Properties of Relativistic Rotating Compact Stars

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Properties of Relativistic Rotating

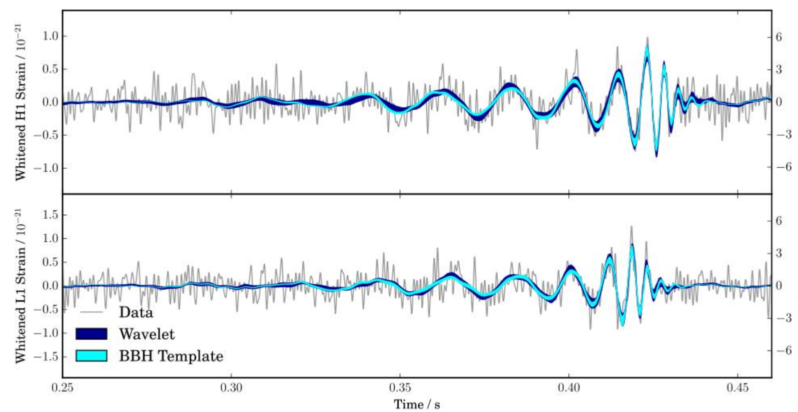
Compact Stars

Pulsar: the most accurate clock in the Universe, periodic pulsation in radio/X-ray/Gamma ray bands.

Why are we interested in pulsars?

➢ **Strong interaction**: $T \sim 0, \ \ \rho > \rho_0 \ \ \rightarrow \ \ \alpha_s > 1$

- Testing different EoS by pulsar observations can give us information about Non-perturbative QCD



➢ **Gravity**: $C_{NS} \sim 0.5 \ C_{BH}$

- Binary pulsar merger produces gravitational wave

The discovery of GW150914 opens a new era of gravitational wave astronomy, by the advantage of which we can also constrain EoS of compact stars. NR needs to be invoked for this quest.

Properties of Relativistic Rotating Compact stars

- Einstein came up with the non-linear (SEP) field equations 100 years ago

$$G_{\mu\nu} = 8\pi T_{\mu\nu},$$

which denies analytical solutions unless certain symmetry conditions are assumed.

- **Cauchy problem**:

$$\frac{\partial^2 \phi_k}{\partial t^2} = F_k(t, x^i; \phi_a, \frac{\partial \phi_a}{\partial t}, \frac{\partial \phi_a}{\partial x^i}, \frac{\partial^2 \phi_a}{\partial t \partial x^i}, \frac{\partial^2 \phi_a}{\partial x^i \partial x^j})$$

is deterministic given the initial data: $\phi_k(t_0, x^i)$ & $\frac{\partial \phi_k}{\partial t}(t_0, x^i)$.

So in order to solve Einstein equation numerically, we have to isolate those equations with 2nd order time derivative (6 evolution equations) and those without (4 constraint equations), which is called 3+1 decomposition.

Properties of Relativistic Rotating Compact stars

- In the NR scheme, solving equilibrium configuration for a system is to build initial data for it.
- COCAL is used in my work. It solved 4 constraint equations and 1 evolution equations in a conformally flat approximation.

Newtonian Gravity

1 Poisson equation

$$\nabla^2 \phi = 4\pi G\rho$$

NR Gravity

5 Poisson equations

$$\nabla^2 \alpha = \cdots$$

$$\nabla^2 \beta_i = \cdots$$

$$\nabla^2 (\alpha\psi) = \cdots$$

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Hydrodynamic Eqs:

Newtonian:
Euler Equations

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \frac{\nabla p}{\rho} = \mathbf{g}$$

$$\frac{\partial e}{\partial t} + \mathbf{u} \cdot \nabla e + \frac{p}{\rho} \nabla \cdot \mathbf{u} = 0$$

GR:
Mass conservation and energy-momentum conservation:

$$\nabla_\alpha (\rho u^\alpha) = 0$$
$$\nabla_\alpha T^{\alpha\beta} = 0$$

We still need EoS to close this system....

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Raw egg vs cooked egg analogy
For neutron star vs strange cluster star

How would distinguish between two objects that has different rigidity (i.e. a stone and a cake on your desk in midnight without light)?
Act a force on it!

But who can do us a favor to act a force on a compact star?
A companion! ---→ Tidal love number
Or suffering an acceleration ---→ Rotating compact star

Modifying COCAL for QS EOS

Piecewise polytrope (Neutron star)

$$p = \kappa_i \rho^{\Gamma_i}$$

- Emden function ($q = p/\rho$) is used as the fluid variable since it has linear relationship with the enthalpy:

$$dh = \frac{\Gamma_i}{\Gamma_i - 1} dq$$

- Surface density/pressure/energy density are all 0. No first order discontinuity.
- Other thermodynamic quantities are also easily related with q and h:

$$\rho = \kappa_i^{\frac{1}{1-\Gamma_i}} q^{\frac{1}{\Gamma_i - 1}}$$

$$p = \kappa_i^{\frac{1}{1-\Gamma_i}} q^{\frac{\Gamma_i}{\Gamma_i - 1}}$$

$$e = \rho h - p$$

Polynomial EoS (Quark star)

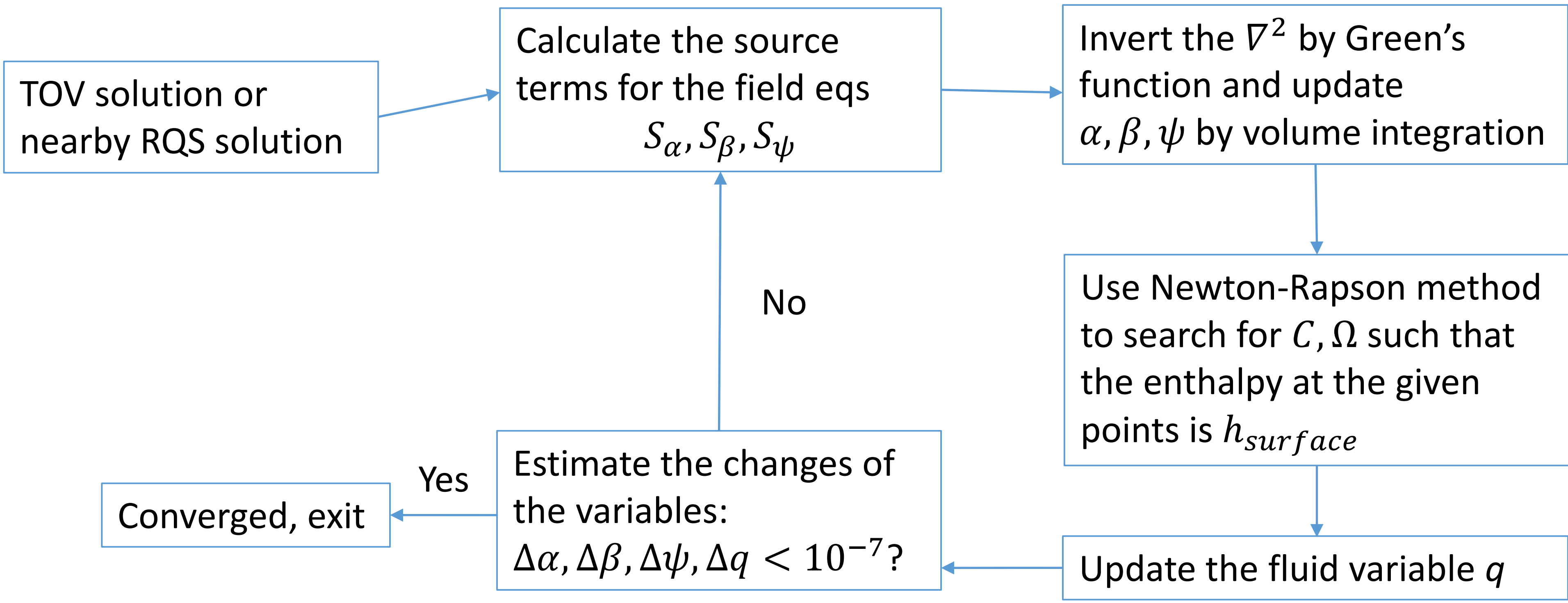
$$p = \sum \kappa_i \rho^{\Gamma_i}$$

- The trivial relation between emden function and enthalpy doesn't hold any more. We choose rest mass density to be the fluid variable and use root finding.
- First order discontinuity on the surface. Surface density/energy density are non-zero.
- We need to create parameter file with more information.

$$e = \sum \frac{\kappa_i}{\Gamma_i - 1} \rho^{\Gamma_i} + \rho + \rho C$$

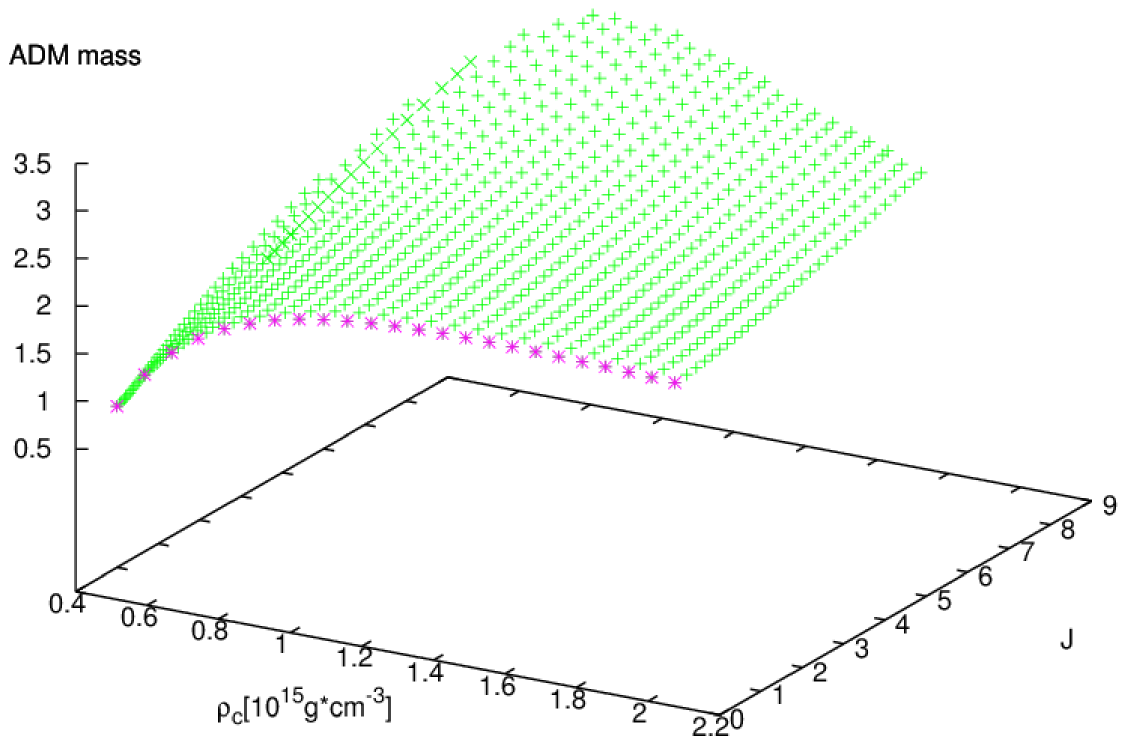
$$h = \frac{p + e}{\rho} = \sum \frac{\Gamma_i \kappa_i}{\Gamma_i - 1} \rho^{\Gamma_i - 1} + 1 + C$$

Iteration procedure



Axisymmetric RQS sequence

- EOS: MIT BAG model with $B = 60\mathrm{MeVfm}^{-3}$

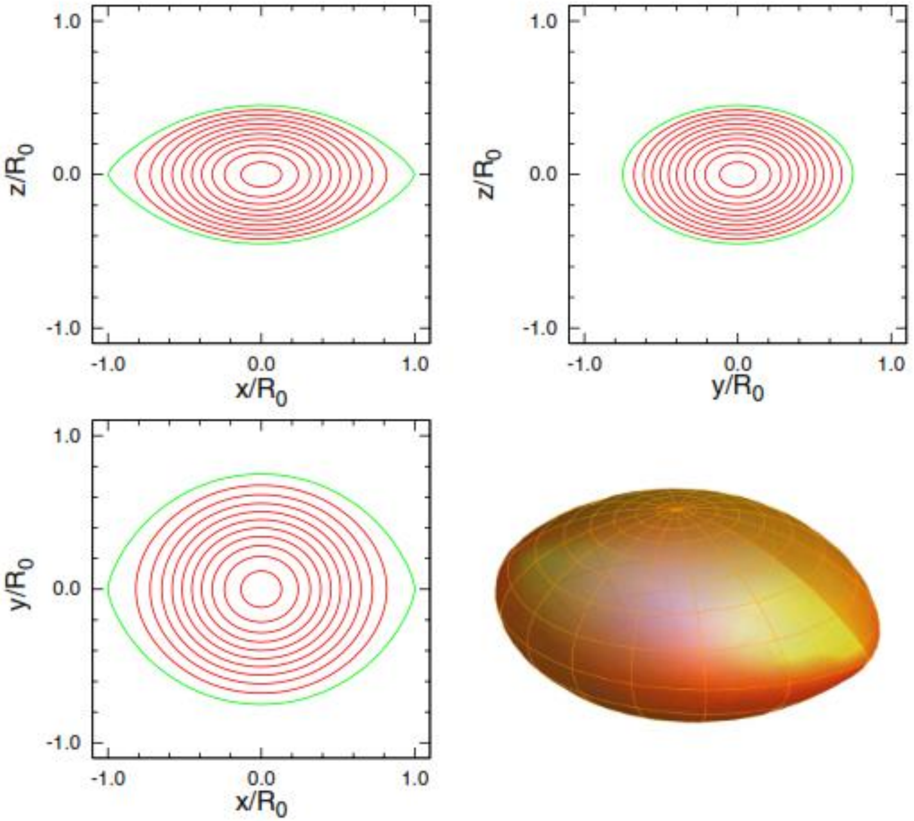


Triaxially RQS sequence

- Secular instability

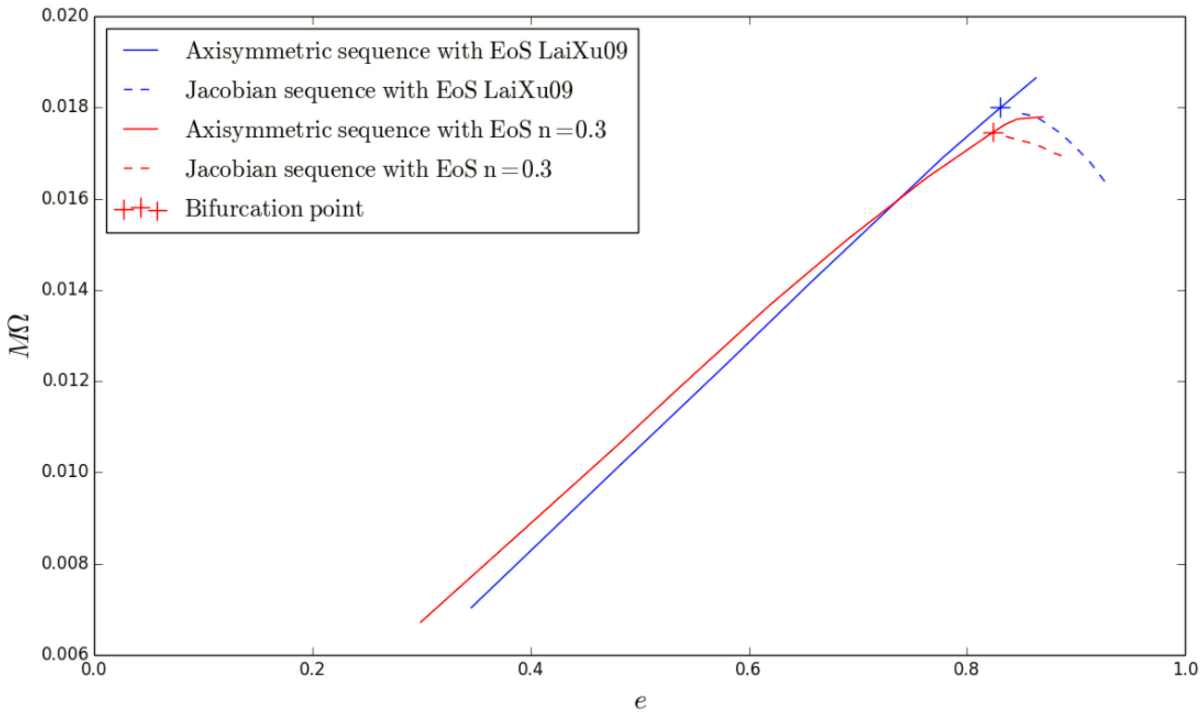
- Newtonian scheme :
  Jacobian ellipsoid

induced @ T/W >0.1375



Triaxially RQS sequence

- Some interesting outputs: the bifurcation from Maclaurin sequence (axisymmetric deformation) to Jacobian sequence (triaxial deformation)

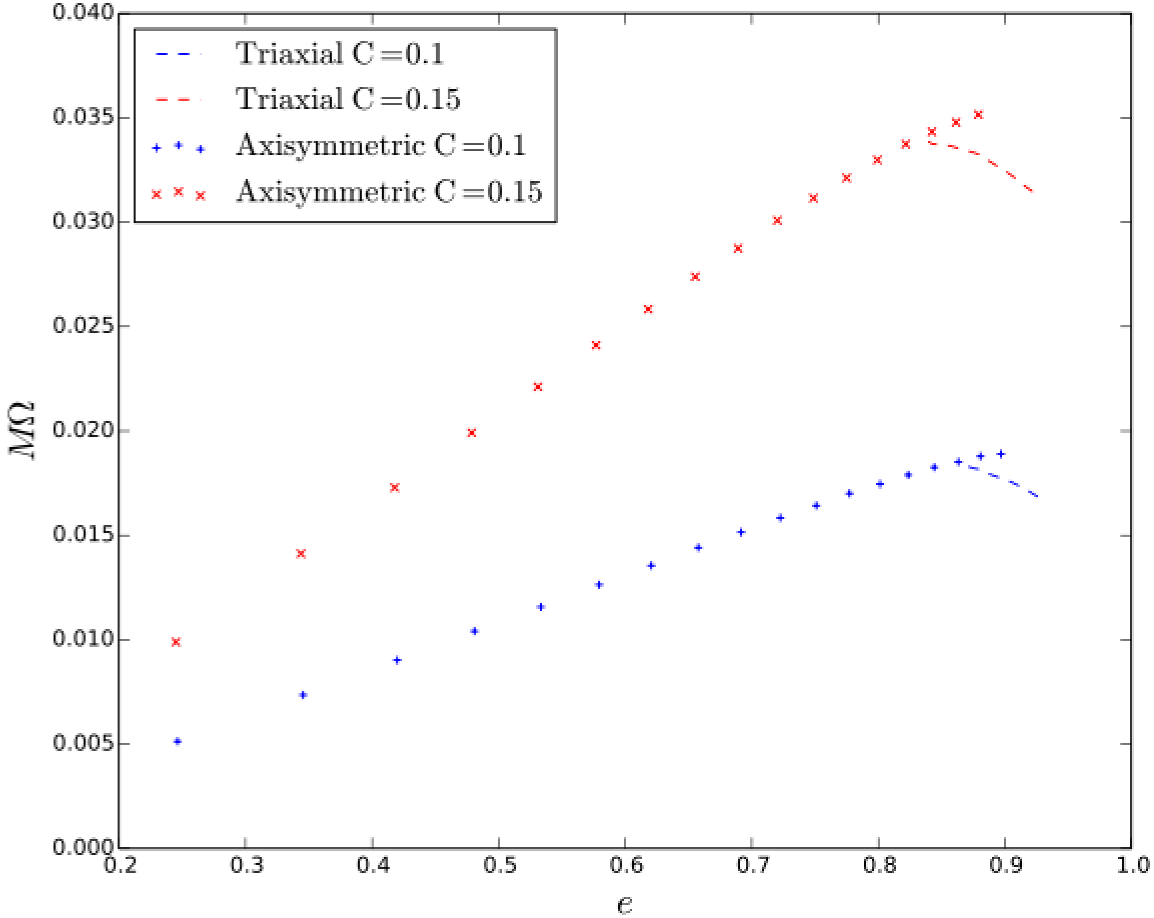


The bifurcation point constrains the maximum spin a compact star can reach when spun up.

From my calculation and Huang et al. 2007, it shows that a neutron star with polytropic index n=0.3 can barely reach 1ms with compactness C=0.1.
While LaiXu09 EoS can reach 0.9ms with a compactness of C=0.1

Remark: Newtonian limit for incompressible fluid $\Omega = 2e\sqrt{\frac{2\pi\rho G}{15}}$

Triaxially RQS sequences



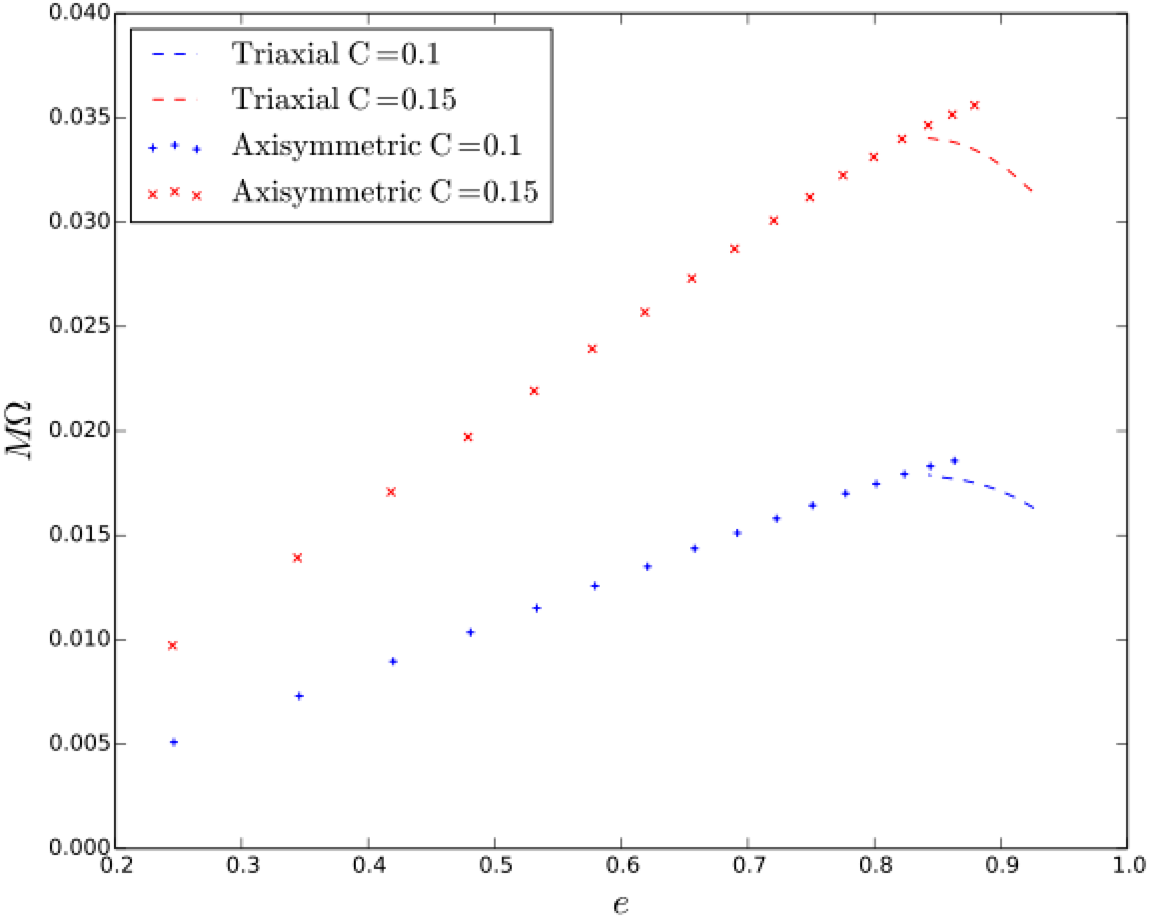
EOS:
MIT BAG model

Bifurcation @

for C=0.15
T/W ~ 0.1342
P ~ 1.04 ms
M ~ 1.18 M_sol

for C=0.1
T/W ~ 0.1317
P ~ 1.10 ms
M ~ 0.65 M_sol

Triaxially RQS sequences



EOS:
Lai&Xu 2009

Bifurcation @

for C=0.15
T/W ~ 0.1379
P ~ 0.85 ms
M~0.980 M_sol

for C=0.1
T/W ~ 0.1377
P ~ 0.89 ms
M ~ 0.537 M_sol

Discussion and Conclusion

- We managed to build up 3-D full GR configuration for rotating quark stars.
- QS with stiffer EoS can reach a faster spin before bifurcating to Jacobian sequence than NS. This difference can be tested by future observations.
- GW radiation from triaxially rotating compact stars will also be an interesting topic as they might be detected in the future. The GW waveforms can be obtained by using the initial data that we have produced.