Unified Description of Nuclear Matter Properties Within the CBF Effective Interaction Approach

Omar Benhar

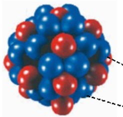
INFN and Department of Physics, “Sapienza” University
I-00185 Roma, Italy

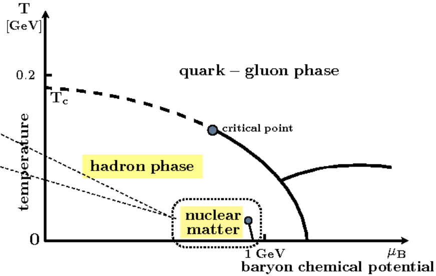
Compact Stars in the QCD phase diagram V
GSSI, May 23-27, 2016

PREAMBLE

⋆ This talk will focus on a *very small*—although arguably very important—region of the QCD phase diagram

nuclei





Scales in **nuclear matter**:

- momentum scale: **Fermi momentum** $\quad p_F \simeq 1.4\ \mathrm{fm}^{-1} \sim 2m_\pi$
- NN distance: $\quad d_{NN} \simeq 1.8\ \mathrm{fm} \simeq 1.3\ m_\pi^{-1}$
- energy per nucleon: $\quad E/A \simeq -16\ \mathrm{MeV}$
- compression modulus: $\quad K = (260 \pm 30)\ \mathrm{MeV} \sim 2m_\pi$

Outline

- ⋆ Modeling Nuclear Dynamics
- ⋆ *Ab initio* Approaches to the Nuclear Many-Body Problem
- ⋆ The CBF Effective Interaction
- ⋆ The Fermi Hard-Sphere System as a Testing Ground
  - ▷ Energy per Particle and Quasi Particle Properties
  - ▷ Momentum Distribution
  - ▷ Transport Coefficients
- ⋆ Nuclear Matter Properties
  - ▷ Energy of Cold β-Stable Matter
  - ▷ Extension to Finite Temperature
  - ▷ Response to Interactions With Low-Energy Neutrinos
- ⋆ Summary & Outlook

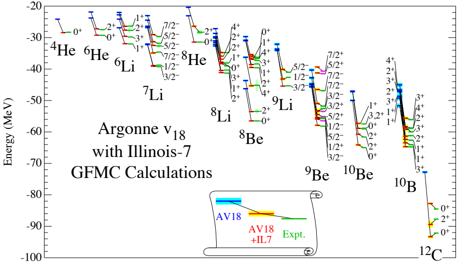
Modeling Nuclear Dynamics

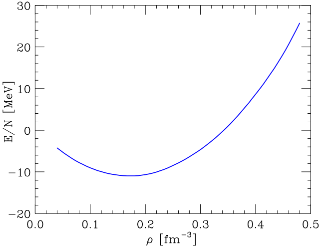
- *ab initio* (bottom-up) approach: nuclear systems are described as a collection of point-like particles, whose dynamics are decribed by the Hamiltonian

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} v_{ijk}$$

  - v_{ij} provides a very accurate descritpion of the two-nucleon system, in both bound and scettering states, and reduces to Yukawa's one-pion-exchange potential at large distances
  - inclusion of v_{ijk} needed to explain the ground-state energies of the three-nucleon systems
  - v_{ij} is spin and isospin dependent, non spherically symmetric, and strongly repulsive at short distance
  - nuclear interactions can not be treated in perturbation theory in the basis of eigenstates of the non interacting system

⋆ Quantum Monte Carlo and variational calculations performed using phenomenological nuclear Hamiltonians explain the energies of the ground-and low-lying excited states of nuclei with mass $A \leq 12$, as well as saturation of the equation of state of cold isospin-symmetric nuclear matter





Correlated Basis Functions (CBF)

- Replace the basis states of the non-interacting system with a set of correlated states

$$|n_0\rangle \to |n\rangle = \frac{F|n_0\rangle}{\langle n_0|F^\dagger F|n_0\rangle^{1/2}} = \frac{1}{\sqrt{\mathcal{N}_n}}\, F\,|n_0\rangle$$

$$F = \mathcal{S}\prod_{j>i} f_{ij}$$

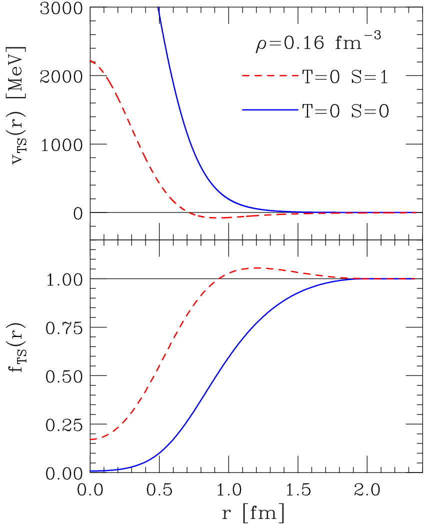
- The structure of the two-nucleon correlation operator reflects the complexity of nuclear dynamics

$$f_{ij} = \sum_{S,T=0,1} [f_{TS}(r_{ij}) + \delta_{S1} f_{tT}(r_{ij}) S_{ij}] P_{ST}$$

$$P_{ST} \ \text{spin} - \text{isospin projector operator} \ , \ S_{ij} = \sigma_i^\alpha \sigma_j^\beta \left(\frac{r_{ij}^\alpha r_{ij}^\beta}{r_{ij}^2} - \delta^{\alpha\beta}\right)$$

- The shapes of $f_{TS}(r_{ij})$ and $f_{tT}(r_{ij})$ are determined form minimization of the ground-state energy

NN Potential and Correlation Functions



CBF Effective Interaction

- In principle, the *complete* set of correlated states can be employed to carry out perturbative calculations, using the *bare* nuclear Hamiltonian
- However, correlated states are *non orthogonal*. Owing to this feature, perturbation theory in the correlated basis involves serious additional difficulties
- Alternatively, the formalism of correlated basis functions can be exploited to obtain a well behaved *effective interaction*, suitable for perturbation theory in the basis of eigenstates of the non-interacting system

- Cluster expansion of the Hamiltonian expectation value in the correlated ground state of isospin-symmetric nuclear matter at density $\rho = 2k_F^3/3\pi^2$

$$\langle H \rangle = \langle 0|H|0\rangle = \frac{3}{5}\frac{k_F^2}{2m} + \sum_{n\geq 2} (\Delta E[f_{TS}, f_{tT}])_n$$

- The shapes of the $f_{TS},\ f_{tT}$ are determined from functional minimisation of $\langle H \rangle$. All relevant cluster terms can be included using the FHNC/SOC summation scheme

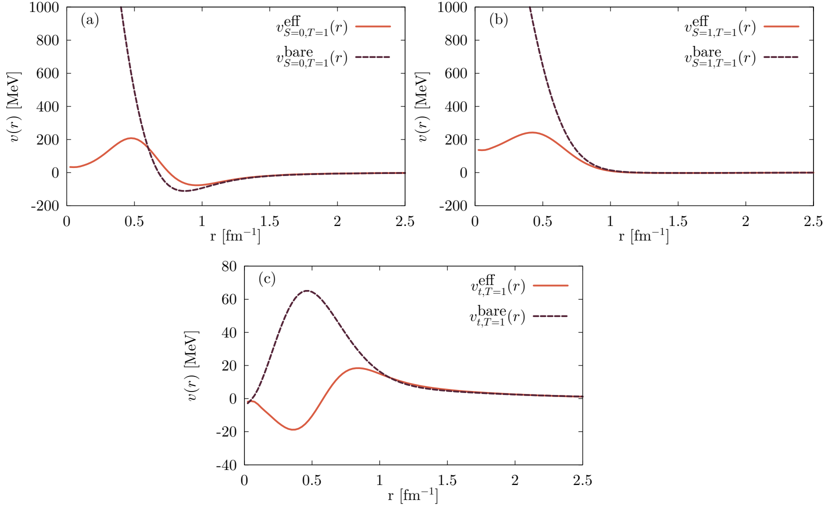
- The CBF effective interaction is *defined* adjusting the correlation functions in such a way as to satisfy the relation

$$\langle H \rangle = \frac{3}{5}\frac{k_F^2}{2m} + \left(\Delta E[\tilde{f}_{TS}, \tilde{f}_{tT}]\right)_2 = \frac{3}{5}\frac{k_F^2}{2m} + \langle 0_{FG}|V_{\rm eff}|0_{FG}\rangle$$

- In the case of central state-independent potential and correlation function

$$V_{\rm eff} = \sum_{j>i} v_{\rm eff}(r_{ij}) \ \ , \ \ v_{\rm eff}(r) = \frac{1}{m}\left(\nabla \tilde{f}(r)\right)^2 + \tilde{f}(r)v(r)\tilde{f}(r)$$

⋆ CBF effective interaction obtained from the Argonne $v_6' + UIX$ nuclear Hamiltonian

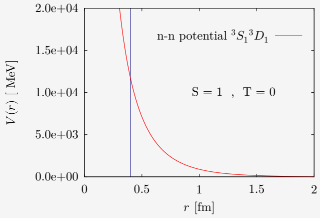


The hard-sphere model

The Fermi hard-sphere model: point-like spin one-half particles

$$v(r) = \begin{cases} \infty & r < a \\ 0 & r > a \end{cases}$$

- ⋆ Valuable model to study properties of nuclear matter.
- ⋆ Purely repulsive potential to prevent the possibility of Cooper pairs formation.
- ⋆ A simple many-body system to investigate the validity and robustness of the assumptions of CBF effective interaction approach.

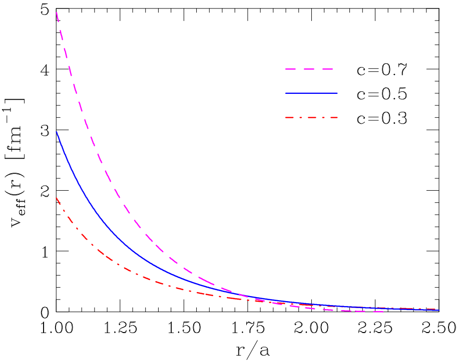


DETERMINATION OF $v_{\rm eff}$

For the hard-sphere system (HS) $f(r \leq a) = 0$, $\lim_{r\to\infty} f(r) = 1$

$$v_{\rm eff}(r) = \frac{1}{m}\left[\boldsymbol{\nabla} f(r)\right]^2 \quad , \quad r > a$$

We adjust the range of $f(r)$ in order to reproduce the ground state energy (FHNC/DMC) at two-body cluster level.

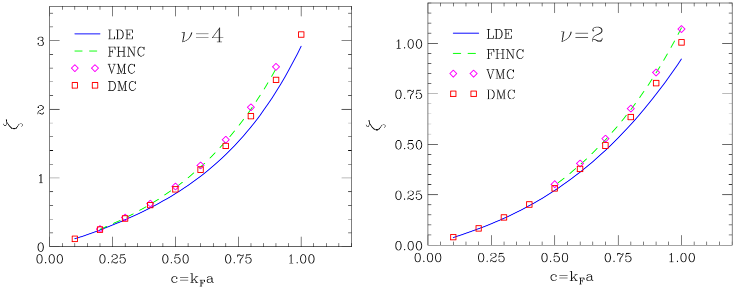


$v_{\rm eff}$

- ⋆ defined from $\langle H\rangle$ in the correlated ground state
- ⋆ employed in calculations of matrix elements involving excited states.

The ground-state energy

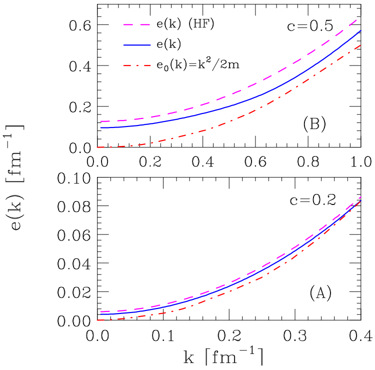
$$E_0 = \frac{3k_F^2}{10m}(1+\zeta)$$

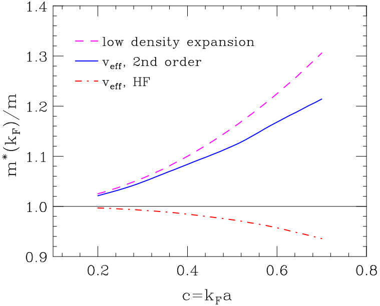


- The accuracy of the variational results depends on the quality of the trial wave function.
- Long-range statistical correlations effects in $f(r)$ much larger for $\nu = 2$ than for $\nu = 4$.
- DMC overcomes the limitations of the variational approach by using a projection technique on the trial wave function.

Quasiparticle spectrum

$$m^\star = \left[\frac{1}{k}\frac{de(k)}{dk}\right]^{-1}$$

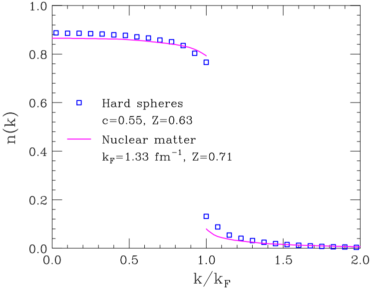




$$\frac{de(k)}{dk} = \left[\frac{k}{m} + \frac{\partial}{\partial k}\mathrm{Re}\Sigma\left(k, E\right)\right]\left[1 - \frac{\partial}{\partial E}\mathrm{Re}\Sigma\left(k, E\right)\right]^{-1}_{E=e(k)}$$

MOMENTUM DISTRIBUTION $\nu = 4$

In comparison with non orthogonal CBF perturbation theory



S. Fantoni and V. R. Pandharipande, Nucl. Phys. A **427**(1984)

Momentum distribution of HS

$$c \equiv k_F a = 0.55$$

corresponds to $n(k)$ of nuclear matter

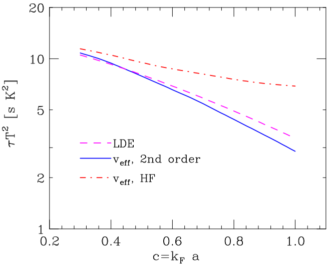
$$\rho_{NM} = 0.16 \text{ fm}^{-3}$$
$$k_F = 1.33 \text{ fm}^{-1}$$

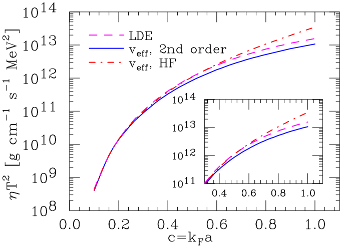
Nucleons in nuclear matter $\sim$ HS of radius $a = 0.55/1.33 \sim 0.4$ fm.

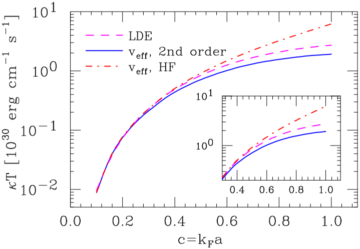
Virtual scattering processes between strongly correlated particles are mainly driven by the short-range repulsive core of the nucleon-nucleon interaction.

LIFETIME AND TRANSPORT COEFFICIENTS

The second order contributions lead to a sharp increase of $m^\star$, which in turn implies a decrease of the shear viscosity coefficient η and the thermal conductivity κ.

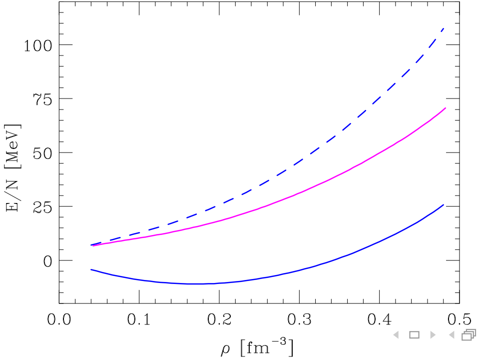




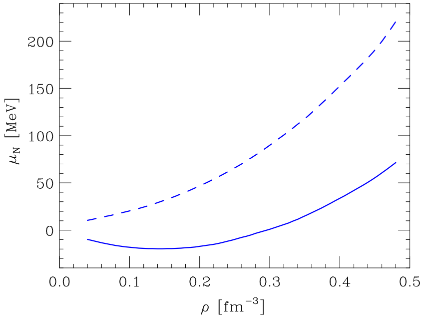


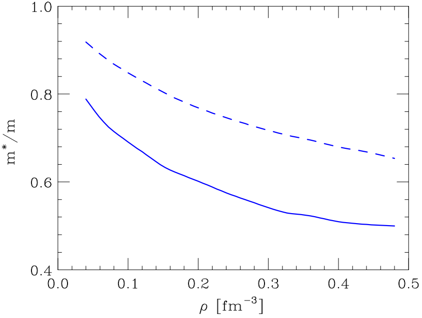
BACK TO NUCLEAR MATTER

- Energy per nucleon in isospin-symmetric matter (solid blue line), pure neutron matter (dashed blue line) and β-stable matter (solid magenta line)
- Calculations carried out at first order in the CBF effective interaction, derived from the Argonne $v_6' + UIX$ Hamiltonian.
- The effect of three-nucleon forces is accounted for through a density-depencence of the CBF effective interaction



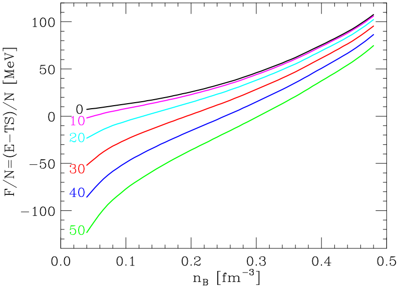
⋆ Nucleon chemical potential (left) and effective mass in units of the nucleon mass (right), in isospin-symmetric matter (solid lines) and pure neutron matter (dashed lines). Calculations performed at first order in the CBF effective interaction

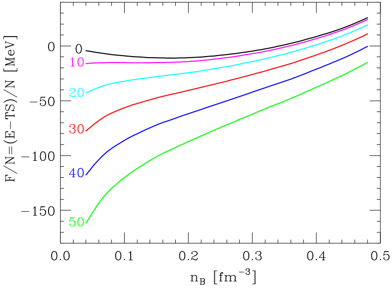




EXTENSION TO $T \neq 0$

- A realistic and *consistent* description of the properties of hot nuclear matter will be needed to perform systematic studies of gravitational-wave emission from protoneutron stars
- Free energy of PNM (left) and SNM (right) at $0 \leq T \leq 50$ MeV





Neutrino Interactions with Nuclear Matter

- Consider, for example, a neutral current process in pure neutron matter

$$\nu + n \to \nu' + n$$

- The mean free path of a low-energy non degenerate neutrino at zero temperature is obtained from

$$\frac{1}{\lambda} = \frac{G_F^2}{4}\,\rho \int \frac{d^3q}{(2\pi)^3}\left[(1+\cos\theta)S(\mathbf{q},\omega) + \mathbf{C_A^2}(\mathbf{3}-\cos\theta)\mathcal{S}(\mathbf{q},\omega)\right]$$

where S and $\mathcal{S}$ are the density (Fermi) and spin (Gamow Teller) response, respectively

Nuclear Matter Responses

- ⋆ Density response

$$\mathcal{S}^{\rho} = \frac{1}{N} \sum_{n} |\langle 0|J_0|n\rangle\langle n|J_0|0\rangle \delta^{(4)}(P_0 + q - P_n)$$

- ⋆ Spin-density response

$$\mathcal{S}^{\rho} = \sum_{\alpha} \mathcal{S}^{\rho}_{\alpha\alpha} \;\; , \;\; (\alpha = 1, 2, 3)$$

$$\mathcal{S}^{\rho}_{\alpha\beta} = \frac{1}{N} \sum_{n} |\langle 0|J_\alpha|n\rangle\langle n|J_\beta|0\rangle \delta^{(4)}(P_0 + q - P_n)$$

- ⋆ Neutral weak current

$$J_0 = \sum_i j_i^0 = \sum_i \mathrm{e}^{i\mathbf{q}\cdot\mathbf{x}_i} \quad , \quad J_\alpha = \sum_i j_i^\mu = \sum_i \mathrm{e}^{i\mathbf{q}\cdot\mathbf{x}_i} \sigma_\alpha$$

Effective Transition Operators

- The CBF states can be exploited to construct effective transition operators, consistent with the effective interaction, suitable for the calculation of the nuclear matter response
- The weak responses can be computed using the Fermi gas states and the corresponding Fermi and Gamow-Teller effective operators, defined through

$$\langle n|J^\mu|0\rangle = \langle n_{FG}|J^\mu_{\text{eff}}|0_{FG}\rangle$$

and

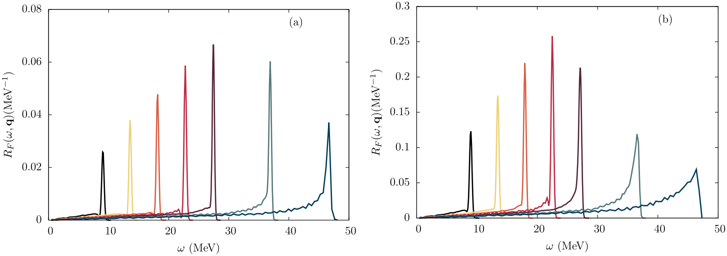
$$J^\mu_{\text{eff}} = FJ^\mu F = J^\mu + \sum_{j>i}\{j^\mu_i + j^\mu_j, g_{ij}\} + \dots$$

with

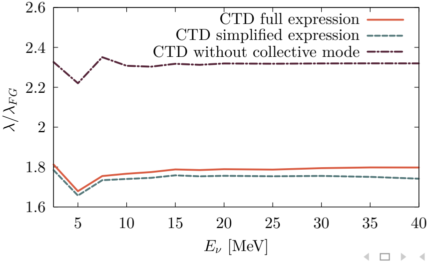
$$g_{ij} = f_{ij} - 1$$

- The correlation functions are the same entering the definition of the effective interaction V_{eff}

⋆ Density (a) and spin-density (b) responses of isospin-symmetric nuclear matter at equilibrium density ($|\mathbf{q}| = 0.1, \ldots 0.50\ \mathrm{fm}^{-1}$)



⋆ Neutrino mean free path in units of the Fermi gas result



SUMMARY & OUTLOOK

- ⋆ The CBF formalism provides a consistent theoretical framework for the *unified* description of equilibrium and non equilibrium properties of strongly interacting many-body systems
- ⋆ The results of the exploratory studies of the fermion hard-sphere system stronlgly support the validity of the assumptions underlying the CBF effective interaction approach
- ⋆ Calculations of a variety of properties of nuclear matter at temperatures $\lesssim 50$ MeV are on their way. Early results of these studies are being used in simulations of GW emission from protoneutron stars.

TWO–POINT GREEN'S FUNCTION

Dyson's equation

$$G(k,E) = G_0(k,E) + G_0(k,E)\Sigma(k,E)G(k,E)$$

Non interacting Green's function

$$G_0(k,E) = \frac{\theta(k-k_F)}{E-e_0(k)+i\eta} + \frac{\theta(k_F-k)}{E-e_0(k)-i\eta}$$

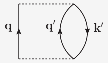
The irreducible (proper) self-energy $\Sigma(\mathbf{k},E)$ (mass operator) takes into account the effect of interactions.
The spectrum is determined by the singularities of $G(k,E)$

$$G(k,E) = \frac{1}{E-e_0(k)-\Sigma(k,E)}$$

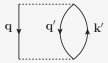
In perturbation theory

$$\Sigma(k,E) = \Sigma^{(1)}(k) + \Sigma^{(2)}(k,E) + \ldots$$

$\Sigma_{HF}(\mathbf{k})$



$\Sigma_{2p1h}(\mathbf{k},E)$



$\Sigma_{2h1p}(\mathbf{k},E)$

The elementary excitation spectrum

- The self energy is responsible for shifting the pole of the Green's function.
- The new poles determine energy $e(k)$ and the damping Γ_k of the quasiparticles state
- For small Γ_k, the propagation of quasiparticle states is described by

$$G(k,E) = \frac{Z_k}{E - e(k) + i\Gamma_k}$$

The energy of quasiparticle

$$e(k) = e_0(k) + \mathrm{Re}\Sigma[k, e(k)]$$

Quasiparticle lifetime

$$\tau_k^{-1} = \Gamma_k = Z_k \mathrm{Im}\Sigma[k, e(k)]$$

The residue of the Green's function

$$Z_k = \left[1 - \frac{\partial}{\partial E}\mathrm{Re}\Sigma[k,E]\right]^{-1}_{E=e(k)}$$

Boltzmann-Landau equation

Shear viscosity η and thermal conductivity κ measure momentum and energy fluxes in response to a gradient of velocity and temperature.

Boltzmann equation for a Fermi liquid:

$$\frac{\partial n_{\mathbf{k}}}{\partial t} + \frac{\partial n_{\mathbf{k}}}{\partial \mathbf{r}} \cdot \frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}} - \frac{\partial n_{\mathbf{k}}}{\partial \mathbf{k}} \cdot \frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{r}} = I[n_{\mathbf{k}}]$$

- $n_{\mathbf{k}}$ is the distribution function
- $\epsilon_{\mathbf{k}}$ is the energy of a quasiparticle carrying momentum $\mathbf{k}$
- $I[n_{\mathbf{k}}]$ is the collision integral, defined in terms of the scattering probability W

Taking into account small deviations from local equilibrium, transport coefficients determined from the collision integral $I[n_{\mathbf{k}}]$.

Abrikosov–Khalatnikov solution

The lifetime

$$\tau = \frac{1}{T^2}\,\frac{8\pi^4}{m^{\star 3}}\,\frac{1}{\langle W\rangle}$$

The transport coefficients

$$\eta = \frac{16}{15}\frac{1}{T^2}\frac{k_F^5}{m^{\star 4}}\;\frac{1}{\langle W\rangle(1-\lambda_\eta)}\;\;,\;\;\kappa = \frac{16}{3}\frac{1}{T}\frac{\pi^2 k_F^3}{m^{\star 4}}\;\frac{1}{\langle W\rangle(3-\lambda_\kappa)}$$

τ, η, κ are expressed in terms of angular averages of W

$$\langle W\rangle\;\;,\;\;\lambda_\eta = \frac{\langle W[1-3\sin^4(\theta/2)\sin^2\phi]\rangle}{\langle W\rangle}\;\;,\;\;\lambda_\kappa = \frac{\langle W[1+2\cos\theta]\rangle}{\langle W\rangle}$$

The angular average is defined as

$$\langle f\rangle \equiv \int \frac{d\Omega}{2\pi}\,\frac{f(\theta,\phi)}{\cos\theta/2}$$

THE SCATTERING PROBABILITY

W is related to the scattering cross section

$$W(\theta, \phi) = \frac{16\pi^3}{m^2} \left(\frac{d\sigma}{d\Omega} \right)$$

- The AK formalism is derived in the frame in which the Fermi sphere is at rest (AK)
- $\frac{d\sigma}{d\Omega}$ expressed in the laboratory or in the center of mass reference frame
- the relative kinetic energy is the same E_{cm} $\forall$ frame

$$E_{\mathrm{cm}} = E_{\mathrm{rel}}^{AK} = \frac{k_F^2}{2m}(1 - \cos\theta)$$

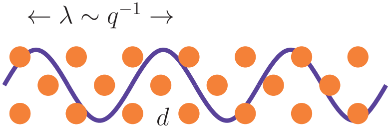
$$\Theta_{\mathrm{cm}} = \phi$$

The in medium scattering probability has been computed within the Born approximation using v_{eff}

$$W(\theta, \phi) = \pi \left| [\mathbf{k}_1', \mathbf{k}_2' | v_{\mathrm{eff}} | \mathbf{k}_1, \mathbf{k}_2] \right|^2$$

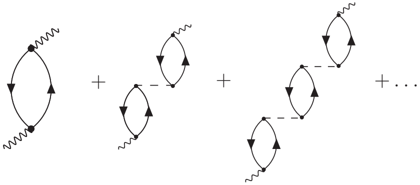
Long-Range Correlations

- At low momentum transfer the space resolusion of the neutrino becomes much larger than the average NN separation distance (~ 1.5 fm), and the interaction involves many nucleons



- Write the nuclear final state as a superposition of 1p1h states (RPA scheme)

$$|n\rangle = \sum_{i=1}^{N} C_i \, |p_i h_i\rangle$$



Tamm-Dancoff (ring) Approximation

- ⋆ Propagation of the particle-hole pair produced at the interaction vertex gives rise to a collective excitation. Replace

$$|ph\rangle \to |n\rangle = \sum_{i=1}^{N} C_i \, |p_i h_i)$$

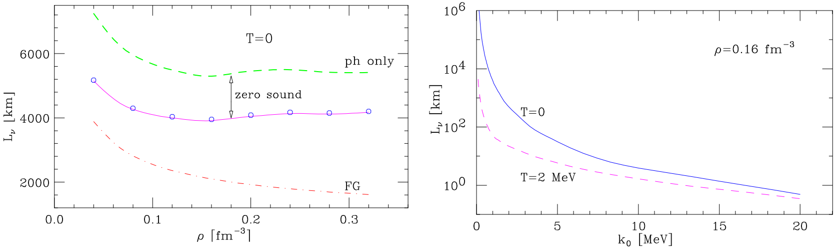
- ⋆ The energy of the state $|n\rangle$ and the coefficients C_i are obtained diagonalizing the hamiltonian matrix

$$H_{ij} = (E_0 + e_{p_i} - e_{h_i})\delta_{ij} + (h_i p_i|V_{\rm eff}|h_j p_j)$$

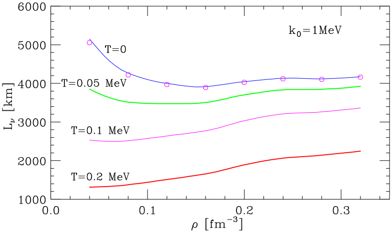
$$e_k = \frac{k^2}{2m} + \sum_{\mathbf{k}'} \langle \mathbf{k}\mathbf{k}'|V_{\rm eff}|\mathbf{k}\mathbf{k}'\rangle_a$$

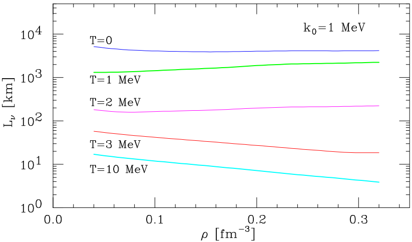
- ⋆ The appearance of an eigenvalue, ω_n, lying outside the particle-hole continuum signals the excitation of a collective mode

⋆ Mean free path of a non degenerate neutrino in neutron matter. Left: density-dependence at $k_0 = 1$ MeV and $T = 0$; Right: energy dependence at $\rho = 0.16\ \mathrm{fm}^{-3}$ and $T = 0, 2$ MeV



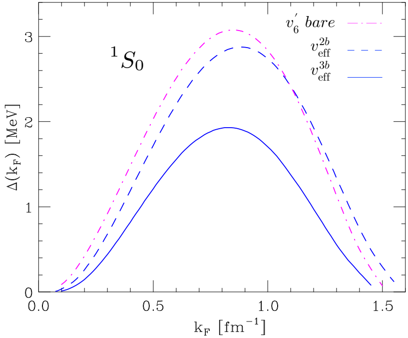
⋆ Density and temperature dependence of the mean free path of a non degenerate neutrino at $k_0 = 1$ MeV and $\rho = 0.16$ fm^{-3}





Superfluid gap in pure neutron matter

$$\Delta(k) = -\frac{1}{\pi}\int dk' k'^2 \frac{v(k,k')\Delta(k')}{[\xi^2(k')+\Delta^2(k')]^{1/2}}$$



$$v(k,k') = \int dr\, r^2 j_0(kr) v_{\rm eff}(r) j_0(k'r)$$

$\Delta(k_F)$ in $^1S_0$ channel

Sizable reduction of the gap, due to the inclusion of three-nucleon interaction in the definition of $v_{\rm eff}$