Equations of State of Different Phases of Dense Quark Matter

Efrain J. Ferrer

The University of Texas at El Paso

CSQCD-V, L'Aquila, Italy, May 23-27, 2016

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OUTLINE

- ✓ ***The Color Superconducting Phase at B=0 & B≠0***

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- ✓ *The Color Superconducting Phase at $B=0$ & $B\neq0$*
- ✓ *The BCS-BEC Crossover at Strong Coupling*
- ✓ *The Neutral Magnetic DCDW Phase*

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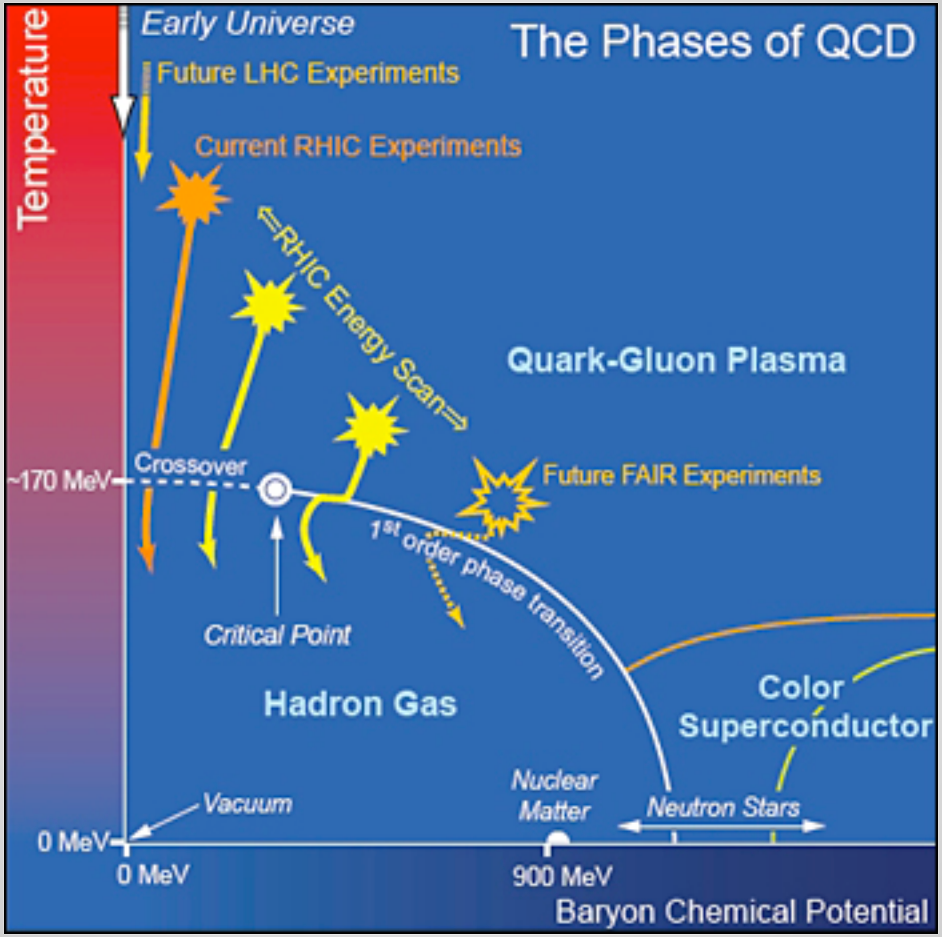
QCD Phase Diagram



Sophie Bushwick, News, July 22, 2010

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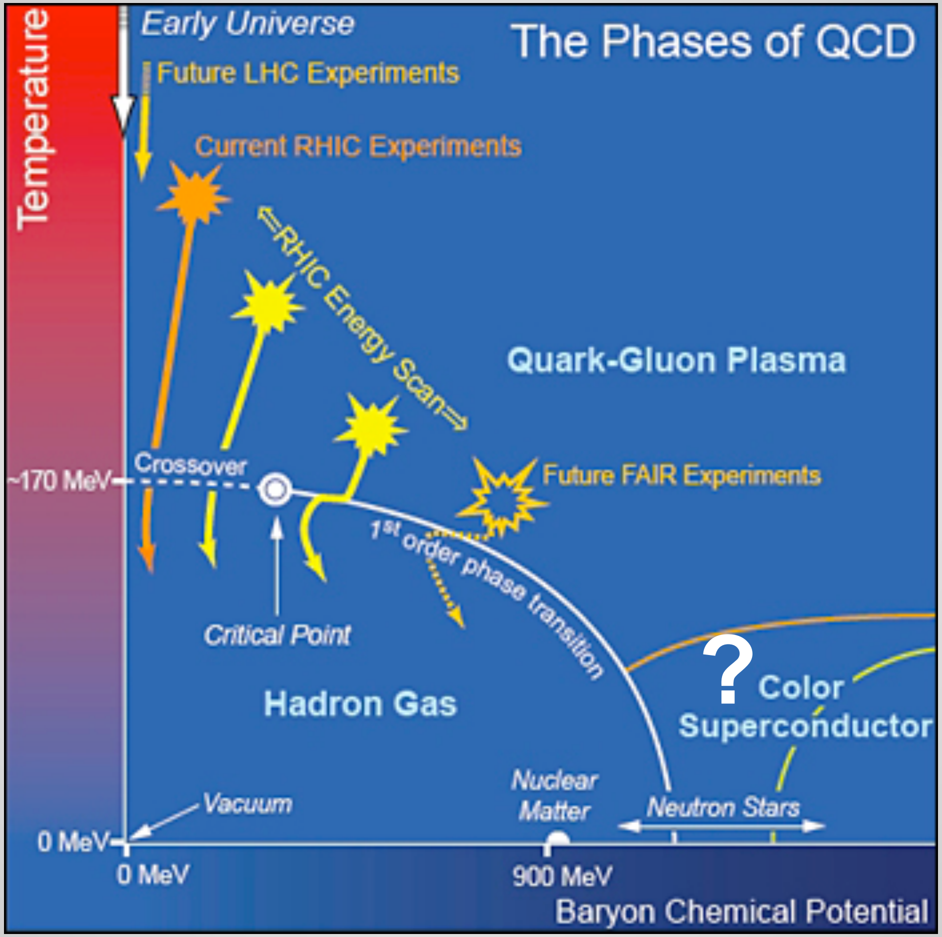


Sophie Bushwick, News, July 22, 2010

Neutron Stars

Diameter:

$R \approx 10\ km$

?

Mass:

$1.25 M_\odot \lesssim M \lesssim 2 M_\odot$

Temperature:

$10\ keV \lesssim T \lesssim 10 MeV$

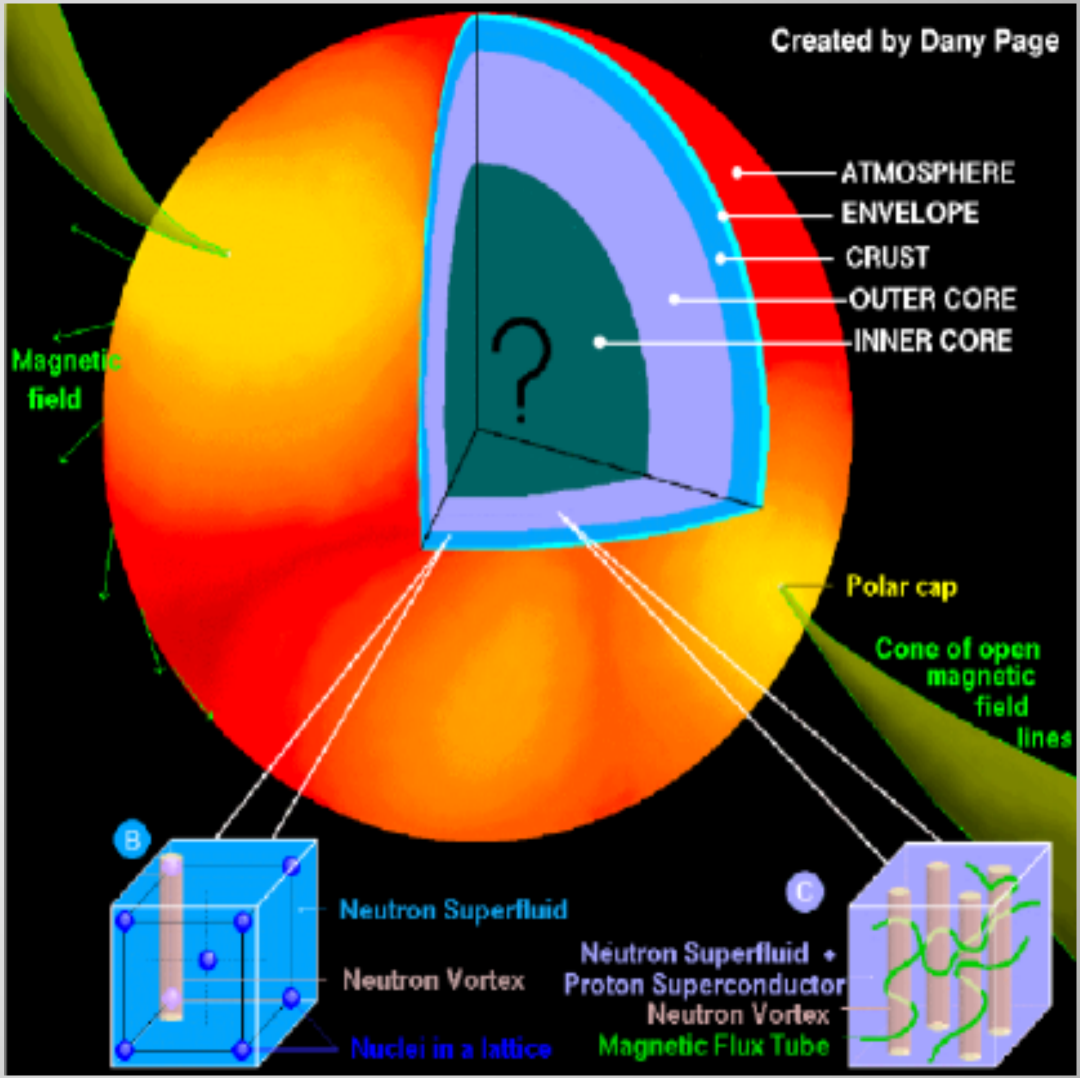
Magnetic fields:

pulsar's surface:

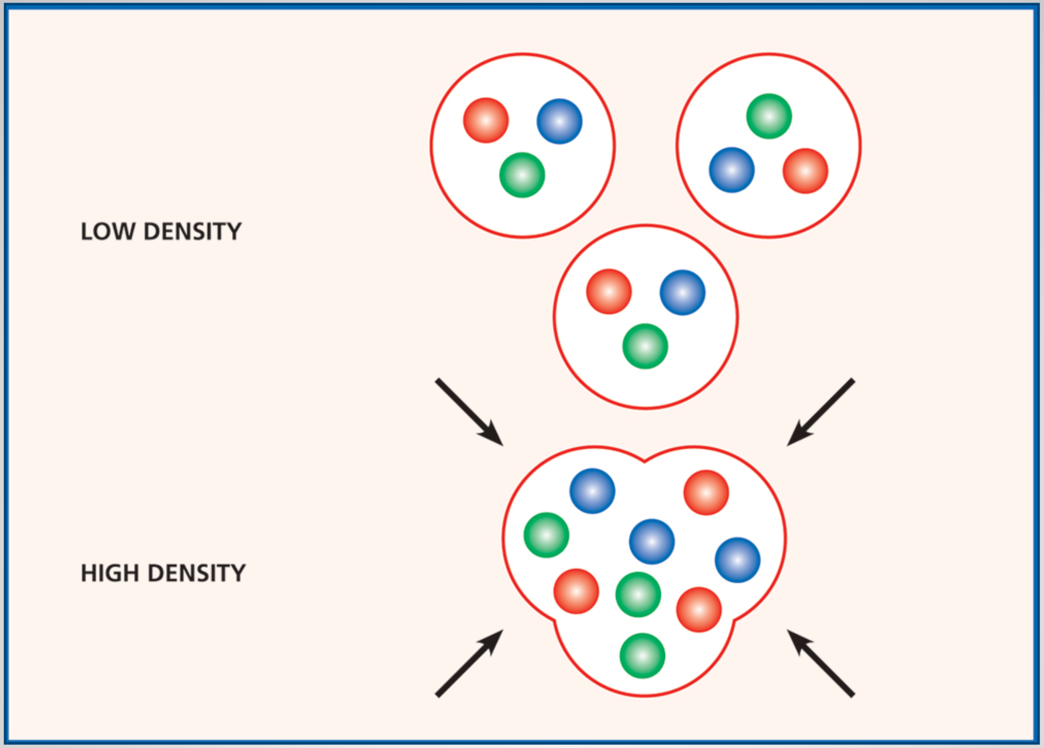
$B \sim 10^{12}$–$10^{13} G$

magnetar's surface:

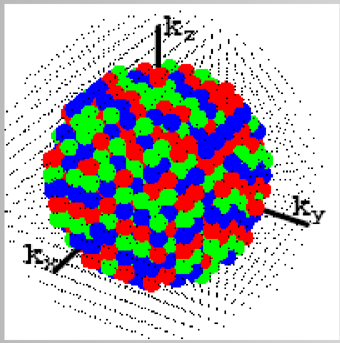
$B \sim 10^{14}$–$10^{15} G$

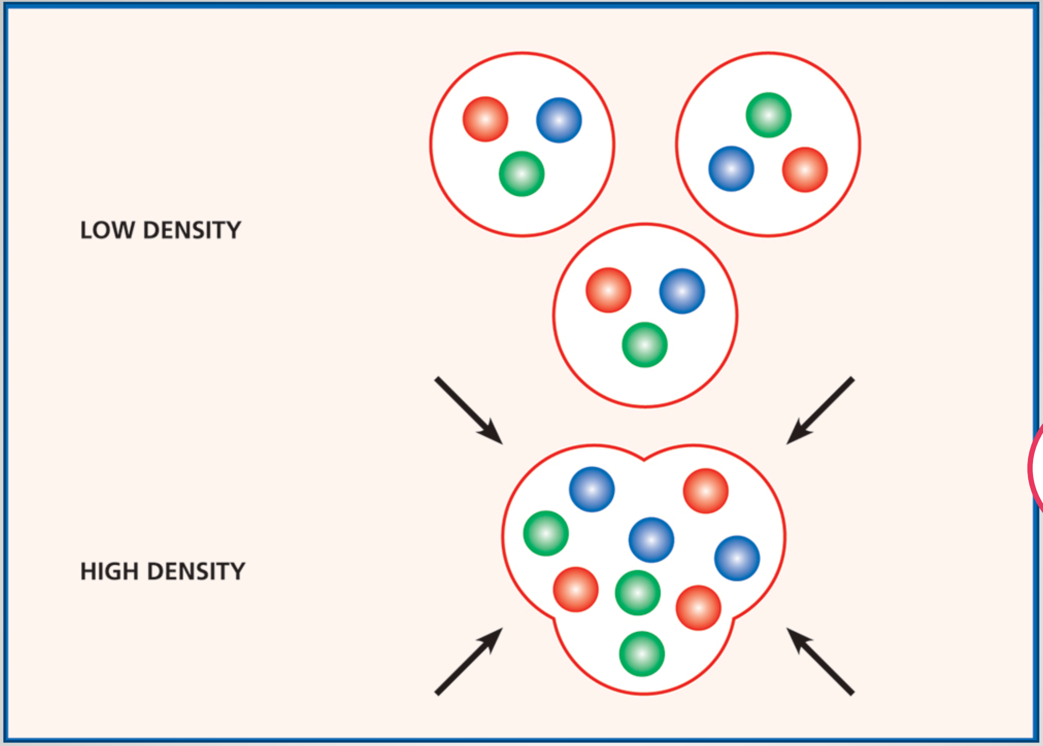


Baryonic Matter at Very High Density & CS



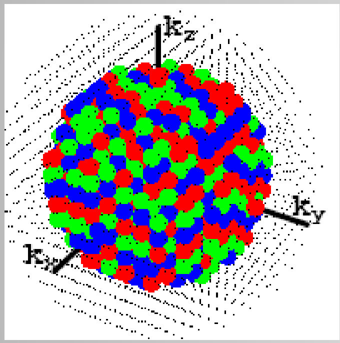
Baryonic Matter at Very High Density & CS

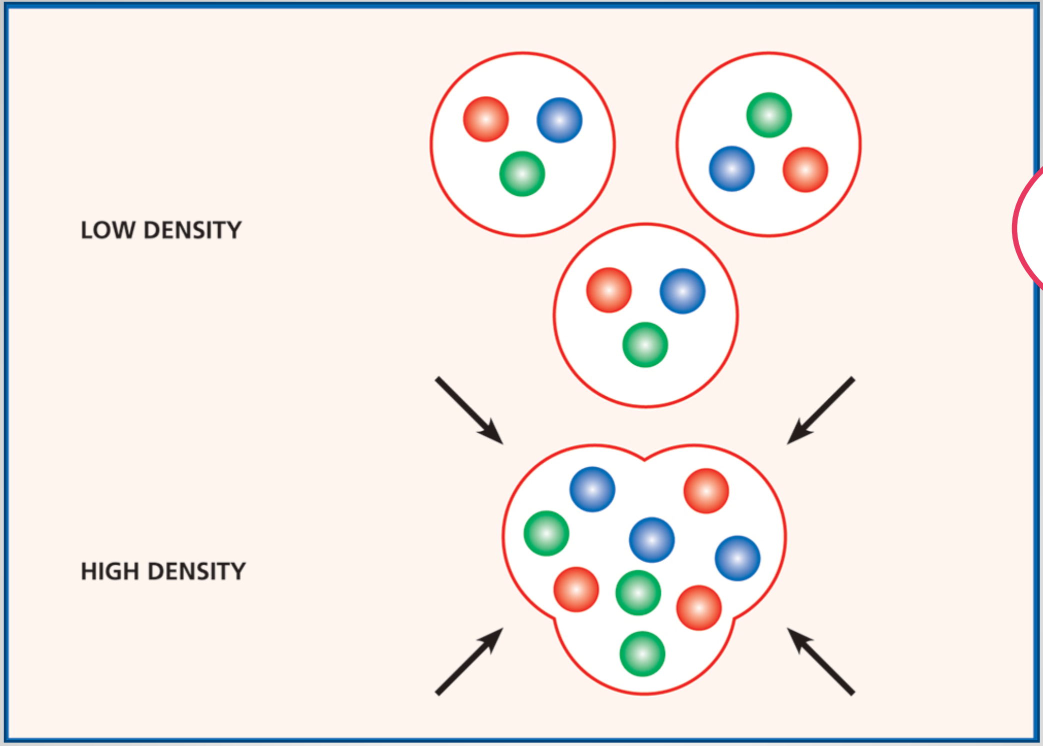




Asymptotic freedom

Baryonic Matter at Very High Density & CS

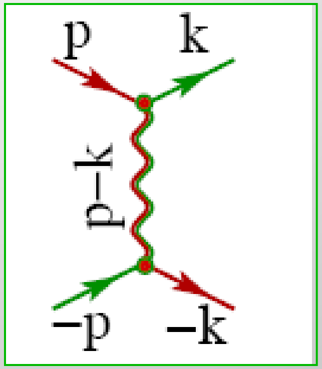




Asymptotic freedom

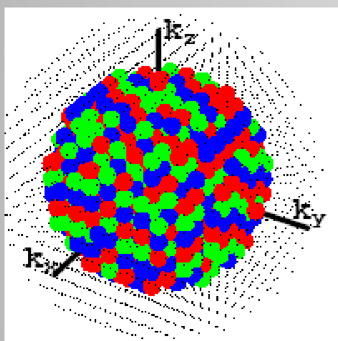
plus

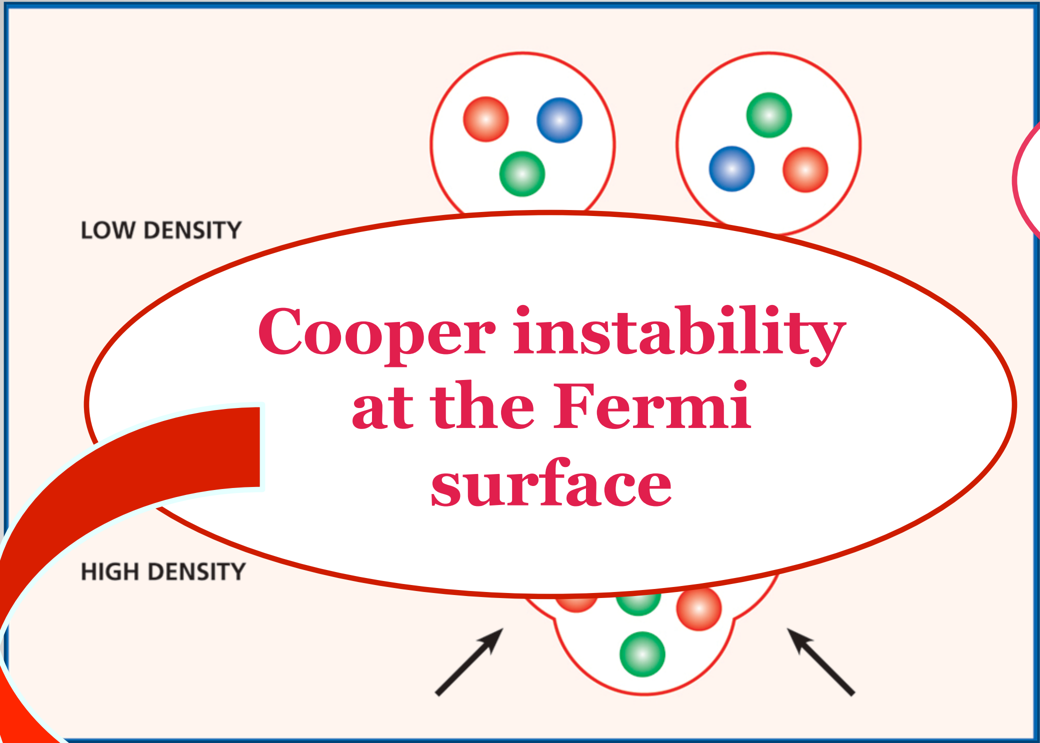
$$T^a{}_{ij}T^a{}_{kl} = -\frac{1}{3}\left(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}\right) + \frac{1}{6}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)$$



Attractive interactions

Baryonic Matter at Very High Density & CS





Cooper instability at the Fermi surface

Asymptotic freedom

plus

Attractive ...actions

$T^a{}_{ij}T^a{}_{kl}$

$= \ldots$

$+ \frac{\ldots}{6}$

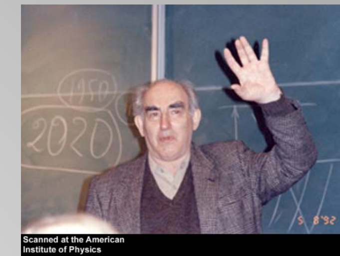
Formation of Quark-Quark Pairs: Color Superconductivity

Bailin & Love, Phys Rep. '84

Cooper Pair Condensation

L. D. Landau

$$F[\Psi] = \int d^3x \left\{ \frac{\hbar^2}{2\hat{m}} \vec{\nabla}\Psi^* \vec{\nabla}\Psi + \alpha(T - T_C)\Psi^*\Psi + \frac{\beta}{2}(\Psi^*\Psi)^2 \right\}$$



V. L. Ginzburg

Electric Superconductivity

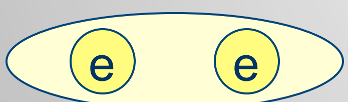
J. Bardeen, L. N. Cooper and J. R. Schrieffer

Color Superconductivity

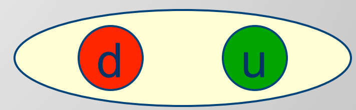
Barrois '77; Frautschi '78;
Bailin and Love'84;
Alford, Rajagopal and Wilczek '98;
Rapp, Schafer, Shuryak and Velkovsky '98

✓ ***Boson: Zero Spin and opposite momenta***

✓ **Electric Charge**



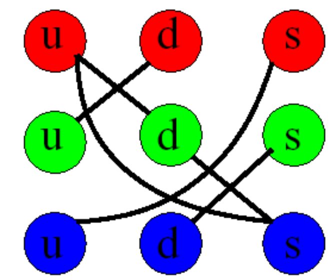
✓ **Color Charge**



✓ **Broken Symmetry : $U(1)_{em}$**

✓ **Broken Symmetry : $SU(3)_{C,}\ U(1)_{em}$**

CFL Phase



$$\Delta_1 = \Delta_2 = \Delta_3$$

$$\mathbf{SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_{e.m}}$$

$$\downarrow$$

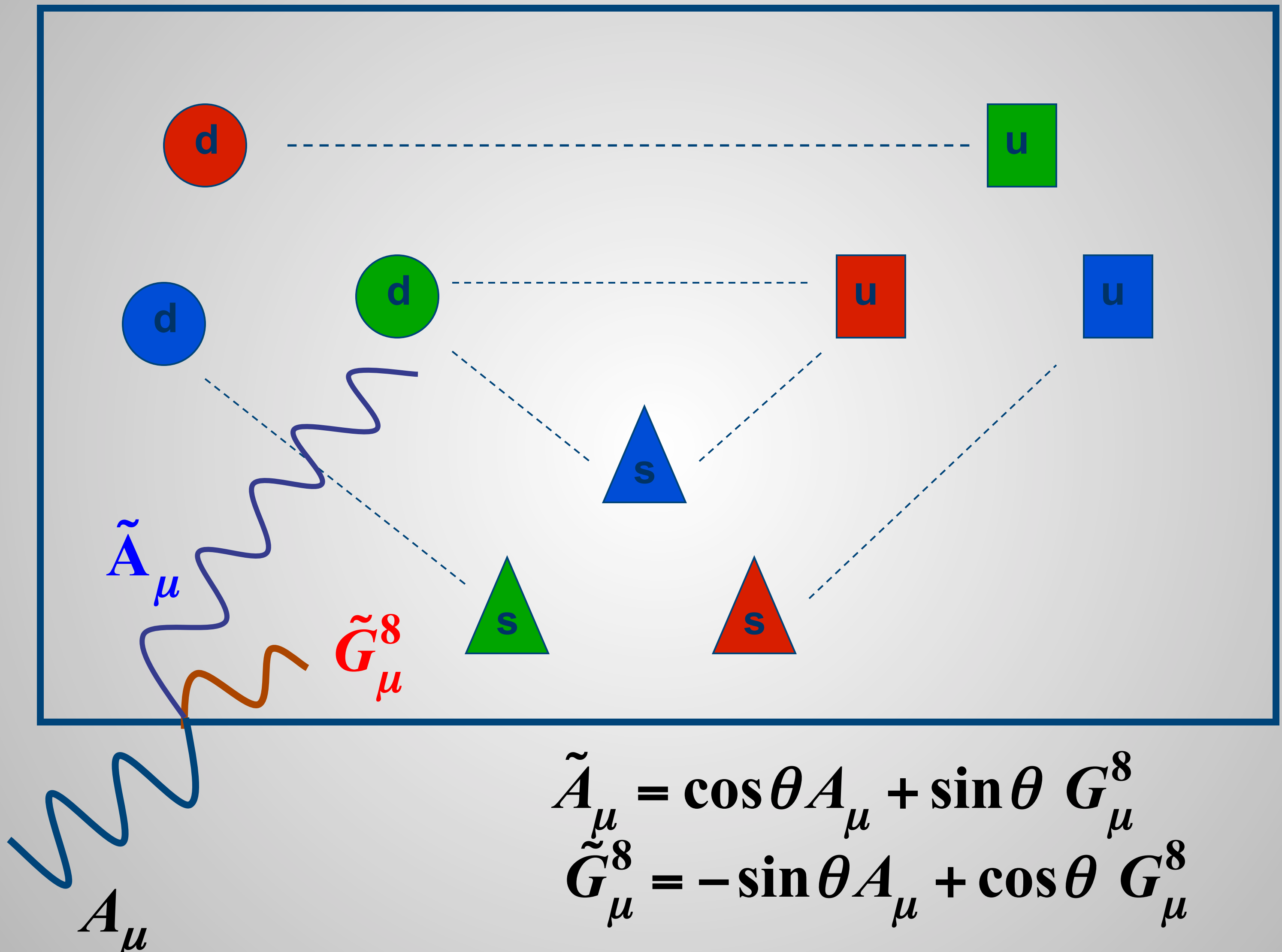
$$SU(3)_{C+L+R} \times \tilde{U}(1)_{em} \rightarrow$$ **9 GB**

Gluon Masses^2

$$m_D^2 = \frac{21 - 8\ln 2}{18} m_g^2, \qquad m_M^2 = \frac{21 - 8\ln 2}{54} m_g^2, \qquad m_g^2 = g^2 \mu^2 N_f / 6\pi^2.$$

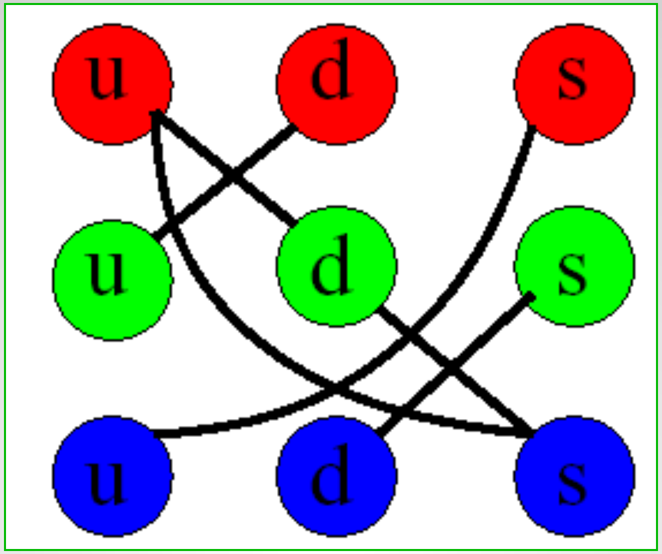
In the CFL phase all quarks pair and all the nonabelian gauge bosons are massive, so the only infrared degrees of freedom are Goldstone bosons and Abelian photon.

In-Medium Magnetic Field in the CFL Phase



$$\tilde{A}_\mu = \cos\theta A_\mu + \sin\theta\ G^8_\mu$$
$$\tilde{G}^8_\mu = -\sin\theta A_\mu + \cos\theta\ G^8_\mu$$

Rotated Charges



The pairs are all $\tilde{Q}$-neutral, but the quarks can be neutral or charged

$\tilde{Q}$ - CHARGES								
s	s	s	d	d	d	u	u	u
0	0	-1	0	0	-1	1	1	0

All $\tilde{Q}$-charged quarks have integer charges

Bag vs NJL Model in the CFL Phase

Bag Model

$$\Omega_{CFL}^{MIT} = \sum_i \Omega_i - \frac{3}{\pi^2}\Delta_{CFL}^2\mu^2 + B$$

$$\Omega_i = \frac{\mu_i^4}{4\pi^2}$$

NJL Model

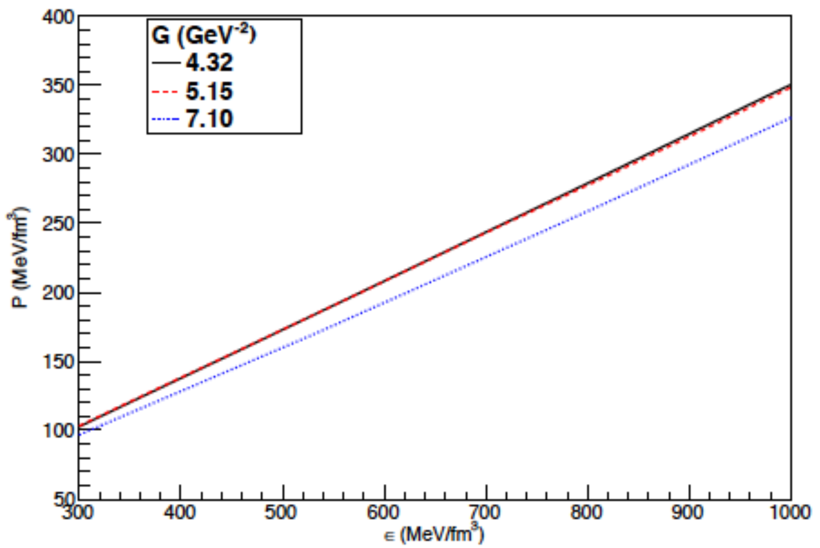
$$\Omega_{CFL}^{NJL} = -\frac{1}{4\pi^2}\int_0^\infty dpp^2 e^{-p^2/\Lambda^2}(16|\epsilon| + 16|\overline{\epsilon}|) + \\ -\frac{1}{4\pi^2}\int_0^\infty dpp^2 e^{-p^2/\Lambda^2}(2|\epsilon'| + 2|\overline{\epsilon'}|) + \frac{3\Delta_{CFL}^2}{G} + B$$

$$\varepsilon = \pm\sqrt{(p-\mu)^2 + \Delta_{CFL}^2}, \quad \overline{\varepsilon} = \pm\sqrt{(p+\mu)^2 + \Delta_{CFL}^2}$$

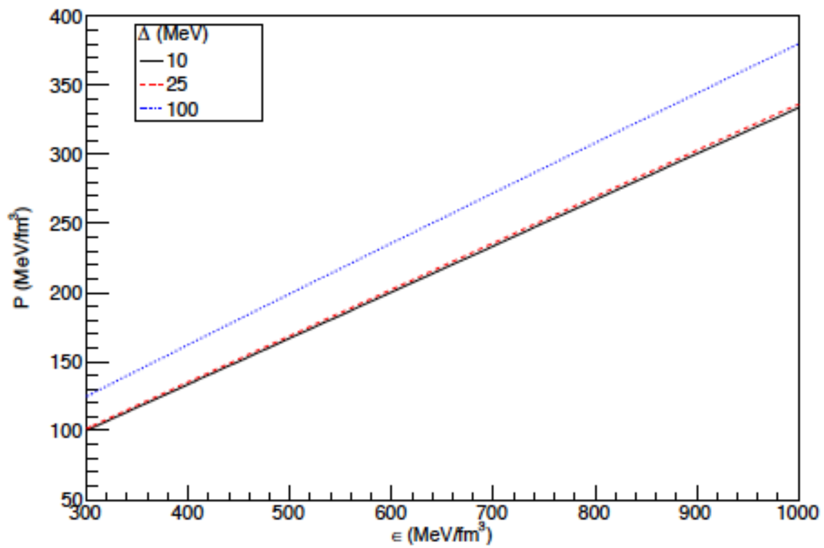
$$\varepsilon' = \pm\sqrt{(p-\mu)^2 + 4\Delta_{CFL}^2}, \quad \overline{\varepsilon'} = \pm\sqrt{(p+\mu)^2 + 4\Delta_{CFL}^2}$$

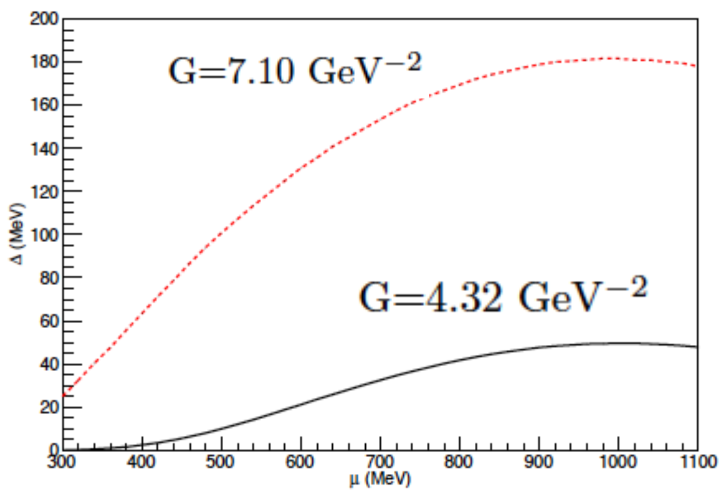
Bag & NJL EoS in the CFL Phase

NJL Model



Bag Model

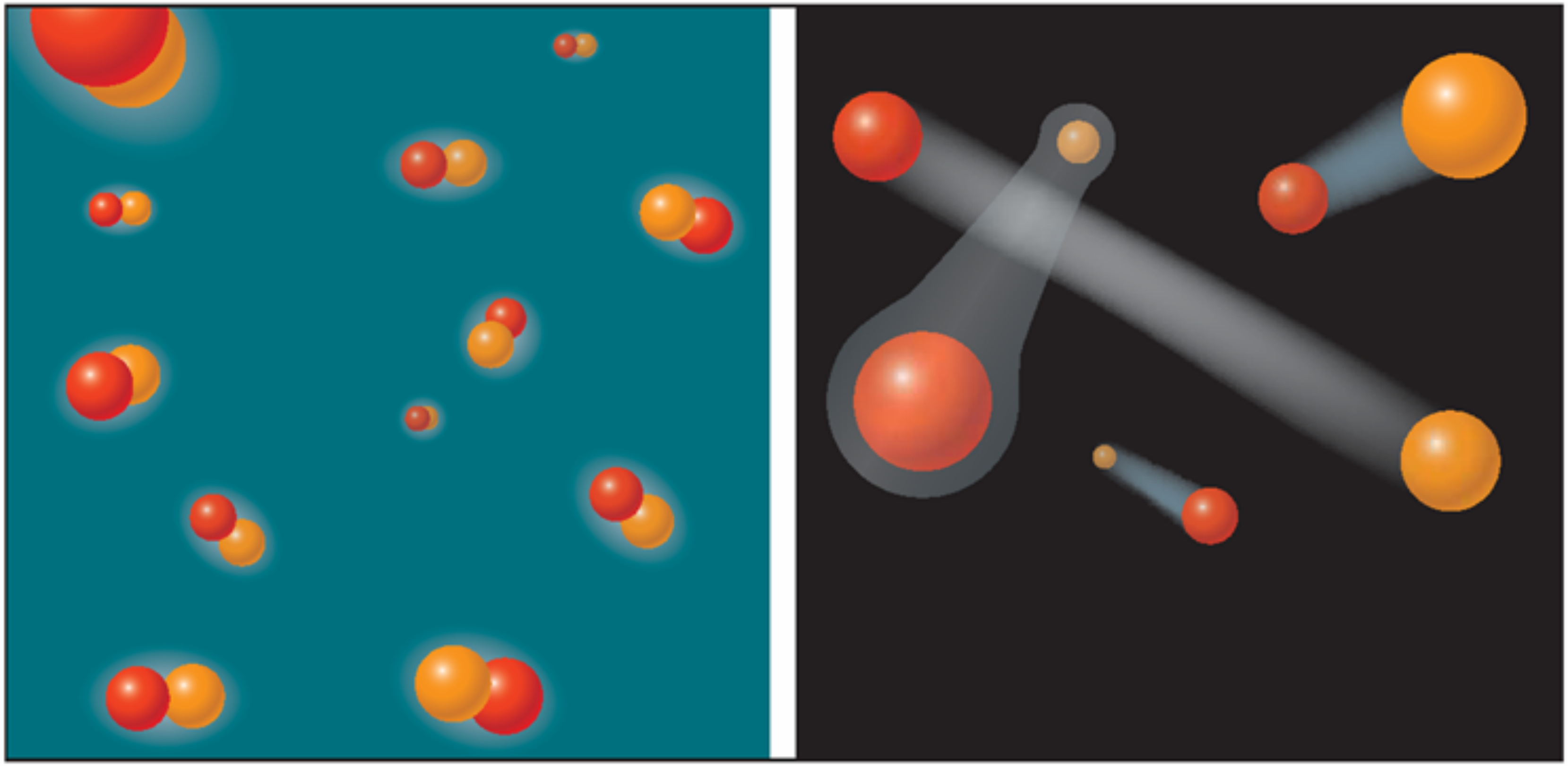




$$\epsilon_{CFL} = \Omega_{CFL} - \mu \frac{\partial \Omega_{CFL}}{\partial \mu},$$

$$P_{CFL} = -\Omega_{CFL}$$

BEC-BSC CROSSOVER



Tango or twist? In a magnetic field, atoms in different spin states can form molecules (*left*). Vary the field, and they might also form loose-knit Cooper pairs. ***Science***

- *Strong coupling*
- *Small pair size*
- *Coherence length << mean interparticle distance*

- *Weak coupling*
- *Large pair size*
- *Coherence length >>mean interparticle distance*

NJL Model with Multi-Fermion Interactions

NJL-Type Model

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_{int}^{dd}$$

$$\mathcal{L}_1 = \bar{\psi}(i\gamma^\mu\partial_\mu + \gamma^0\mu)\psi + \frac{G_{\rm S}}{4}(\bar{\psi}\psi)^2 + \frac{G_{\rm D}}{4}(\bar{\psi}i\gamma_5 C\bar{\psi}^T)(\psi^T C i\gamma_5\psi)$$

$$\mathcal{L}_{int}^{dd} = \lambda\left[(\bar{\psi}i\gamma_5 C\bar{\psi}^T)(\psi^T C i\gamma_5\psi)\right]^2$$

Condensates

$$m = -\frac{G_{\rm S}}{2}\langle\bar{\psi}\psi\rangle, \quad \Delta = -\frac{G_{\rm D}}{2}\langle\psi^T C\gamma_5\psi\rangle$$

Thermodynamic Potential

$$\Omega_0 = -\sum_{e=\pm1}\int_\Lambda \frac{d^3k}{(2\pi)^3}\epsilon_k^e + \chi'\frac{|\Delta|^2}{G_{\rm D}} + \frac{m^2}{G_{\rm S}}$$

$$\epsilon_k^e = \sqrt{(\epsilon_k - e\mu)^2 + \chi^2|\Delta|^2}, \qquad \epsilon_k = \sqrt{k^2+m^2}, \qquad e = \pm1$$

$$\chi = (1 + 32\lambda|\Delta|^2/G_{\rm D}^3), \qquad \chi' = (1 + 48\lambda|\Delta|^2/G_{\rm D}^3)$$

Gap Equations at Fixed Particle Number Density

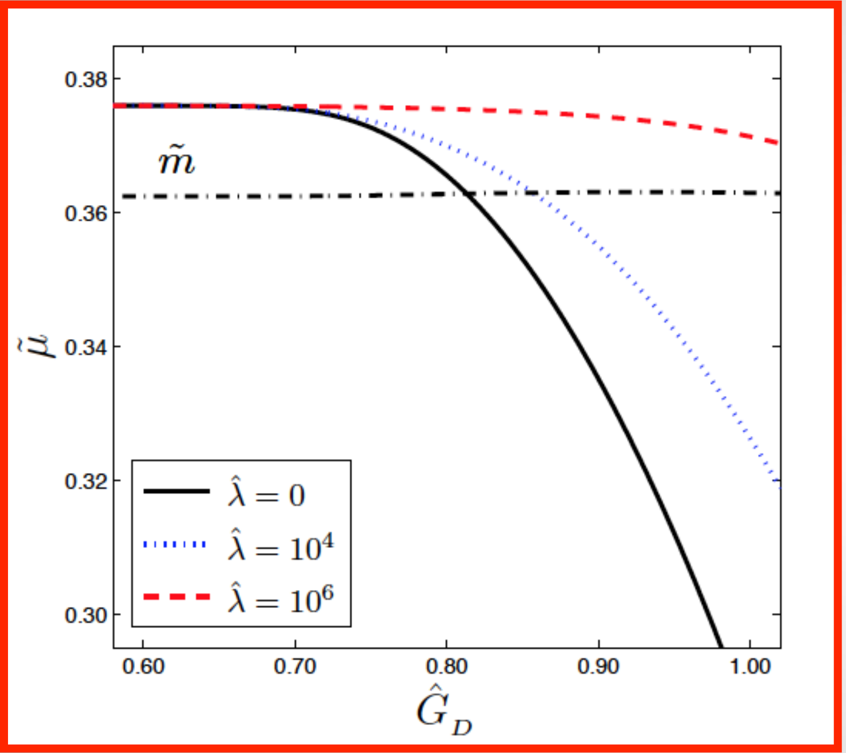
$$\frac{\partial \Omega_0}{\partial \Delta} = 0 \quad \Rightarrow \quad \int_\Lambda \frac{d^3k}{(2\pi)^3}\left(\frac{1}{\epsilon_k^+} + \frac{1}{\epsilon_k^-}\right) = \frac{2(2\chi' - 1)}{G_D(3\chi^2 - 2\chi)}$$

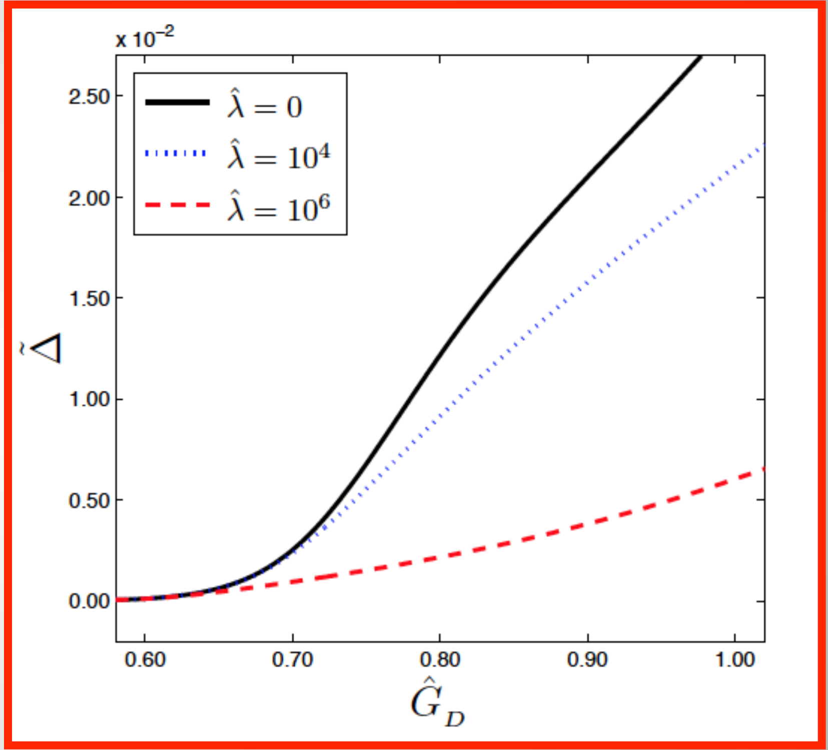
$$\frac{\partial \Omega_0}{\partial m} = 0 \quad \Rightarrow \quad \int_\Lambda \frac{d^3k}{(2\pi)^3}\frac{1}{2\epsilon_k}\left(\frac{\xi_k^+}{\epsilon_k^+} + \frac{\xi_k^-}{\epsilon_k^-}\right) = \frac{1}{G_S}$$

$$\xi_k^\pm = \epsilon_k \mp \mu$$

$$n_F = \frac{P_F^3}{3\pi^2} = -\int_\Lambda \frac{d^3k}{(2\pi)^3}\left(\frac{\xi_k^+}{\epsilon_k^+} - \frac{\xi_k^-}{\epsilon_k^-}\right)$$

BCS-BEC Crossover

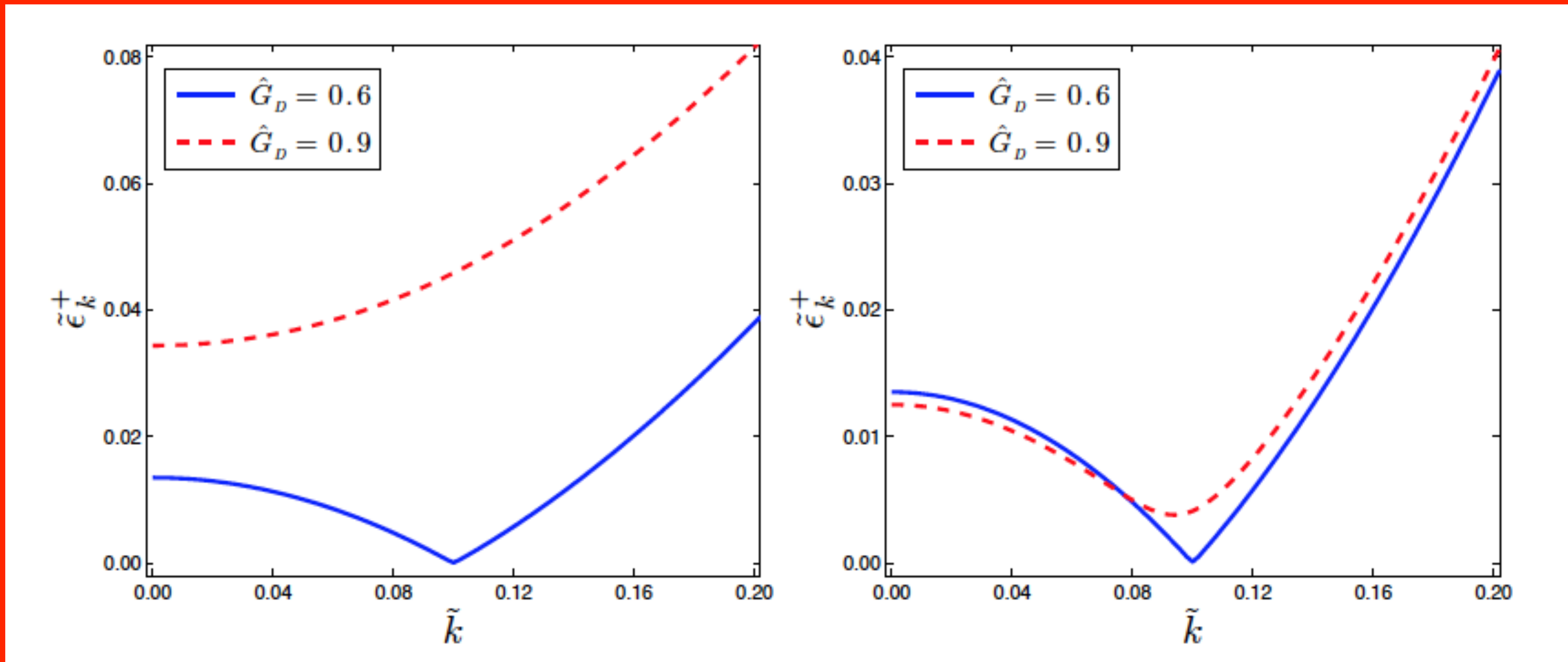




The graphs were taken at $G_S\Lambda^2$=1.20 and P_F/Λ=0.10
The value of the critical coupling at λ=0 is $G^{cr}_D\Lambda^2$=0.81.

Quasiparticles' Dispersion Relations

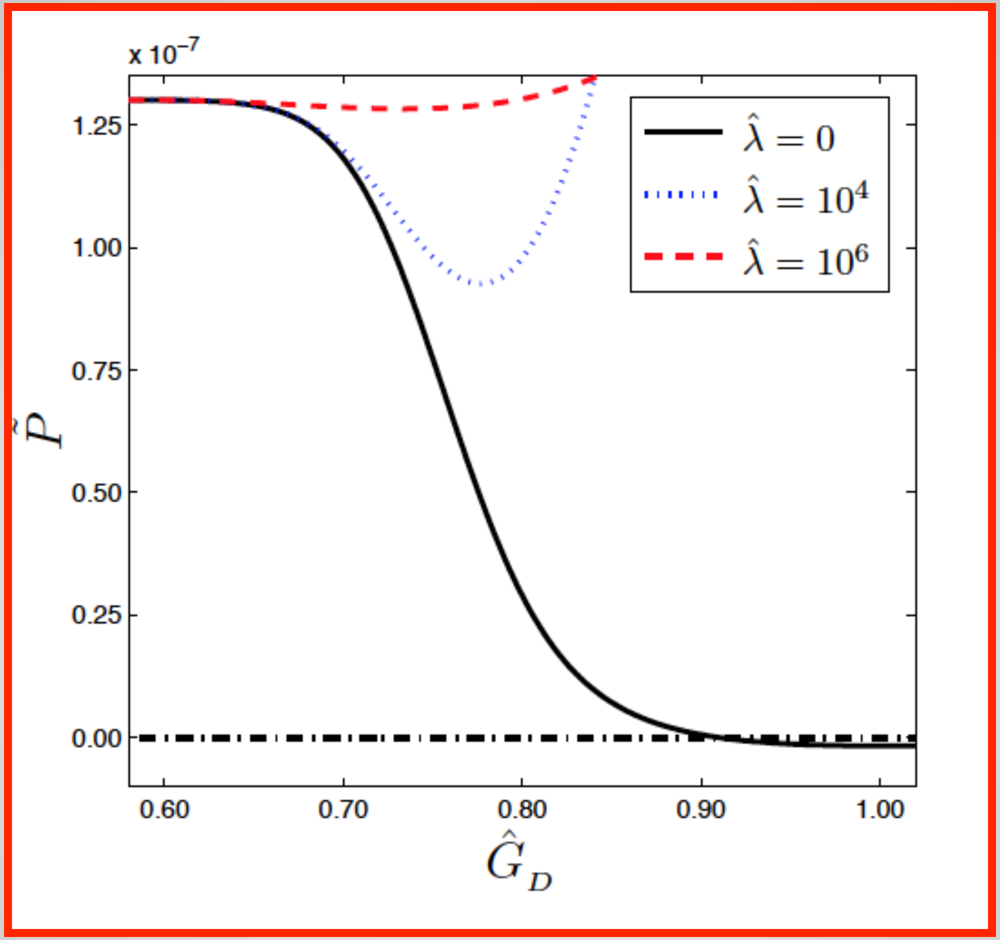
λ=0 **λ≠0**



The graphs are taken at values of the critical coupling smaller and larger than the critical one$G^{cr}_D\Lambda^2$=0.81.

We note that at λ=0 for $G_D > G^{cr}_D$ the quasiparticle spectrum is bosonic, while for $G_D < G^{cr}_D$ is fermionic. At $\lambda\Lambda^8=10^6$ both spectra are fermionic.

EoS in the Crossover



The graphs were taken at $G_S\Lambda^2$=1.20 and P_F/Λ=0.10
The value of the critical coupling at λ=0 is $G^{cr}_D\Lambda^2$=0.81.

In the strong coupling regime only if there exists a strong diquark-diquark repulsion the system pressure can increase.

Gluons in the CFL Phase

Gauged-NJL Model with vector interaction

$$\mathcal{L} = -\bar{\psi}(\gamma^\mu D_\mu + \mu\gamma^0)\psi - G_V(\bar{\psi}\gamma_\mu\psi)^2 + G_S\sum_{k=0}^{8}\left[\left(\bar{\psi}\lambda_k\psi\right)^2 + \left(\bar{\psi}i\gamma_5\lambda_k\psi\right)^2\right]$$
$$-K\left[det_f\left(\bar{\psi}(1+\gamma_5)\psi\right) + det_f\left(\bar{\psi}(1+\gamma_5)\psi\right)\right] + \frac{G_D}{4}\sum_\eta(\bar{\psi}P_\eta\bar{\psi}^T)(\psi^T P_\eta\psi) + \mathcal{L}_G,$$

$$\mathcal{L}_G = -\frac{1}{4}G^A_{\mu\nu}G^{\mu\nu}_A + \mathcal{L}_{gauge} + \mathcal{L}_{ghost},$$

$$G^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + gf^{ABC}G^B_\mu G^C_\nu,$$

$$\mathcal{L}_{ghost} = -\eta^{A\dagger}\partial^\mu(\partial_\mu\eta^A + gf^{ABC}G^B_\mu\eta^C)$$

Condensates

$$\Delta_\eta = \langle\psi^T P_\eta\psi\rangle \qquad \rho = \langle\bar{\psi}\gamma_0\psi\rangle \;\rightarrow\; \tilde{\mu} = \mu - 2G_V\rho$$

Gluon Contribution

System Effective Action

$$\Gamma(\mathbf{G}) = \beta V \Omega_0 + \sum_{n=2}^{\infty} \int dx_1 ... dx_n \Pi^{A_1, A_2, ..., A_n}_{\mu_1, \mu_2, ... \mu_n}(x_1, x_2, ... x_n) G^{\mu_1}_{A_1}(x_1) ... G^{\mu_n}_{A_n}(x_n)$$

Gluon Effective Action Leading Contribution

$$\int dx^4 [-\frac{1}{4}(\partial_\mu G^A_\nu - \partial_\nu G^A_\mu)^2 - \frac{\xi^2}{2}(\partial_\mu G^A_\mu)^2 + \frac{1}{2} G^A_\mu \Pi^{\mu\nu}_{AB} G^B_\nu]$$

In Covariant Gauge: $F_\xi = \xi \partial^\mu G^A_\mu = 0$

In the Hard-Loop Approximation:

$$\Pi^{AB}_{\mu\nu}(p_0 = 0, \mathbf{p} \to 0) = [\hat{\tilde{m}}^2_D \delta_{\mu 0} \delta_{\nu 0} + \hat{\tilde{m}}^2_M \delta_{\mu i} \delta_{\nu i}] \delta^{AB}$$

with

$$\hat{\tilde{m}}_D = \tilde{m}_D \theta(\Delta - p) + \sqrt{3} \tilde{m}_g \theta(\tilde{\mu} - p) \theta(p - \Delta)$$

$$\hat{\tilde{m}}_M = \tilde{m}_M \theta(\Delta - p)$$

Thermodynamic Potential of the CFL-Gluonic Phase

$$\Omega = \Omega_q + \Omega_g - \Omega_{vac}$$

$$\Omega_{vac} \equiv \Omega(\mu = 0, \Delta = 0)$$

$$\Omega_{CFL}(T=0) = -\frac{1}{4\pi^2}\int_0^{\Lambda} dp p^2 (16|\epsilon| + 16|\overline{\epsilon}|) -$$

$$-\frac{1}{4\pi^2}\int_0^{\Lambda} dp p^2 (2|\epsilon'| + 2|\overline{\epsilon'}|) + \frac{3\Delta^2}{G_D} - G_V \rho^2,$$

$$\varepsilon = \pm\sqrt{(p-\tilde{\mu})^2 + \Delta^2}, \quad \overline{\varepsilon} = \pm\sqrt{(p+\tilde{\mu})^2 + \Delta^2},$$

$$\varepsilon' = \pm\sqrt{(p-\tilde{\mu})^2 + 4\Delta^2}, \quad \overline{\varepsilon}' = \pm\sqrt{(p+\tilde{\mu})^2 + 4\Delta^2}.$$

$$\Omega_G(T=0) = \frac{2}{\pi^2}\int_0^{\Lambda} dp p^2 \left(\sqrt{p^2 + \tilde{m}_D^2\theta(\Delta - p) + 3\tilde{m}_g^2\theta(\tilde{\mu} - p)\theta(p - \Delta)} + \right.$$

$$\left. + 3\sqrt{p^2 + \tilde{m}_M^2\theta(\Delta - p)}\right)$$

EoS of the CFL-Gluonic Phase

Gap Equations

$$\frac{\partial \Omega}{\partial \Delta} = 0, \quad \rho = -\frac{\partial \Omega_q}{\partial \tilde{\mu}}$$

Equations of State

$$\epsilon = \Omega_q + \Omega_g - \Omega_{vac} + \tilde{\mu}\rho - (B - B_0)$$

$$P = -(\Omega_q + \Omega_g - \Omega_{vac}) + (B - B_0)$$

B_0 is introduced to ensure that ε=P=0 in vacuum.

Dynamical Bag Constant

M. Buballa and M. Oertel, Phys. Lett. B 457 (1999) 261

$$B = \sum_{i=u,d,s}\left[\frac{3}{\pi^2}\int_0^\Lambda p^2dp\left(\sqrt{m_i^2+p^2}-\sqrt{p^2}\right)-2G_S\langle\overline{\psi}_i\psi_i\rangle\right]+4K\langle\overline{\psi}_u\psi_u\rangle\langle\overline{\psi}_d\psi_d\rangle\langle\overline{\psi}_s\psi_s\rangle$$

Quark Condensates

$$\langle\overline{\psi}_i\psi_i\rangle = -\frac{3}{\pi^2}\int_{p_{Fi}}^\Lambda p^2dp\frac{m_i}{\sqrt{m_i^2+p^2}},$$

Dynamical Masses

$$m_i = 4G_S\frac{3}{\pi^2}\int_{p_{Fi}}^\Lambda p^2dp\frac{m_i}{\sqrt{m_i^2+p^2}}+2K\frac{9}{\pi^4}\int_{p_{Fi}}^\Lambda p^2dp\frac{m_j}{\sqrt{m_j^2+p^2}}\int_{p_{Fi}}^\Lambda p^2dp\frac{m_k}{\sqrt{m_k^2+p^2}}$$

Vacuum Bag Constant

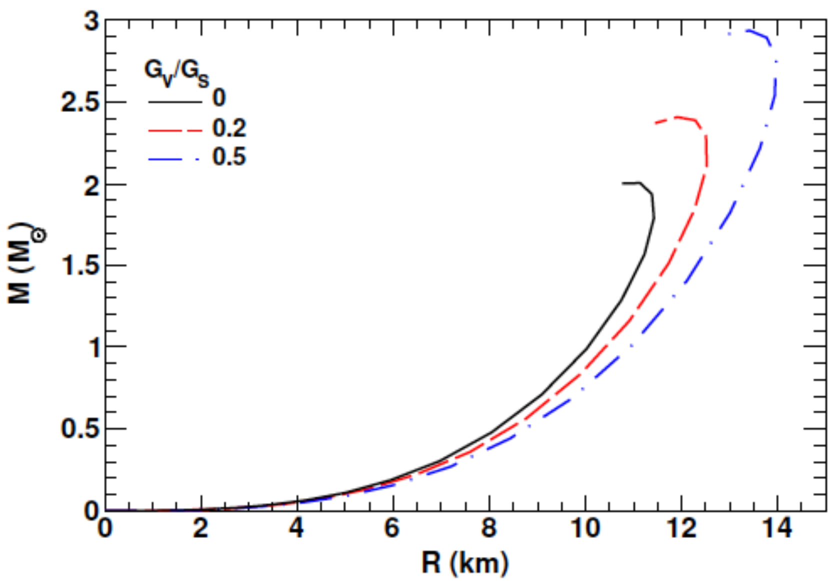
$$B_0 = B|_{\rho_u=\rho_d=\rho_s=0}$$

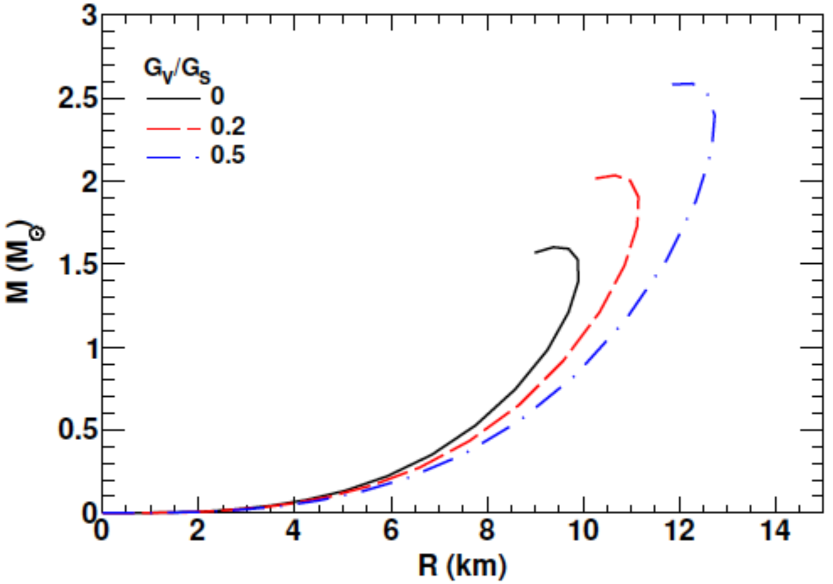
$$B_0 = \frac{9}{\pi^2}\left[\int_0^\Lambda p^2dp\left(\sqrt{m^2+p^2}-\sqrt{p^2}+\frac{2G_Sm}{\sqrt{m^2+p^2}}\right)\right]-4K\left(\frac{3}{\pi^2}\right)^3\left[\int_0^\Lambda dpp^2\frac{m}{\sqrt{m^2+p^2}}\right]^3$$

$$1 = 4G_S\frac{3}{\pi^2}\int_0^\Lambda p^2dp\frac{1}{\sqrt{m^2+p^2}}+2K\frac{9}{\pi^4}\left[\int_0^\Lambda p^2dp\frac{m}{\sqrt{m^2+p^2}}\right]^2$$

Mass-Radius Relationship

Without Gluon Contribution | **With Gluon Contribution**



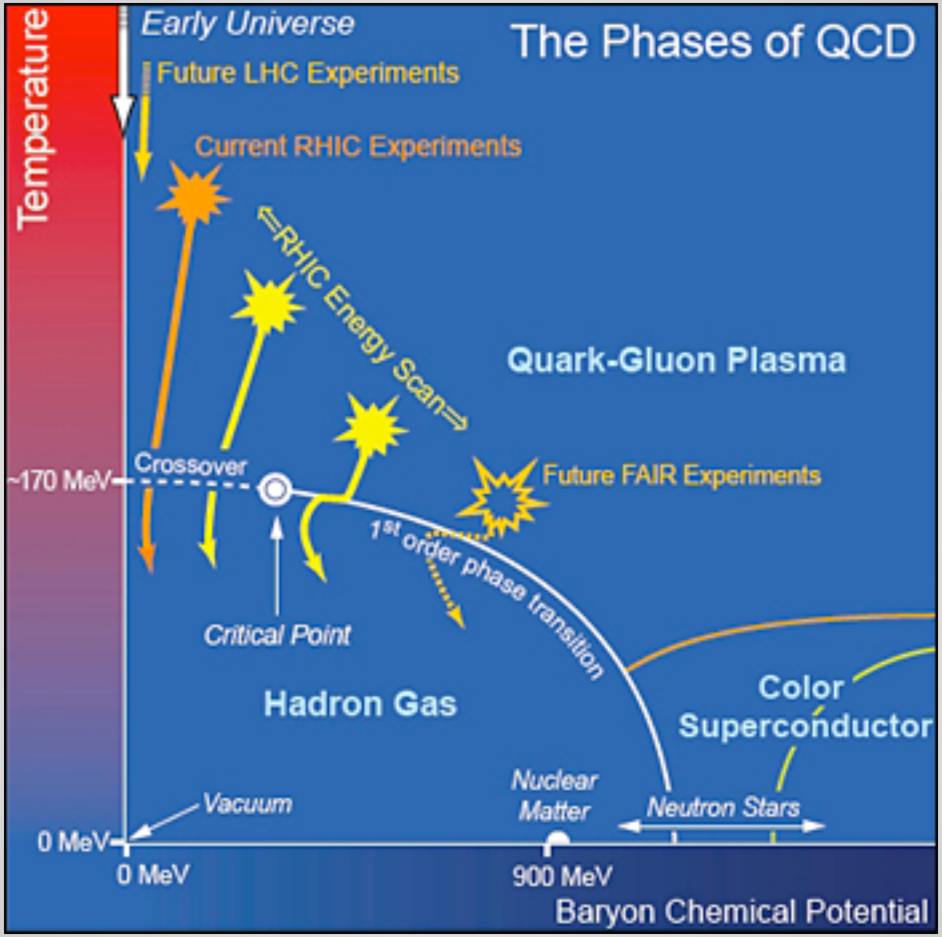


$$\frac{dM}{dR} = 4\pi R^2 \epsilon$$

$$\frac{dP}{dR} = -\frac{\epsilon M}{R^2}\left(1+\frac{P}{\epsilon}\right)\left(1+\frac{4\pi R^3 P}{M}\right)\left(1-\frac{2M}{R}\right)^{-1}$$

The Gluons decrease the maximum stellar mass for each sequence up to 20%. Sequence including gluons cannot reach two solar masses if G_V/G_S<0.2.

QCD Phase Diagram



Sophie Bushwick, News, July 22, 2010

Pressure Anisotropy in the MCFL Phase

EJF, *V. de la Incera, J. Keith, I. Portillo and P. Springsteen, PRC 82 (2010) 065802*

$$\frac{1}{\beta V}\langle\widetilde{\tau}^{\mu\nu}\rangle = \Omega_H \eta^{\mu\nu} + (\mu N + TS) u^\mu u^\nu + HM\eta_\perp^{\mu\nu}$$

$$\Omega_H = \Omega_{MCFL} + B + \frac{\widetilde{H}^2}{2}$$

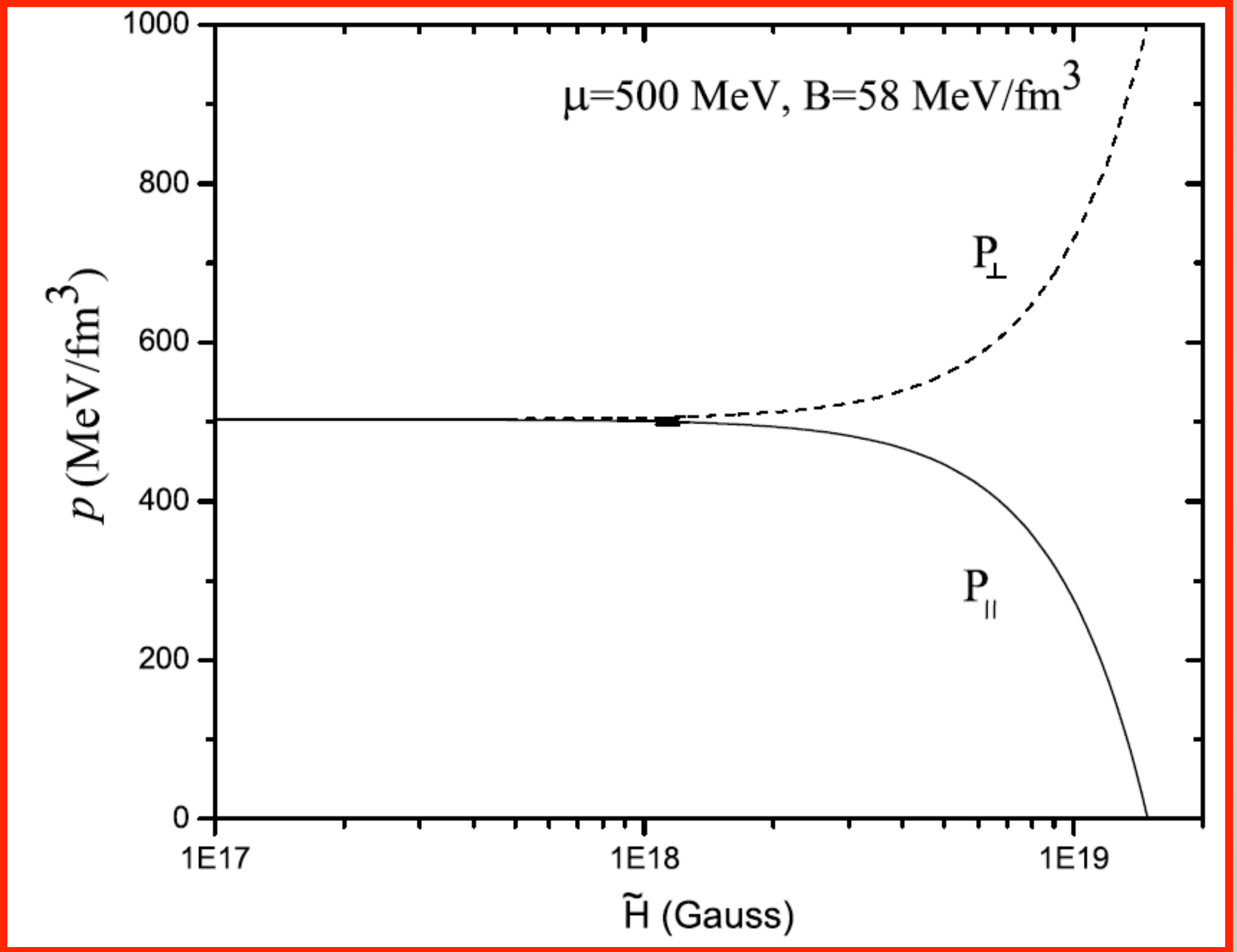
$$\eta_\perp^{\mu\nu} = \widehat{F}^{\mu\rho}\widehat{F}_\rho^{\nu}$$

$$\varepsilon_{MCFL} = \Omega_H - \mu\frac{\partial\Omega_H}{\partial\mu},$$

$$p_{MCFL}^{\parallel} = -\Omega_H, \quad p_{MCFL}^{\perp} = -\Omega_H + \widetilde{H}\frac{\partial\Omega_H}{\partial\widetilde{H}}$$

Pressure Anisotropy in the MCFL Phase

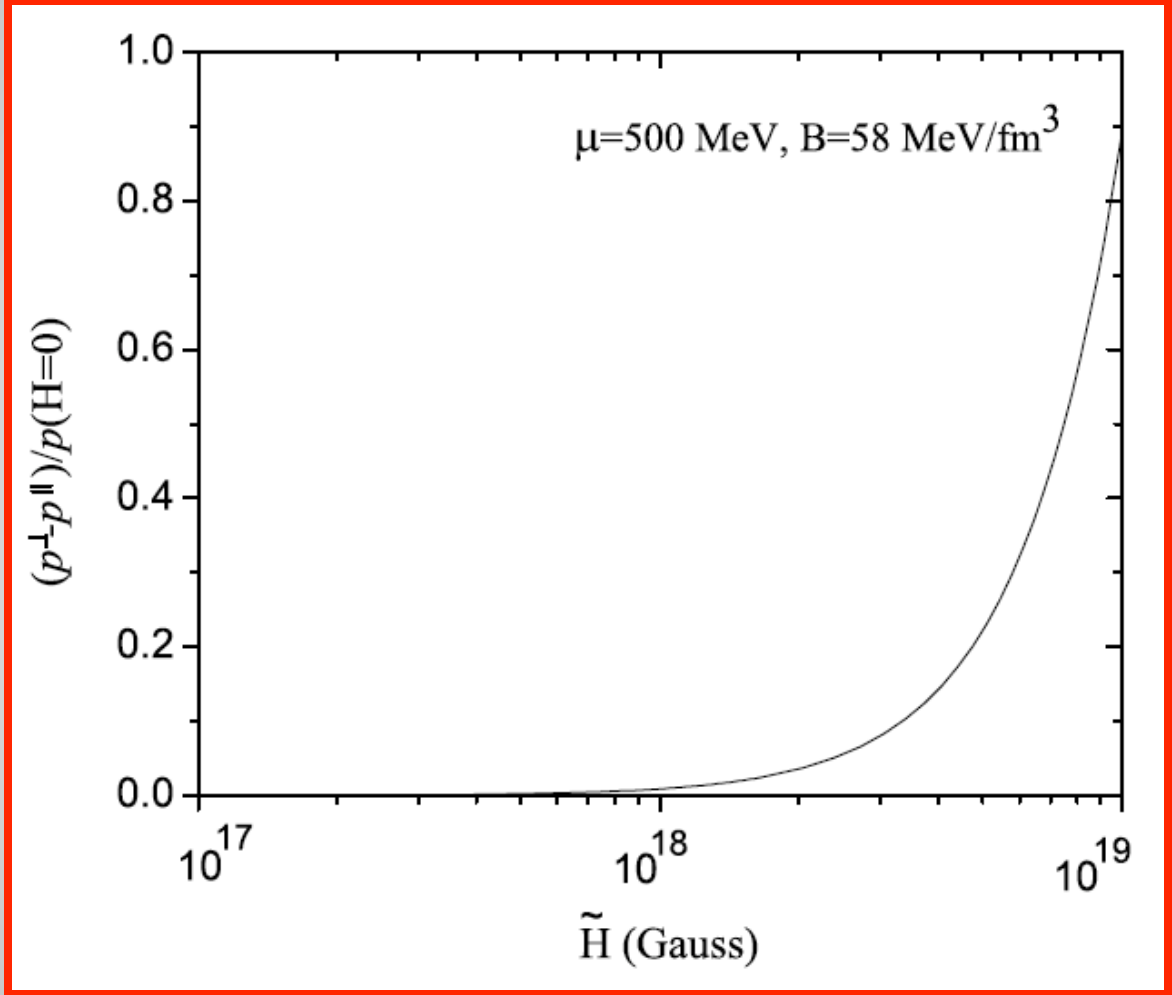
L. Paulucci, EJF, de la Incera, J.E. Horvath, PRD (2011) 04300



$$p_{\parallel}{}^{0} = -\Omega_f^0 - \frac{H^2}{2} - B, \qquad p_{\perp}{}^{0} = -\Omega_f^0 - HM^0 + \frac{H^2}{2} - B$$

Pressure Splitting vs B

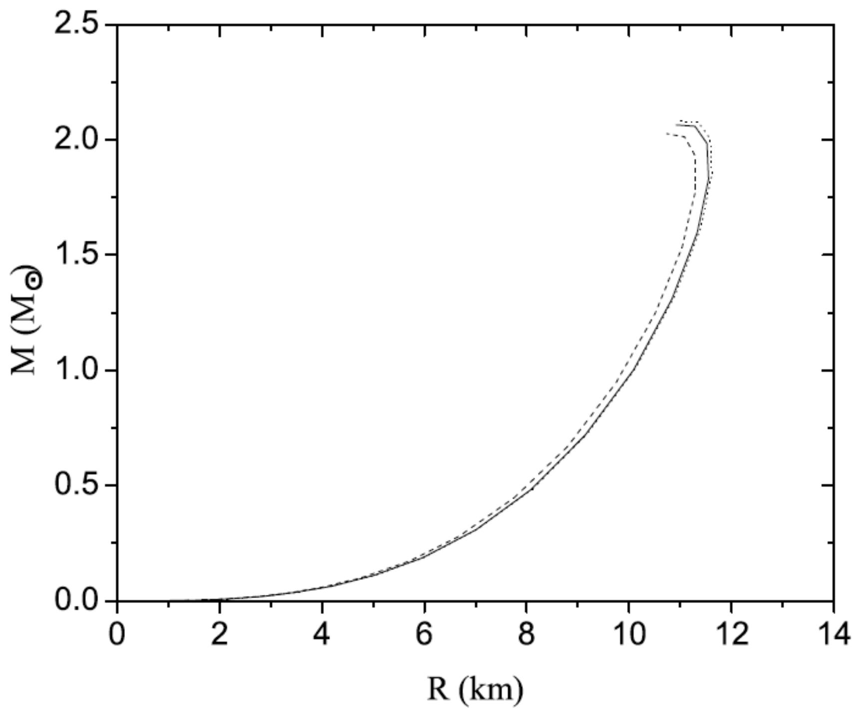
L. Paulucci, EJF, de la Incera, J.E. Horvath, PRD (2011) 04300



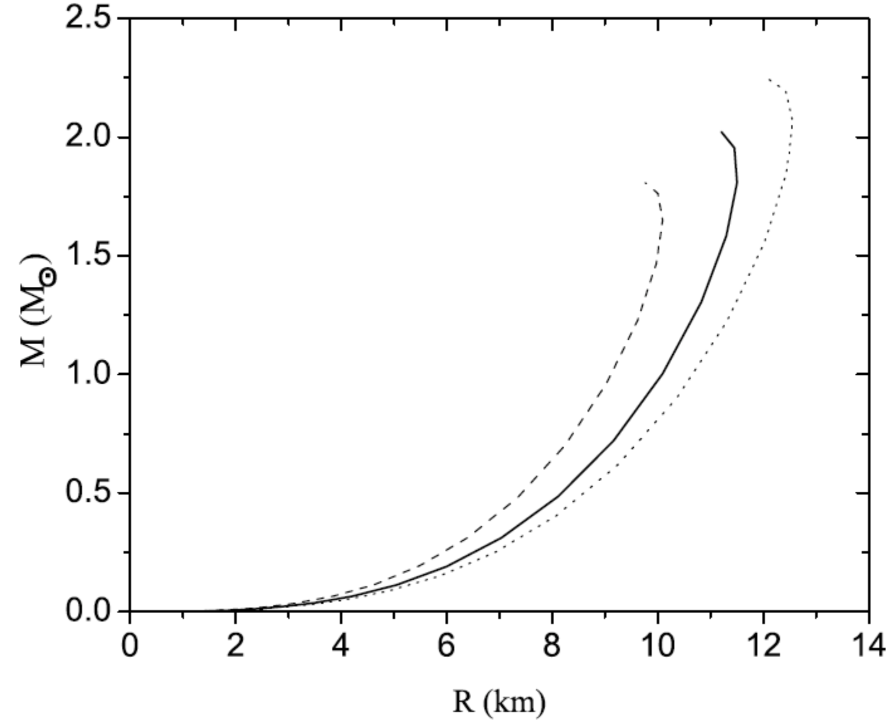
At μ=500MeV, for H≈3×10^{18} G the splitting is 10% of the isotropic value at H=0

Mass-Radius Relationship for MCFL Stars

L. Paulucci, EJF, de la Incera, J.E. Horvath, PRD (2011) 04300

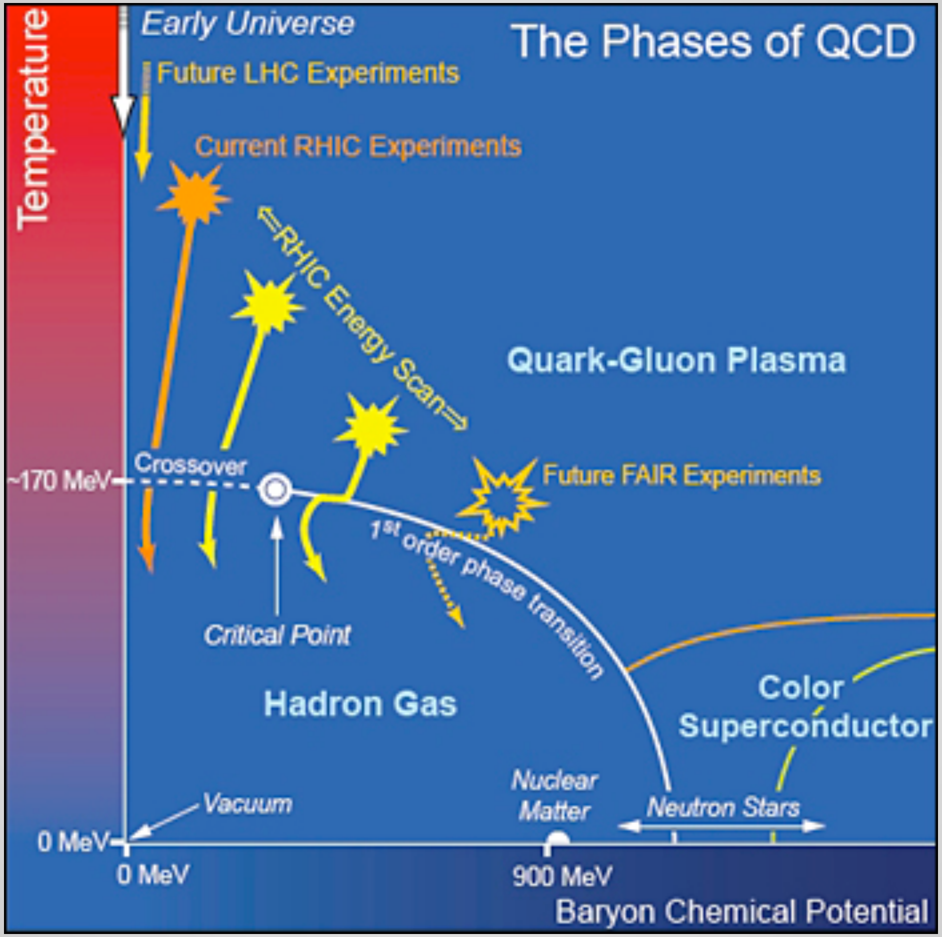


At μ=500MeV, for H≈1.7×10^{17} G



At μ=500MeV, for H≈3×10^{18} G

QCD Phase Diagram



Sophie Bushwick, News, July 22, 2010

The Neutral Inhomogeneous Condensate at B≠0

$$\mathcal{L}^{(2f)} = \bar{\psi}\left(i\gamma^\mu D_\mu + \mu\gamma^0 - m_q\right)\psi + \bar{\psi}_e\left(i\gamma^\mu D_\mu^{(e)} - m_e\right)\psi_e + \mathcal{L}_{int}$$

$$D_\mu = \partial_\mu + iQA_\mu^{ext}$$

$$Q = \mathrm{diag}(e_u, e_d) = \mathrm{diag}(\tfrac{2}{3}e, -\tfrac{1}{3}e)$$

$$\mathcal{L}_{int} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_V$$

$$\mathcal{L}_1 = G_1\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2 + (\bar{\psi}\tau^a\psi)^2 + (\bar{\psi}i\gamma^5\tau^a\psi)^2\right],$$

$$\mathcal{L}_2 = G_2\left[(\bar{\psi}\psi)^2 - (\bar{\psi}i\gamma^5\psi)^2 - (\bar{\psi}\tau^a\psi)^2 + (\bar{\psi}i\gamma^5\tau^a\psi)^2\right],$$

$$\mathcal{L}_V = -G_V\left[(\bar{\psi}\gamma_\mu\psi)^2 + (\bar{\psi}\gamma^5\gamma_\mu\psi)^2 + (\bar{\psi}\gamma_\mu\tau^a\psi)^2 + (\bar{\psi}\gamma^5\gamma_\mu\tau^a\psi)^2\right]$$

Condensates

$$-4G_S\langle\bar{\psi}_u\psi_u\rangle = \Delta_u\cos(q_u z)\,, \quad -4G_S\langle\bar{\psi}_u i\gamma_5\psi_u\rangle = \Delta_u\sin(q_u z)\,,$$
$$-4G_S\langle\bar{\psi}_d\psi_d\rangle = \Delta_d\cos(q_d z)\,, \quad -4G_S\langle\bar{\psi}_d i\gamma_5\psi_d\rangle = \Delta_d\sin(q_d z)\,.$$

$$\langle\bar{\psi}_u\gamma_0\psi_u\rangle = \rho_u, \quad \langle\bar{\psi}_d\gamma_0\psi_d\rangle = \rho_d$$

Thermodynamic Potential

$$\Omega^{(2f)} = \Omega_e + N_c \sum_{f=u,d} \Omega_f + \sum_{f=u,d} \left[\frac{\Delta_f^2}{8G_S} - \frac{(\widetilde{\mu}_f - \mu_f)^2}{8G_V} \right]$$

$$\widetilde{\mu}_u = \mu_u - 4G_V \rho_u \,, \qquad \widetilde{\mu}_d = \mu_d - 4G_V \rho_d$$

$$\mu_u = \mu - \frac{2}{3}\mu_e \,, \qquad \mu_d = \mu + \frac{1}{3}\mu_e$$

$$b_f = \frac{q_f}{2}$$

$$\Omega_f = \Omega_f^{vac} + \Omega_f^{med} \,,$$

$$\Omega_f^{vac} = \frac{1}{4\sqrt{\pi}} \frac{|e_f H|}{(2\pi)^2} \int_{-\infty}^{\infty} dp_3 \int_{1/\Lambda^2}^{\infty} \frac{ds}{s^{3/2}} \left(\sum_{\epsilon} e^{-sE_{f,0}^2} + \sum_{n>0,\zeta,\epsilon} e^{-sE_{f,n}^2} \right)$$

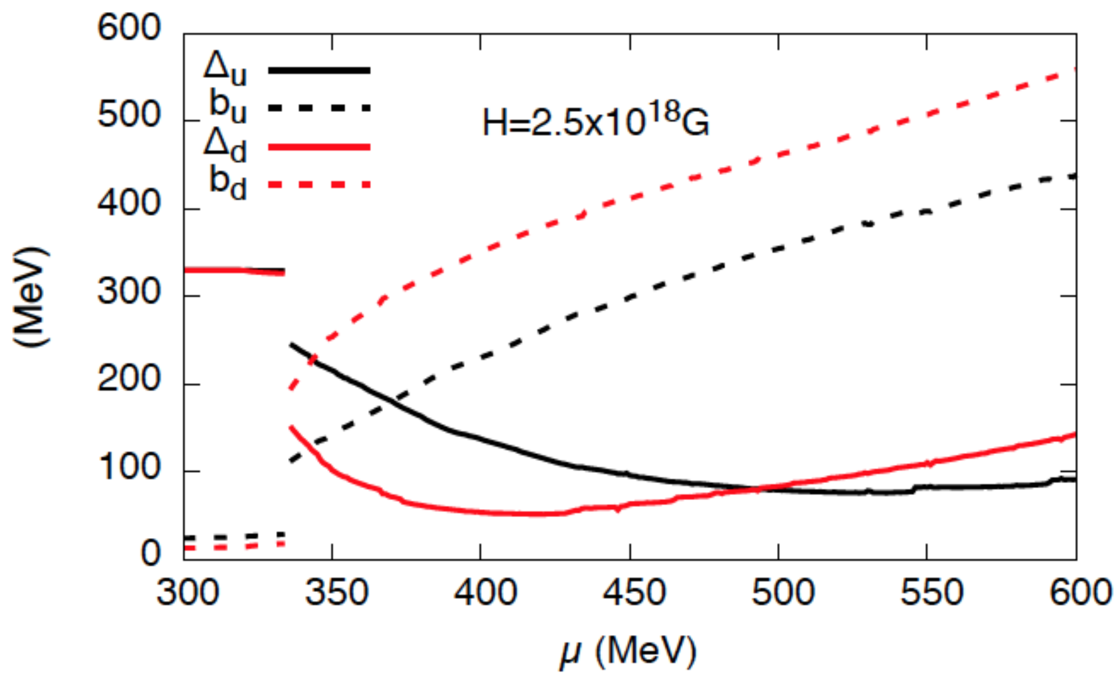
$$\Omega_f^{med} = -\frac{|e_f H|}{2\pi^2} \widetilde{\mu}_f b_f - \frac{|e_f H|}{8\pi^2} \int_{-\infty}^{\infty} dp_3 \sum_{\epsilon} (|E_{f,0} - \widetilde{\mu}_f| - |E_{f,0}|)|_{reg}$$

$$- \frac{|e_f H|}{4\pi^2} \int_{-\infty}^{\infty} dp_3 \sum_{n>0,\zeta} (\widetilde{\mu}_f - E_{f,n}) \Theta(\widetilde{\mu}_f - E_{f,n})|_{\epsilon=1} ,$$

$$E_{f,0} = \epsilon \sqrt{\Delta_f^2 + p_3^2} + b_f \,, \qquad \epsilon = \pm \,, \, n = 0 \,,$$

$$E_{f,n} = \epsilon \sqrt{\left(\zeta \sqrt{\Delta_f^2 + p_3^2} + b_f \right)^2 + 2|e_f H| n} \,, \quad \epsilon = \pm \,, \, \zeta = \pm \,, \, n > 0$$

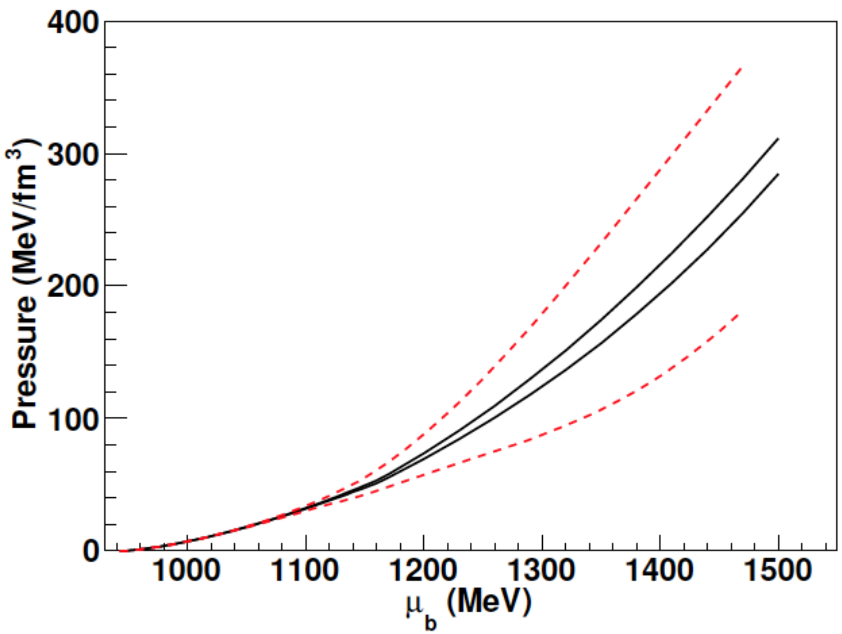
Gap Equations and Neutrality Condition



$$\frac{\partial \Omega^{(2f)}}{\partial \Delta_f} = 0, \qquad \frac{\partial \Omega^{(2f)}}{\partial b_f} = 0, \qquad \frac{\partial \Omega^{(2f)}}{\partial \tilde{\mu}_f} = 0\,, \quad f \in \{u, d\}\,,$$

$$\frac{\partial \Omega^{(2f)}}{\partial \mu_e} = 0$$

Pressure Splitting and Maxwell Construction



Parallel (lower curves), perpendicular (upper curves). Central magnetic field of 2.5×10^{18} G (solid lines) and 6.8×10^{18} G (dashed lines).

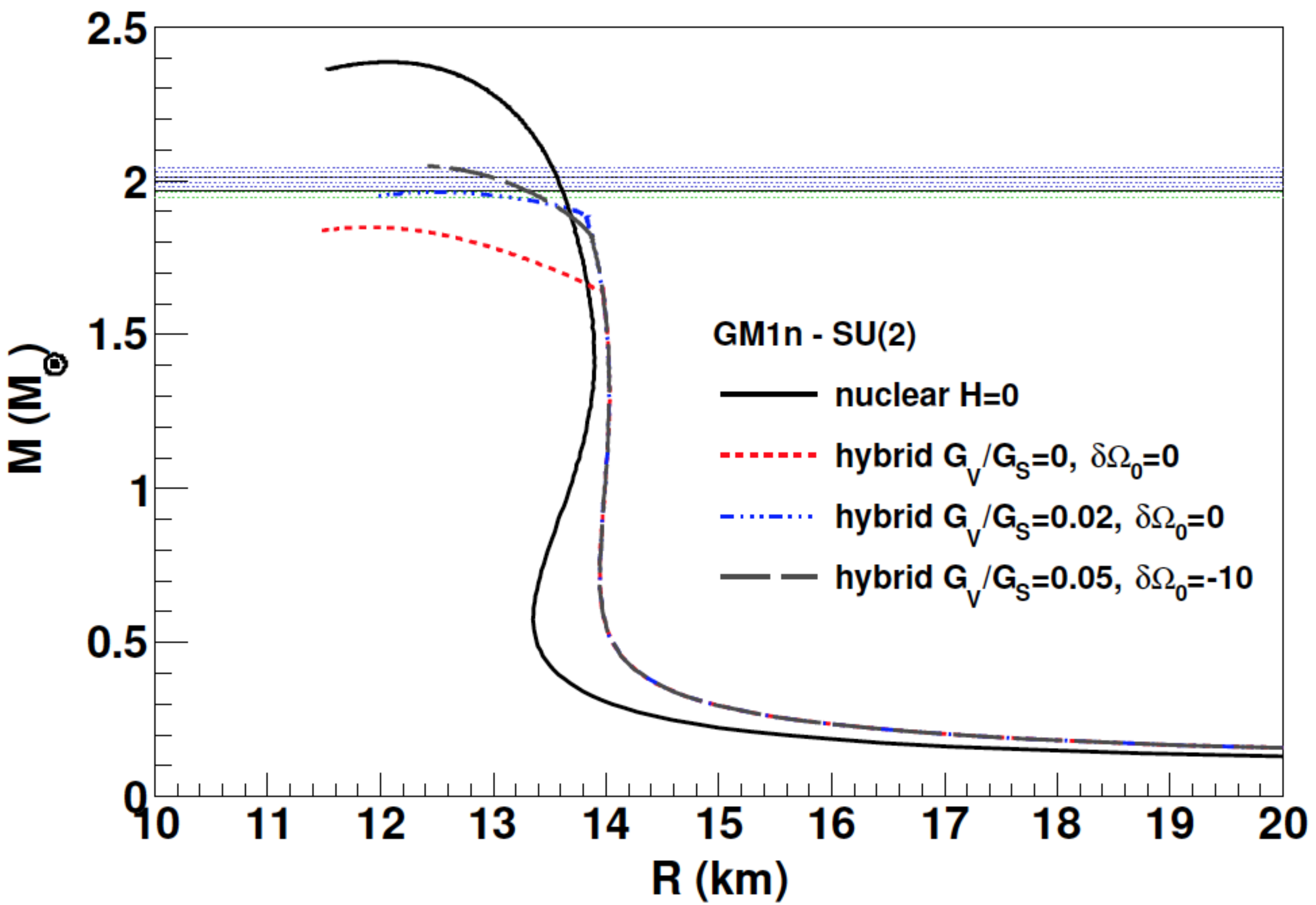
$$P^{\parallel} = -\Omega - \frac{H^2}{2},$$

$$P^{\perp} = -\Omega - H\mathcal{M} + \frac{H^2}{2},$$

$$\varepsilon = \Omega + \mu\rho + \frac{H^2}{2},$$

$$\Omega_{\text{nuclear}}(\mu_{tr}) = \Omega_{\text{quark}}(\mu_{tr})$$

Mass-Radius Relationship



Conclusions

- *In the strong coupling region the EoS of quark matter becomes softer due to the BCS-BEC crossover.*
- *Gluons in the CFL phase of color-superconductivity decrease the stellar maximum mass in 20%. Only for a sufficiently high coupling G_V the value of 2 solar masses can be reached.*
- *In the magnetic DCDW phase the 2 solar mass value can be only reached is the coupling G_V is sufficiently high.*