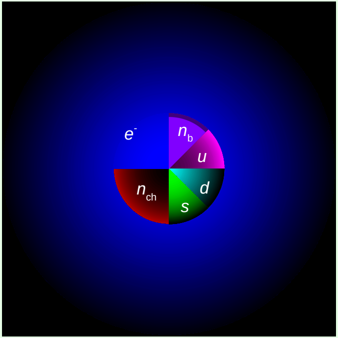
From strangelets to strange stars: A unified description

Xia_Peng_Zhao_Zhou2016_PRD93-085025

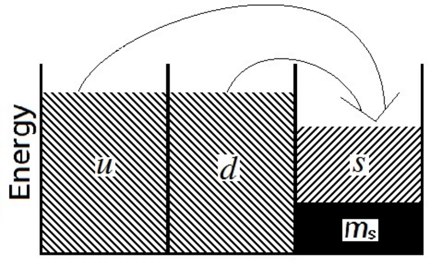
Xia Cheng-Jun (夏铖君)

Key Laboratory of Theoretical Physics,
Institute of Theoretical Physics,
Chinese Academy of Sciences

Collaborators: Peng Guang-Xiong, Zhao En-Guang, Zhou Shan-Gui



Strange quark matter (SQM)



- Bodmer first suggested a low energy nuclear state called "collapsed nuclei". (Bodmer1971_PRD4-1601)
- Witten reported on the stability of SQM consisting of approximately equal numbers of u, d and s quarks, suggesting that SQM could indeed be stable even at zero external pressure. (Witten1984_PRD30-272)

If Witten-Bodmer hypothesis is true, there exists stable lumps of SQM with the baryon number $A \approx 2 \sim 10^{57}$:

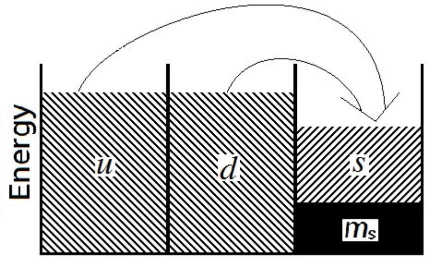
Strangelets ($A \lesssim 10^7$)

Comparing with nuclei, strangelets have: lower charge-to-mass ratio; larger mass; smaller radius; spherical shape; ...

Nuclearites (Rujula_Glashow1984_Nature312-734); Meteorlike Compact Ultradense Objects (CUDO) (Rafelski_Labun_Birrell2013_PRL110-111102); ...

Strange stars ($A \approx 10^{57}$)

Comparing with traditional neutron stars, strange stars have: no crust; different mass-radius relations; smaller radii; higher rotational frequencies; ...



- Bodmer first suggested a low energy nuclear state called "collapsed nuclei". (Bodmer1971_PRD4-1601)
- Witten reported on the stability of SQM consisting of approximately equal numbers of u, d and s quarks, suggesting that SQM could indeed be stable even at zero external pressure. (Witten1984_PRD30-272)

If Witten-Bodmer hypothesis is true, there exists stable lumps of SQM with the baryon number $A \approx 2 \sim 10^{57}$:

Strangelets ($A \lesssim 10^7$)

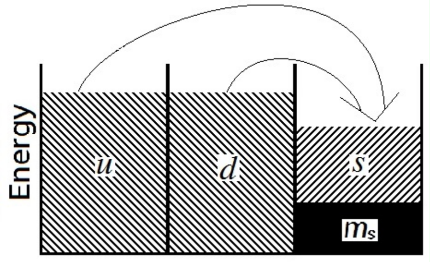
Comparing with nuclei, strangelets have: lower charge-to-mass ratio; larger mass; smaller radius; spherical shape; . . .

Nuclearites (Rujula_Glashow1984_Nature312-734); Meteorlike Compact Ultradense Objects (CUDO) (Rafelski_Labun_Birrell2013_PRL110-111102); . . .

Strange stars ($A \approx 10^{57}$)

Comparing with traditional neutron stars, strange stars have: no crust; different mass-radius relations; smaller radii; higher rotational frequencies; . . .

Strange quark matter (SQM)



- Bodmer first suggested a low energy nuclear state called "collapsed nuclei". (Bodmer1971_PRD4-1601)
- Witten reported on the stability of SQM consisting of approximately equal numbers of u, d and s quarks, suggesting that SQM could indeed be stable even at zero external pressure. (Witten1984_PRD30-272)

If Witten-Bodmer hypothesis is true, there exists stable lumps of SQM with the baryon number $A \approx 2 \sim 10^{57}$:

Strangelets ($A \lesssim 10^7$)

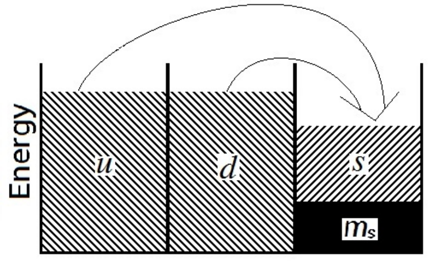
Comparing with nuclei, strangelets have: lower charge-to-mass ratio; larger mass; smaller radius; spherical shape; . . .

Nuclearites (Rujula_Glashow1984_Nature312-734); Meteorlike Compact Ultradense Objects (CUDO) (Rafelski_Labun_Birrell2013_PRL110-111102); . . .

Strange stars ($A \approx 10^{57}$)

Comparing with traditional neutron stars, strange stars have: no crust; different mass-radius relations; smaller radii; higher rotational frequencies; . . .

Strange quark matter (SQM)



- Bodmer first suggested a low energy nuclear state called "collapsed nuclei". (Bodmer1971_PRD4-1601)
- Witten reported on the stability of SQM consisting of approximately equal numbers of u, d and s quarks, suggesting that SQM could indeed be stable even at zero external pressure. (Witten1984_PRD30-272)

If Witten-Bodmer hypothesis is true, there exists stable lumps of SQM with the baryon number $A \approx 2 \sim 10^{57}$:

Strangelets ($A \lesssim 10^7$)

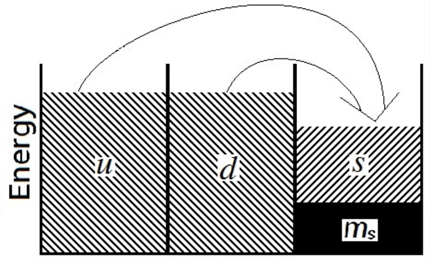
Comparing with nuclei, strangelets have: lower charge-to-mass ratio; larger mass; smaller radius; spherical shape; . . .

Nuclearites (Rujula_Glashow1984_Nature312-734); Meteorlike Compact Ultradense Objects (CUDO) (Rafelski_Labun_Birrell2013_PRL110-111102); . . .

Strange stars ($A \approx 10^{57}$)

Comparing with traditional neutron stars, strange stars have: no crust; different mass-radius relations; smaller radii; higher rotational frequencies; . . .

Strange quark matter (SQM)



- Bodmer first suggested a low energy nuclear state called "collapsed nuclei". (Bodmer1971_PRD4-1601)
- Witten reported on the stability of SQM consisting of approximately equal numbers of u, d and s quarks, suggesting that SQM could indeed be stable even at zero external pressure. (Witten1984_PRD30-272)

If Witten-Bodmer hypothesis is true, there exists stable lumps of SQM with the baryon number $A \approx 2 \sim 10^{57}$:

Strangelets ($A \lesssim 10^7$)

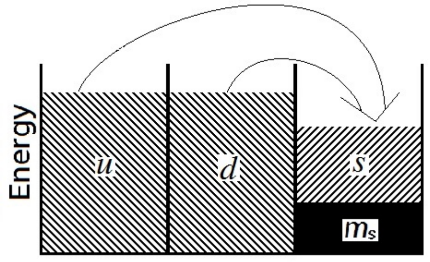
Comparing with nuclei, strangelets have: lower charge-to-mass ratio; larger mass; smaller radius; spherical shape; ...

Nuclearites (Rujula_Glashow1984_Nature312-734); Meteorlike Compact Ultradense Objects (CUDO) (Rafelski_Labun_Birrell2013_PRL110-111102); ...

Strange stars ($A \approx 10^{57}$)

Comparing with traditional neutron stars, strange stars have: no crust; different mass-radius relations; smaller radii; higher rotational frequencies; ...

Strange quark matter (SQM)



- Bodmer first suggested a low energy nuclear state called "collapsed nuclei". (Bodmer1971_PRD4-1601)
- Witten reported on the stability of SQM consisting of approximately equal numbers of *u*, *d* and *s* quarks, suggesting that SQM could indeed be stable even at zero external pressure. (Witten1984_PRD30-272)

If Witten-Bodmer hypothesis is true, there exists stable lumps of SQM with the baryon number $A \approx 2 \sim 10^{57}$:

Strangelets ($A \lesssim 10^7$)

Comparing with nuclei, strangelets have: lower charge-to-mass ratio; larger mass; smaller radius; spherical shape; . . .

Nuclearites (Rujula_Glashow1984_Nature312-734); Meteorlike Compact Ultradense Objects (CUDO) (Rafelski_Labun_Birrell2013_PRL110-111102); . . .

Strange stars ($A \approx 10^{57}$)

Comparing with traditional neutron stars, strange stars have: no crust; different mass-radius relations; smaller radii; higher rotational frequencies; . . .

Experimental searches of SQM: Cosmic rays, CUDO, etc.

Reviewed in Klingenberg1999_JPG25-R273, Finch2006_JPG32-S251, Burdin et al.2015_PR582-1, . . .

No compelling evidence is found!!!

The extreme complexity of an SQM system:

- involves all the fundamental interactions, i.e., the strong, weak, electromagnetic, and gravitational interactions;
- important effects such as the quark depletion, charge screening, and inhomogeneous particle distributions;
- . . .

Experimental searches of SQM: Cosmic rays, CUDO, etc.

Reviewed in Klingenberg1999_JPG25-R273, Finch2006_JPG32-S251, Burdin et al.2015_PR582-1, . . .

No compelling evidence is found!!!

The extreme complexity of an SQM system:

- involves all the fundamental interactions, i.e., the strong, weak, electromagnetic, and gravitational interactions;
- important effects such as the quark depletion, charge screening, and inhomogeneous particle distributions;
- . . .

Experimental searches of SQM: Cosmic rays, CUDO, etc.

Reviewed in Klingenberg1999_JPG25-R273, Finch2006_JPG32-S251, Burdin et al.2015_PR582-1, . . .

No compelling evidence is found!!!

The extreme complexity of an SQM system:

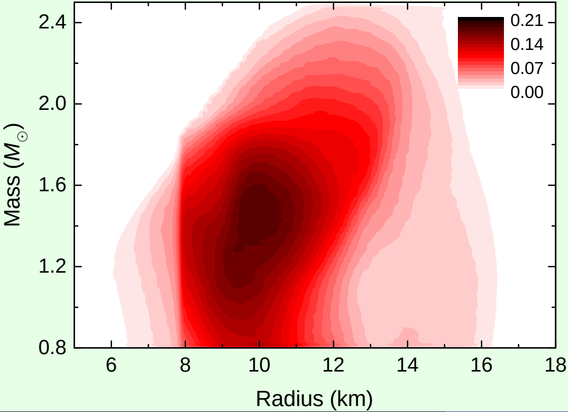
- involves all the fundamental interactions, i.e., the strong, weak, electromagnetic, and gravitational interactions;
- important effects such as the quark depletion, charge screening, and inhomogeneous particle distributions;
- . . .

Experimental searches of SQM: Pulsars

Hints

1. Black body spectrum observed in pulsars: RX J1856.5-3754 (Drake et al.2002_ApJ572-996, Burwitz et al.2003_AA399-1109);
2. Small mass-radius probability: 4U 1746-37 (Li et al.2015_ApJ798-56);
3. The hyperon puzzle for the traditional neutron stars;...

~2500 (observed), ~70 (mass), $\gtrsim 10$ (mass-radius probability distributions):



Mass-radius probability distributions obtained with PRE bursts and QLMXBs (Lattimer2012_ARNPS62-485).

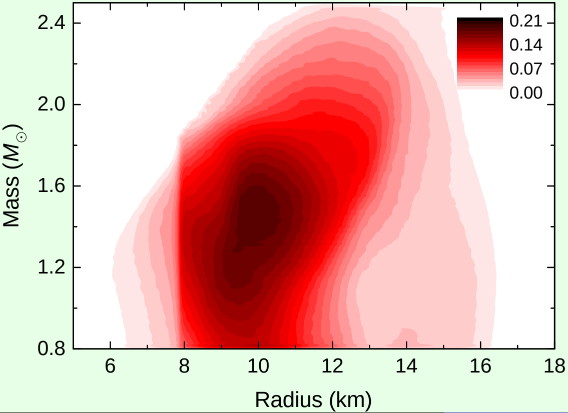
Future: the Lightweight Asymmetry and Magnetism Probe (LAMP) project; the Five-hundred-meter Aperture Spherical Telescope (FAST); the Neutron star Interior Composition Explorer (NICER); the Square Kilometre Array (SKA);...

Experimental searches of SQM: Pulsars

Hints

1. Black body spectrum observed in pulsars: RX J1856.5-3754 (Drake et al.2002_ApJ572-996, Burwitz et al.2003_AA399-1109);
2. Small mass-radius probability: 4U 1746-37 (Li et al.2015_ApJ798-56);
3. The hyperon puzzle for the traditional neutron stars;. . .

$\sim$2500 (observed), $\sim$70 (mass), $\gtrsim$ 10 (mass-radius probability distributions):



Mass-radius probability distributions obtained with PRE bursts and QLMXBs (Lattimer2012_ARNPS62-485).

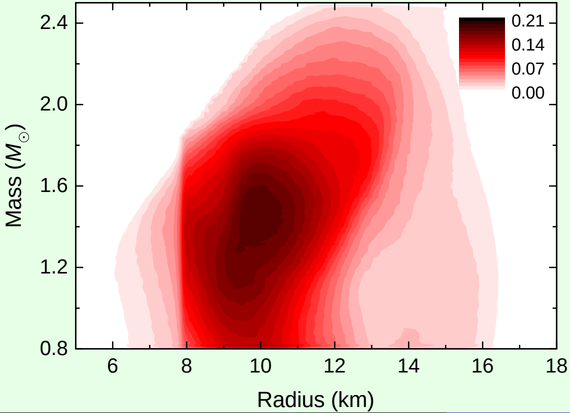
Future: the Lightweight Asymmetry and Magnetism Probe (LAMP) project; the Five-hundred-meter Aperture Spherical Telescope (FAST); the Neutron star Interior Composition Explorer (NICER); the Square Kilometre Array (SKA);. . .

Experimental searches of SQM: Pulsars

Hints

1. Black body spectrum observed in pulsars: RX J1856.5-3754 (Drake et al.2002_ApJ572-996, Burwitz et al.2003_AA399-1109);
2. Small mass-radius probability: 4U 1746-37 (Li et al.2015_ApJ798-56);
3. The hyperon puzzle for the traditional neutron stars;...

$\sim$2500 (observed), $\sim$70 (mass), $\gtrsim$ 10 (mass-radius probability distributions):



Mass-radius probability distributions obtained with PRE bursts and QLMXBs (Lattimer2012_ARNPS62-485).

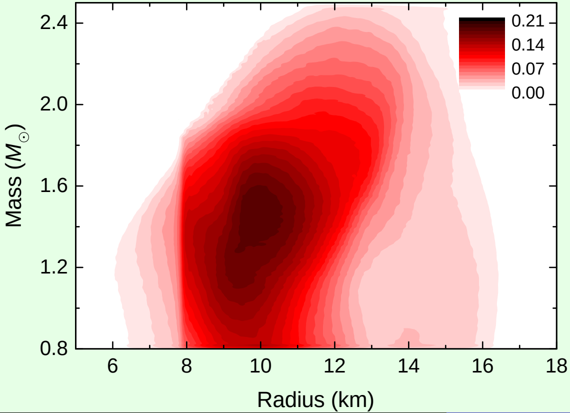
Future: the Lightweight Asymmetry and Magnetism Probe (LAMP) project; the Five-hundred-meter Aperture Spherical Telescope (FAST); the Neutron star Interior Composition Explorer (NICER); the Square Kilometre Array (SKA); ...

Experimental searches of SQM: Pulsars

Hints

1. Black body spectrum observed in pulsars: RX J1856.5-3754 (Drake et al.2002_ApJ572-996, Burwitz et al.2003_AA399-1109);
2. Small mass-radius probability: 4U 1746-37 (Li et al.2015_ApJ798-56);
3. The hyperon puzzle for the traditional neutron stars;...

$\sim$2500 (observed), $\sim$70 (mass), $\gtrsim$ 10 (mass-radius probability distributions):



Mass-radius probability distributions obtained with PRE bursts and QLMXBs (Lattimer2012_ARNPS62-485).

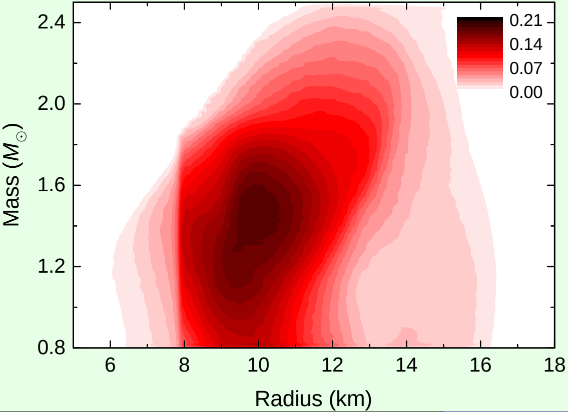
Future: the Lightweight Asymmetry and Magnetism Probe (LAMP) project; the Five-hundred-meter Aperture Spherical Telescope (FAST); the Neutron star Interior Composition Explorer (NICER); the Square Kilometre Array (SKA); ...

Experimental searches of SQM: Pulsars

Hints

1. Black body spectrum observed in pulsars: RX J1856.5-3754 (Drake et al.2002_ApJ572-996, Burwitz et al.2003_AA399-1109);
2. Small mass-radius probability: 4U 1746-37 (Li et al.2015_ApJ798-56);
3. The hyperon puzzle for the traditional neutron stars;. . .

$\sim$2500 (observed), $\sim$70 (mass), $\gtrsim$ 10 (mass-radius probability distributions):



Mass-radius probability distributions obtained with PRE bursts and QLMXBs (Lattimer2012_ARNPS62-485).

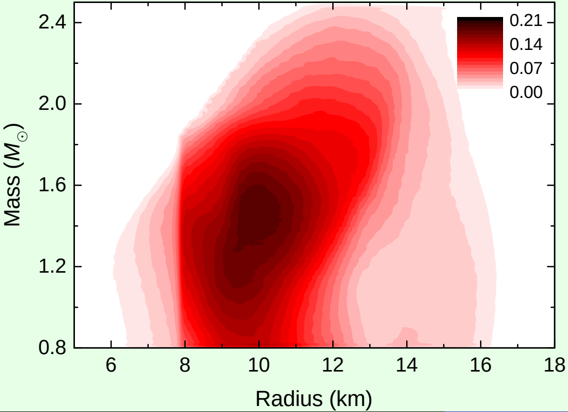
Future: the Lightweight Asymmetry and Magnetism Probe (LAMP) project; the Five-hundred-meter Aperture Spherical Telescope (FAST); the Neutron star Interior Composition Explorer (NICER); the Square Kilometre Array (SKA); . . .

Experimental searches of SQM: Pulsars

Hints

1. Black body spectrum observed in pulsars: RX J1856.5-3754 (Drake et al.2002_ApJ572-996, Burwitz et al.2003_AA399-1109);
2. Small mass-radius probability: 4U 1746-37 (Li et al.2015_ApJ798-56);
3. The hyperon puzzle for the traditional neutron stars;. . .

$\sim$2500 (observed), $\sim$70 (mass), $\gtrsim$ 10 (mass-radius probability distributions):



Mass-radius probability distributions obtained with PRE bursts and QLMXBs (Lattimer2012_ARNPS62-485).

Future: the Lightweight Asymmetry and Magnetism Probe (LAMP) project; the Five-hundred-meter Aperture Spherical Telescope (FAST); the Neutron star Interior Composition Explorer (NICER); the Square Kilometre Array (SKA); . . .

Theoretical descriptions

Approximations in previous descriptions

- strangelets: 1. electrons were ignored; 2. isotropic particle distribution.
- strange stars: local charge neutrality.

Important effects

1. charge screening (Heiselberg1993_PRD48-1418, ...);
2. electron-positron pair creation (Madsen2008_PRL100-151102);
3. nonzero charge in strange stars (Alcock_Farhi_Olinto1986_ApJ310-261, Negreiros_Weber_Malheiro_Usov2009_PRD80-083006, ...);
4. ...

Theoretical descriptions

Approximations in previous descriptions

- strangelets: 1. electrons were ignored; 2. isotropic particle distribution.
- strange stars: local charge neutrality.

Important effects

1. charge screening (Heiselberg1993_PRD48-1418, . . .);
2. electron-positron pair creation (Madsen2008_PRL100-151102);
3. nonzero charge in strange stars (Alcock_Farhi_Olinto1986_ApJ310-261, Negreiros_Weber_Malheiro_Usov2009_PRD80-083006, . . .);
4. . . .

Metric

Based on Thomas-Fermi approximation, we study a spherically symmetric static object considering the gravity and electrostatic interaction on the macroscopic scale while incorporating the strong and weak interactions locally. The metric for a spherically symmetric, charged, and static object

$$\mathrm{d}s^2 = \mathrm{e}^{\nu}\mathrm{d}t^2 - \mathrm{e}^{\lambda}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2). \tag{1}$$

The metric elements are obtained by solving the Einstein's field equations

$$\frac{\mathrm{e}^{-\lambda}}{r^2}\left(r\frac{\mathrm{d}\lambda}{\mathrm{d}r} - 1\right) + \frac{1}{r^2} = 8\pi G\left(E + \frac{\alpha Q^2}{8\pi r^4}\right), \tag{2}$$

$$\frac{\mathrm{e}^{-\lambda}}{r^2}\left(r\frac{\mathrm{d}\nu}{\mathrm{d}r} + 1\right) - \frac{1}{r^2} = 8\pi G\left(P - \frac{\alpha Q^2}{8\pi r^4}\right). \tag{3}$$

Here $Q(r)$ is defined as the total charge enclosed in a sphere with radius r. By solving Eq. (2) and Eq. (3) we have

$$\mathrm{e}^{-\lambda} = 1 - \frac{2G}{r}M_{\mathrm{t}}(r), \tag{4}$$

$$\frac{\mathrm{d}\nu}{\mathrm{d}r} = \frac{2G\mathrm{e}^{\lambda}}{r^2}\left[4\pi r^3\left(P - \frac{\alpha Q^2}{8\pi r^4}\right) + M_{\mathrm{t}}\right], \tag{5}$$

where the total mass is $M_{\mathrm{t}}(r) = \int_0^r 4\pi r^2\left(E + \frac{\alpha Q^2}{8\pi r^4}\right)\mathrm{d}r$.

Metric

Based on Thomas-Fermi approximation, we study a spherically symmetric static object considering the gravity and electrostatic interaction on the macroscopic scale while incorporating the strong and weak interactions locally. The metric for a spherically symmetric, charged, and static object

$$\mathrm{d}s^2 = \mathrm{e}^{\nu}\mathrm{d}t^2 - \mathrm{e}^{\lambda}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2). \tag{1}$$

The metric elements are obtained by solving the Einstein's field equations

$$\frac{\mathrm{e}^{-\lambda}}{r^2}\left(r\frac{\mathrm{d}\lambda}{\mathrm{d}r} - 1\right) + \frac{1}{r^2} = 8\pi G\left(E + \frac{\alpha Q^2}{8\pi r^4}\right), \tag{2}$$

$$\frac{\mathrm{e}^{-\lambda}}{r^2}\left(r\frac{\mathrm{d}\nu}{\mathrm{d}r} + 1\right) - \frac{1}{r^2} = 8\pi G\left(P - \frac{\alpha Q^2}{8\pi r^4}\right). \tag{3}$$

Here $Q(r)$ is defined as the total charge enclosed in a sphere with radius r. By solving Eq. (2) and Eq. (3) we have

$$\mathrm{e}^{-\lambda} = 1 - \frac{2G}{r}M_{\mathrm{t}}(r), \tag{4}$$

$$\frac{\mathrm{d}\nu}{\mathrm{d}r} = \frac{2G\mathrm{e}^{\lambda}}{r^2}\left[4\pi r^3\left(P - \frac{\alpha Q^2}{8\pi r^4}\right) + M_{\mathrm{t}}\right], \tag{5}$$

where the total mass is $M_{\mathrm{t}}(r) = \int_0^r 4\pi r^2\left(E + \frac{\alpha Q^2}{8\pi r^4}\right)\mathrm{d}r$.

Metric

Based on Thomas-Fermi approximation, we study a spherically symmetric static object considering the gravity and electrostatic interaction on the macroscopic scale while incorporating the strong and weak interactions locally. The metric for a spherically symmetric, charged, and static object

$$\mathrm{d}s^2 = \mathrm{e}^{\nu}\mathrm{d}t^2 - \mathrm{e}^{\lambda}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2). \tag{1}$$

The metric elements are obtained by solving the Einstein's field equations

$$\frac{\mathrm{e}^{-\lambda}}{r^2}\left(r\frac{\mathrm{d}\lambda}{\mathrm{d}r} - 1\right) + \frac{1}{r^2} = 8\pi G\left(E + \frac{\alpha Q^2}{8\pi r^4}\right), \tag{2}$$

$$\frac{\mathrm{e}^{-\lambda}}{r^2}\left(r\frac{\mathrm{d}\nu}{\mathrm{d}r} + 1\right) - \frac{1}{r^2} = 8\pi G\left(P - \frac{\alpha Q^2}{8\pi r^4}\right). \tag{3}$$

Here $Q(r)$ is defined as the total charge enclosed in a sphere with radius r.
By solving Eq. (2) and Eq. (3) we have

$$\mathrm{e}^{-\lambda} = 1 - \frac{2G}{r}M_{\mathrm{t}}(r), \tag{4}$$

$$\frac{\mathrm{d}\nu}{\mathrm{d}r} = \frac{2G\mathrm{e}^{\lambda}}{r^2}\left[4\pi r^3\left(P - \frac{\alpha Q^2}{8\pi r^4}\right) + M_{\mathrm{t}}\right], \tag{5}$$

where the total mass is $M_{\mathrm{t}}(r) = \int_0^r 4\pi r^2\left(E + \frac{\alpha Q^2}{8\pi r^4}\right)\mathrm{d}r$.

Metric

Based on Thomas-Fermi approximation, we study a spherically symmetric static object considering the gravity and electrostatic interaction on the macroscopic scale while incorporating the strong and weak interactions locally. The metric for a spherically symmetric, charged, and static object

$$\mathrm{d}s^2 = \mathrm{e}^{\nu}\mathrm{d}t^2 - \mathrm{e}^{\lambda}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2). \tag{1}$$

The metric elements are obtained by solving the Einstein's field equations

$$\frac{\mathrm{e}^{-\lambda}}{r^2}\left(r\frac{\mathrm{d}\lambda}{\mathrm{d}r} - 1\right) + \frac{1}{r^2} = 8\pi G\left(E + \frac{\alpha Q^2}{8\pi r^4}\right), \tag{2}$$

$$\frac{\mathrm{e}^{-\lambda}}{r^2}\left(r\frac{\mathrm{d}\nu}{\mathrm{d}r} + 1\right) - \frac{1}{r^2} = 8\pi G\left(P - \frac{\alpha Q^2}{8\pi r^4}\right). \tag{3}$$

Here $Q(r)$ is defined as the total charge enclosed in a sphere with radius r. By solving Eq. (2) and Eq. (3) we have

$$\mathrm{e}^{-\lambda} = 1 - \frac{2G}{r}M_{\mathrm{t}}(r), \tag{4}$$

$$\frac{\mathrm{d}\nu}{\mathrm{d}r} = \frac{2G\mathrm{e}^{\lambda}}{r^2}\left[4\pi r^3\left(P - \frac{\alpha Q^2}{8\pi r^4}\right) + M_{\mathrm{t}}\right], \tag{5}$$

where the total mass is $M_{\mathrm{t}}(r) = \int_0^r 4\pi r^2\left(E + \frac{\alpha Q^2}{8\pi r^4}\right)\mathrm{d}r$.

Distribution functions

The particle number: $N_i(r) = \int_0^r 4\pi n_i(r) \mathrm{e}^{\lambda/2} r^2 \mathrm{d}r$;
The total charge: $Q(r) \equiv \sum_i q_i N_i(r)$;
The entropy: $\bar{S}(r) = \int_0^r 4\pi S(r) \mathrm{e}^{\lambda/2} r^2 \mathrm{d}r$.

By minimizing the mass $M = M_{\mathrm{t}}(\infty)$ with respect to the particle distribution $N_i(r)$ and entropy distribution $\bar{S}(r)$ at fixed the total particle number $N_i(\infty)$ and entropy $\bar{S}(\infty)$, we immediately have

$$\frac{\mathrm{d}\mu_i}{\mathrm{d}r} = \frac{\alpha Q}{r^2} q_i \mathrm{e}^{\lambda/2} - \frac{\mu_i}{2}\frac{\mathrm{d}\nu}{\mathrm{d}r}, \tag{6}$$

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{T}{2}\frac{\mathrm{d}\nu}{\mathrm{d}r}. \tag{7}$$

They are equivalent to the constancy of the generalized chemical potential

$$\bar{\mu}_i = \mu_i \mathrm{e}^{\nu/2} + q_i \varphi(r) = \mathrm{constant} \tag{8}$$

and gravitationally red-shifted temperature

$$\bar{T} = T\mathrm{e}^{\nu/2} = \mathrm{constant} \tag{9}$$

with $\frac{\mathrm{d}\varphi}{\mathrm{d}r} = -\frac{\alpha Q}{r^2}\mathrm{e}^{(\lambda+\nu)/2}$ (Klein1949_RMP21-531, Tolman1930_PR35-904, Olson_Bailyn1975_PRD12-3030, Kodama_Yamada1972_PTP47-444).

Distribution functions

The particle number: $N_i(r) = \int_0^r 4\pi n_i(r) \mathrm{e}^{\lambda/2} r^2 \mathrm{d}r$;
The total charge: $Q(r) \equiv \sum_i q_i N_i(r)$;
The entropy: $\bar{S}(r) = \int_0^r 4\pi S(r) \mathrm{e}^{\lambda/2} r^2 \mathrm{d}r$.

By minimizing the mass $M = M_{\mathrm{t}}(\infty)$ with respect to the particle distribution $N_i(r)$ and entropy distribution $\bar{S}(r)$ at fixed the total particle number $N_i(\infty)$ and entropy $\bar{S}(\infty)$, we immediately have

$$\frac{\mathrm{d}\mu_i}{\mathrm{d}r} = \frac{\alpha Q}{r^2} q_i \mathrm{e}^{\lambda/2} - \frac{\mu_i}{2}\frac{\mathrm{d}\nu}{\mathrm{d}r}, \tag{6}$$

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{T}{2}\frac{\mathrm{d}\nu}{\mathrm{d}r}. \tag{7}$$

They are equivalent to the constancy of the generalized chemical potential

$$\bar{\mu}_i = \mu_i \mathrm{e}^{\nu/2} + q_i\varphi(r) = \mathrm{constant} \tag{8}$$

and gravitationally red-shifted temperature

$$\bar{T} = T\mathrm{e}^{\nu/2} = \mathrm{constant} \tag{9}$$

with $\frac{\mathrm{d}\varphi}{\mathrm{d}r} = -\frac{\alpha Q}{r^2}\mathrm{e}^{(\lambda+\nu)/2}$ (Klein1949_RMP21-531, Tolman1930_PR35-904, Olson_Bailyn1975_PRD12-3030, Kodama_Yamada1972_PTP47-444).

Distribution functions

The particle number: $N_i(r) = \int_0^r 4\pi n_i(r) \mathrm{e}^{\lambda/2} r^2 \mathrm{d}r$;
The total charge: $Q(r) \equiv \sum_i q_i N_i(r)$;
The entropy: $\bar{S}(r) = \int_0^r 4\pi S(r) \mathrm{e}^{\lambda/2} r^2 \mathrm{d}r$.

By minimizing the mass $M = M_{\mathrm{t}}(\infty)$ with respect to the particle distribution $N_i(r)$ and entropy distribution $\bar{S}(r)$ at fixed the total particle number $N_i(\infty)$ and entropy $\bar{S}(\infty)$, we immediately have

$$\frac{\mathrm{d}\mu_i}{\mathrm{d}r} = \frac{\alpha Q}{r^2} q_i \mathrm{e}^{\lambda/2} - \frac{\mu_i}{2}\frac{\mathrm{d}\nu}{\mathrm{d}r}, \tag{6}$$

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{T}{2}\frac{\mathrm{d}\nu}{\mathrm{d}r}. \tag{7}$$

They are equivalent to the constancy of the generalized chemical potential

$$\bar{\mu}_i = \mu_i \mathrm{e}^{\nu/2} + q_i \varphi(r) = \mathrm{constant} \tag{8}$$

and gravitationally red-shifted temperature

$$\bar{T} = T\mathrm{e}^{\nu/2} = \mathrm{constant} \tag{9}$$

with $\frac{\mathrm{d}\varphi}{\mathrm{d}r} = -\frac{\alpha Q}{r^2}\mathrm{e}^{(\lambda+\nu)/2}$ (Klein1949_RMP21-531, Tolman1930_PR35-904, Olson_Bailyn1975_PRD12-3030, Kodama_Yamada1972_PTP47-444).

Distribution functions

The particle number: $N_i(r) = \int_0^r 4\pi n_i(r) \mathrm{e}^{\lambda/2} r^2 \mathrm{d}r$;
The total charge: $Q(r) \equiv \sum_i q_i N_i(r)$;
The entropy: $\bar{S}(r) = \int_0^r 4\pi S(r) \mathrm{e}^{\lambda/2} r^2 \mathrm{d}r$.

By minimizing the mass $M = M_\mathrm{t}(\infty)$ with respect to the particle distribution $N_i(r)$ and entropy distribution $\bar{S}(r)$ at fixed the total particle number $N_i(\infty)$ and entropy $\bar{S}(\infty)$, we immediately have

$$\frac{\mathrm{d}\mu_i}{\mathrm{d}r} = \frac{\alpha Q}{r^2} q_i \mathrm{e}^{\lambda/2} - \frac{\mu_i}{2}\frac{\mathrm{d}\nu}{\mathrm{d}r}, \tag{6}$$

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{T}{2}\frac{\mathrm{d}\nu}{\mathrm{d}r}. \tag{7}$$

They are equivalent to the constancy of the generalized chemical potential

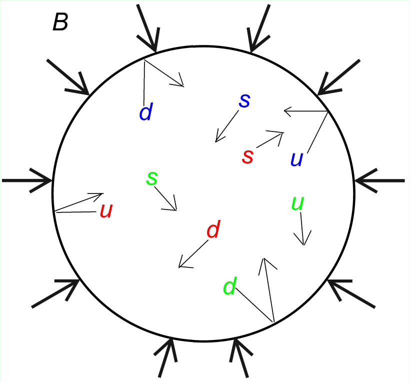
$$\bar{\mu}_i = \mu_i \mathrm{e}^{\nu/2} + q_i \varphi(r) = \mathrm{constant} \tag{8}$$

and gravitationally red-shifted temperature

$$\bar{T} = T\mathrm{e}^{\nu/2} = \mathrm{constant} \tag{9}$$

with $\frac{\mathrm{d}\varphi}{\mathrm{d}r} = -\frac{\alpha Q}{r^2}\mathrm{e}^{(\lambda+\nu)/2}$ (Klein1949_RMP21-531, Tolman1930_PR35-904, Olson_Bailyn1975_PRD12-3030, Kodama_Yamada1972_PTP47-444).

Local properties



Strong interaction (Bag Model)

The thermodynamic potential density at zero temperature:

$$\Omega = -\sum_i \frac{g_i}{24\pi^2}\left[\mu_i\nu_i(\nu_i^2 - \frac{3}{2}m_i^2) + \frac{3}{2}m_i^4 \ln\frac{\mu_i+\nu_i}{m_i}\right] + B, \quad (10)$$

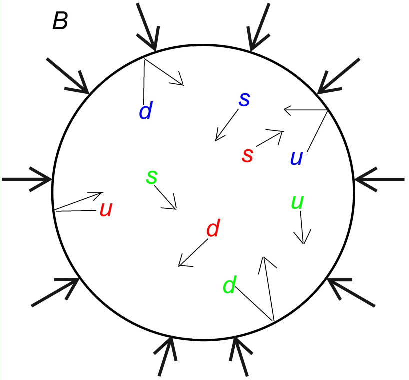
with $\mu_i = \sqrt{\nu_i^2 + m_i^2}$. The particle number density, energy density and pressure are given by $n_i = -\frac{\partial\Omega}{\partial\mu_i} = \frac{g_i\nu_i^3}{6\pi^2}$, $E = \Omega + \sum_i \mu_i n_i$, and $P = -\Omega$.

Weak interaction

Weak reactions: $d, s \leftrightarrow u + e + \bar{\nu}_e$, $s + u \leftrightarrow u + d$, ...

The weak equilibrium is reached: $\mu_u + \mu_e = \mu_d = \mu_s$.

Local properties



Strong interaction (Bag Model)

The thermodynamic potential density at zero temperature:

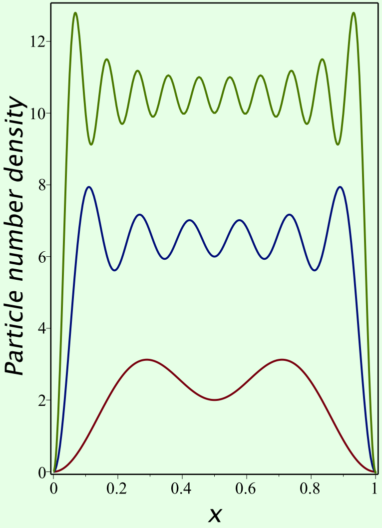
$$\Omega = -\sum_i \frac{g_i}{24\pi^2}\left[\mu_i\nu_i(\nu_i^2 - \frac{3}{2}m_i^2) + \frac{3}{2}m_i^4 \ln\frac{\mu_i+\nu_i}{m_i}\right] + B, \quad (10)$$

with $\mu_i = \sqrt{\nu_i^2 + m_i^2}$. The particle number density, energy density and pressure are given by $n_i = -\frac{\partial\Omega}{\partial\mu_i} = \frac{g_i\nu_i^3}{6\pi^2}$, $E = \Omega + \sum_i \mu_i n_i$, and $P = -\Omega$.

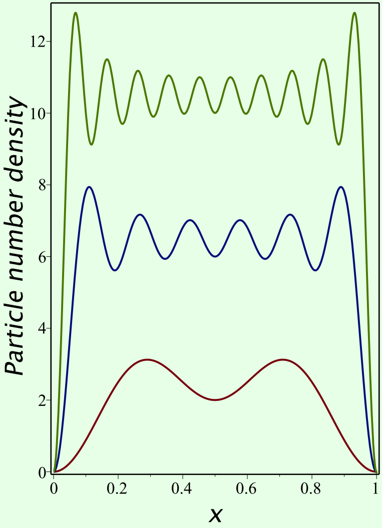
Weak interaction

Weak reactions: $d, s \leftrightarrow u + e + \bar{\nu}_e$, $s + u \leftrightarrow u + d$, ...
The weak equilibrium is reached: $\mu_u + \mu_e = \mu_d = \mu_s$.

Surface effects

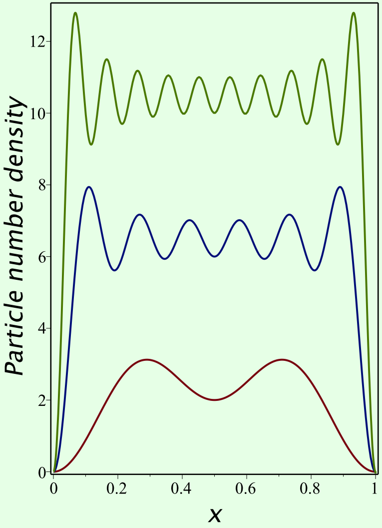


- **The multiple reflection expansion (MRE) method:** The average effects due to quark depletion are treated with a modification to the density of states, i.e., $\frac{\mathrm{d}N_i^{\mathrm{surf}}}{\mathrm{d}p_i} = -\frac{g_i R^2 p_i}{\pi}\arctan\left(\frac{m_i}{p_i}\right) + \frac{2g_i R}{3\pi}\left[1-\frac{3p_i}{2m_i}\arctan\left(\frac{m_i}{p_i}\right)\right]$ with the contribution to the total energy $\bar{E}_i^{\mathrm{surf}} = \int_0^{\nu_i(R)} \sqrt{p_i^2+m_i^2}\,\frac{\mathrm{d}N_i^{\mathrm{surf}}}{\mathrm{d}p_i} dp_i$. (Berger_Jaffe1987_PRC35-213, Madsen1994_PRD50-3328, ...)
- **Constant surface tension:** The contributions to the total energy due to the interface effects are taken into account by a surface tension σ with the modification to the total energy $4\pi R^2\sigma$. (Heiselberg1993_PRL70-1355, Jaikumar_Reddy_Steiner2006_PRL96-041101,...)



- **The multiple reflection expansion (MRE) method:** The average effects due to quark depletion are treated with a modification to the density of states, i.e., $\frac{\mathrm{d}N_i^{\mathrm{surf}}}{\mathrm{d}p_i} = -\frac{g_i R^2 p_i}{\pi} \arctan\left(\frac{m_i}{p_i}\right) + \frac{2g_i R}{3\pi}\left[1 - \frac{3p_i}{2m_i}\arctan\left(\frac{m_i}{p_i}\right)\right]$ with the contribution to the total energy $\bar{E}_i^{\mathrm{surf}} = \int_0^{\nu_i(R)} \sqrt{p_i^2 + m_i^2}\frac{\mathrm{d}N_i^{\mathrm{surf}}}{\mathrm{d}p_i} dp_i$. (Berger_Jaffe1987_PRC35-213, Madsen1994_PRD50-3328, ...)
- **Constant surface tension:** The contributions to the total energy due to the interface effects are taken into account by a surface tension σ with the modification to the total energy $4\pi R^2\sigma$. (Heiselberg1993_PRL70-1355, Jaikumar_Reddy_Steiner2006_PRL96-041101,...)

Surface effects



- **The multiple reflection expansion (MRE) method:** The average effects due to quark depletion are treated with a modification to the density of states, i.e., $\frac{dN_i^{\mathrm{surf}}}{dp_i} = -\frac{g_i R^2 p_i}{\pi}\arctan\left(\frac{m_i}{p_i}\right) + \frac{2g_i R}{3\pi}\left[1 - \frac{3p_i}{2m_i}\arctan\left(\frac{m_i}{p_i}\right)\right]$ with the contribution to the total energy $\bar{E}_i^{\mathrm{surf}} = \int_0^{\nu_i(R)} \sqrt{p_i^2 + m_i^2}\,\frac{dN_i^{\mathrm{surf}}}{dp_i} dp_i$. (Berger_Jaffe1987_PRC35-213, Madsen1994_PRD50-3328, ...)
- **Constant surface tension:** The contributions to the total energy due to the interface effects are taken into account by a surface tension σ with the modification to the total energy $4\pi R^2\sigma$. (Heiselberg1993_PRL70-1355, Jaikumar_Reddy_Steiner2006_PRL96-041101,...)

Parameters

Particle masses (in MeV)

The electron mass: $m_e = 0.511$;
Quark masses: $m_u = 2.3$, $m_d = 4.8$, and $m_s = 95$.
Obtained from Particle Data Group2014_CPC38-090001.

The bag constant B (according to the Witten-Bodmer hypothesis)

- Lower limit (2f-quark matter: $\left.\frac{E}{n_\mathrm{b}}\right|_\mathrm{min} > 930$ MeV): $B^{1/4} > 144.37$ MeV;
- Upper limit (3f-quark matter: $\left.\frac{E}{n_\mathrm{b}}\right|_\mathrm{min} < 930$ MeV): $B^{1/4} < 159.26$ MeV.

We take $B^{1/4} = 152 \pm 7$ MeV.

The surface tension σ

Evaluated with lattice QCD, the linear sigma model, the Nambu-Jona-Lasinio model, the three-flavor Polyakov-quark-meson model, and the quasiparticle model etc.: $\sigma \approx 5\text{-}300\ \mathrm{MeV\ fm}^{-2}$
Note: No adjustable parameters for the MRE method.

Parameters

Particle masses (in MeV)

The electron mass: $m_e = 0.511$;
Quark masses: $m_u = 2.3$, $m_d = 4.8$, and $m_s = 95$.
Obtained from Particle Data Group2014_CPC38-090001.

The bag constant B (according to the Witten-Bodmer hypothesis)

- Lower limit (2f-quark matter: $\left.\frac{E}{n_\mathrm{b}}\right|_\mathrm{min} > 930$ MeV): $B^{1/4} > 144.37$ MeV;
- Upper limit (3f-quark matter: $\left.\frac{E}{n_\mathrm{b}}\right|_\mathrm{min} < 930$ MeV): $B^{1/4} < 159.26$ MeV.

We take $B^{1/4} = 152 \pm 7$ MeV.

The surface tension σ

Evaluated with lattice QCD, the linear sigma model, the Nambu-Jona-Lasinio model, the three-flavor Polyakov-quark-meson model, and the quasiparticle model etc.: $\sigma \approx 5\text{-}300\ \mathrm{MeV\ fm}^{-2}$
Note: No adjustable parameters for the MRE method.

Parameters

Particle masses (in MeV)

The electron mass: $m_e = 0.511$;
Quark masses: $m_u = 2.3$, $m_d = 4.8$, and $m_s = 95$.
Obtained from Particle Data Group2014_CPC38-090001.

The bag constant B (according to the Witten-Bodmer hypothesis)

- Lower limit (2f-quark matter: $\left.\frac{E}{n_{\rm b}}\right|_{\rm min} > 930$ MeV): $B^{1/4} > 144.37$ MeV;
- Upper limit (3f-quark matter: $\left.\frac{E}{n_{\rm b}}\right|_{\rm min} < 930$ MeV): $B^{1/4} < 159.26$ MeV.

We take $B^{1/4} = 152 \pm 7$ MeV.

The surface tension σ

Evaluated with lattice QCD, the linear sigma model, the Nambu-Jona-Lasinio model, the three-flavor Polyakov-quark-meson model, and the quasiparticle model etc.: $\sigma \approx 5\text{-}300\ \mathrm{MeV\ fm^{-2}}$
Note: No adjustable parameters for the MRE method.

Parameters

Particle masses (in MeV)

The electron mass: $m_e = 0.511$;
Quark masses: $m_u = 2.3$, $m_d = 4.8$, and $m_s = 95$.
Obtained from Particle Data Group2014_CPC38-090001.

The bag constant B (according to the Witten-Bodmer hypothesis)

- Lower limit (2f-quark matter: $\left.\frac{E}{n_\mathrm{b}}\right|_\mathrm{min} > 930$ MeV): $B^{1/4} > 144.37$ MeV;
- Upper limit (3f-quark matter: $\left.\frac{E}{n_\mathrm{b}}\right|_\mathrm{min} < 930$ MeV): $B^{1/4} < 159.26$ MeV.

We take $B^{1/4} = 152 \pm 7$ MeV.

The surface tension σ

Evaluated with lattice QCD, the linear sigma model, the Nambu-Jona-Lasinio model, the three-flavor Polyakov-quark-meson model, and the quasiparticle model etc.: $\sigma \approx 5\text{-}300\ \mathrm{MeV\ fm^{-2}}$

Note: No adjustable parameters for the MRE method.

Parameters

Particle masses (in MeV)

The electron mass: $m_e = 0.511$;
Quark masses: $m_u = 2.3$, $m_d = 4.8$, and $m_s = 95$.
Obtained from Particle Data Group2014_CPC38-090001.

The bag constant B (according to the Witten-Bodmer hypothesis)

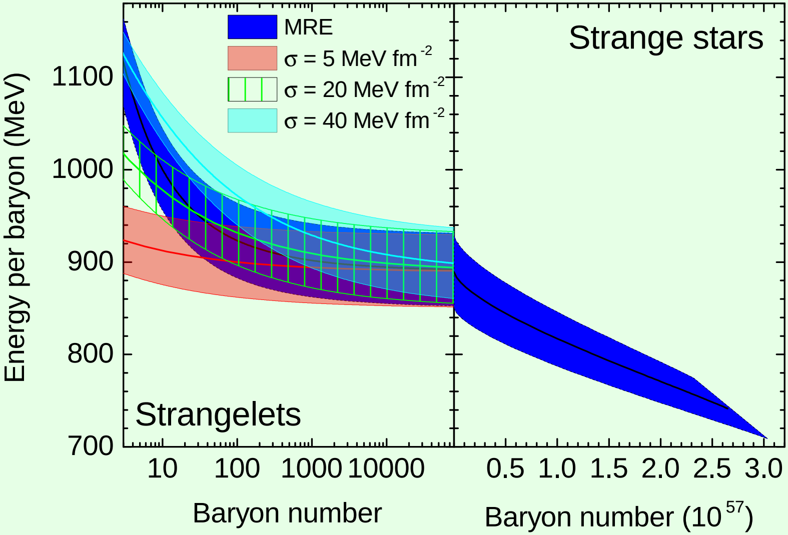
- Lower limit (2f-quark matter: $\left.\frac{E}{n_{\mathrm{b}}}\right|_{\mathrm{min}} > 930$ MeV): $B^{1/4} > 144.37$ MeV;
- Upper limit (3f-quark matter: $\left.\frac{E}{n_{\mathrm{b}}}\right|_{\mathrm{min}} < 930$ MeV): $B^{1/4} < 159.26$ MeV.

We take $B^{1/4} = 152 \pm 7$ MeV.

The surface tension σ

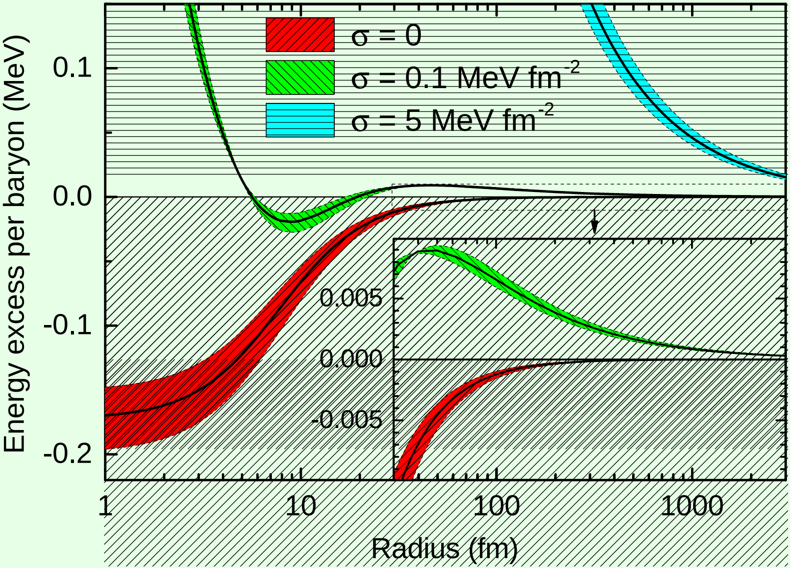
Evaluated with lattice QCD, the linear sigma model, the Nambu-Jona-Lasinio model, the three-flavor Polyakov-quark-meson model, and the quasiparticle model etc.: $\sigma \approx 5\text{-}300\ \mathrm{MeV\ fm}^{-2}$
Note: No adjustable parameters for the MRE method.

Energy per baryon



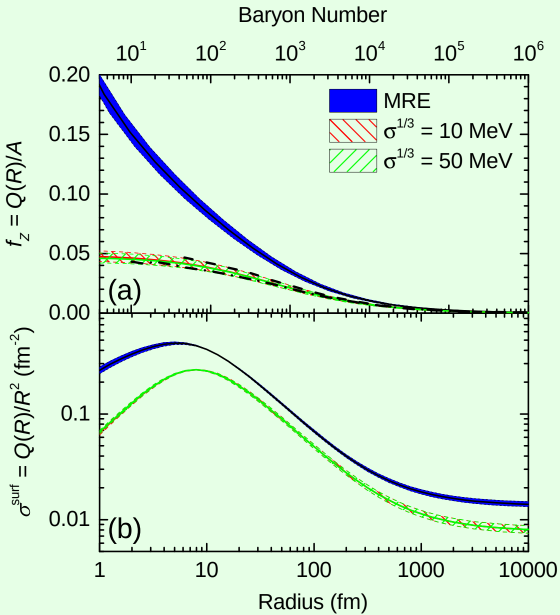
The MRE method: $\sigma \approx$ 10-45 $\mathrm{MeV\ fm}^{-2}$

Energy excess per baryon



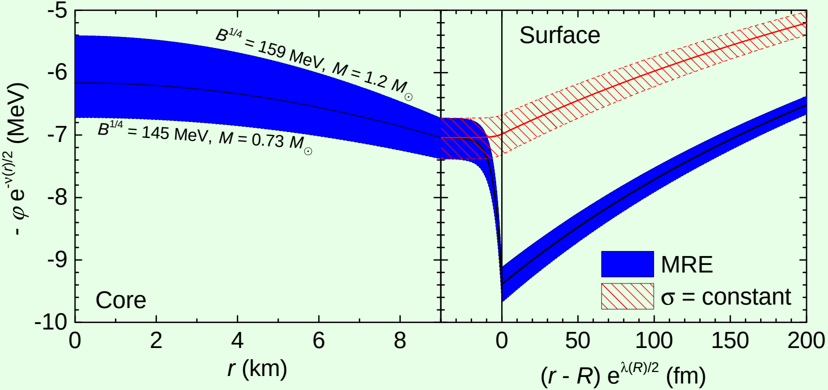
Constant surface tension cases.

Charge properties



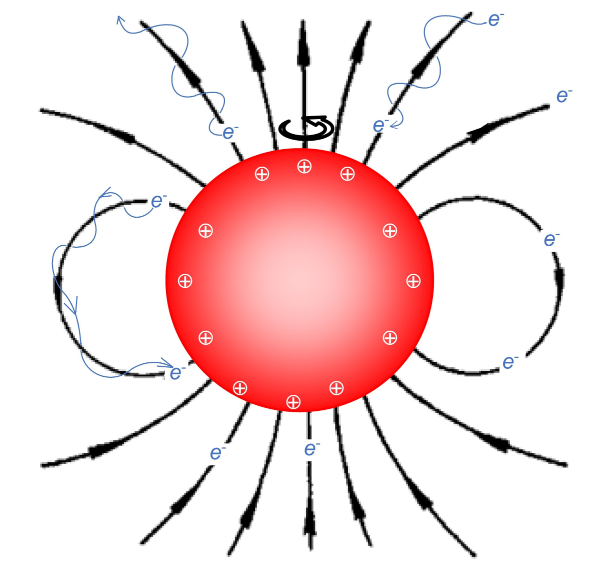
- (a): The charge-to-mass ratio of the SQM core of strangelets (dashed line: Heiselberg1993_PRD48-1418; thick dash-dotted line: Berger_Jaffe1987_PRC35-213)
- (b): The surface charge density σ^{surf} of the SQM core for strangelets.

Electric potential



The electric potential from the SQM core to the surface of strange stars with $R = 9$ km.

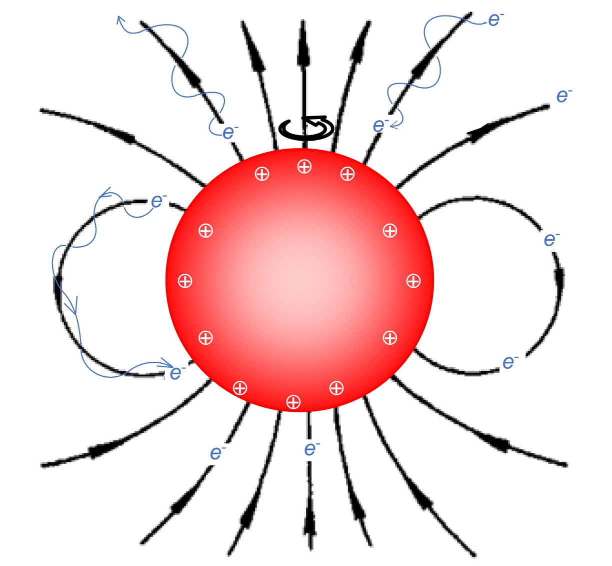
The magnetic field of a rotating strange star



The magnetic field formed at the pole area on the surface of a rotating SQM core (Negreiros et al.2010_PRD82-103010):
$B_{\rm surf} = \frac{1}{3} u_0 \sigma^{\rm surf} R f$.
(Xia2016_SciSinPMA46-012021)

$\sigma^{\rm surf} \approx$
$0.0135 \pm 0.0008\ \rm fm^{-2}$ (MRE);
$0.0081 \pm 0.0007\ \rm fm^{-2}$ (Constant σ).

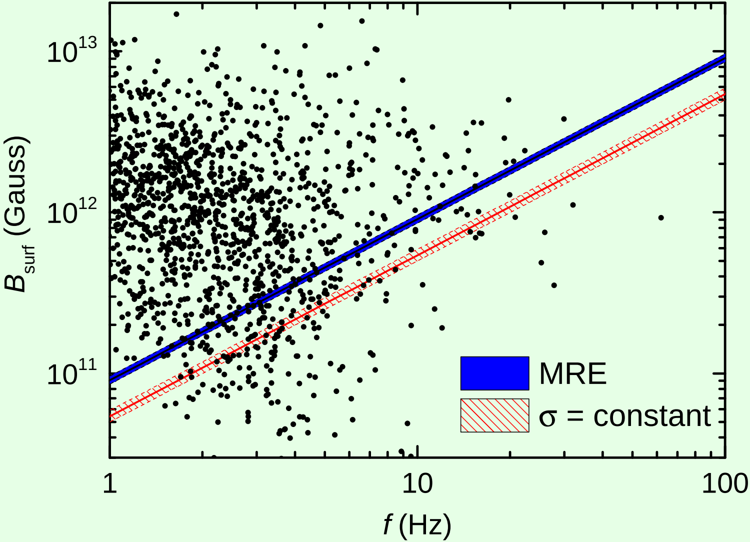
The magnetic field of a rotating strange star



The magnetic field formed at the pole area on the surface of a rotating SQM core (Negreiros et al.2010_PRD82-103010):
$B_{\rm surf} = \frac{1}{3} u_0 \sigma^{\rm surf} R f$.
(Xia2016_SciSinPMA46-012021)

$\sigma^{\rm surf} \approx$
$0.0135 \pm 0.0008\ {\rm fm}^{-2}$ (MRE);
$0.0081 \pm 0.0007\ {\rm fm}^{-2}$ (Constant σ).

The magnetic field of a rotating strange star



The binary pulsars are excluded since their magnetic fields may be dampened by mass accretion (Bhattacharya_van den Heuvel1991_PR203-1).

Summary

We proposed a unified description to study SQM objects with all the fundamental interactions and important effects considered.

1. The energy per baryon of an SQM object is most likely decreasing with increasing size, and the local minimum may not exist.
2. There exist nonzero charge inside an SQM object, i.e., the charge located on the quark-vacuum interface due to the separation of electrons from SQM and the gravity induced charge throughout the object.
3. The properties of strangelets as well as the surface profiles of strange stars depend crucially on the surface treatments.
4. A magnetic field comparable to the typical values of pulsars for a rotating strange star.
5. ...

Summary

We proposed a unified description to study SQM objects with all the fundamental interactions and important effects considered.

1. The energy per baryon of an SQM object is most likely decreasing with increasing size, and the local minimum may not exist.
2. There exist nonzero charge inside an SQM object, i.e., the charge located on the quark-vacuum interface due to the separation of electrons from SQM and the gravity induced charge throughout the object.
3. The properties of strangelets as well as the surface profiles of strange stars depend crucially on the surface treatments.
4. A magnetic field comparable to the typical values of pulsars for a rotating strange star.
5. ...

Summary

We proposed a unified description to study SQM objects with all the fundamental interactions and important effects considered.

1. The energy per baryon of an SQM object is most likely decreasing with increasing size, and the local minimum may not exist.
2. There exist nonzero charge inside an SQM object, i.e., the charge located on the quark-vacuum interface due to the separation of electrons from SQM and the gravity induced charge throughout the object.
3. The properties of strangelets as well as the surface profiles of strange stars depend crucially on the surface treatments.
4. A magnetic field comparable to the typical values of pulsars for a rotating strange star.
5. ...

Summary

We proposed a unified description to study SQM objects with all the fundamental interactions and important effects considered.

1. The energy per baryon of an SQM object is most likely decreasing with increasing size, and the local minimum may not exist.
2. There exist nonzero charge inside an SQM object, i.e., the charge located on the quark-vacuum interface due to the separation of electrons from SQM and the gravity induced charge throughout the object.
3. The properties of strangelets as well as the surface profiles of strange stars depend crucially on the surface treatments.
4. A magnetic field comparable to the typical values of pulsars for a rotating strange star.
5. ...

Summary

We proposed a unified description to study SQM objects with all the fundamental interactions and important effects considered.

1. The energy per baryon of an SQM object is most likely decreasing with increasing size, and the local minimum may not exist.
2. There exist nonzero charge inside an SQM object, i.e., the charge located on the quark-vacuum interface due to the separation of electrons from SQM and the gravity induced charge throughout the object.
3. The properties of strangelets as well as the surface profiles of strange stars depend crucially on the surface treatments.
4. A magnetic field comparable to the typical values of pulsars for a rotating strange star.
5. ...

Thank You!!!