^2C_q^2+B_q^2},$$

$$B_\mu = m + \frac{4N_c}{9m_G^2}n_s(T,\mu^*,B),$$

$$\widetilde{p}_4^2 C_p = \widetilde{p}_4^2 + \frac{8N_c}{9m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{\widetilde{p}_4\widetilde{q}_4C_q}{\vec{q}^2A_q^2+\widetilde{q}_4^2C_q^2+B_q^2},$$

$$\mu = \mu^* - \frac{2N_c}{9m_G^2}n_v(T,\mu^*,B),$$

$$\widetilde{p}_4 C = p_4 + i(\mu+\omega_\mu) \equiv \hat{p}_4$$

Thermodynamical Potential

DS: steepest descent

$$P[S] = \mathrm{Tr}\ln[S^{-1}] - \frac{1}{2}\mathrm{Tr}[\Sigma S].$$

$$P_{FG} = \mathrm{Tr}\ln S^{-1} = 2N_c \int_\Lambda \frac{d^4p}{(2\pi)^4} \ln(\vec{p}^2 + \hat{p}_4^2 + B_\mu^2)$$

$$P_I = -\frac{1}{2}\mathrm{Tr}\Sigma S = \frac{3}{4} m_G^2 \omega_\mu^2 - \frac{3}{8} m_G^2 \phi_\mu^2$$

Compare to NJL type model with following Lagrangian (interaction part only):

$$\mathcal{L}_\mathrm{I} = \mathcal{L}_\mathrm{S} + \mathcal{L}_\mathrm{V} = G_s \sum_{a=0}^{8} (\bar{q}\tau_a q)^2 + G_v (\bar{q} i \gamma_0 q)^2.$$

$$\Omega_q = \Omega_q^0 + \frac{\phi^2}{4G_s} - \frac{\omega^2}{2G_v} - \Omega_q(T = \mu = 0)$$

$$\phi_\mu = \quad 2G_s N_c n_s(T, m_f^*, \mu_f^*)$$

$$\omega_\mu = -2G_s N_c n_v(T, m_f^*, \mu_f^*)$$

$$\frac{\partial \Omega_q}{\partial \phi_\mu} = \frac{\partial \Omega_q}{\partial \omega_\mu} = 0.$$

Thermodynamical Potential

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$$P[S] = \mathrm{Tr}\ln[S^{-1}] - \frac{1}{2}\mathrm{Tr}[\Sigma S].$$

NJL model is easily understood as a particular approximation of QCD's DS gap equations

$$P_{FG} = \mathrm{Tr}\ln S^{-1} = 2N_c \int_\Lambda \frac{d^4p}{(2\pi)^4} \ln(\vec{p}^2 + \hat{p}_4^2 + B_\mu^2)$$

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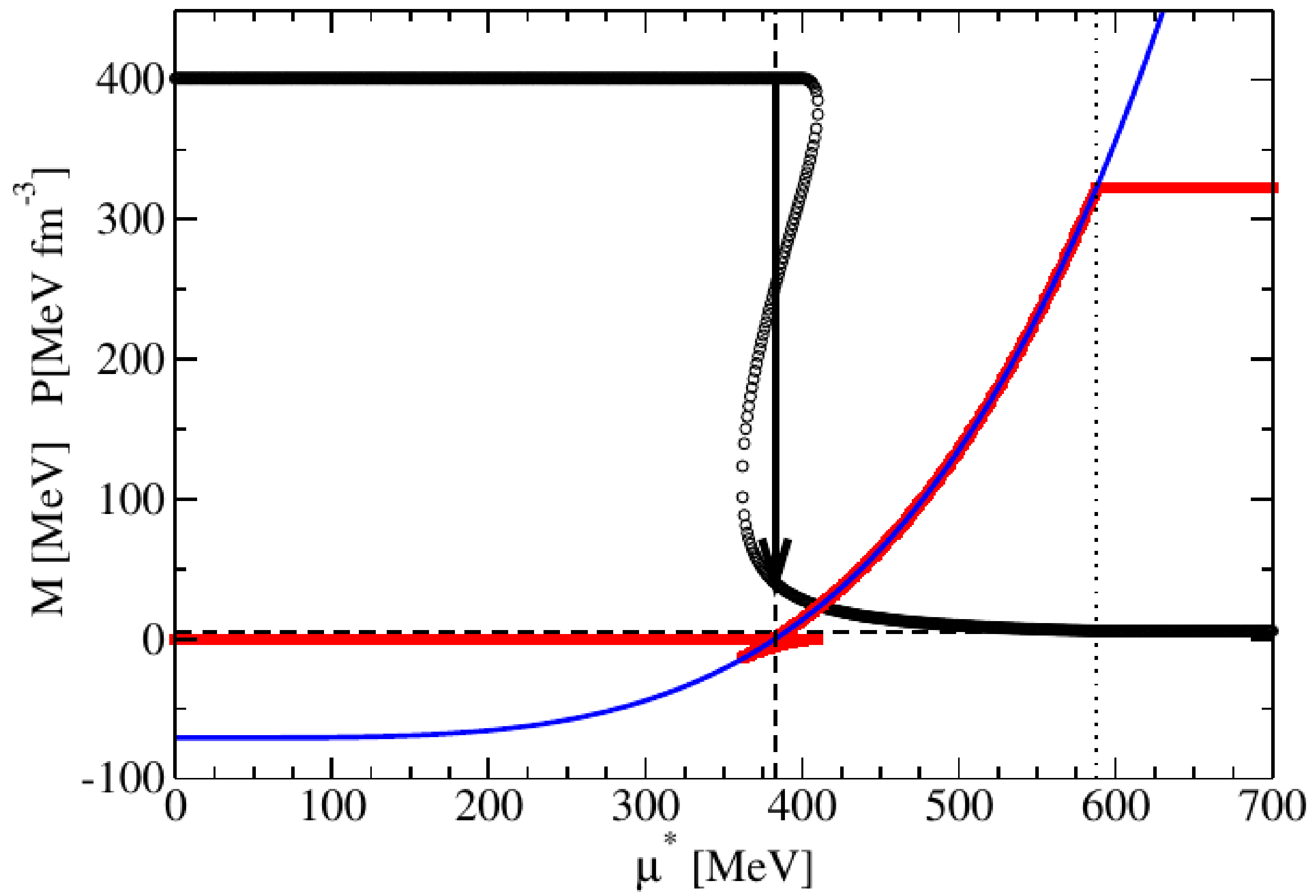
Bag Model from NJL perspective (TK, T.Fischer, ApJ , 2015)

obvious differences between NJL and Bag:

- **DχSB**
- confinement
- vector interaction



u,d-quark

Mass

Pressure NJL

Pressure Ideal Gas - Bag

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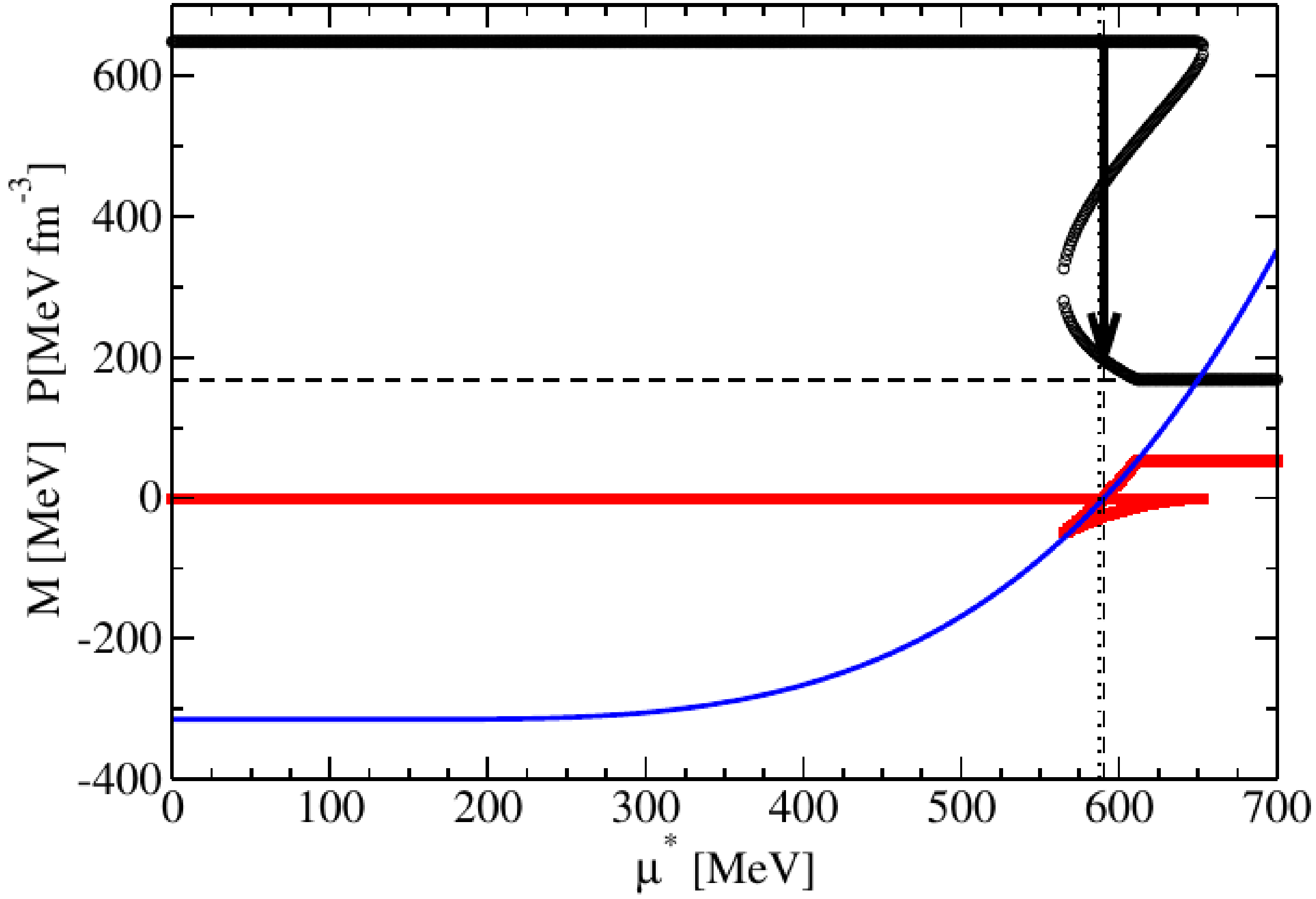

s-quark

Mass

Pressure NJL

Pressure Ideal Gas - Bag

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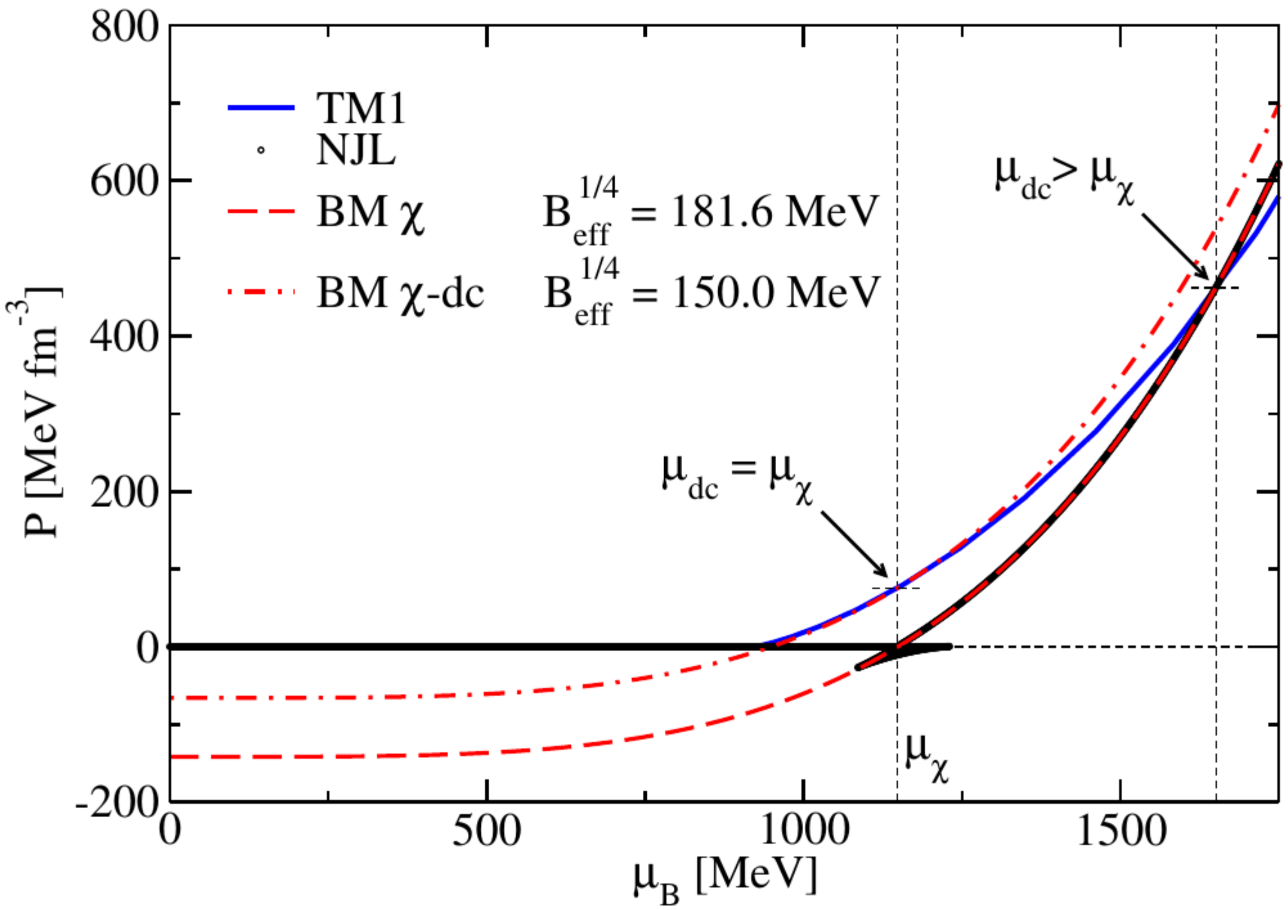

confinement

Pressure Quark NJL/Bag

Pressure Nuclear Matter

Pressure not zero at χ transition

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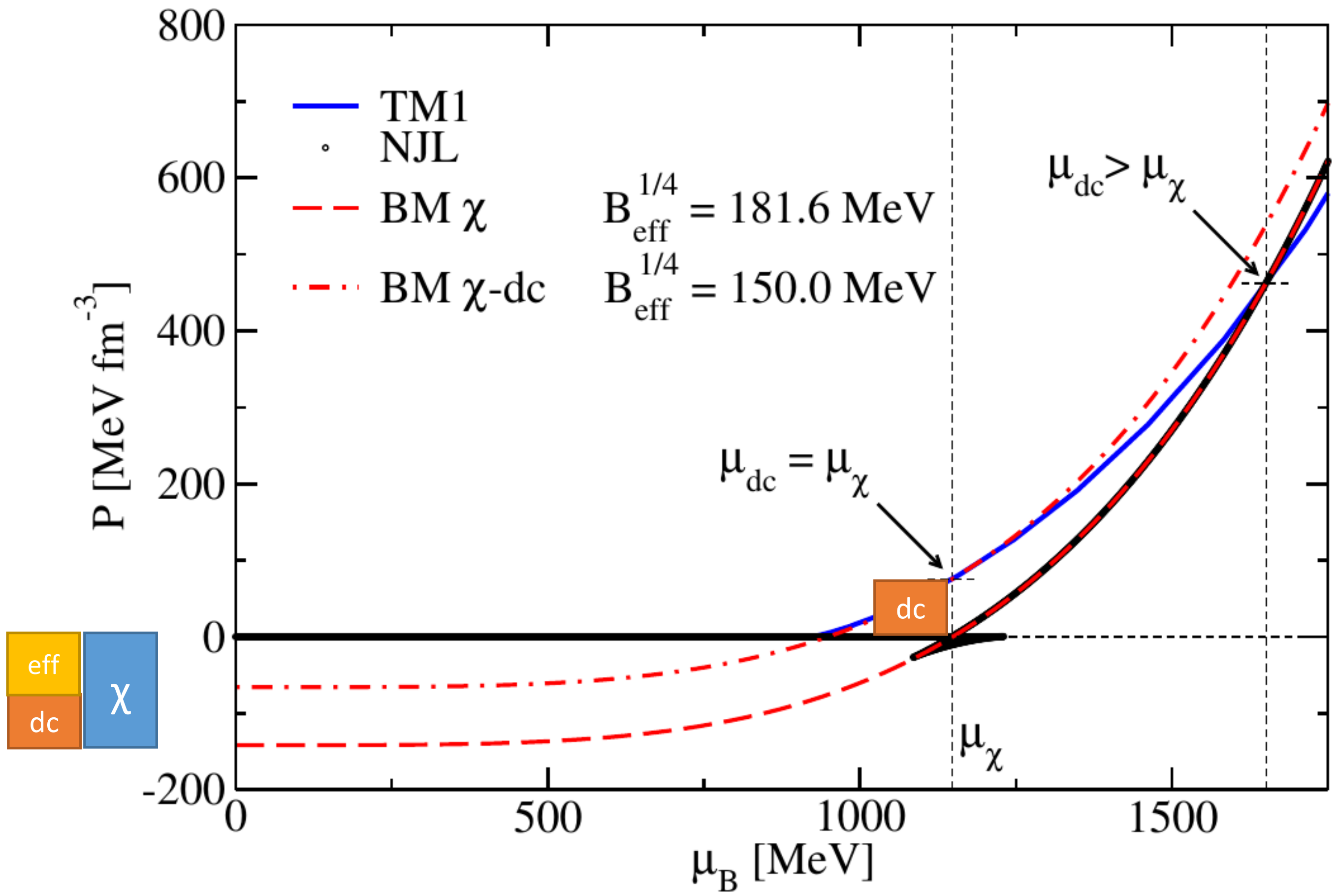

confinement

Pressure Quark NJL/Bag

Pressure Nuclear Matter

Pressure not zero at χ transition

Reduce χ bag pressure to match to nuclear EoS

Bag Model from NJL perspective

obvious differences between NJL and Bag:

- DχSB
- confinement
- **vector interaction**

$$B_\mu = m + \frac{4N_c}{9m_G^2} n_s(T,\mu^*,B),$$

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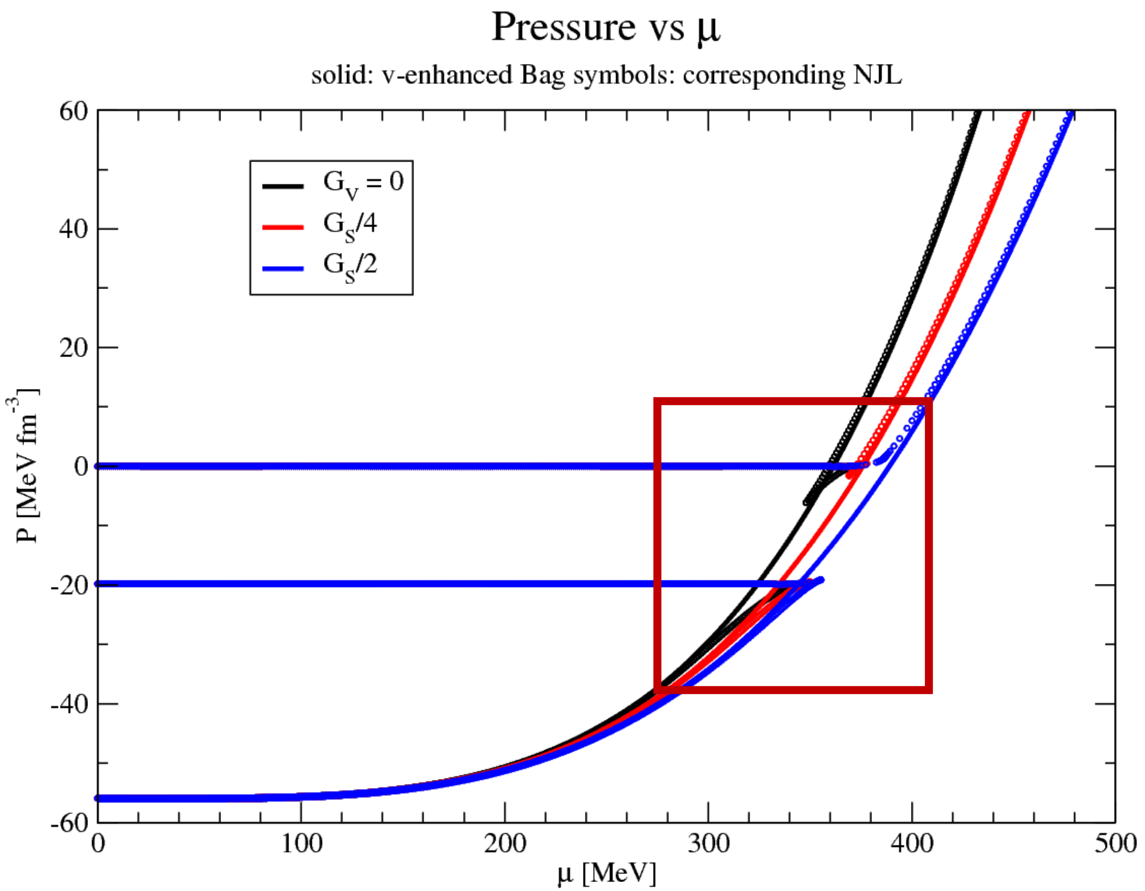
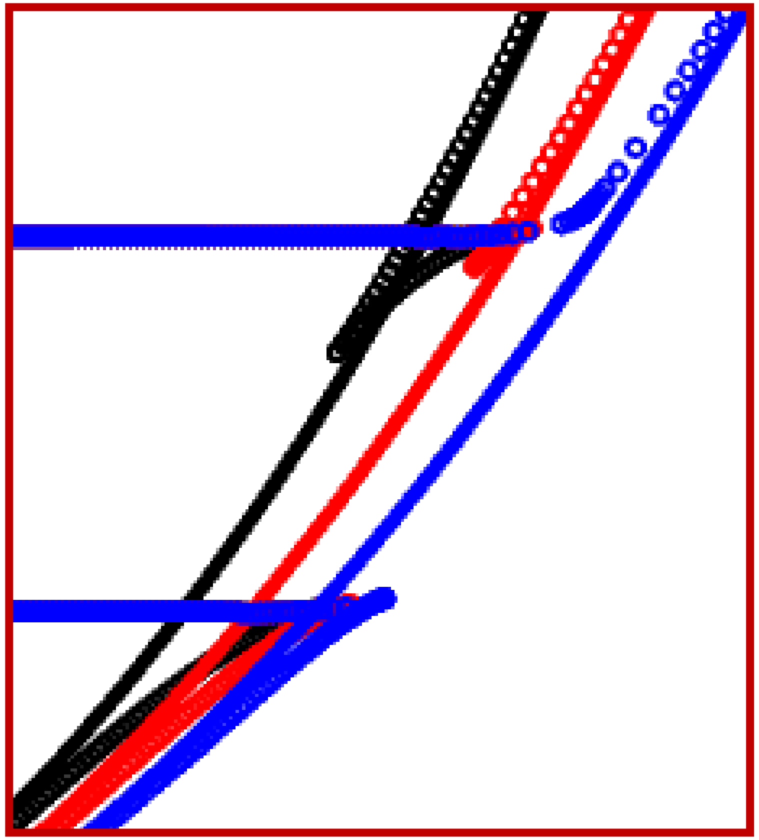




vBag: vector interaction enhanced bag model

Chiral + Vector:

$$P^i_{BM}(\mu_i) = P_{kin}(\mu_i^*) + \frac{K_v}{2} n_v^2(\mu_i^*) - P^i_{BAG}$$

$$\varepsilon^i_{BM}(\mu_i) = \varepsilon_{kin}(\mu_i^*) + \frac{K_v}{2} n_v^2(\mu_i^*) + P^i_{BAG}$$

$$\mu_i = \mu_i^* + K_v n_v(T, \mu_i^*)$$

'Confinement':

$$P = \sum_f P_f^{kin} - B_{eff} \text{ with } B_{eff} = \sum_f B_\chi^f - B_{\text{dc}}$$

And, of course, chiral+vector+'confinement' (Klahn & Fischer arXiv:1503.07442 ApJ 2015)

Conclusions Part I

Vector enhanced bag like model can be motivated from NJL - which can be obtained from DS gap equations

Bag model character: bare quark masses
effective <u>bag pressure</u>

Difference: <u>chiral</u> bag pressure as consequence of DχSB, flavor dependence
<u>confining</u> bag pressure with opposite sign (binding energy)
accounts for vector interaction -> stiff EoS, promising for astrophysical applications

What NJL couldn't: reduced chiral bag pressure due to confinement -> by hand, no harm to td consistence

Advantage of the model: extremely simple to use, no regularization required, Fermi gas expressions, bare masses
no (obvious) gap equation

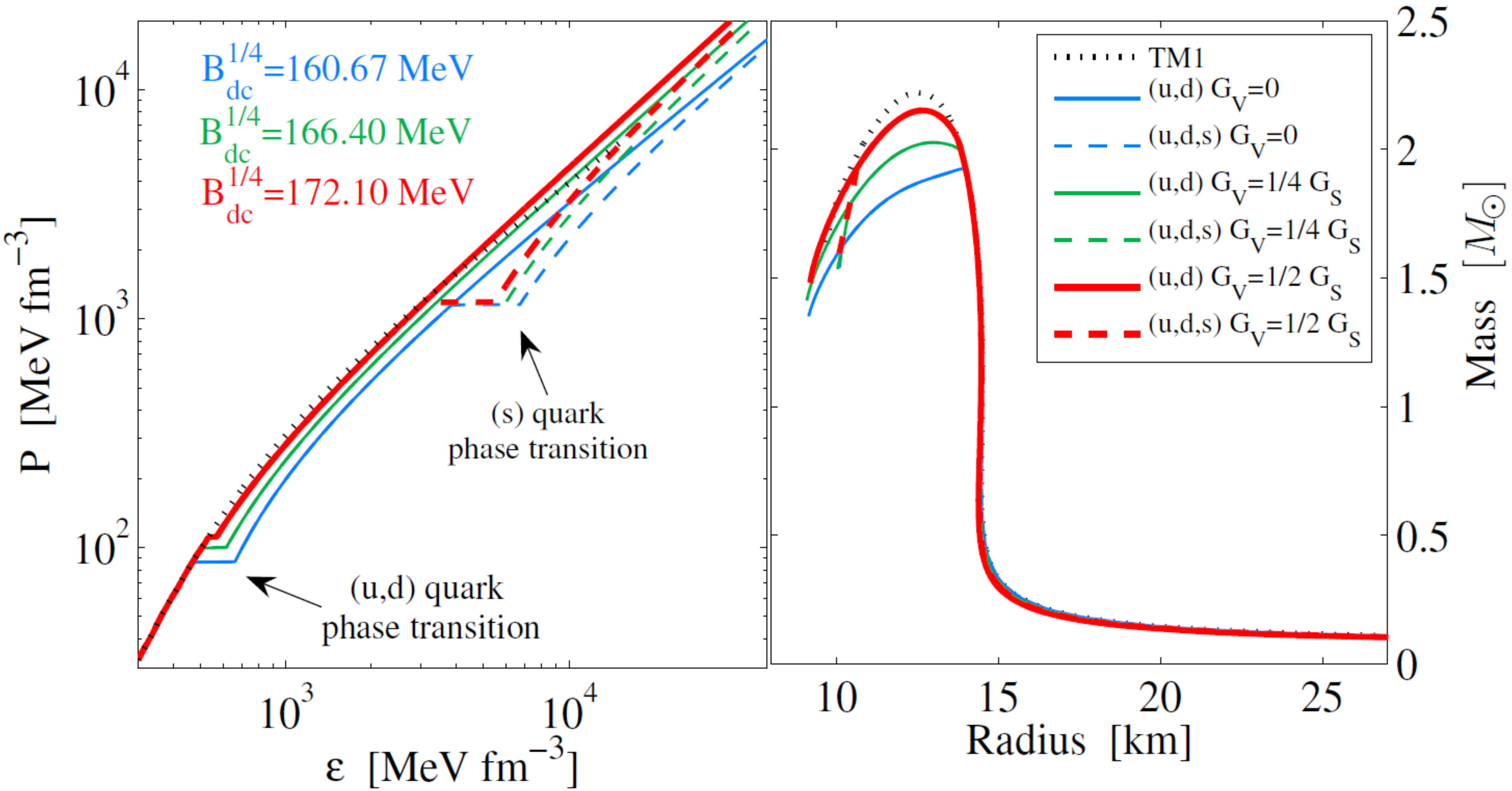
$$P^i_{BM}(\mu_i) = P_{kin}(\mu_i^*) + \frac{K_v}{2} n_v^2(\mu_i^*) - P^i_{BAG}$$

$$P = \sum_f P_f^{kin} - B_{eff} \text{ with } B_{eff} = \sum_f B_\chi^f - B_{\rm dc}$$

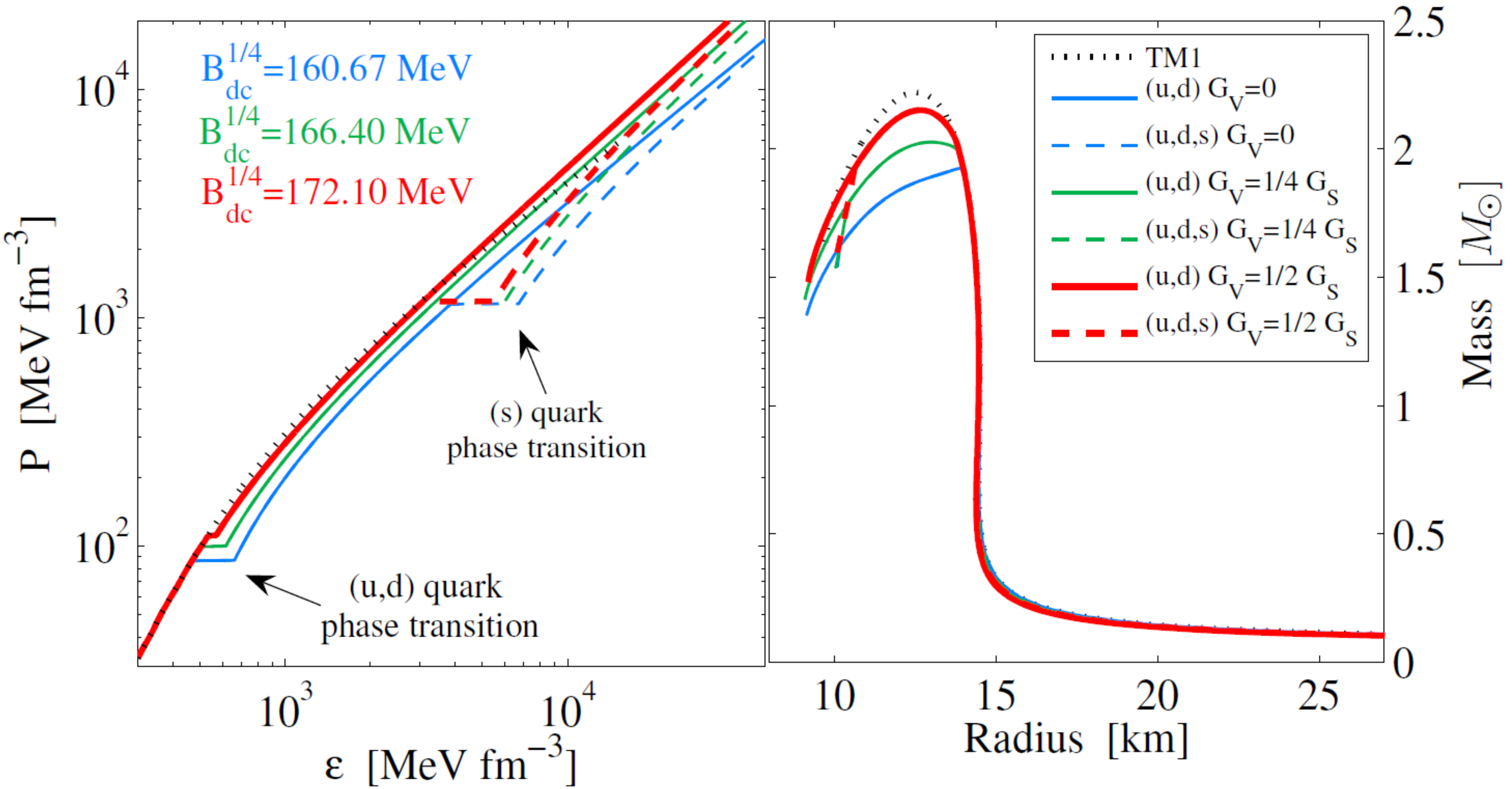
$$\varepsilon^i_{BM}(\mu_i) = \varepsilon_{kin}(\mu_i^*) + \frac{K_v}{2} n_v^2(\mu_i^*) + P^i_{BAG}$$

$$\mu_i = \mu_i^* + K_v n_v(T, \mu_i^*)$$

Neutron Stars with QM core – vBAG vs BAG



Neutron Stars with QM core – vBAG vs BAG



Absolutely Stable Strange Matter?

(very) brief review:

Three essential papers:

PHYSICAL REVIEW D VOLUME 9, NUMBER 12 15 JUNE 1974

New extended model of hadrons*

A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 25 March 1974)

We propose that a strongly interacting particle is a finite region of space to which fields are confined. The confinement is accomplished in a Lorentz-invariant way by endowing the finite region with a constant energy per unit volume , B. We call this finite region a "bag." The contained fields may be either fermions or bosons and may have any spin; they may or may not be coupled to one another. Equations of motion and boundary conditions are obtained from a variational principle. The confining region has no dynamical freedom but constrains the fields inside: There are no excitations of the coordinates determining the confining region. The model possesses many desirable features of hadron dynamics: (i) a parton

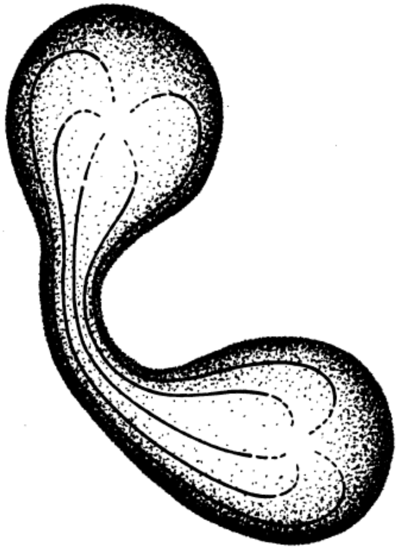


FIG. 1. A color-singlet bag attempting to fission into two bags which are not color singlets. The flux lines of the colored gluon field are shown explicitly.

Key assumptions: Bag is a given, massless colored quark and gluon fields, boundary conditions ensure confinement

Absolutely Stable Strange Matter?

(very) brief review:

Three essential papers:

PHYSICAL REVIEW D VOLUME 30, NUMBER 2 15 JULY 1984

Cosmic separation of phases

Edward Witten*
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 9 April 1984)

A first-order QCD phase transition that occurred reversibly in the early universe would lead to a surprisingly rich cosmological scenario. Although observable consequences would not necessarily survive, it is at least conceivable that the phase transition would concentrate most of the quark excess in dense, invisible quark nuggets, providing an explanation for the dark matter in terms of QCD effects only. This possibility is viable only if quark matter has energy per baryon less than 938 MeV. Two related issues are considered in appendices: the possibility that neutron stars generate a quark-matter component of cosmic rays, and the possibility that the QCD phase transition may have produced a detectable gravitational signal.

The average quark kinetic energy is proportional to μ, so (with a common pressure in the two cases) it is smaller in the three-flavor case by a factor

$$\tilde{\mu}/(\tfrac{1}{3}\mu+\tfrac{2}{3}2^{1/3}\mu)=[3/(1+2^{4/3})]^{3/4}\simeq 0.89 \ .$$

strange-quark mass will reduce this effect, but it is still plausible that strange quarks lower the energy per baryon of quark matter by 50–70 MeV per baryon. This is

Absolutely Stable Strange Matter?

(very) brief review:

Three essential papers:

PHYSICAL REVIEW D VOLUME 30, NUMBER 11 1 DECEMBER 1984

Strange matter

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(Received 9 May 1984)

We explore the properties of quark matter in equilibrium with the weak interactions, containing comparable numbers of up, down, and strange quarks. Witten has recently conjectured that this "strange matter" may be absolutely stable. Using a Fermi-gas model including $O(\alpha_c)$ corrections we establish the conditions under which strange matter in bulk is stable and describe its characteristics. Augmenting our model with surface-tension and Coulomb effects we study strange matter with intermediate baryon number, $10^2 \lesssim A \lesssim 10^7$. For low baryon numbers $A \lesssim 10^2$, we replace the Fermi gas by the bag model and study shell effects and the approach to the bulk limit. Finally, we discuss the phenomenology of strange matter in all its forms.

Absolutely Stable Strange Matter?

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Strange matter

In Sec. II, we investigate the properties of stable strange matter in bulk. Our study rests on several plausible assumptions. The first, as we have already mentioned, is that the system is well approximated by a Fermi gas separated from the vacuum by a phase boundary. We further assume that the effects of dynamical chiral-symmetry breakdown (e.g., dynamical quark masses, Goldstone pions) can be ignored in the quark gas so quarks are characterized by their current-algebra masses. Finally, we assume that the properties of the quark Fermi gas can be computed using renormalization-group-improved QCD perturbation theory. Unfortunately, at the momentum scale typical of the problem at hand (roughly $M_N/3$) α_c is not small. Other methods (e.g., lattice Monte Carlo simulations of QCD) may eventually yield information about quark matter; at present, perturbative QCD is the only tool available. Our study of strange matter in bulk is

Three important statements:

1. Limiting case of original (MIT) bag model (bag is filled with relativistic Fermi gas) -> thermodynamic bag model

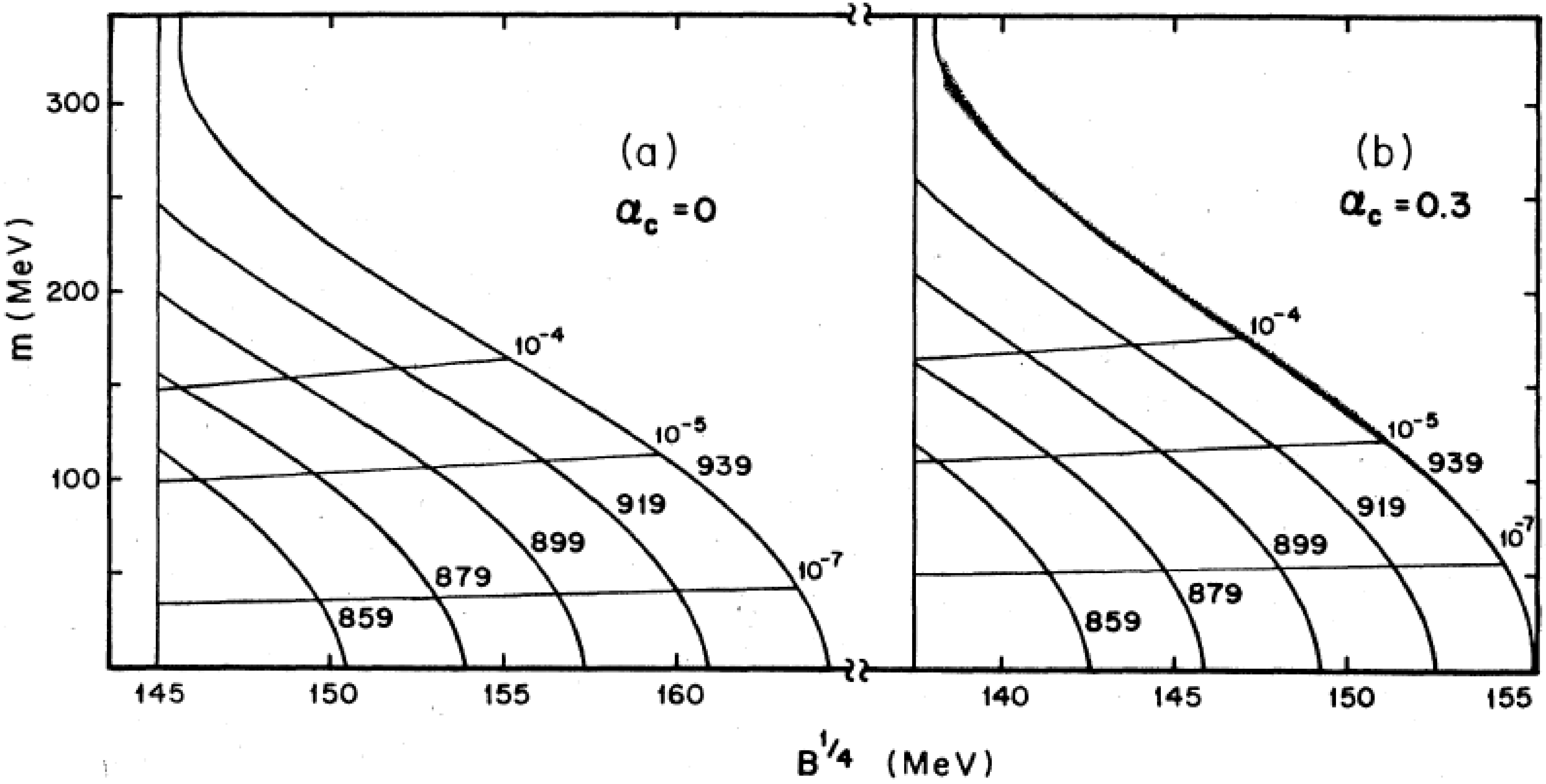
2. Chiral symmetry is restored bare quark masses

3. Perturbation theory applicable (more or less)

2. and 3. are related.

Absolutely Stable Strange Matter?

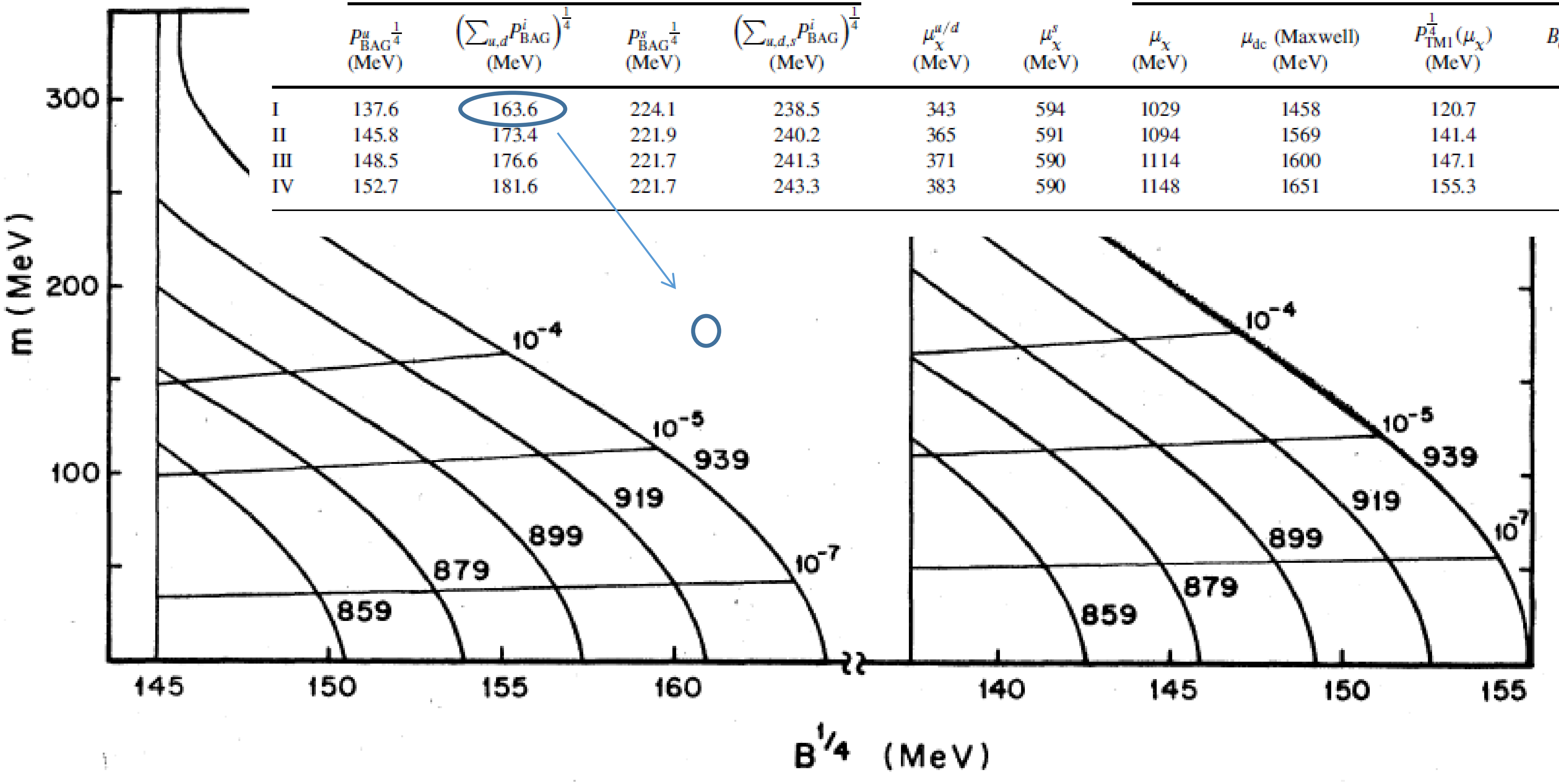
EDWARD FARHI AND R. L. JAFFE



Absolutely Stable Strange Matter?

Table 2
Single Flavor, Effective Two-flavor Chiral Bag Constants, and $\mu_{\chi/\mathrm{dc}}$ for the Parameterizations of Table 1

	Chiral Bag Model Parameters						Phase Transition TM1 $\rightarrow$ 2f QM (symmetric)			
	${P^u_{\mathrm{BAG}}}^{\frac{1}{4}}$ (MeV)	$\left(\sum_{u,d} P^i_{\mathrm{BAG}}\right)^{\frac{1}{4}}$ (MeV)	${P^s_{\mathrm{BAG}}}^{\frac{1}{4}}$ (MeV)	$\left(\sum_{u,d,s} P^i_{\mathrm{BAG}}\right)^{\frac{1}{4}}$ (MeV)	$\mu^{u/d}_{\chi}$ (MeV)	μ^{s}_{χ} (MeV)	μ_{χ} (MeV)	μ_{dc} (Maxwell) (MeV)	$P^{\frac{1}{4}}_{\mathrm{TM1}}(\mu_{\chi})$ (MeV)	$B^{\frac{1}{4}}_{\mathrm{eff}}$ (χ-dc) (MeV)
I	137.6	163.6	224.1	238.5	343	594	1029	1458	120.7	149.8
II	145.8	173.4	221.9	240.2	365	591	1094	1569	141.4	149.8
III	148.5	176.6	221.7	241.3	371	590	1114	1600	147.1	149.9
IV	152.7	181.6	221.7	243.3	383	590	1148	1651	155.3	150.0



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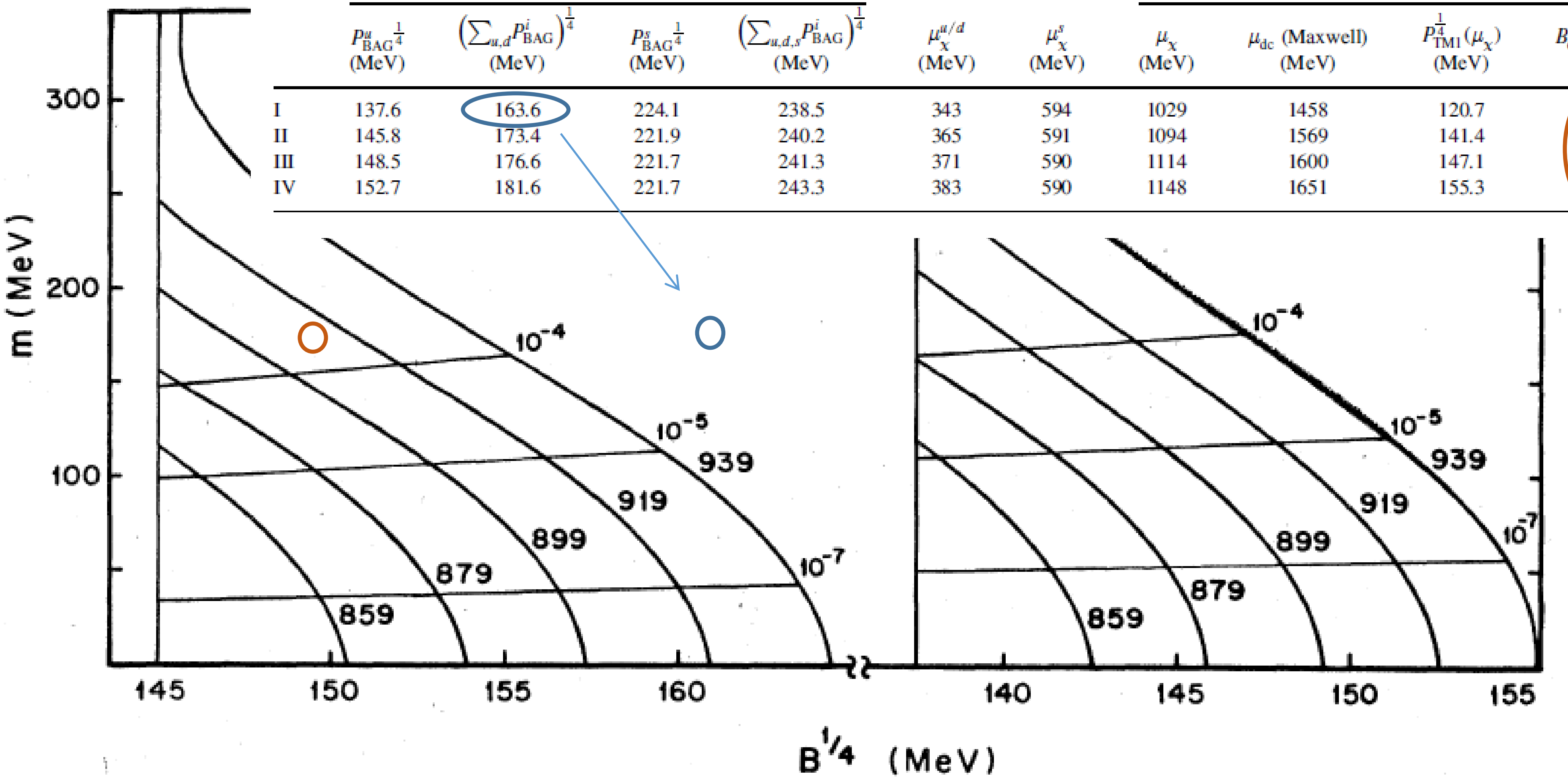
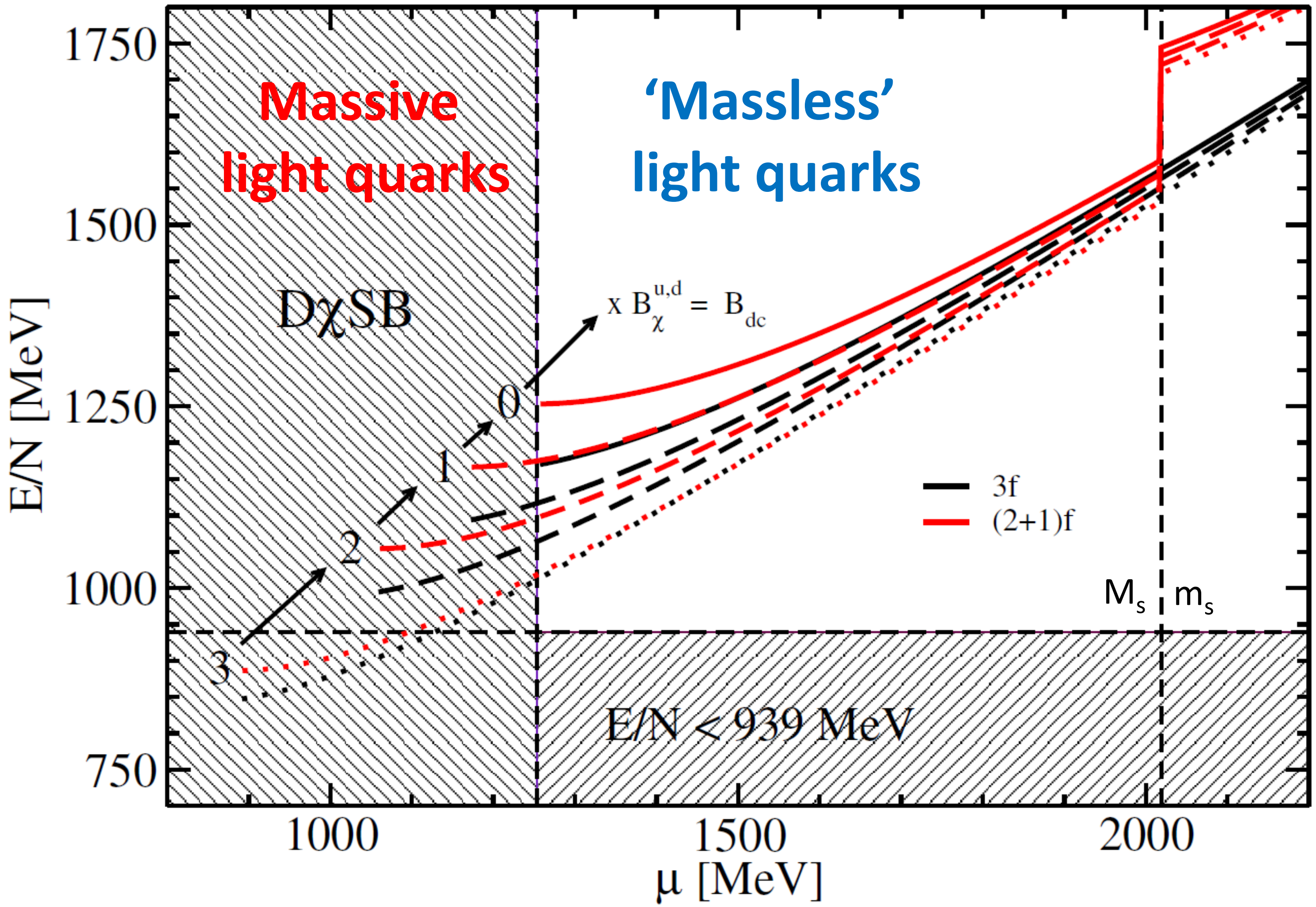


Absolutely Stable Strange Matter?



prediction of absolutely stable strange quark matter crucially relies on neglecting dynamical chiral symmetry breaking <u>for light quarks</u>

Difficult to confirm even if one assumes no DCSB for strange quarks at all

Conclusions Part II

vBAG:

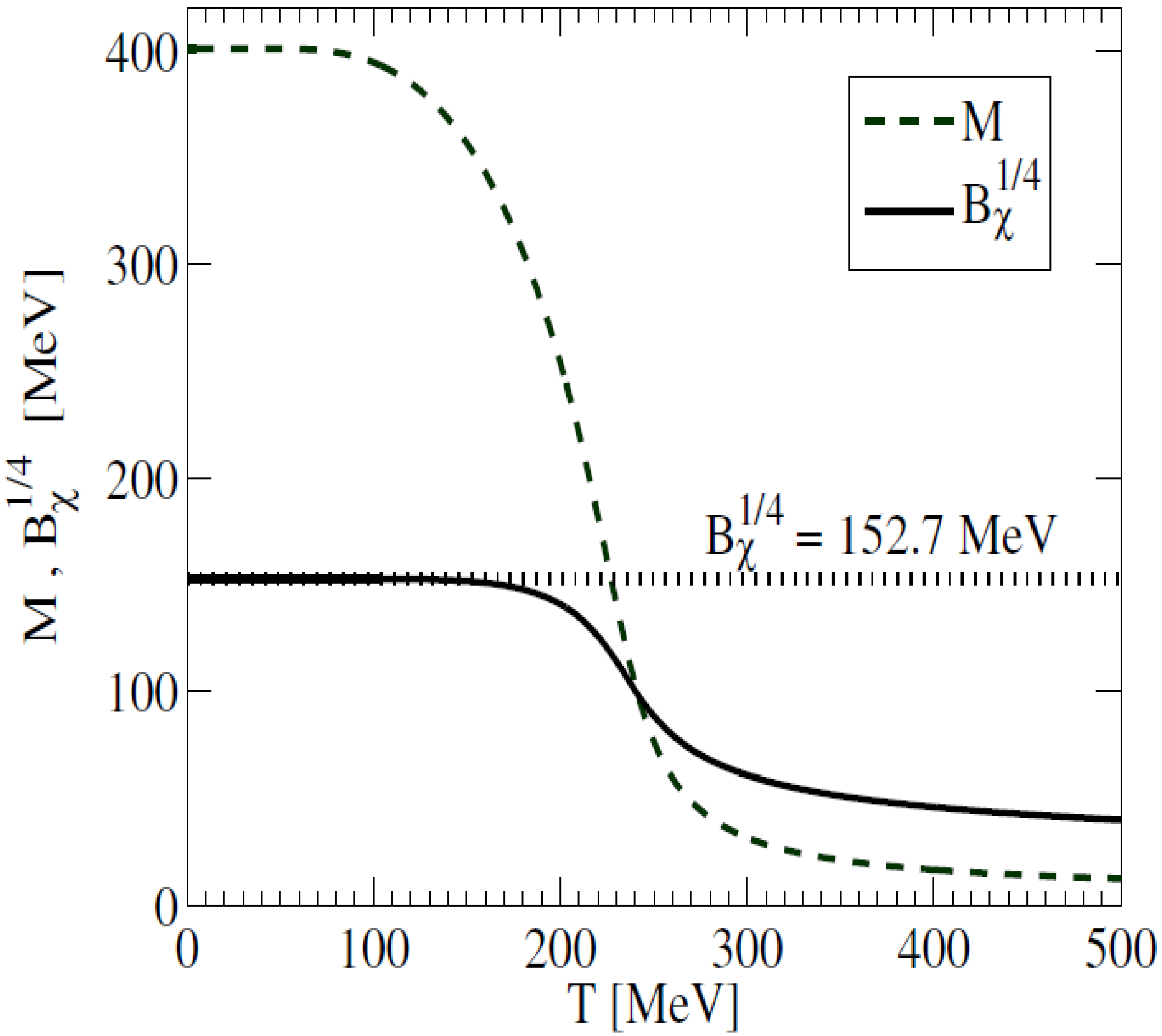
- vector interaction resolves the problem of too soft bag model EoS w/o perturbative corrections
- No problem at all to obtain stable hybrid neutron star configurations
- Standard BAG models bag constant is understood to mimic confinement, DχSB is absent
- vBAG introduces effective bag constant with similar values to original BAG

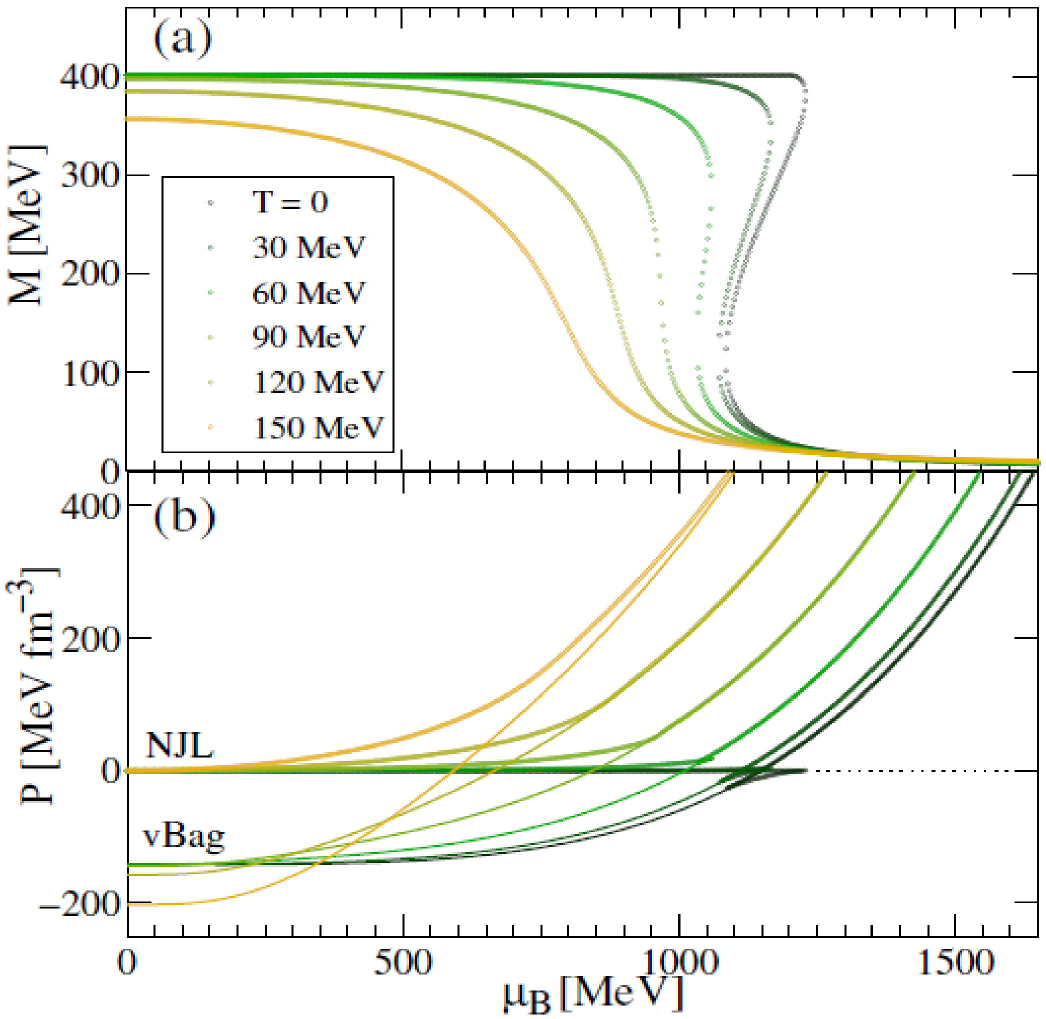
$$B_{eff} = \sum_f B_\chi^f - B_{\rm dc}$$

- However, positive value due to chiral symmetry breaking, (de)confinement reduces B
- Absolutely stable strange matter hypothesis is not trivial to hold up accounting for DχSB

- NJL and partially Bag model result from particular approximation within Dyson-Schwinger approach
  rainbow approximation (quark-gluon vertex) + contact interaction (gluon propagator)
- Consequence: both models lack momentum dependent gap solutions

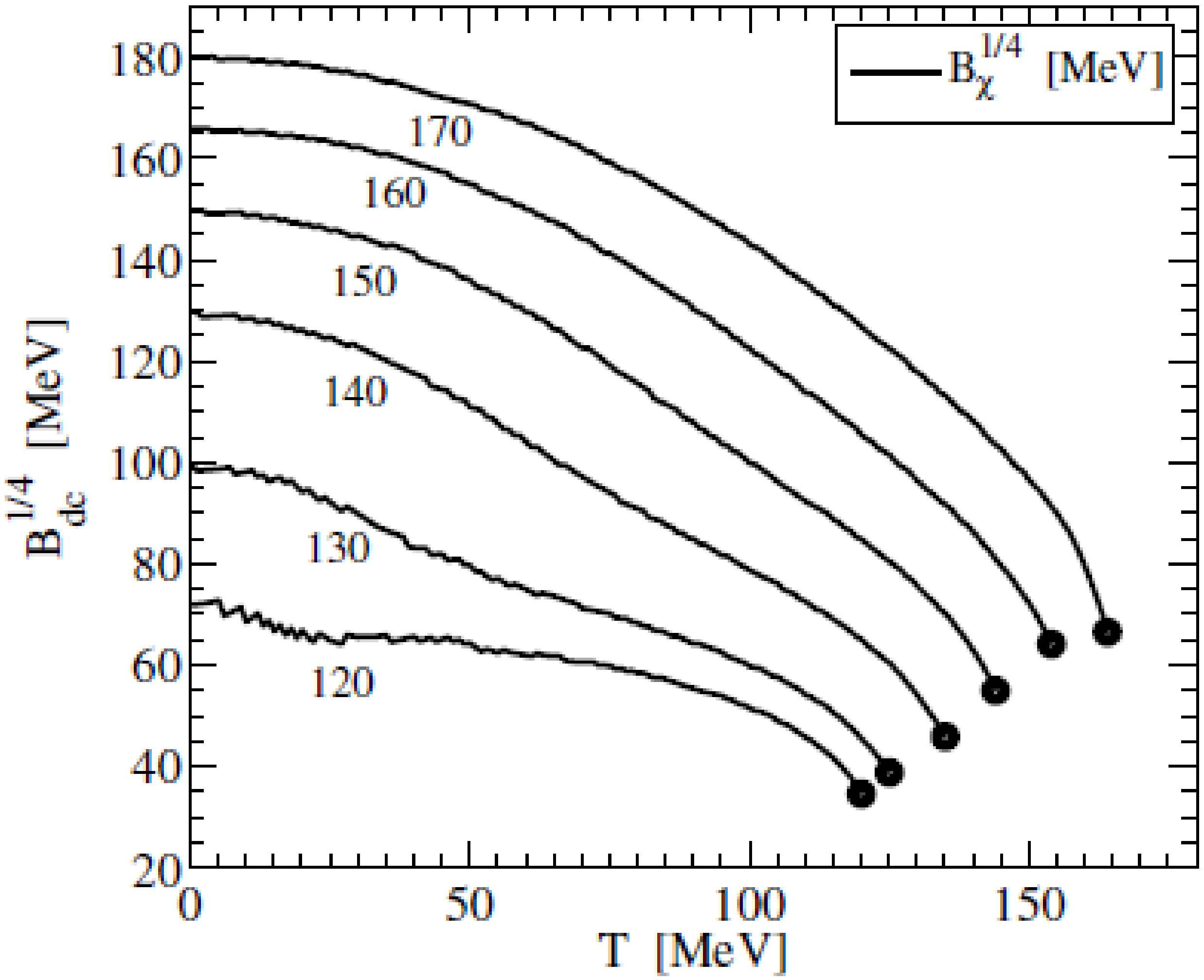
Finite Temperature





TK, T.Fischer, M.Hempel arXiv:1603.03679, ApJ (subm)

Medium Corrections



Coherent picture:

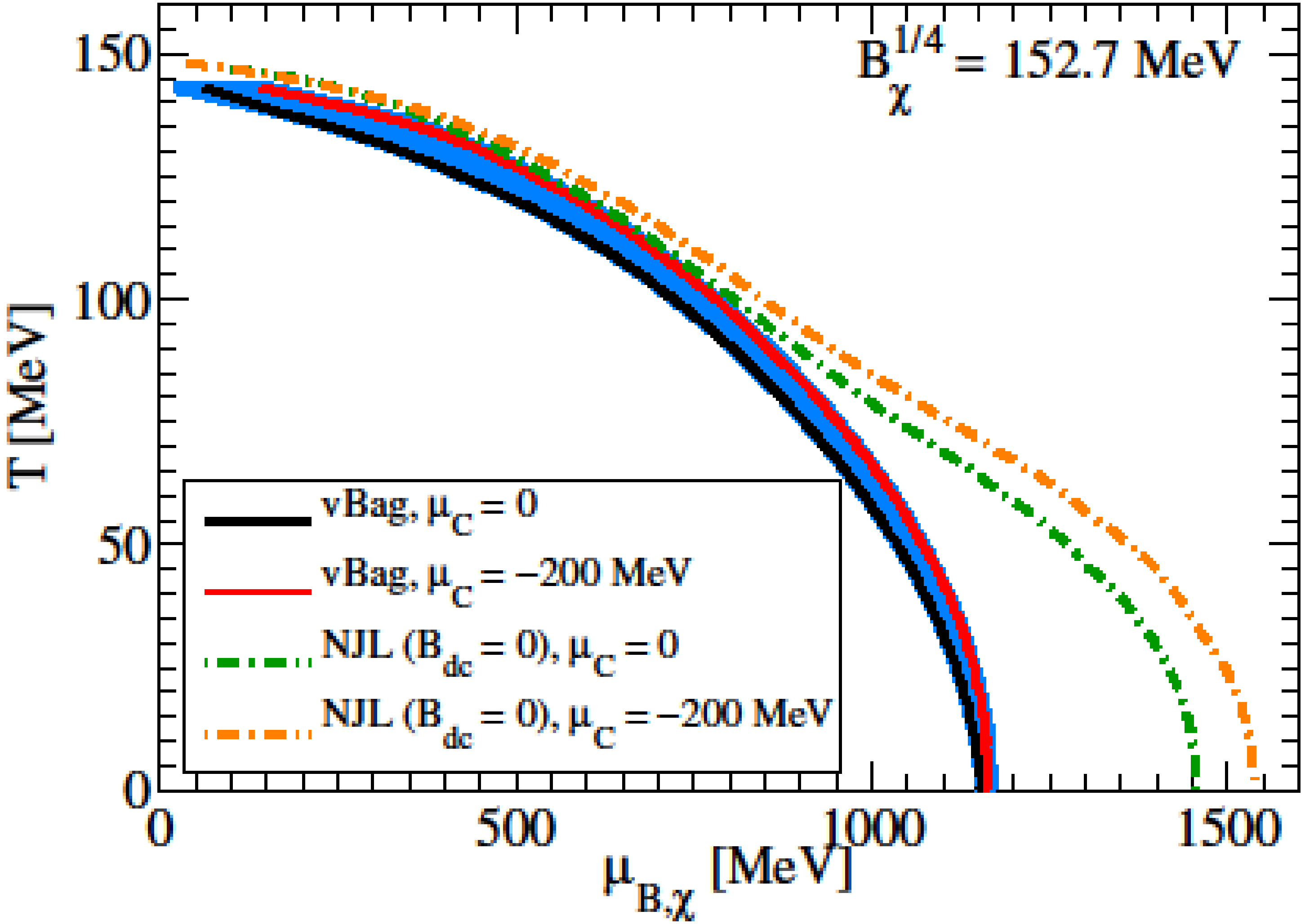
(de)confinement bag constant reduces
with temperature

-> nuclear and chiral quark matter become similar
-> indicates cross-over behaviour

Careful:
Model is not able to actually describe crossover
1st order phase transition is 'hardwired' :
NM and QM EoS are modeled independently
NM EoS doesn't know about quarks

TK, T.Fischer, M.Hempel arXiv:1603.03679, ApJ (subm)

Phase Diagram



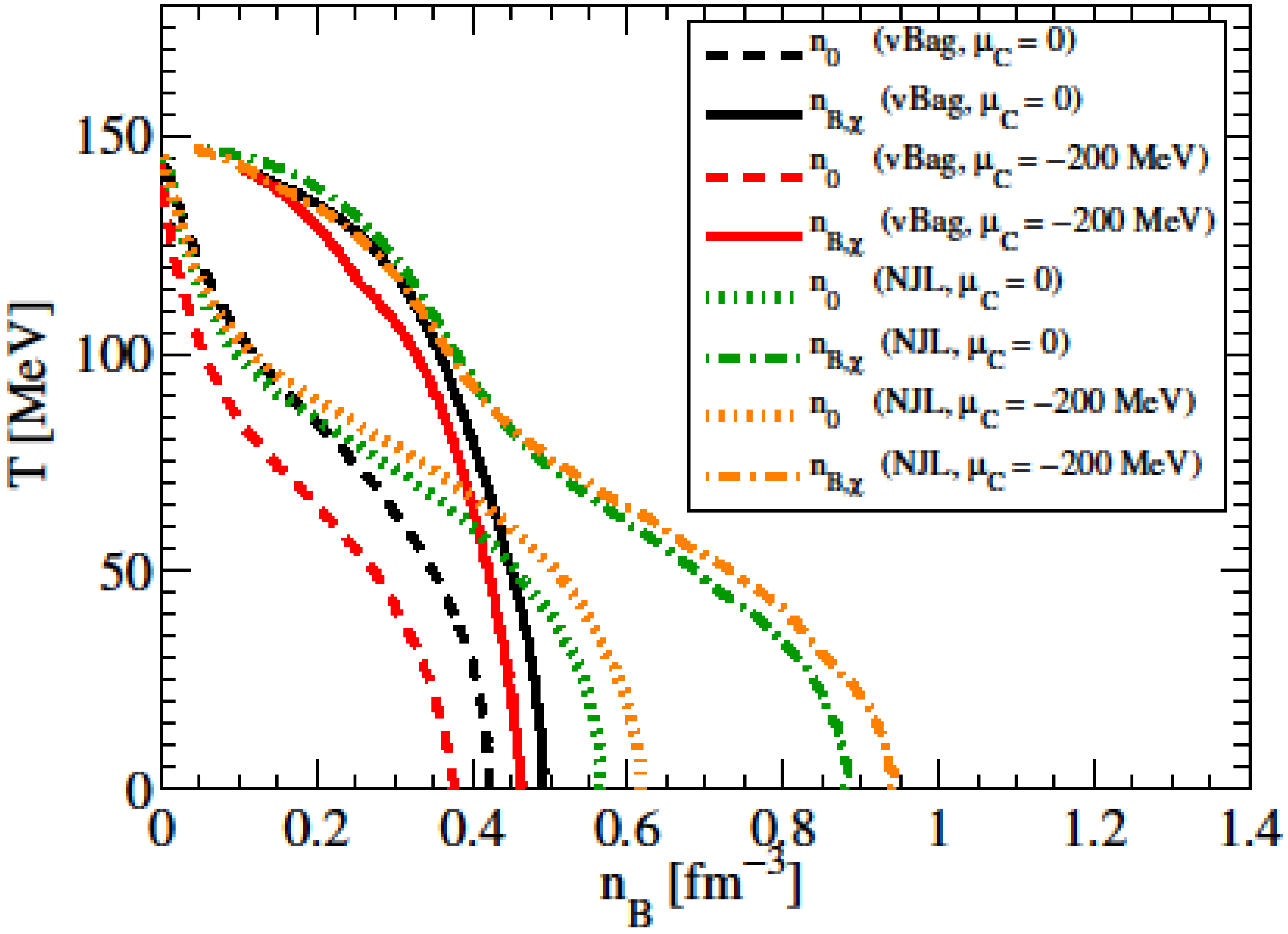
<u>Location of transition line</u>

vBag:
defined by chiral transition
does not depend on hadronic EoS
'low' μ

NJL(+Maxwell):
changes with NM EoS
'high' μ

TK, T.Fischer, M.Hempel arXiv:1603.03679, ApJ (subm)

Phase Diagram



<u>Location of transition line</u>

Onset of coexistence domain:
Depends on NM EoS for both

Onset of pure quark phase:
vBag:
defined by chiral transition
does not depend on hadronic EoS

NJL(+Maxwell):
changes with NM EoS
'high' μ

TK, T.Fischer, M.Hempel arXiv:1603.03679, ApJ (subm)

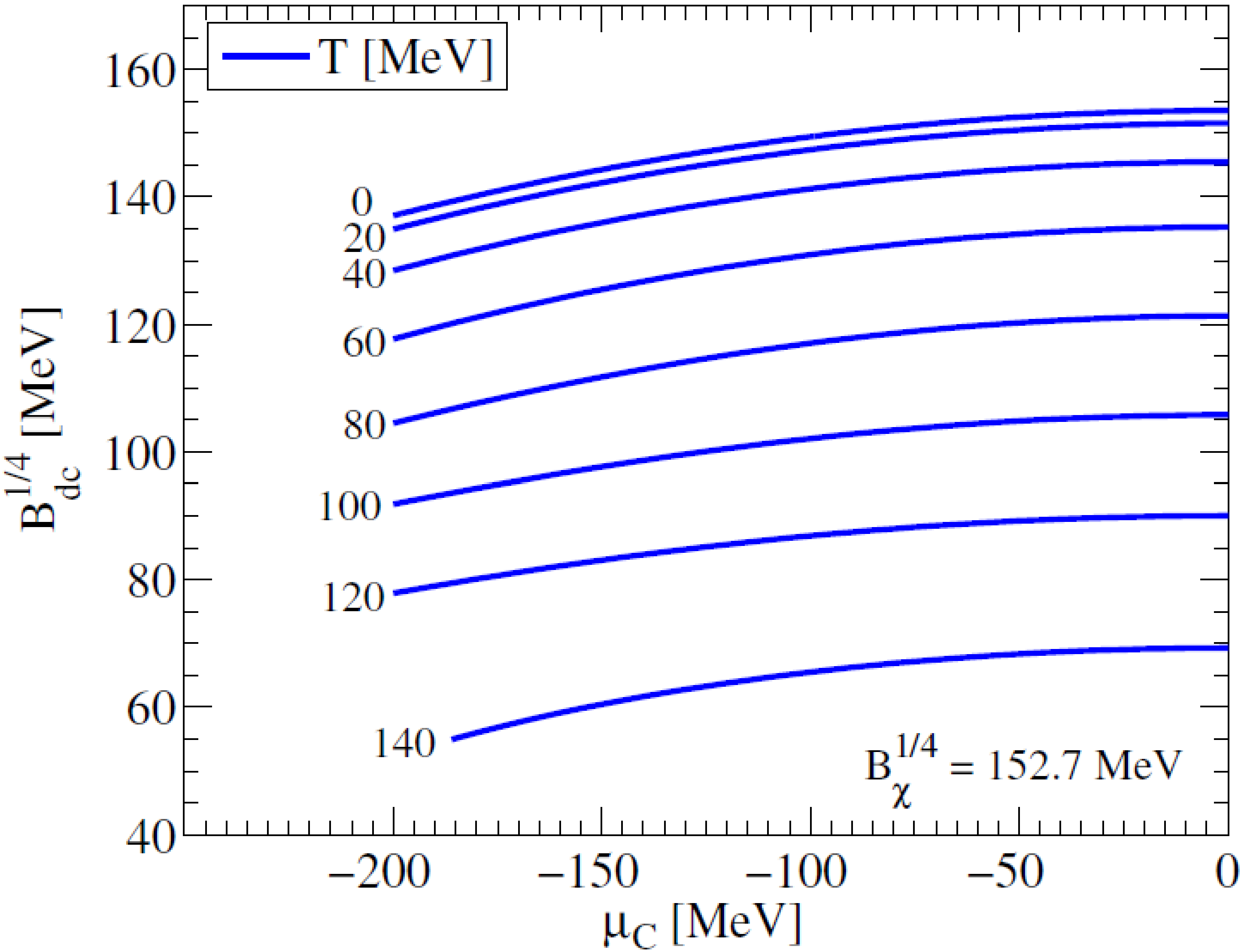
Medium Corrections

TK, T.Fischer, M.Hempel arXiv:1603.03679, ApJ (subm)

$$\frac{\partial B_{\rm dc}}{\partial \mu_C} =: n_{C,{\rm dc}}(T,\mu_C)$$
$$= \frac{\partial}{\partial \mu_C}\left\{P^H\left(T,\mu_C,\mu_B=\mu_{B,\chi}(T,\mu_C)\right)\right\}$$
$$= \frac{\partial P^H}{\partial \mu_C}(T,\mu_C,\mu_{B,\chi}) + \frac{\partial \mu_{B,\chi}}{\partial \mu_C}\frac{\partial P^H}{\partial \mu_B}(T,\mu_C,\mu_{B,\chi})$$
$$= n_C^H(T,\mu_C,\mu_{B,\chi}) - \tilde{n}_C^Q(T,\mu_C,\mu_{B,\chi})\frac{n_B^H(T,\mu_C,\mu_{B,\chi})}{n_B^Q(T,\mu_C,\mu_{B,\chi})}$$

$$\frac{\partial B_{\rm dc}}{\partial T} =: s_{\rm dc}(T,\mu_C)$$
$$= \frac{\partial}{\partial T}\left\{P^H\left(T,\mu_C,\mu_B=\mu_{B,\chi}(T,\mu_C)\right)\right\}$$
$$= \frac{\partial P^H}{\partial T}(T,\mu_C,\mu_{B,\chi}) + \frac{\partial \mu_{B,\chi}}{\partial T}\frac{\partial P^H}{\partial \mu_B}(T,\mu_C,\mu_{B,\chi})$$
$$= s^H(T,\mu_C,\mu_{B,\chi}) - \tilde{s}^Q(T,\mu_C,\mu_{B,\chi})\frac{n_B^H(T,\mu_C,\mu_{B,\chi})}{n_B^Q(T,\mu_C,\mu_{B,\chi})}$$

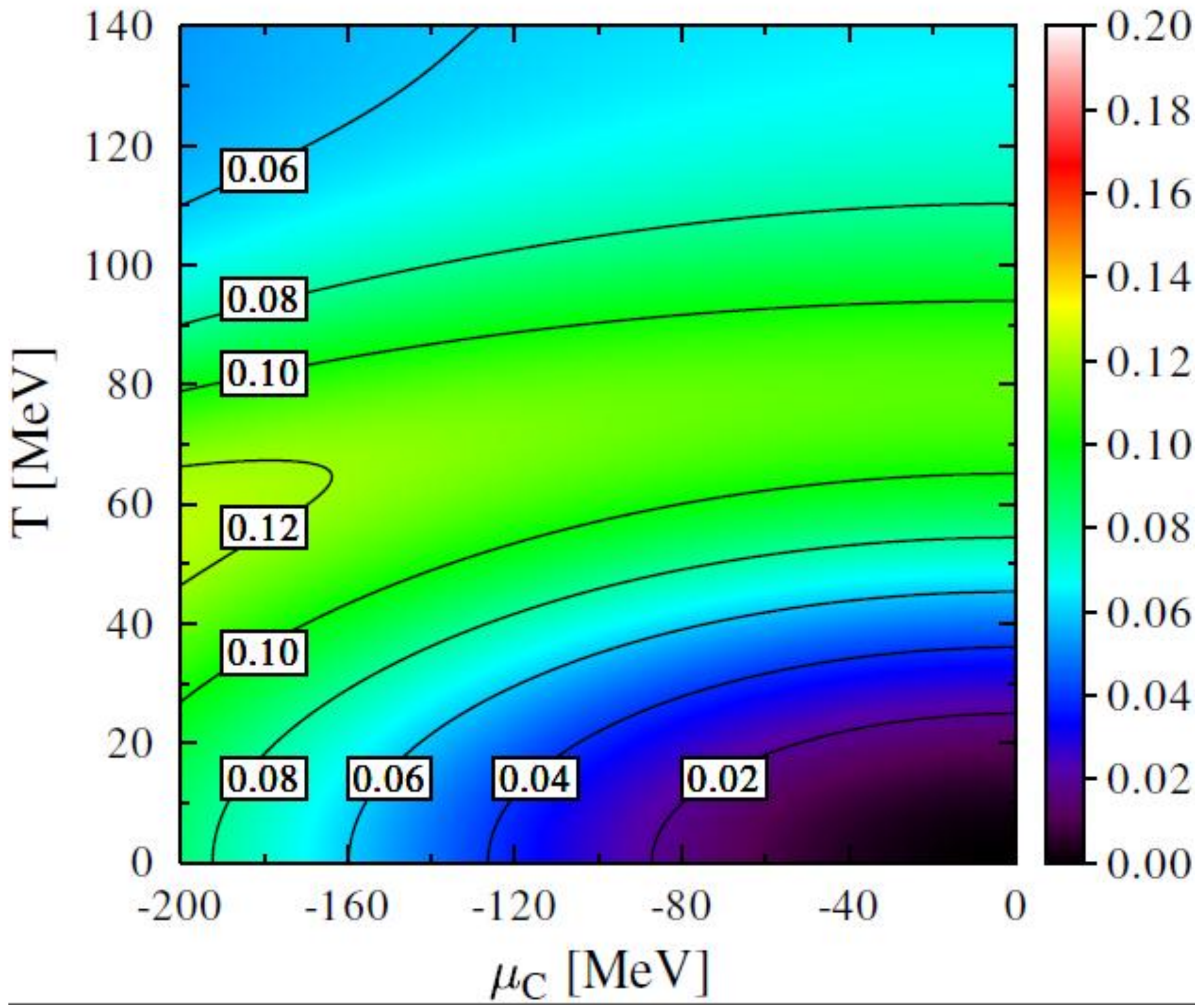
$$\boxed{\begin{aligned} n_C^Q(T,\mu_C,\mu_B) &= \tilde{n}_C^Q(T,\mu_C,\mu_B) + \frac{\partial B_{\rm dc}}{\partial \mu_C} \\ s^Q(T,\mu_C,\mu_B) &= \tilde{s}^Q(T,\mu_C,\mu_B) + \frac{\partial B_{\rm dc}}{\partial T} \end{aligned}}$$



Medium Corrections

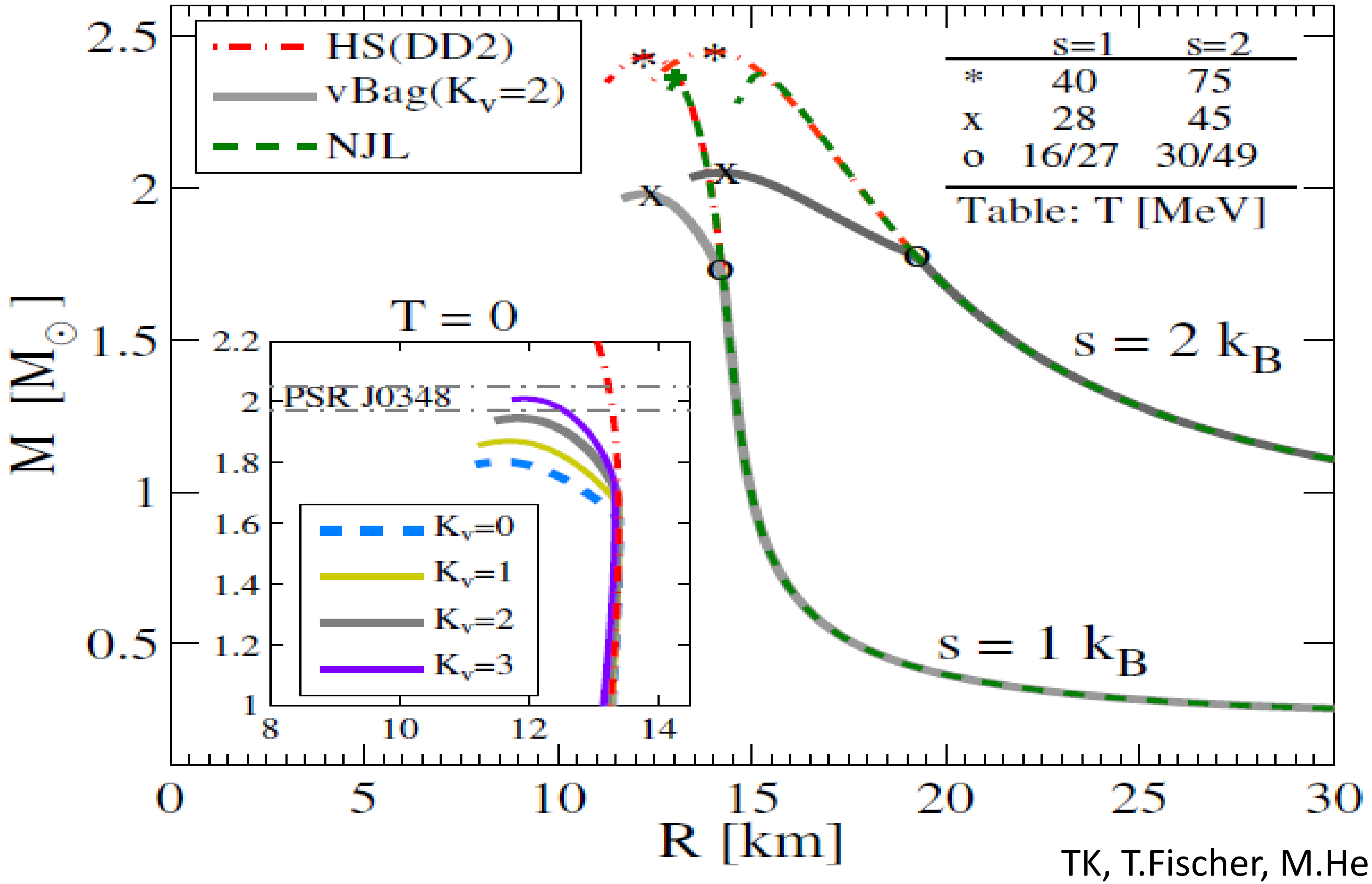
$$R_\varepsilon = \varepsilon_{\rm dc}/\varepsilon^Q$$

$$\varepsilon^Q(T,\mu_C,\mu_B) = \tilde{\varepsilon}^Q(T,\mu_C,\mu_B) - B_{\rm dc}(T,\mu_C) + Ts_{\rm dc}(T,\mu_C) + \mu_C n_{C,{\rm dc}}(T,\mu_C)$$



TK, T.Fischer, M.Hempel arXiv:1603.03679, ApJ (subm)

Proto Neutron Star Configurations



TK, T.Fischer, M.Hempel arXiv:1603.03679, ApJ (subm)

Conclusions

QCD in medium (near critical line):

- Task is difficult
- Not addressable by LQCD
- Not addressable by pQCD
- DSE are promising tool to tackle non-perturbative in-medium QCD
- Qualitatively very different results depending on effective gluon coupling
- Bag model mostly a simple limiting case of NJL model
- NJL model a simple contact interaction model in the gluon sector
- vBag connects them, other models exist

- S: DCSB
- V: renormalizes μ
- D: diquarks → 2SC, CFL
- TD Potential minimized
  in mean-field approximation
- Effective model by its nature;
  can be motivated (1g-exchange)
  doesn't have to though and can
  be extended (KMT, PNJL)
- possible to describe hadrons

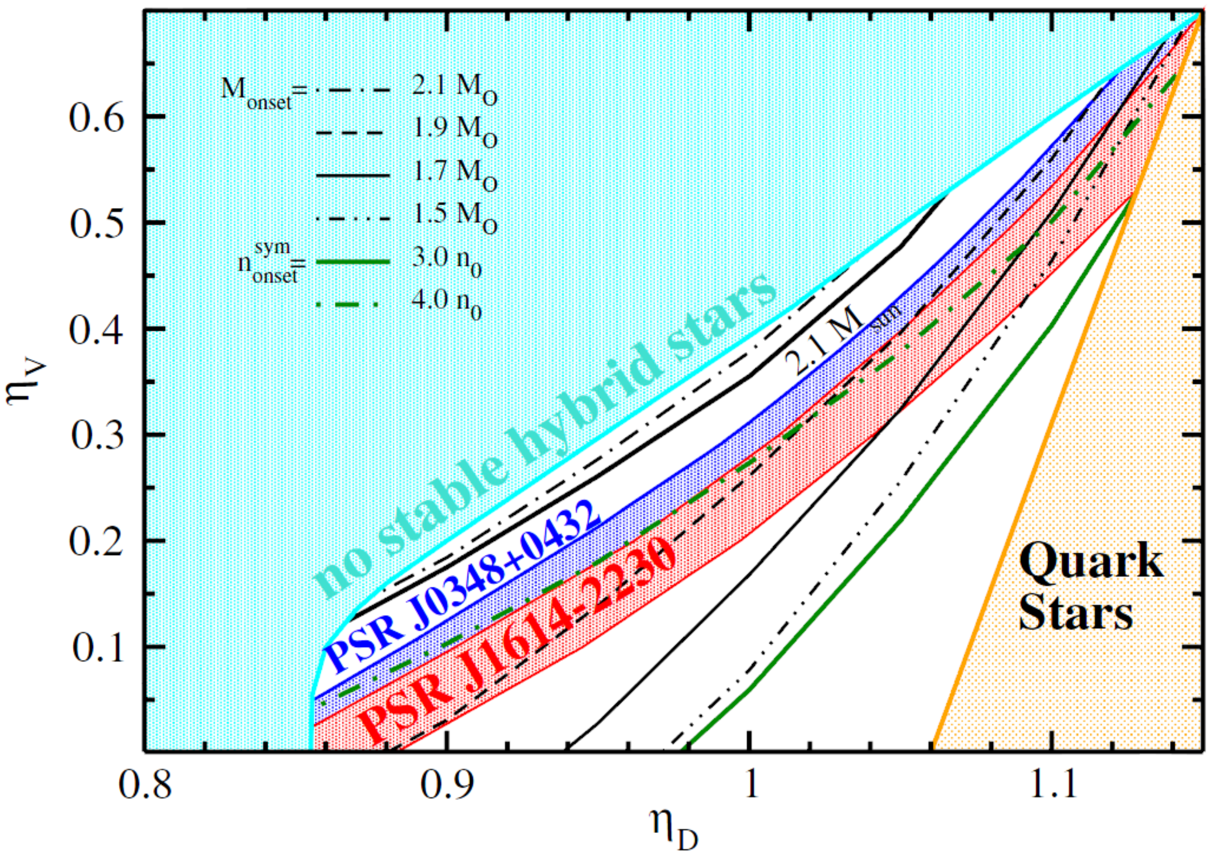
Effective Lagrangian

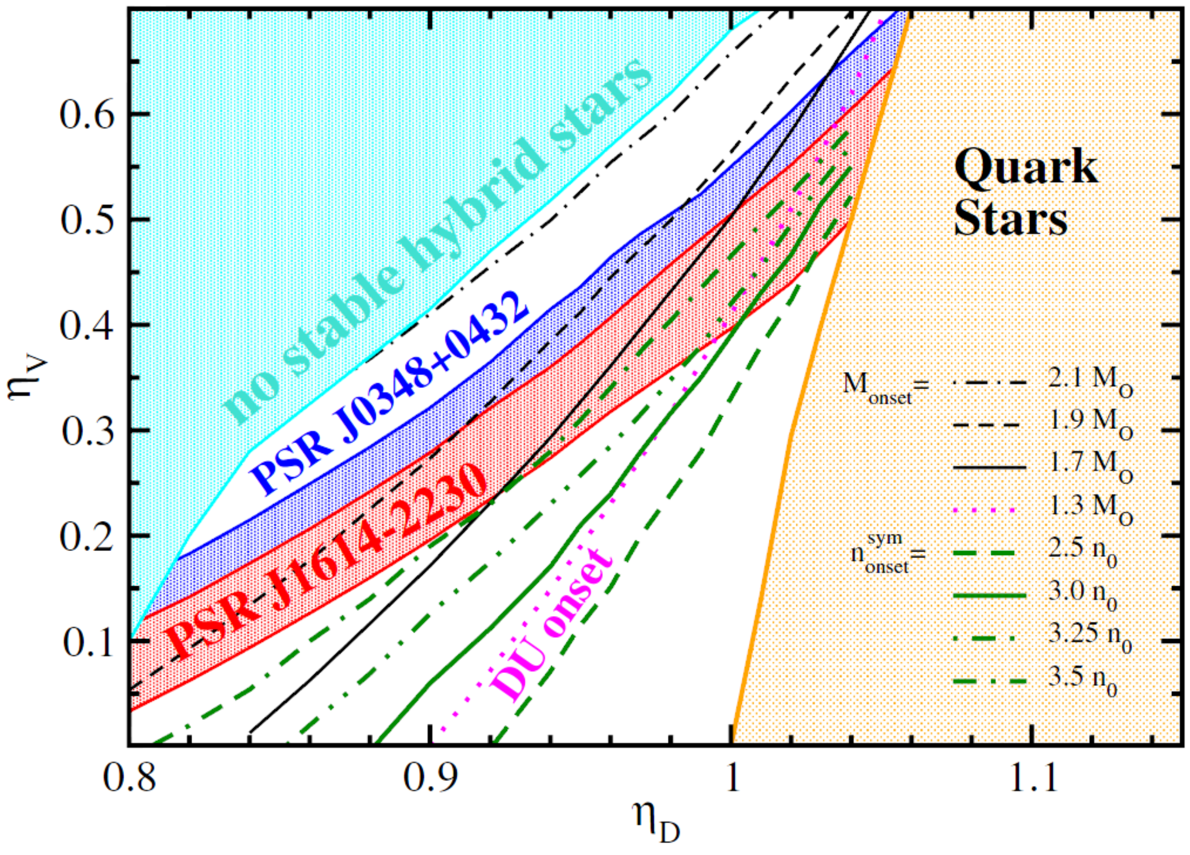
$$\mathcal{L}_{int} = G_S \eta_D \sum_{a,b=2,5,7} (\bar{q} i\gamma_5 \tau_a \lambda_b C \bar{q}^T)(q^T C i\gamma_5 \tau_a \lambda_a q)$$
$$+ \; G_S \sum_{a=0}^{8} \left[(\bar{q}\tau_a q)^2 + \eta_V (\bar{q} i\gamma_0 q)^2\right]$$

Thermodynamical potential

$$\Omega(T,\mu) = \frac{\phi_u^2+\phi_d^2+\phi_s^2}{8G_S} - \frac{\omega_u^2+\omega_d^2+\omega_s^2}{8G_V} + \frac{\Delta_{ud}^2+\Delta_{us}^2+\Delta_{ds}^2}{4G_D}$$
$$- \int \frac{d^3p}{(2\pi)^3} \sum_{n=1}^{18} \left[E_n + 2T ln\left(1+e^{-E_n/T}\right)\right] + \Omega_l - \Omega_0$$

NJL model study for NS (TK, R.Łastowiecki, D.Blaschke, PRD **88**, 085001 (2013))





Conclusion: NS may or may not support a significant QM core.
additional interaction channels won’t change this if coupling strengths are not precisely known.