Compact stars in the braneworld: a new branch of stellar configurations with arbitrarily large mass

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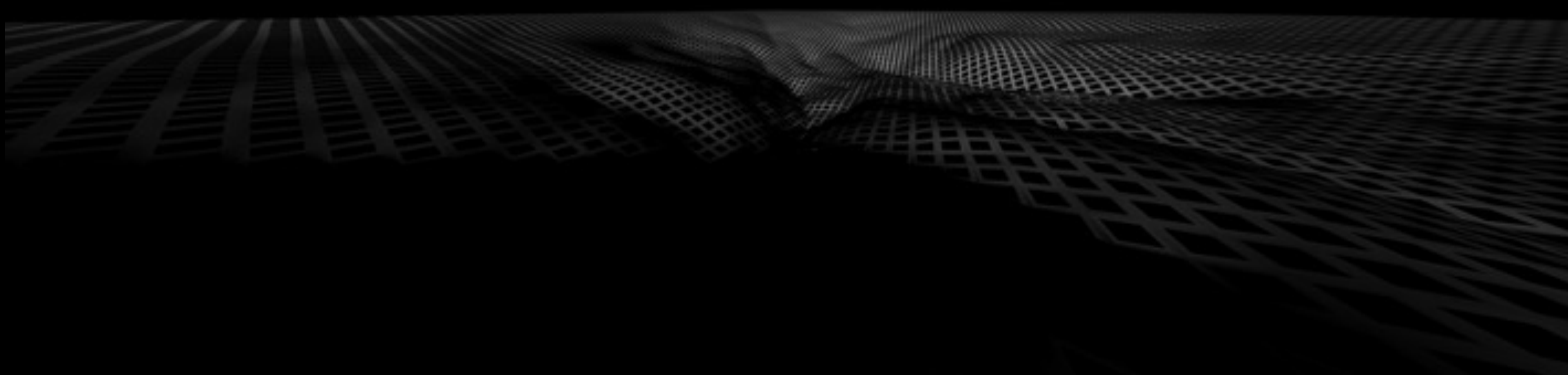


Compact Stars in the QCD phase diagram V
23-27 May 2016 GSSI and LNGS (L'Aquila, Italy)

Summary of the talk:

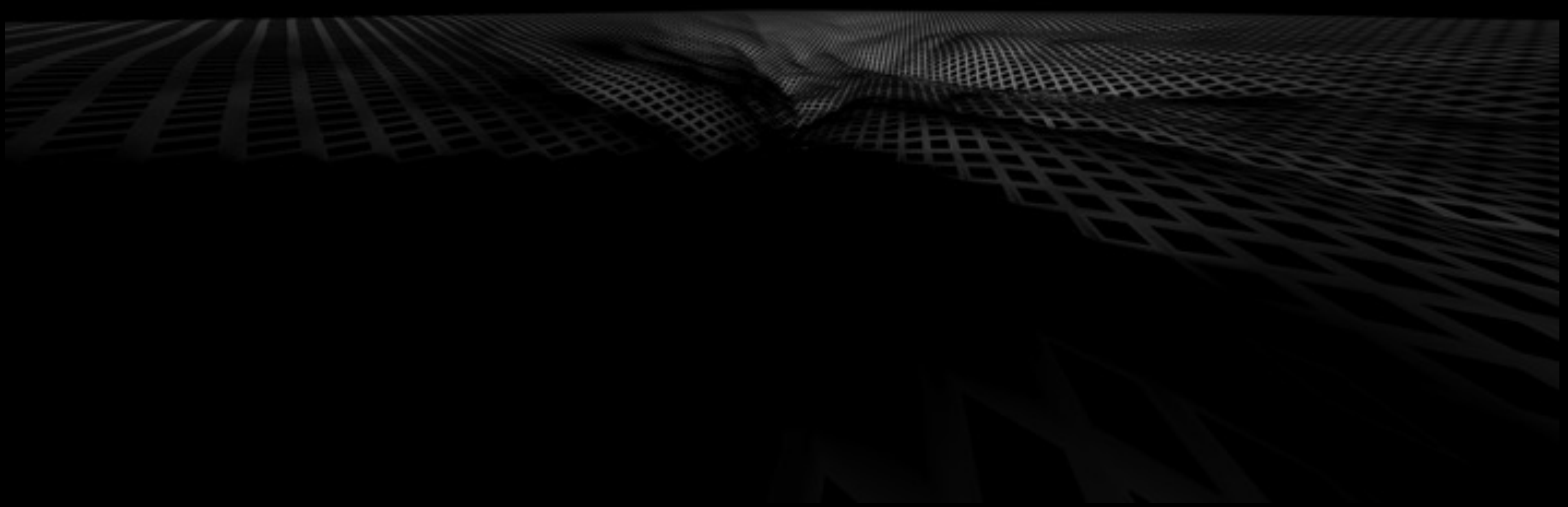
1. Introduction to BraneWorld Models (BWM)

2. Stellar structure in BWM
   a. Structure equations
   b. Boundary conditions

3. Our Work:
   a. The causal limit
   b. Stellar models for hadronic and quark stars
   c. Stability of the configurations

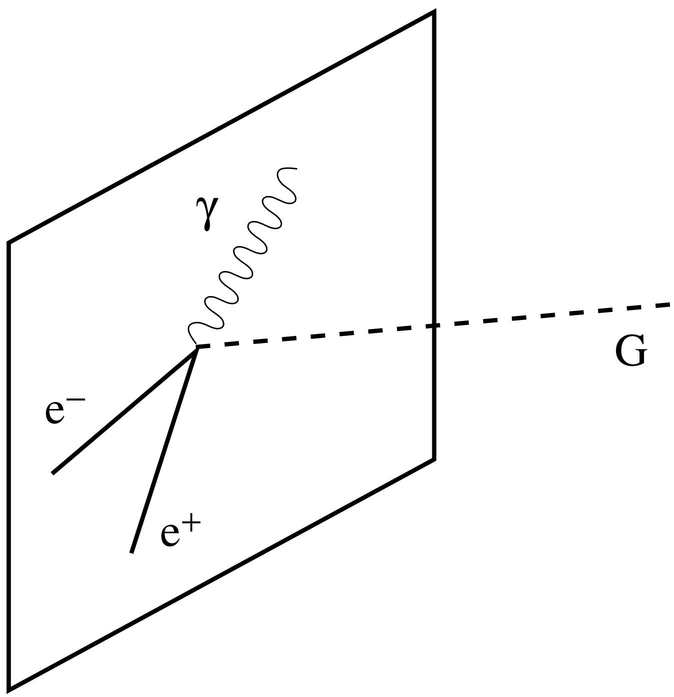
4. Conclusions

1. INTRODUCTION TO BRANEWORLD MODELS



Introduction

- At high enough energies, Einstein's theory of general relativity breaks down, and will be superseded by a quantum gravity theory.
- There is as yet no generally accepted (pre-)quantum gravity theory.
- One candidate is string theory. It removes the infinities of quantum field theory and unifies the fundamental interactions, including gravity. But there is a price → there should exist several extra dimensions.



We can imagine the universe as a **3D brane** where elementary particles live embedded in a **higher-dimensional space-time called the bulk** (only accessed by gravity).

The weakness of gravity is due to the fact that it "spreads" into extra dimensions and only a part of it is felt in 4 dimensions.

- Due to the complexity of string theory it is very difficult to implement it in astrophysics and cosmology.

- This motivates the development **brane-world models** which are inspired by ideas from string theory, but do not attempt to impose the full machinery of the theory.

- Thanks to the simplifications introduced in BW models, we can construct astrophysical and cosmological models and analyse the gravitational effect of extra-dimensions.

- Two well known examples of brane-world models are:
  - Dvali-Gabadadze-Porrati (DGP). IR modifications to GR
  - Randall-Sundrum (RS). UV modifications to GR. Significant deviations from Einstein's theory occur at very high energies, e.g. early universe or gravitational collapse.

In the present work we focus on Randall-Sundrum brane-world models.

Randall Sundrum brane-worlds

see e.g. Randall & Sundrum PRL 1999; Maartens, PRD 2000; Shiromizu et al PRD 2000

➤ 5-dimensional model: we effectively assume that all the extra dimensions in the "parent" string theory may be represented by 1 extra dimension.

➤ In RS models the bulk is a portion of a 5D anti-de Sitter spacetime (AdS_5); i.e. the extra dimension is curved or "warped" rather than flat.

➤ At low energies, gravity doesn't "leak" into the extra dimension due to a negative bulk cosmological constant, $\Lambda_5 = -6/\ell^2$ where ℓ is the curvature radius of AdS_5.

➤ The brane gravitates with self-gravity in the form of a brane tension λ, where

$$\lambda = \frac{3M_p^2}{4\pi\ell^2} \qquad , \qquad M_p^2 = M_5^3\ell$$

➤ On the brane, the negative Λ_5 is counterbalanced by the positive brane tension λ.

➤ The **Einstein field eqs. on the brane** are

$$G_{\mu\nu} = 8\pi G T^{\mathrm{eff}}_{\mu\nu}$$

where $G_{\mu\nu}$ is the usual Einstein's tensor and the standard $T_{\mu\nu}$ is replaced by an effective energy-momentum tensor.

➤ The **effective energy-momentum tensor** has the form

$$T^{\mathrm{eff}}_{\mu\nu} = T_{\mu\nu} + \frac{6}{\lambda} S_{\mu\nu} - \frac{1}{8\pi G} \mathcal{E}_{\mu\nu}$$

1st term: standard energy momentum tensor; e.g. for a perfect fluid we have $T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p h_{\mu\nu}$

2nd and 3rd term: include modifications with respect to the standard 4D Einstein's field eqs. (Maartens and Koyama 2010). Two contributions: local (2nd term) and non-local (3rd term).

- Local correction $6S_{\mu\nu}/\lambda$ where λ is the brane tension. For a perfect fluid

$$S_{\mu\nu} = \frac{1}{12}\rho^2 u_\mu u_\nu + \frac{1}{12}\rho(\rho + 2p)h_{\mu\nu}$$

  - u_μ is the four velocity, and
  - $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ is the projection orthogonal to u_μ

- Non-local correction $\varepsilon_{\mu\nu}/8\pi G$ where, assuming static spherical symmetry
  - r_μ is a unit radial vector,
  - $\mathcal{U}$ and $\mathcal{P}$ **non-local (or dark) energy density and pressure** on the brane

$$\mathcal{E}_{\mu\nu} = -\frac{6}{8\pi G\lambda}\left[\mathcal{U}u_\mu u_\nu + \mathcal{P}r_\mu r_\nu + \frac{(\mathcal{U}-\mathcal{P})}{3}h_{\mu\nu}\right]$$

When the brane tension $\lambda \to \infty$, both corrections vanish and we recover General Relativity.

2. STELLAR STRUCTURE IN BWM

Structure Equations in brane-world models

Germani & Maartens (2001) derived the braneworld generalization of the static stellar structure equations (TOV). Solving the Einstein's equations on the brane they found

$$
\begin{aligned}
\frac{dm}{dr} &= 4\pi r^2 \rho_{\rm eff}\,, \\
\frac{dp}{dr} &= -(\rho + p)\frac{d\phi}{dr}\,, \\
\frac{d\phi}{dr} &= \frac{Gm + 4\pi G r^3 \left(p_{\rm eff} + \frac{4\mathcal{P}}{(8\pi G)^2\lambda}\right)}{r(r-2Gm)}\,, \\
\frac{d\mathcal{U}}{dr} + (4\mathcal{U} + 2\mathcal{P})\frac{d\phi}{dr} &= -2(4\pi G)^2(\rho + p)\frac{d\rho}{dr} - 2\frac{d\mathcal{P}}{dr} - \frac{6}{r}\mathcal{P}\,,
\end{aligned}
$$

where

$$
\rho_{\rm eff} = \rho + \frac{\rho^2}{2\lambda} + \frac{6}{(8\pi G)^2\lambda}\mathcal{U}\,, \qquad p_{\rm eff} = p + \frac{p\rho}{\lambda} + \frac{\rho^2}{2\lambda} + \frac{2}{(8\pi G)^2\lambda}\mathcal{U}\,.
$$

To solve the equations we need an equation of state p = p(ρ) and a relation of the form $\mathcal{P} = \mathcal{P}(\mathcal{U})$ relating the nonlocal radiation and pressure.

Boundary conditions

Two of the boundary conditions are the same as for the TOV equations in GR. Specifically,

$$m(r=0)=0$$

at the center of the star the enclosed mass is zero

$$p(R)=0$$

at the surface of the object the pressure vanishes

The remaining boundary condition is determined by the Israel-Darmois matching condition $[G_{\mu\nu}\, r^\nu]_\Sigma = 0$ or $[T_{\mu\nu}^{\mathrm{eff}}\, r^\nu]_\Sigma = 0$ at the surface Σ of the object, where $[f]_\Sigma \equiv f(R^+) - f(R^-)$. This leads to

$$(4\pi G)^2\rho^2(R) + \mathcal{U}^-(R) + 2\mathcal{P}^-(R) = \mathcal{U}^+(R) + 2\mathcal{P}^+(R)$$

which holds for any static spherical star with vanishing pressure at the surface.

In BW models, the Schwarzschild solution is no longer the unique asymptotically flat vacuum exterior. Other exterior solutions are possible depending on the 5D solution.

Here we focus on a class of models that satisfy the following properties:

- We consider a Schwarzschild exterior solution ($\mathcal{U}^+ = \mathcal{P}^+ = 0$).
- We assume $\mathcal{P}^- = 0$, which is consistent with the isotropy of the physical pressure in the star.

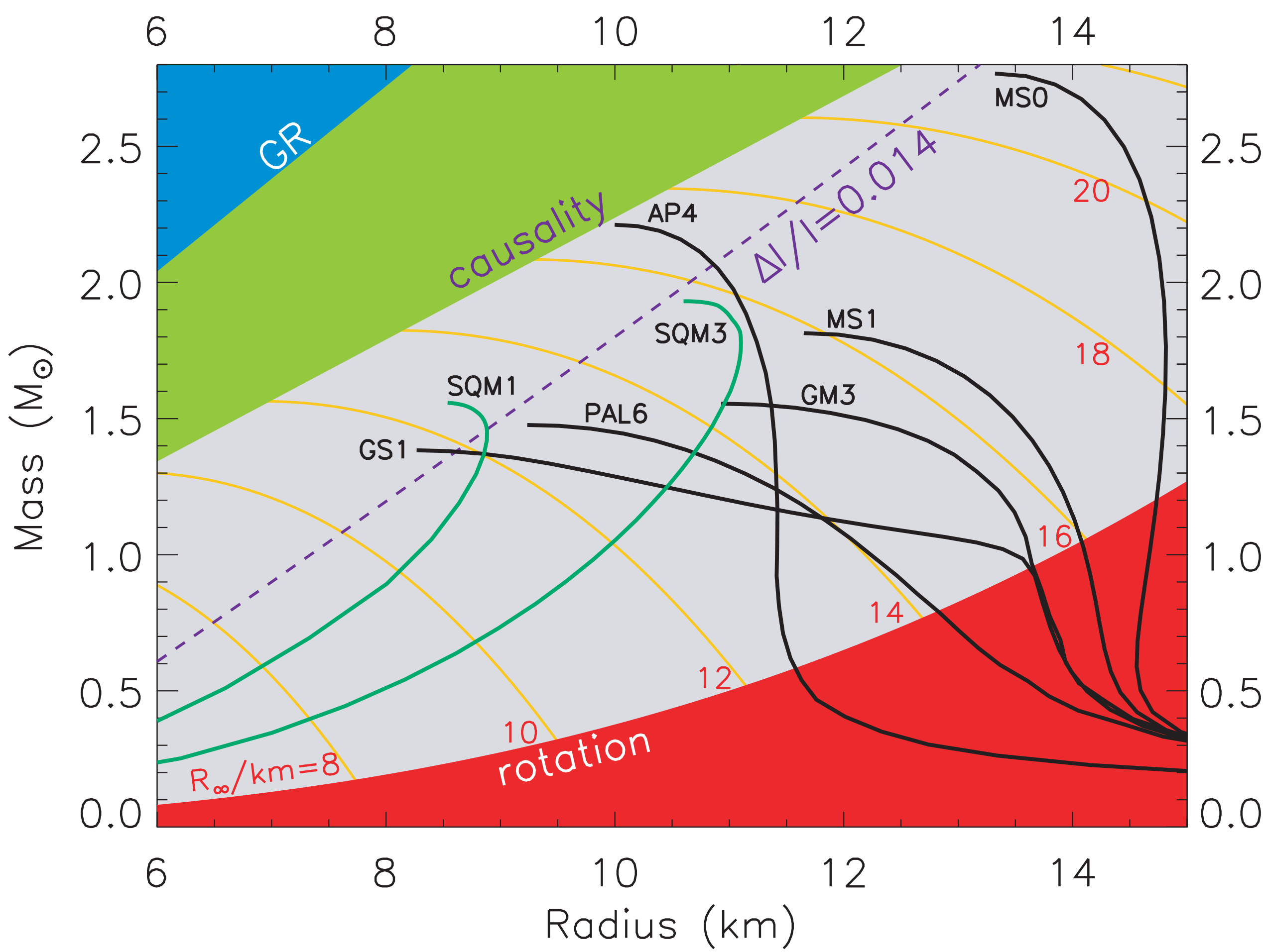
As a consequence, the interior must have a nonvanishing nonlocal energy density ($\mathcal{U}^- \neq 0$). Therefore, the boundary condition for $\mathcal{U}$ at r=R reads:

$$(4\pi G)^2 \rho^2(R) + \mathcal{U}^-(R) = 0.$$

Note on the numerical method: Less straightforward than for standard TOV equations. Since the boundary condition for $\mathcal{U}^-(r)$ is given at the star's surface, a shooting method is used in order to match the latter boundary condition.

Limits in the M-R diagram

Typical Mass vs. Radius diagram in GR (Lattimer & Prakash 2012).



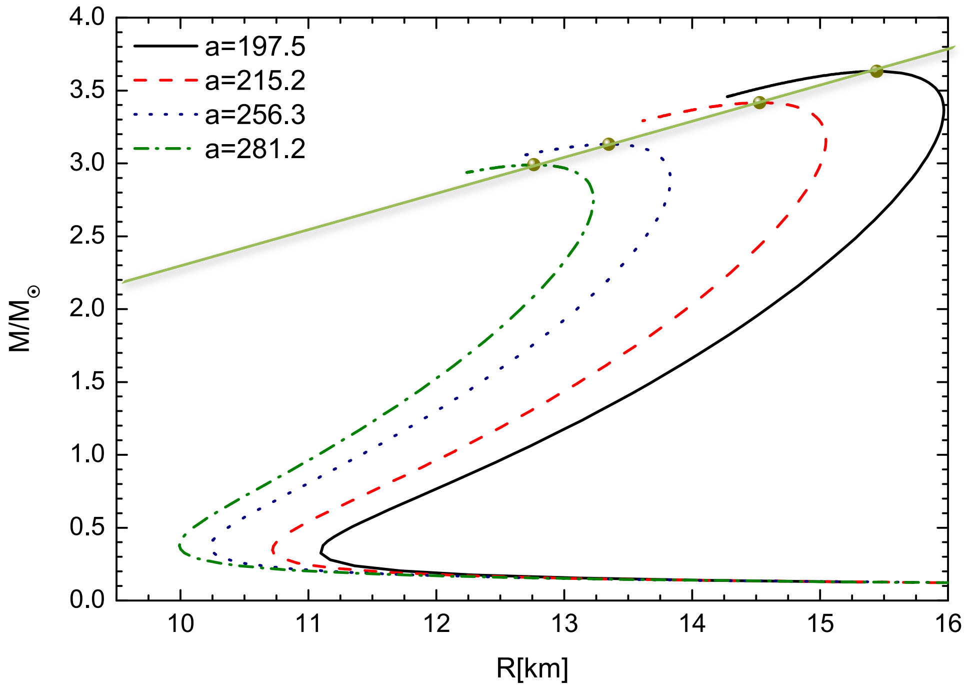
GR	→ Schwarzschild limit	M = 0.5 R	(M=R/2)
Causality	→ next slides	M ≈ 0.34 R	

Causality limit in the mass-radius diagram: GR

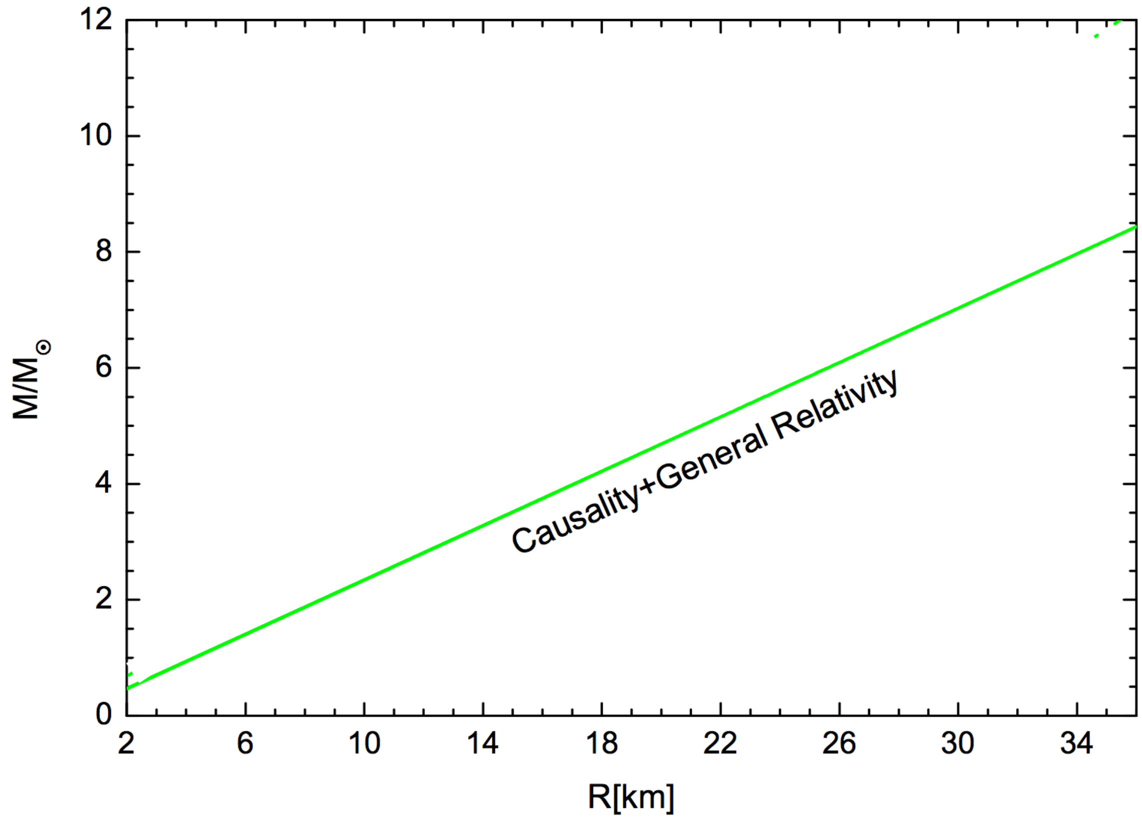
- We adopt the following EOS:
  - the well established BPS EOS for ρ below a fiducial density ρ_t
  - a causal equation of state $p = \rho - a$ above ρ_t

  where $a = \rho_t - p_t$, where ρ_t and p_t also fulfill the BPS EOS.

- We integrate numerically the structure equations for different values of **a** and <u>identify the maximum masses</u>.



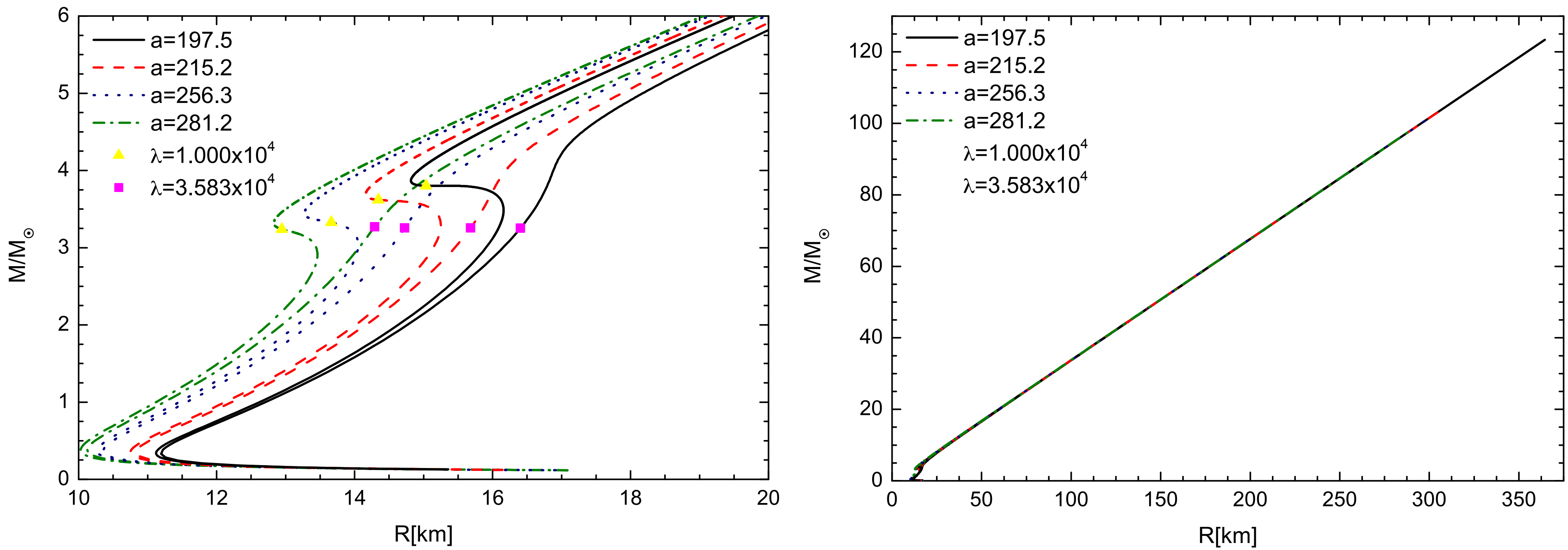
Based on these maximum masses and their respective radii, the region excluded by causality in the M − R diagram is given by $M \gtrsim 0.34\ R$.



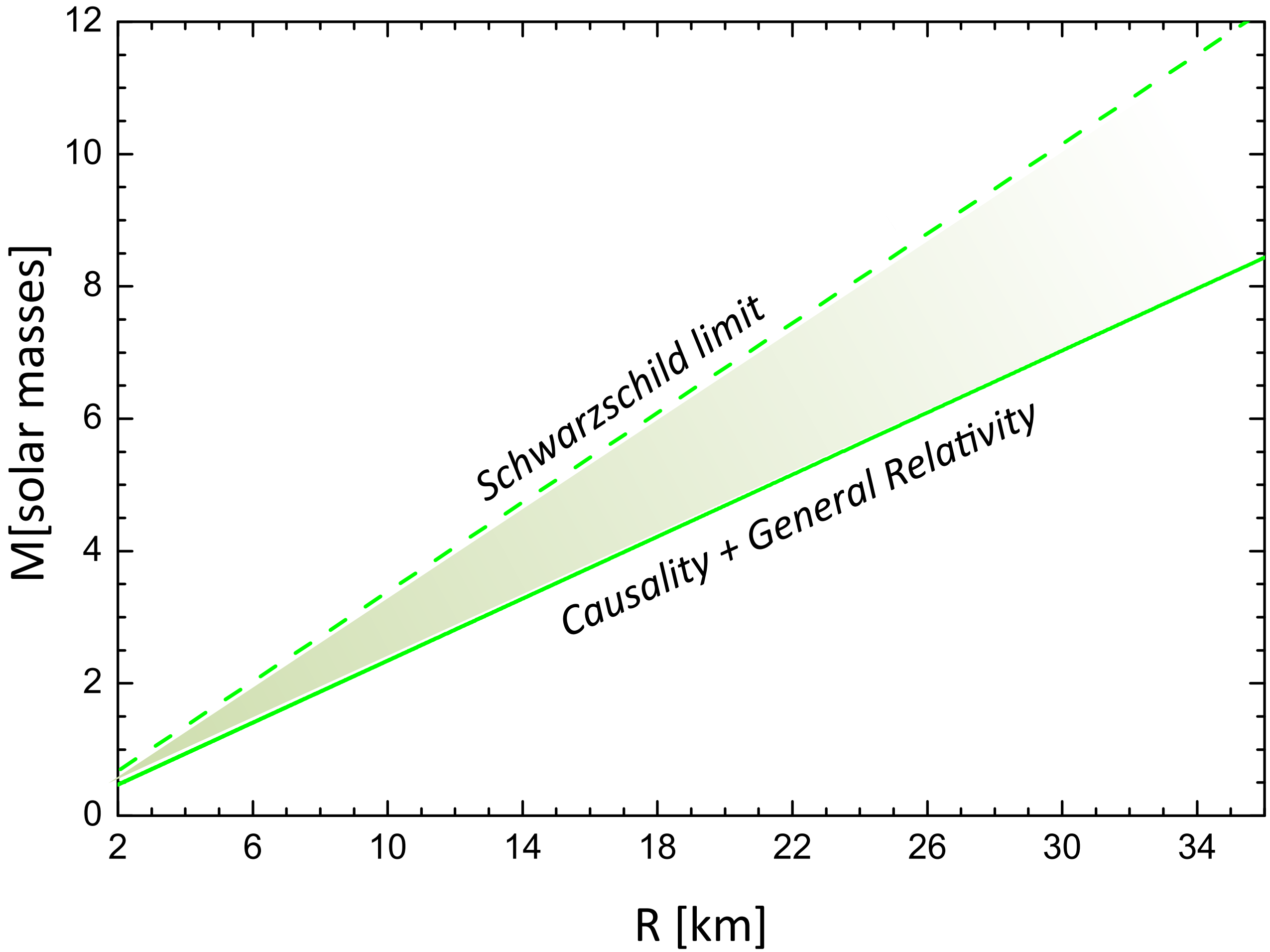
3. PRESENT WORK

Causality limit in the M-R diagram: braneworld model

- We adopt BPS EOS + causal EOS (p = ρ − a).
- We numerically integrate the structure eqs. for different values of a and of the brane tension λ.



- **No maximum mass!!** As the mass increases, the radius becomes larger.
- All the curves tend asymptotically to M = R /2 (Schwarzschild limit)



The causality limit in the braneworld model is coincident with the Schwarzschild limit.

The shaded region in the M-R plane is forbidden in GR for causal EOSs but **it is not within BW models**.

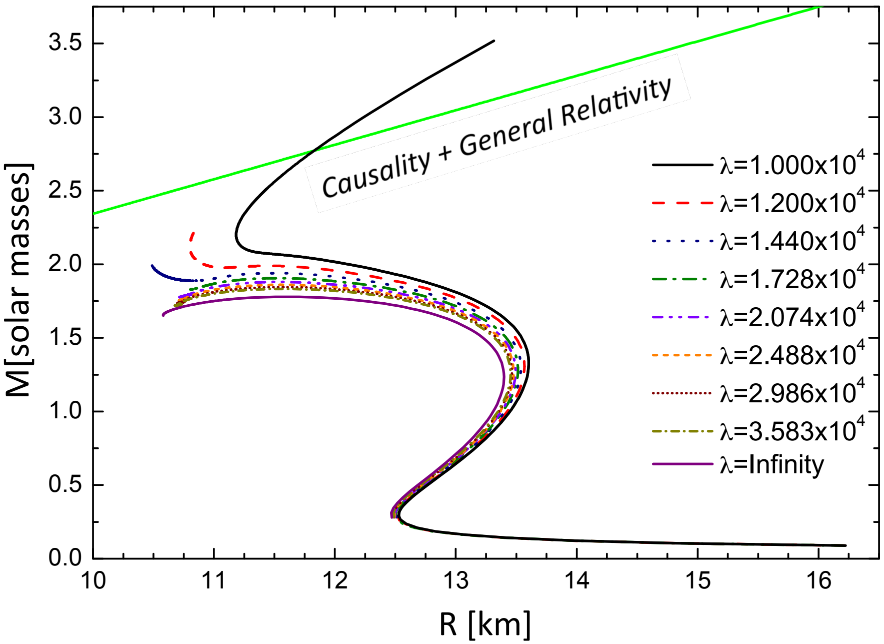
Models for hadronic and quark stars

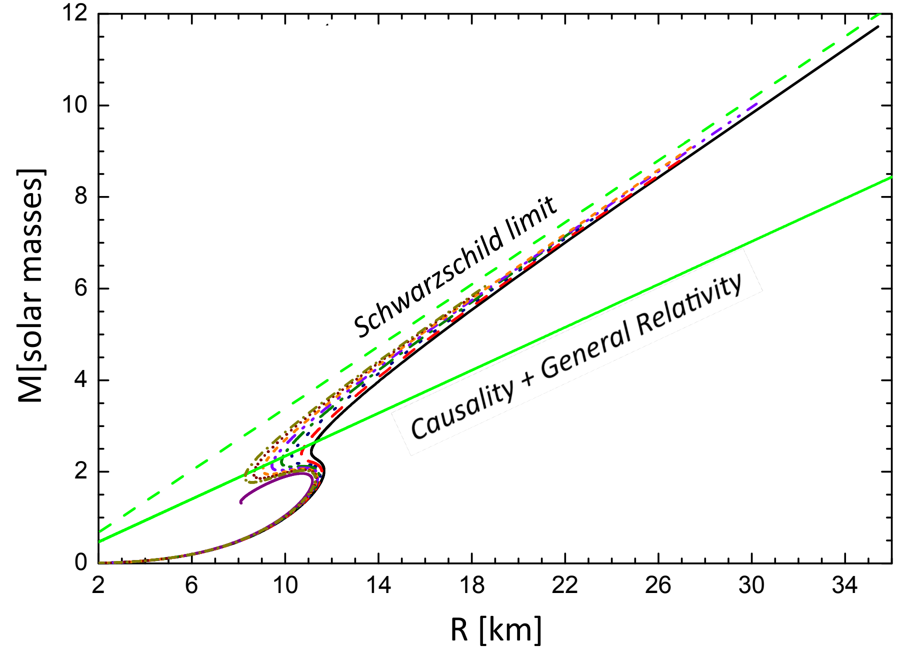
There is a large amount of high density EOS that fulfill present experimental constrains.

Our purpose is not making an exhaustive survey of all the available EOSs, but rather to explore the qualitative properties of hadronic and strange quark stars in BW models.

We use standard equations of state:

- **quark matter:** MIT bag model with zero strong coupling constant and massless quarks $\Rightarrow \rho = 3\,p + 4\,B$.
- **hadronic matter:** non-linear relativistic mean-field Walecka model; Glendenning & Moszkowski parametrisation GM1; nucleons and electrons; BPS model at low densities.





For small masses ⟶ same behaviour as in GR: e.g. very small mass hadronic stars have very large radii, while strange stars follow roughly $M \sim R^3$.

For large mass objects ⟶ significant deviations with respect to GR.

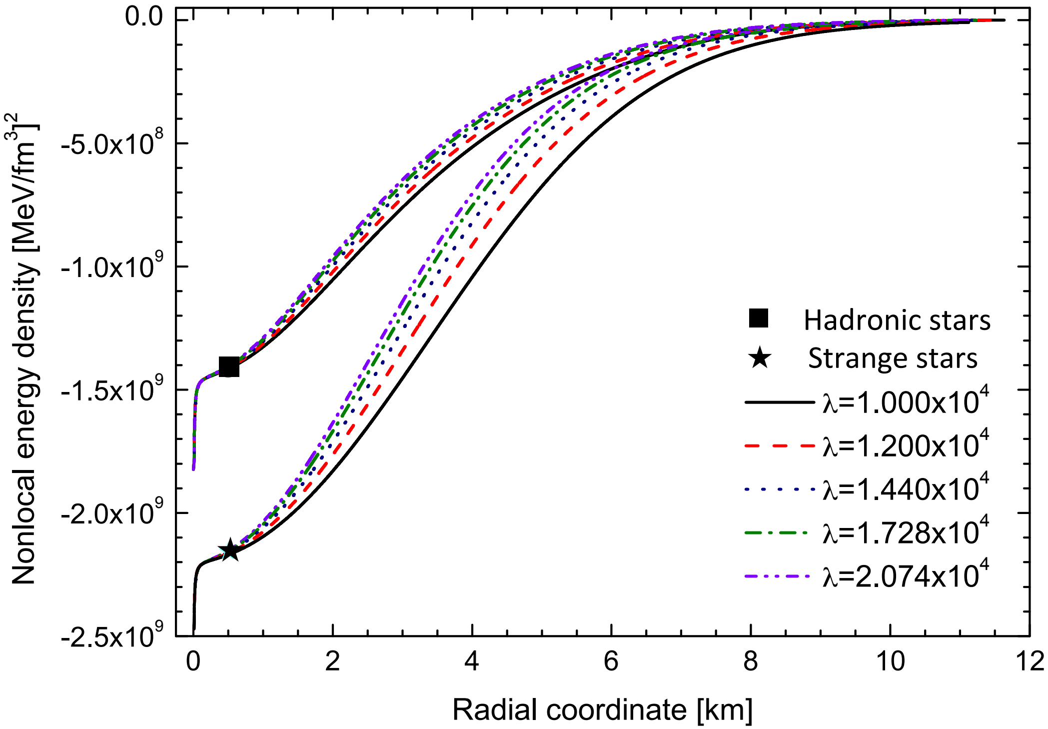
✓ At around $1.5 - 2M_\odot$ the M(R) curves bend anticlockwise as in GR.

✓ In some cases → **local maximum** in M(R); then the curves bend clockwise.

✓ In other cases → **no local maximum** in M(R). The curves also bend clockwise.

For sufficiently large mass the M(R) curves violate the GR causality limit and then they approach the Schwarzschild limit asymptotically.

Nonlocal energy density $\mathcal{U}^{-}(r)$ versus the radial coordinate r

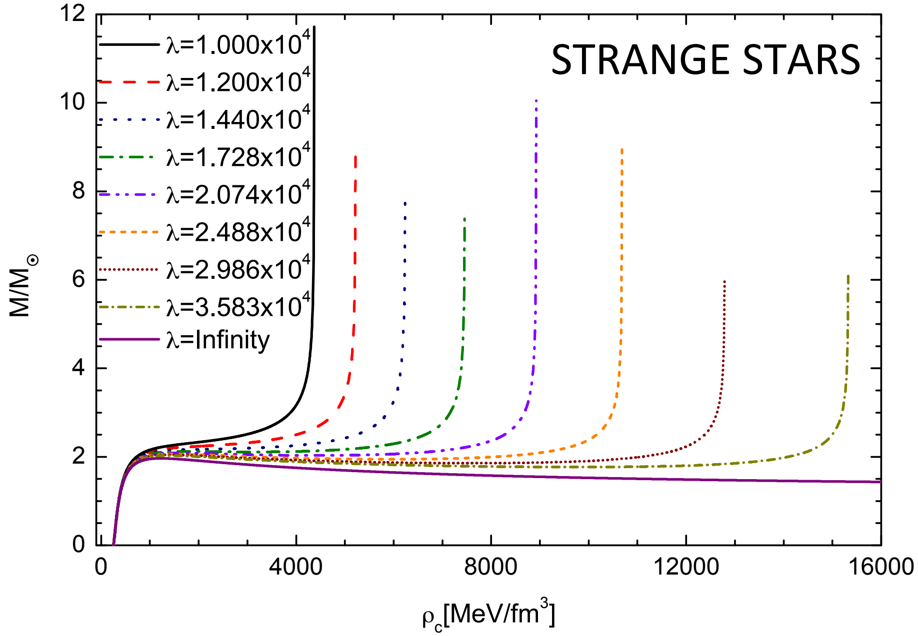


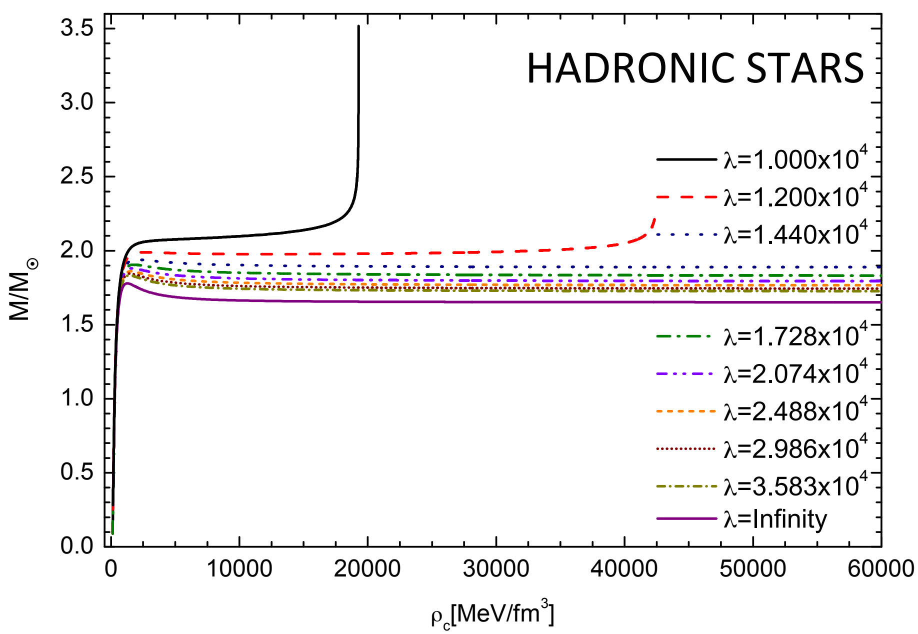
ρ_c = 2500 MeV/fm^3.
λ in MeV/fm^3.

➤ The nonlocal energy density starts at a large negative value and grows monotonically towards the surface of the star.

➤ The negative value of $\mathcal{U}^{-}$ means that it acts as an effective pressure helping against the collapse → a star with a more negative $\mathcal{U}^{-}$ admits more mass.

Mass versus central energy density ρ_c



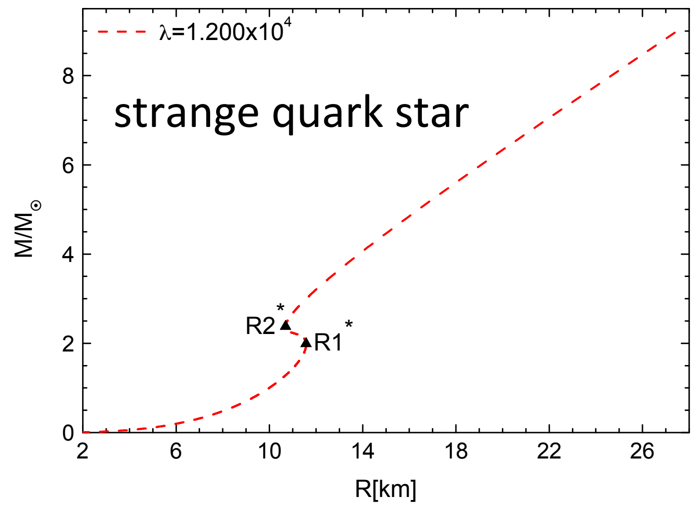


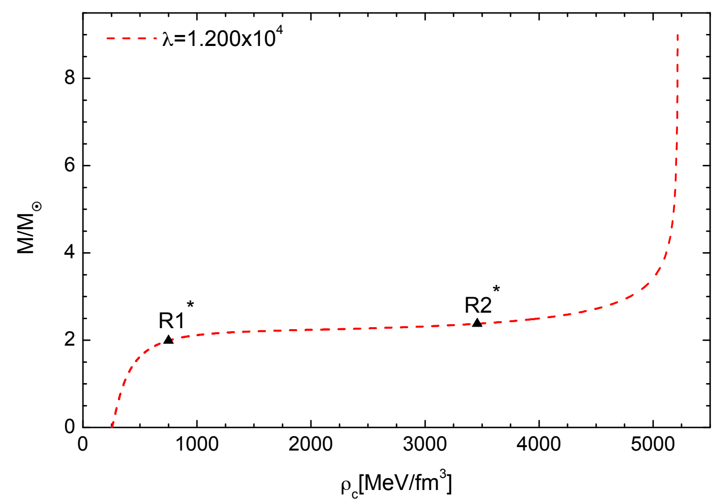
➤For small ρ_c: same behaviour as in GR.

➤ Remarkable feature: **there is a ρ_c for which M diverges** → The reason is that for large enough M the nonlocal energy density $\mathcal{U}^-$ supports the star against collapse.

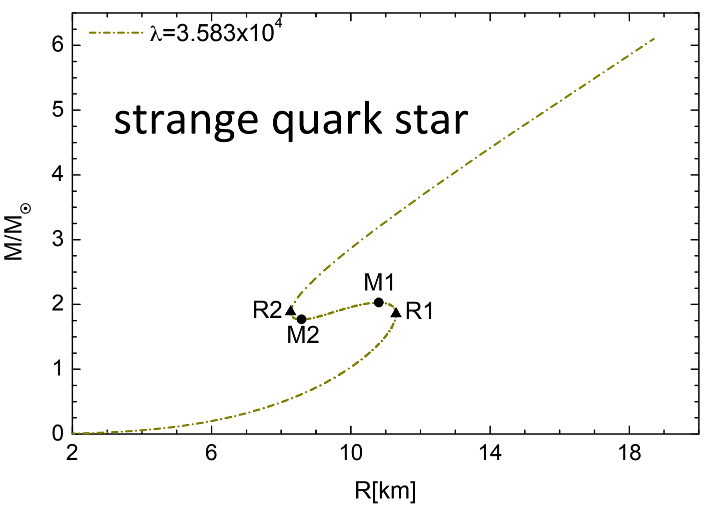
➤ The maximum value of ρ_c increases with λ. In particular, as we approach GR ($\lambda \to \infty$) the maximum ρ_c is shifted to ∞.

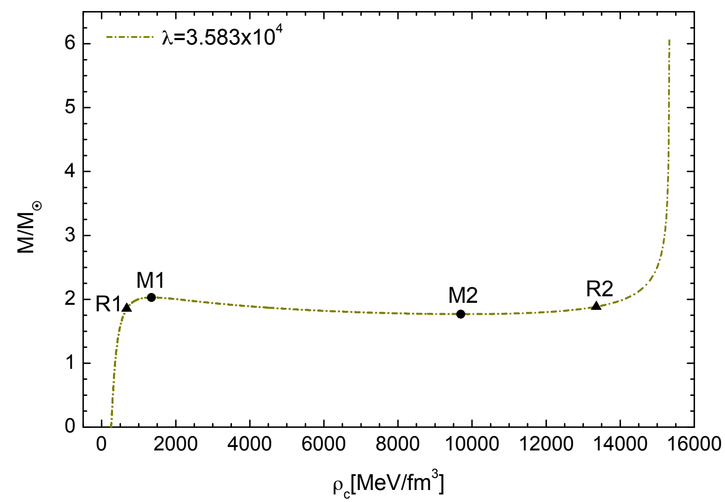
STABILITY: necessary condition $dM/d\rho_c > 0$





$dM/d\rho_c > 0$ not sufficient condition for stability.





We have two qualitatively different types of M-R curves:

Models **with a local maximum** in M(R) and $M(\rho_c)$: instability between points M1 and M2.

Models **without a local maximum** in M(R) and $M(\rho_c)$: necessary condition is always verified.

Stability of radial oscillation modes

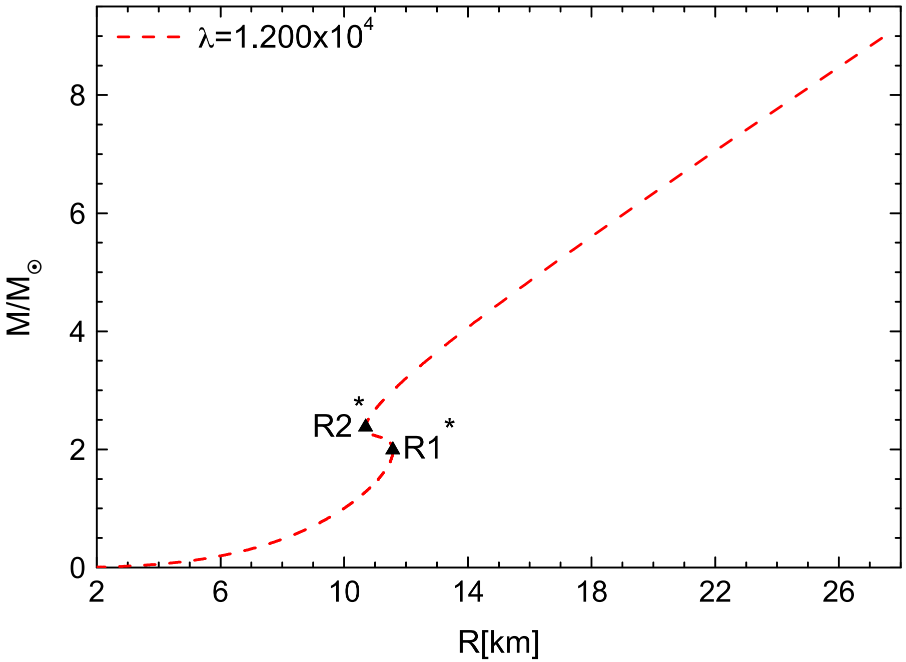
A more detailed study ⇒ analysis of the radial oscillations (future work).

Instead, we use a criterion that allows to determine **the precise number of unstable normal radial modes** using the M(R) curve [Harrison, Thorne, Wakano & Wheeler (1965); Haensel, Potekhin & Yakovlev (2007)]:

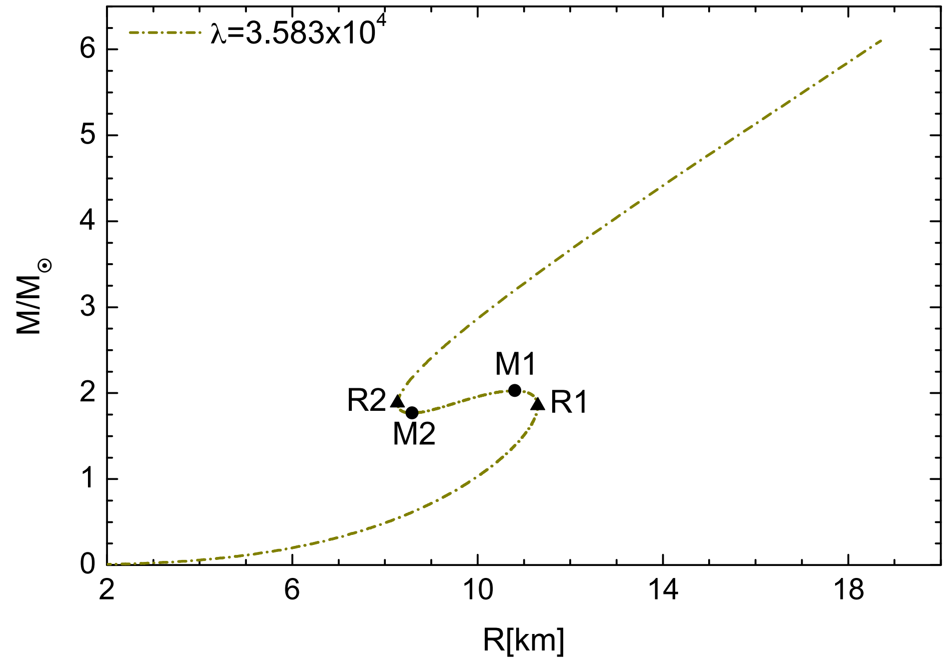
- ✓ at each **critical point** of the M(R) curve one and only one normal radial mode changes its stability (from stable to unstable, or vice versa).
- ✓ There are no changes of stability associated with radial pulsations at other points of the M(R) curves.
- ✓ One mode becomes unstable **if and only if** the M(R) curve bends **counterclockwise** at the critical point.
- ✓ One mode becomes stable **if and only if** the M(R) curve bends **clockwise** at the critical point.

We have two qualitatively different types of M-R curves

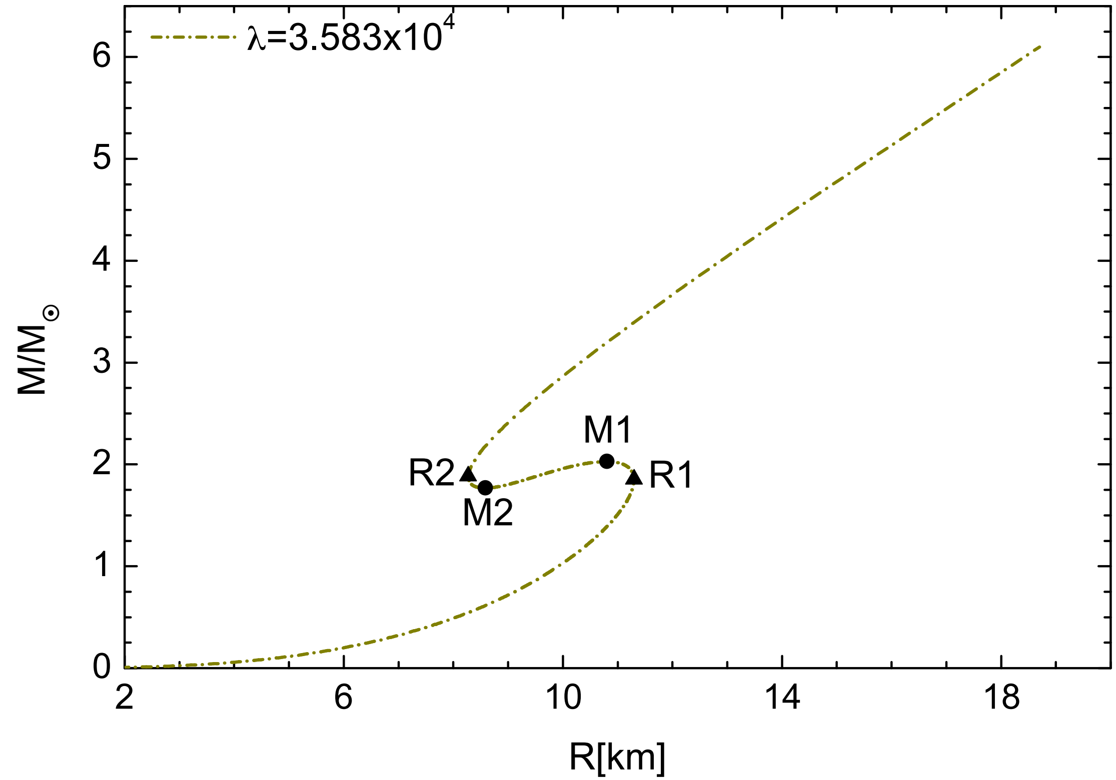
without critical points



two critical points

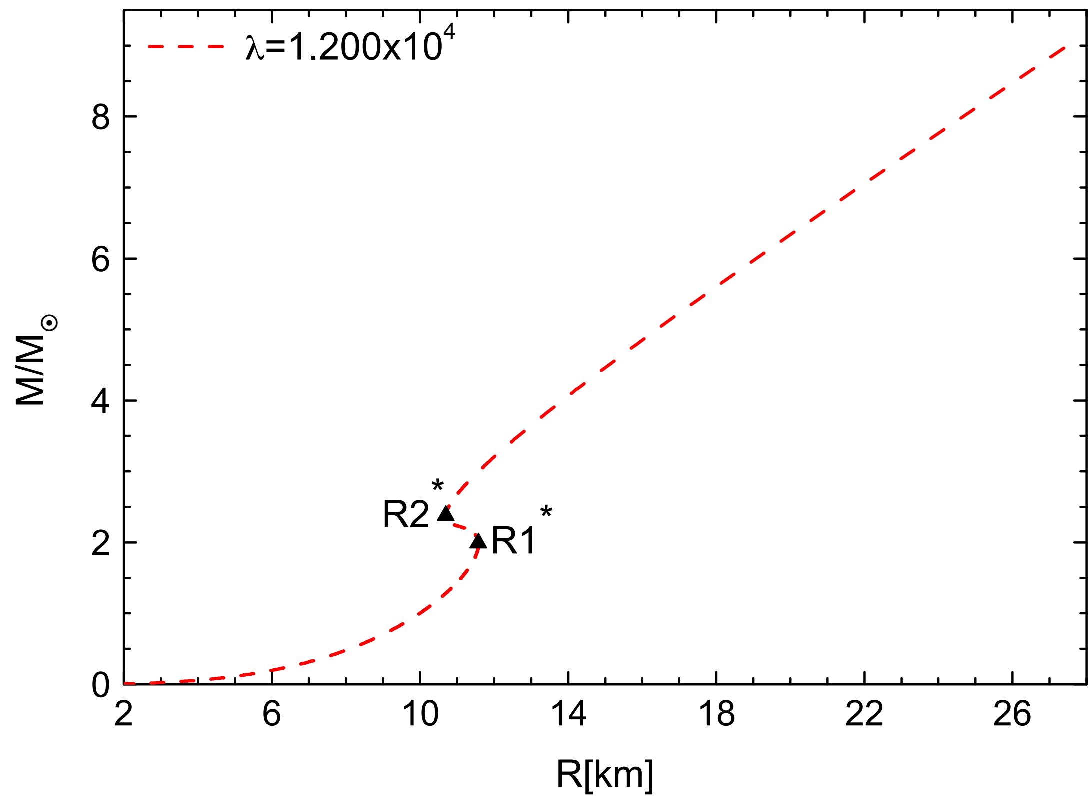


Case with TWO critical points



- We assume that the lowest density segment (below point M1) is stable for all radial modes, as it is in the GR case.
- At the critical point (local maximum) M1 the M(R) curve bends counterclockwise and the fundamental mode becomes unstable.
- At the critical point (local minimum) M2 the fundamental mode becomes stable again because the curve bends clockwise there. **Beyond M2 there are no more critical points and all the radial modes remain stable**.

Case WITHOUT critical points

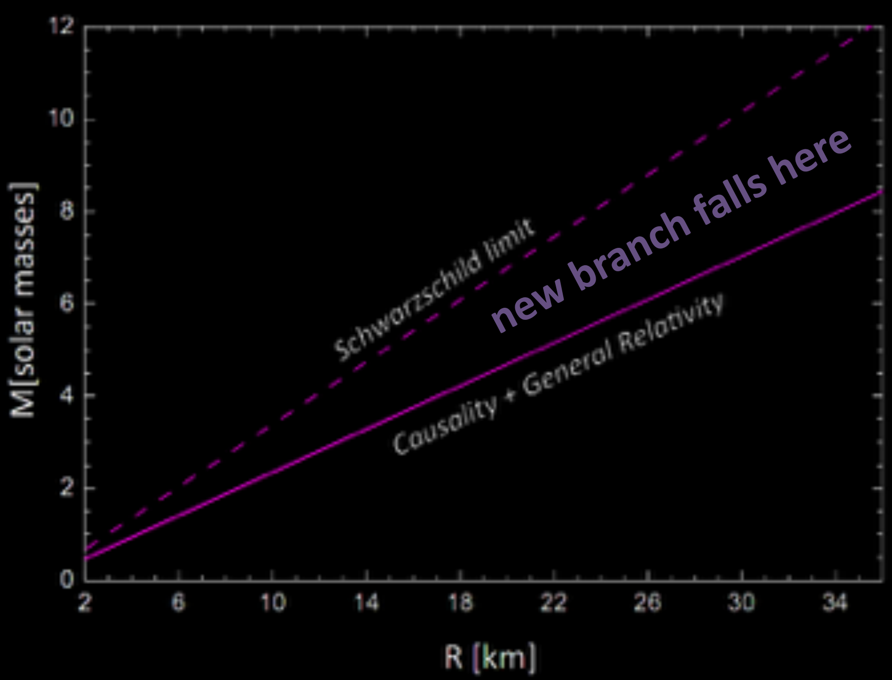


- Again, we assume that for low enough M the stellar configurations are stable for all radial modes, as it is in the GR case.
- Since there are no critical points **the whole sequence remains stable for all radial modes.**

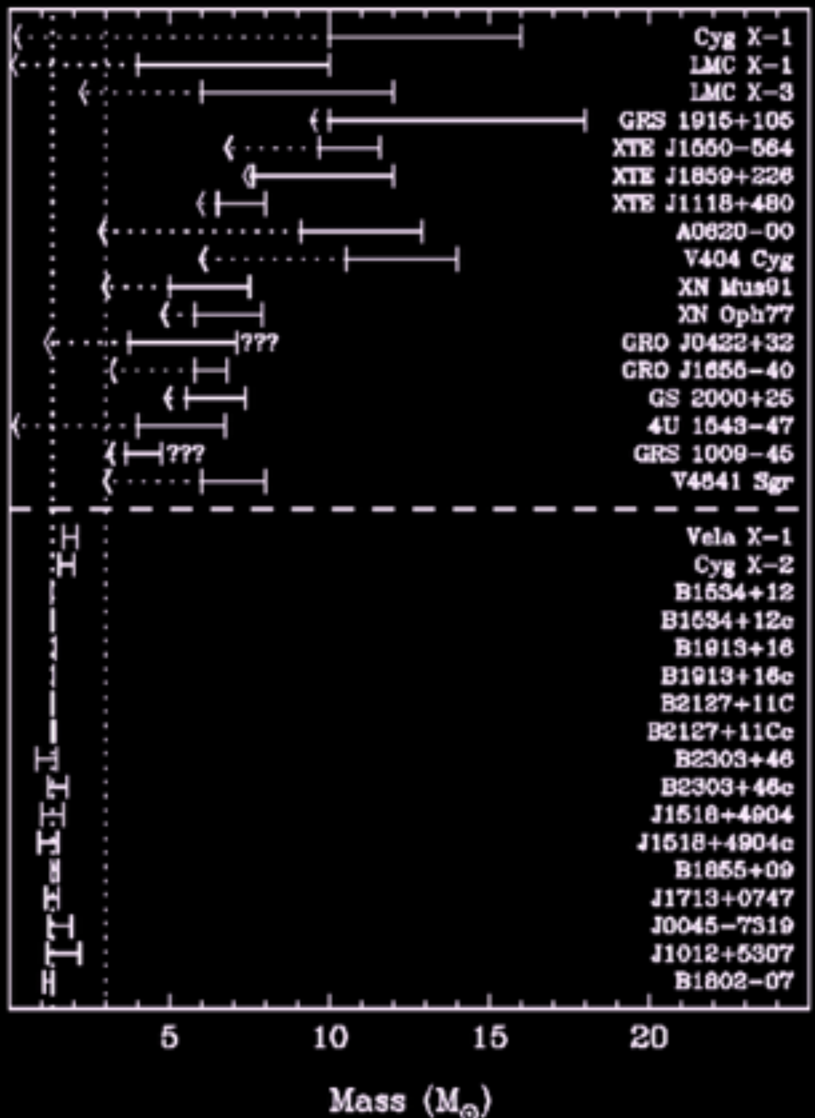
Conclusions

- Within BW models we obtain the usual branch of compact star configurations already known from GR calculations.

- We also find a **new branch** that violates the GR causality limit.
  - it approaches asymptotically to the Schwarzschild limit
  - it is always stable under small radial perturbations.
  - stellar configurations of arbitrarily large mass are possible.
  - supported against collapse by the nonlocal effects of the bulk on the brane.



- If they exist in Nature, such large mass BW stars may be hidden among the population of black hole candidates.



Black hole candidates (Mirabel 2016)

- BH are still possible within BW models. Stellar configurations that asymptotically approach to the Schwarzschild limit are stable under small perturbations, but not necessarily under large ones. → a very large mass braneworld compact star could collapse into a BH if strongly perturbed in a catastrophic astrophysical event, e.g. in a binary stellar merging.

- The existence of the new branch of large mass objects can be tested through the observation of M and R of compact stars. If found, such objects could be an astrophysical manifestation of the existence of extra dimensions.

Thanks !