Anomalous Transport Properties of Dense QCD in a Magnetic Field*

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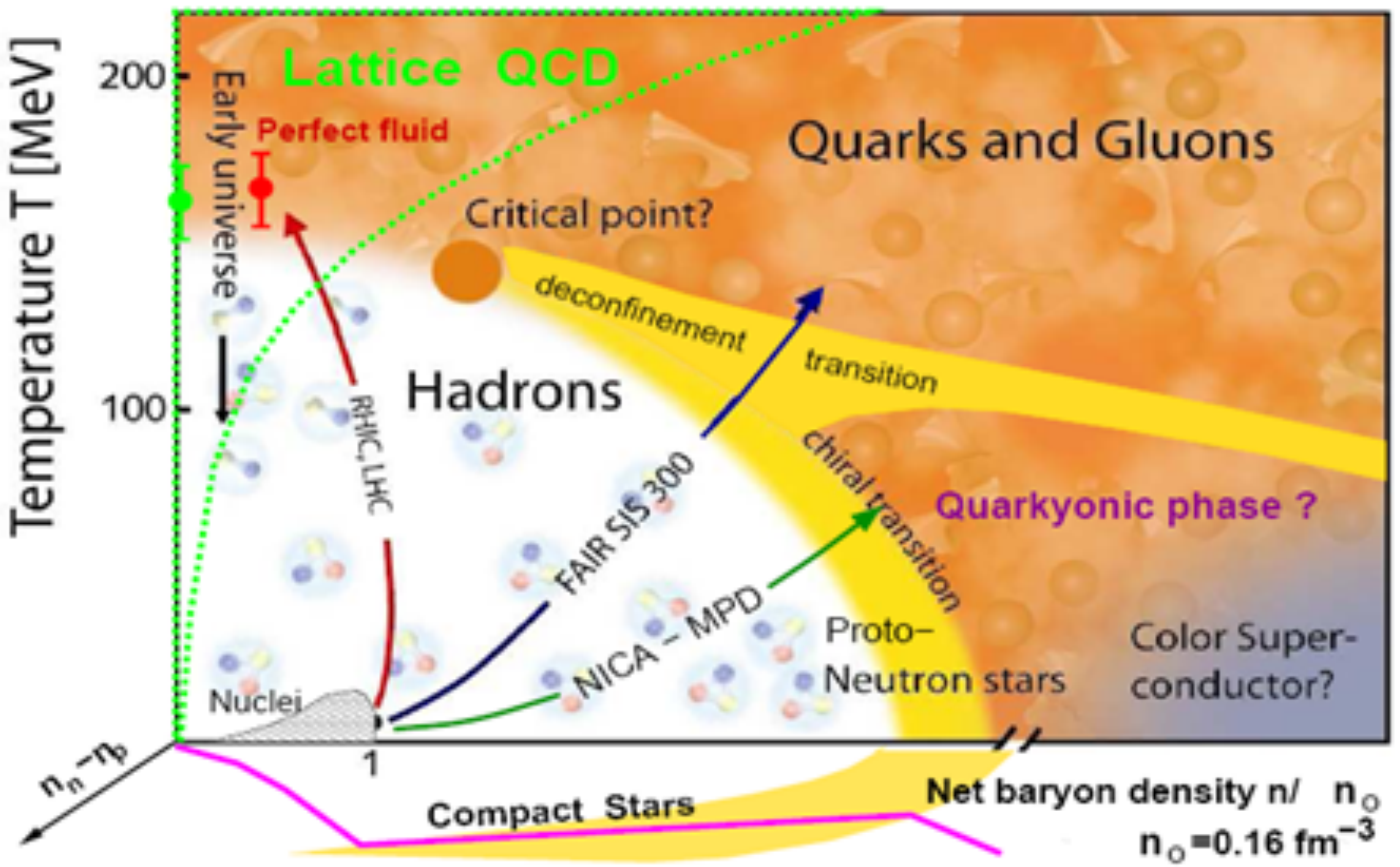
Outline

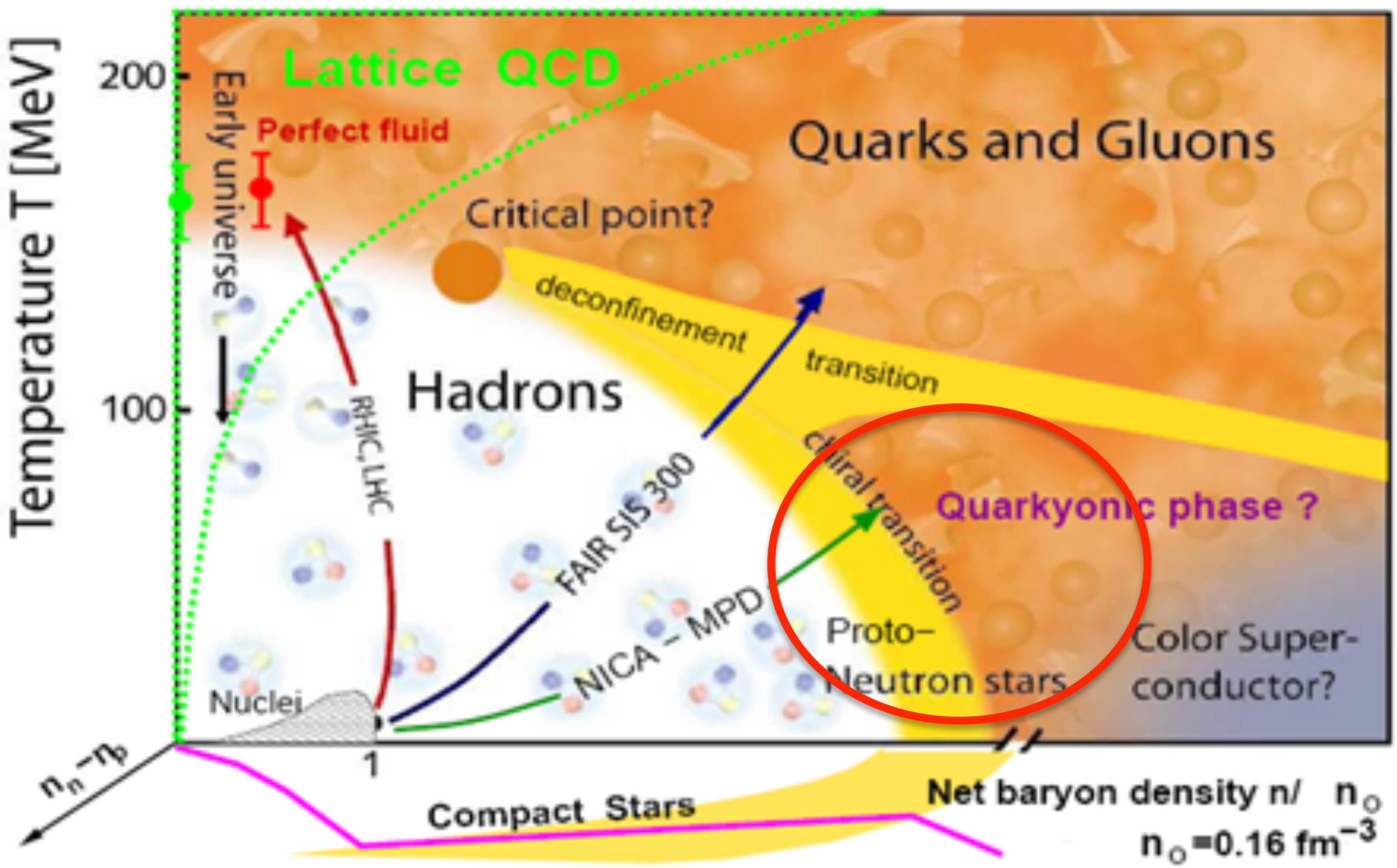
- Motivation
- DCDW in a Magnetic Field: spectral asymmetry, anomalous quark number density
- Axion electrodynamics in QCD at high density in B
- Anomalous electric charge and dissipationless Hall current
- Axion Polariton
- Role of B at finite density: no B, no topology
- Topology and IR physics
- Outlook

Based on arXiv:1512.02972, in collaboration with E.J. Ferrer

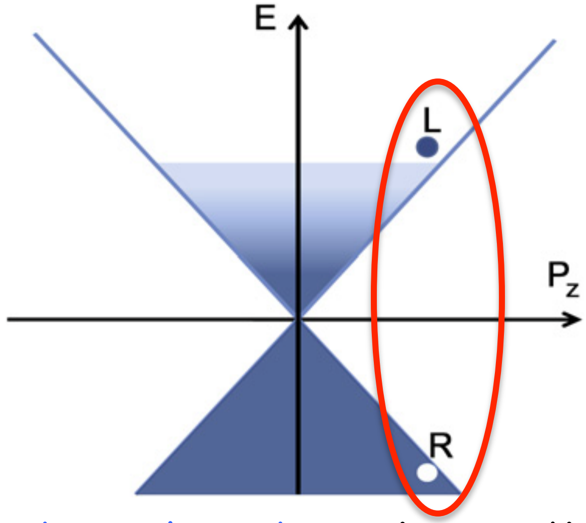
QCD Map & Planned Experiments



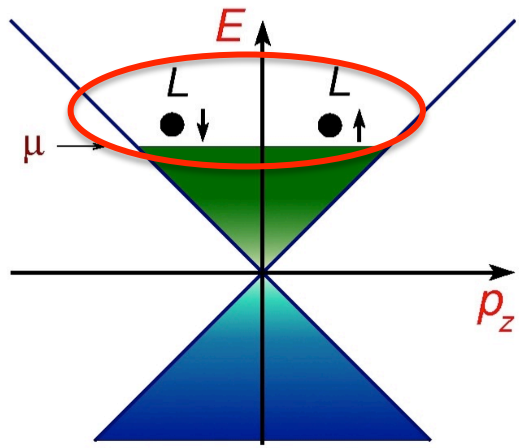
Getting Ready for Exploring the Denser Region



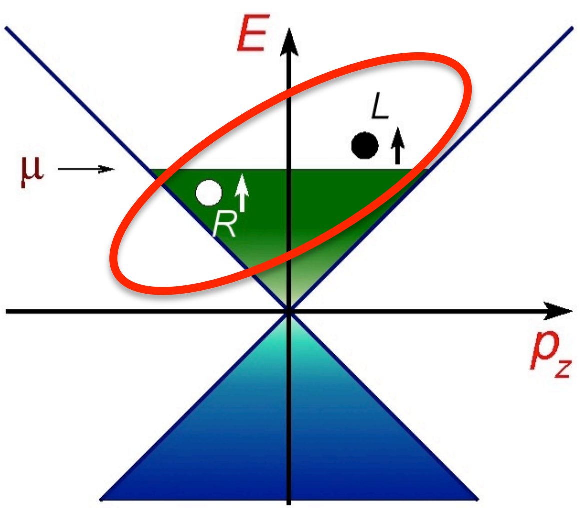
What to Expect at Finite Density?



Chiral Condensate: pairs particle and antiparticle with opposite momentum (homogeneous condensate).

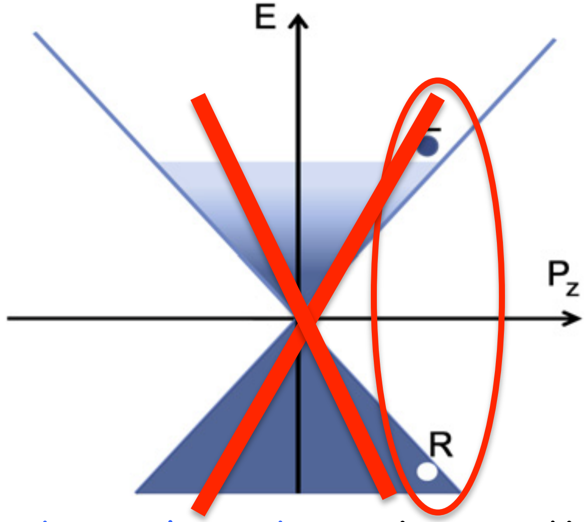


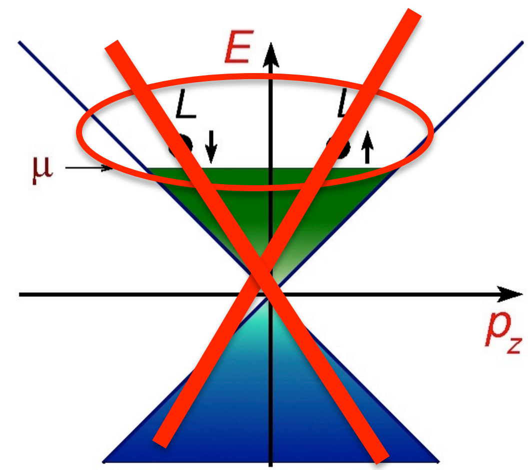
Cooper Pairing: pairs two quarks with opposite spins and momenta.



Density wave pairing: pairs particle and hole with parallel momenta (nonzero net momentum)

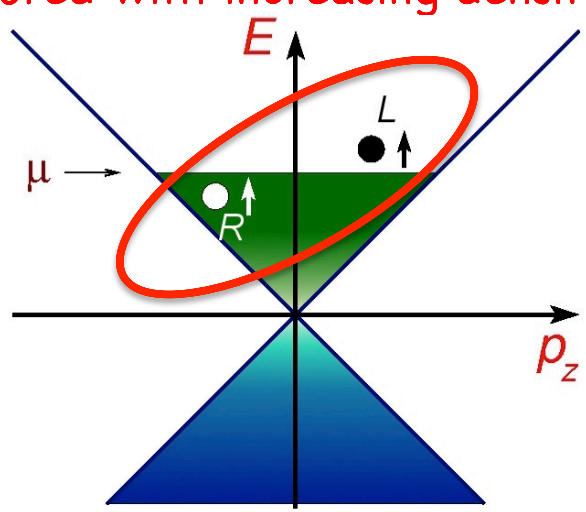
What to Expect at Finite Density?





Chiral Condensate: pairs particle and antiparticle with opposite momentum (homogeneous condensate). Not favored with increasing density.

Cooper Pairing: pairs two quarks with opposite spins and momenta. Favored at very high densities, but suffers from Fermi surface mismatch at intermediate densities and chromomagnetic instabilities.



Density wave pairing: pairs particle and hole with parallel momenta (nonzero net momentum). No Fermi surface mismatch. Favored over homogeneous chiral condensate.

Favored over CS at large Nc.

Dual Chiral Density Wave

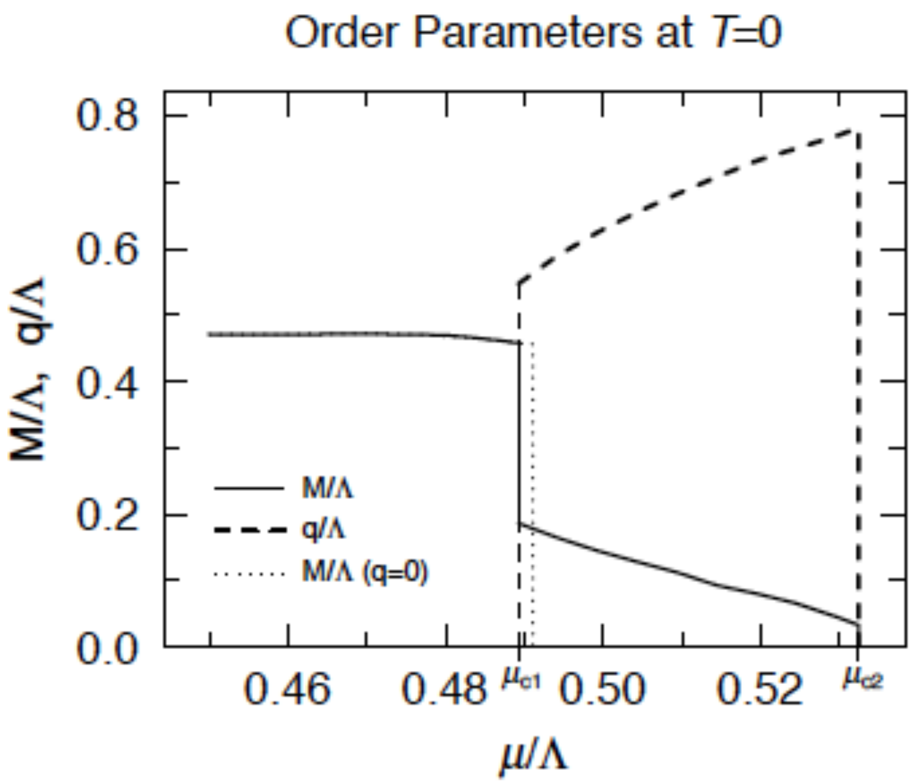
Nakano & Tatsumi, PRD71, 2005

NJL model at finite baryon density

$$\mathcal{L} = \bar{\psi}\left(\gamma^\mu(i\partial_\mu + \mu\delta_{\mu 0})\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\tau\gamma_5\psi)^2\right]\right.$$

DCDW condensate:

$$\begin{aligned}\langle\bar{\psi}\psi\rangle &= \Delta\cos(\mathbf{q}\cdot\mathbf{r})\\ \langle\bar{\psi}i\gamma_5\tau_3\psi\rangle &= \Delta\sin(\mathbf{q}\cdot\mathbf{r}),\end{aligned}$$



DCDW is favored at intermediate densities over the chirally restored ground state, but not favored over Nickel's real kink solution.

DCDW in a Magnetic Field

Frolov, Zhukovsky, & Klimenko PRD82, 2010
Tatsumi, Nishiyama, & Karasawa, PLB 743, 2015

MF Lagrangian

$$\mathcal{L}_{MF} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^\mu(\partial_\mu - i\mu\delta_{\mu 0} + iQA_\mu)]\psi - m\bar{\psi}e^{i\tau_3\gamma_5 qz}\psi - \frac{m^2}{4G},$$

Using the local chiral transformation

$$\psi \to \psi' = U_A\psi = e^{-i\tau_3\gamma_5\frac{qz}{2}}\psi$$

$$\bar{\psi} \to \bar{\psi}' = \bar{\psi}\bar{U}_A = \bar{\psi}e^{-i\tau_3\gamma_5\frac{qz}{2}}$$

$$\mathcal{L}_{MF} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^\mu(\partial_\mu - i\mu\delta_{\mu 0} + iQA_\mu - i\tau_3\gamma_5\delta_{\mu 3}\frac{q}{2}) - m]\psi - \frac{m^2}{4G}$$

$$\lambda_{n=0,p,\epsilon} = \epsilon\sqrt{m^2 + k_3^2} + q/2$$

Asymmetric LLL Spectrum

$$\lambda_{n,p,\zeta,\epsilon} = \epsilon\sqrt{\left(\zeta\sqrt{m^2 + k_3^2} + q/2\right)^2 + 2eBn}, n = 1, 2, ...,$$

Anomalous Quark Number Density

Nontrivial topology appears due to the spectral asymmetry of the LLL

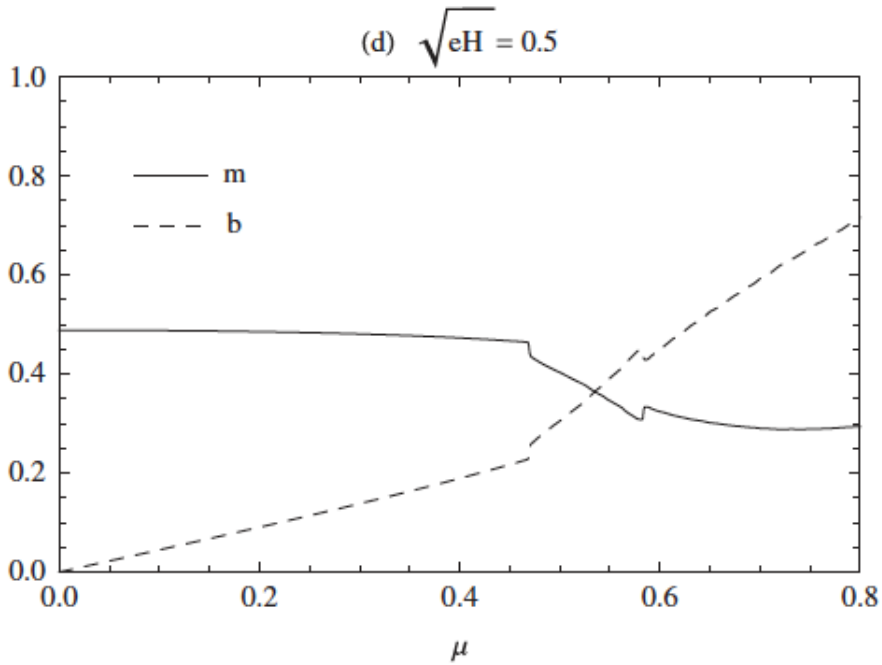
Tatsumi, Nishiyama, & Karasawa, PLB 743, 2015

$$\eta_H = \lim_{s\to 0} \sum_k sgn(\lambda_k)|\lambda_k|^{-s}$$

$$\lambda_{n=0,p,\epsilon} = \epsilon\sqrt{m^2 + k_3^2} + q/2$$

Anomalous quark number density

$$\rho_B^{anom} = -\frac{\partial \Omega_{anom}}{\partial \mu} = N_c \sum_f \frac{|e_f|}{4\pi^2} \vec{\nabla}\theta \cdot \vec{B} = 3\frac{|e|}{4\pi^2} qB$$



The DCDW solution is favored due to the anomaly

Frolov, Zhukovsky, & Klimenko PRD82, 2010

Axion Term in the DCDW in B Theory

Ferrer & VI, 1512.03972 [nucl-th]

However, the fermion measure is not invariant under the chiral transformation U_A

$$D\bar{\psi}D\psi \to (\det U_A)^{-2} D\bar{\psi}D\psi \qquad (\det U_A)^{-2} = e^{i\int d^4x \frac{\kappa}{4}\theta F_{\mu\nu}\tilde{F}^{\mu\nu}}$$

Using Fujikawa's method one can show that the change in the measure leads to an axion term in $\mathcal{L}^{eff}_{MF}$

$$Z = \int DAD\bar{\psi}D\psi e^{i\int \mathcal{L}^{eff}_{MF} d^4x}$$

$$\mathcal{L}^{eff}_{MF} = \bar{\psi}[i\gamma^\mu(\partial_\mu - i\mu\delta_{\mu 0} + iQA_\mu - i\tau_3\gamma_5\delta_{\mu 3}\partial_\mu\theta) - m]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{4}\theta F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{m^2}{4G}$$

$\theta = qz/2.$ Axion field

$$\kappa = \frac{N_c}{2\pi^2}[e_u^2 - e_d^2] = \frac{e^2}{2\pi^2}$$

Integrating in the fermion fields and expanding in A, we obtain the effective action for the electromagnetic field:

$$\Gamma(A) = \int d^4x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu}\right] + \Gamma_{matt}(A)$$

$$\Gamma_{matt}(A) = V\Omega - \int d^4x A_\mu(x) J^\mu(x) + \frac{1}{2}\int d^4x d^4x' A_\mu(x)\Pi^{\mu\nu}(x,x')A_\nu(x') + \cdots$$

With fermion four-current $J_\mu = J^u_{mu} + J^d_{mu}$

$$J^f_\mu = (ie_f)\frac{|e_f B| N_c}{(2\pi)^2\beta}\sum_l \sum_{p_4}\int_{-\infty}^{\infty} dp_3 tr\left[\gamma^\mu i G_l(p)\right]$$

$G_l(p)$ the quark Green function of Landau level l. For the LLL

$$G_0^{(sgn[e_f B])}(p) = \frac{\gamma^\mu_{\parallel}\widetilde{p}^{\parallel}_\mu + m}{(\widetilde{p}_0 + b)^2 - \epsilon^2}\Delta(sgn[e_f B])$$

The electric charge density can be found as

$$-\frac{\partial \Gamma(A)}{\partial A_0} = \kappa \nabla \theta \cdot \mathbf{B} + J_0$$

Ferrer & VI, 1512.03972 [nucl-th]

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It has an anomalous electric charge density

$$J_0^{anom} = \kappa\nabla\theta\cdot\mathbf{B} = \frac{e^2}{4\pi^2}qB$$

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And the ordinary electric charge density coming from the tadpole diagrams, whose LLL contribution is

$$J_0^{LLL} = \Theta(|\mu-b|-m)\sum_{e_f}\frac{e_f|e_fB|N_c}{2\pi^2}\sqrt{(\mu-b)^2-m^2}$$

$$= \frac{e^2B}{2\pi^2}\sqrt{(\mu-b)^2-m^2}\,\Theta(|\mu-b|-m)$$

Ferrer & VI, 1512.03972 [nucl-th]

LLL Origin of the Anomalous Electric Charge

Spectral asymmetry → anomalous baryon charge density

$$\rho_B^{anom} = -\frac{\partial \Omega_{anom}}{\partial \mu} = N_c \sum_f \frac{|e_f|}{4\pi^2} \vec{\nabla}\theta \cdot \vec{B} = 3\frac{|e|}{4\pi^2} qB$$

↓

Anomalous electric charge density

$$J_0^{anom} = N_c \sum_f \frac{e_f |e_f|}{4\pi^2} \mathbf{B} \cdot \nabla(\mathbf{q} \cdot \mathbf{x}) = \frac{e^2}{4\pi^2} qB$$

Axion Electrodynamics in DCDW in B

Ferrer & VI, 1512.03972 [nucl-th]

Anomalous E.C. due to the spectral asymmetry

$$\nabla\cdot\mathbf{E} = J_0 + \kappa\nabla\theta\cdot\mathbf{B},$$

$$\nabla\times\mathbf{B} - \frac{\partial\mathbf{E}}{\partial t} = \mathbf{J}_V + \kappa(\frac{\partial\theta}{dt}\mathbf{B} + \nabla\theta\times\mathbf{E}),$$

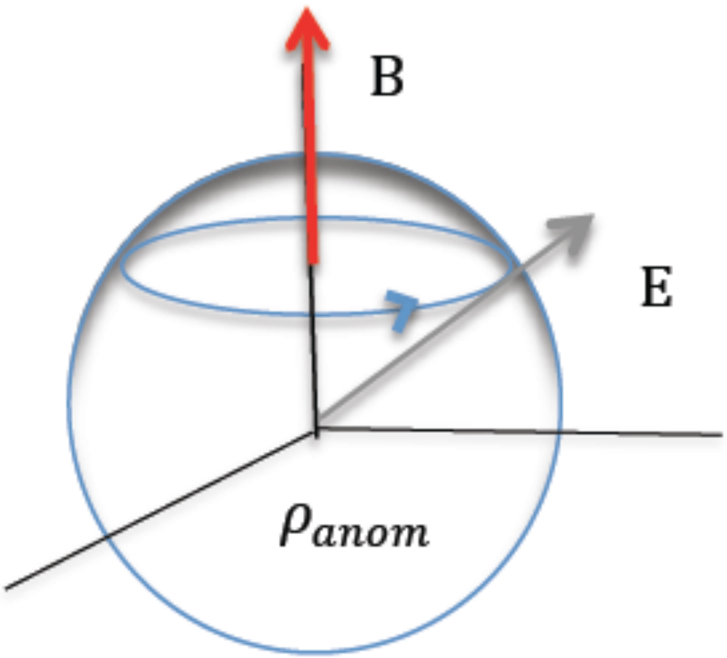
$$\nabla\cdot\mathbf{B} = 0, \quad \nabla\times\mathbf{E} + \frac{\partial\mathbf{B}}{\partial t} = 0,$$

Anomalous Hall current: dissipationless, ⊥ to both B and E

$$\theta = qz/2.$$

$$\kappa = \frac{e^2}{2\pi^2}$$

Anomalous Transport: Dissipantionless Hall Current



The Hall current is perpendicular to E and B. Could be present at the boundary between the quark core & the crust of a NS in the globally neutral case.

- How to connect it to observables in NS?
- Observable effects in future high baryon HIC experiments?

Axion Polariton in Magnetic Topological Insulators

Li, Wang, Qi & Zhang, Nature Physics 2010

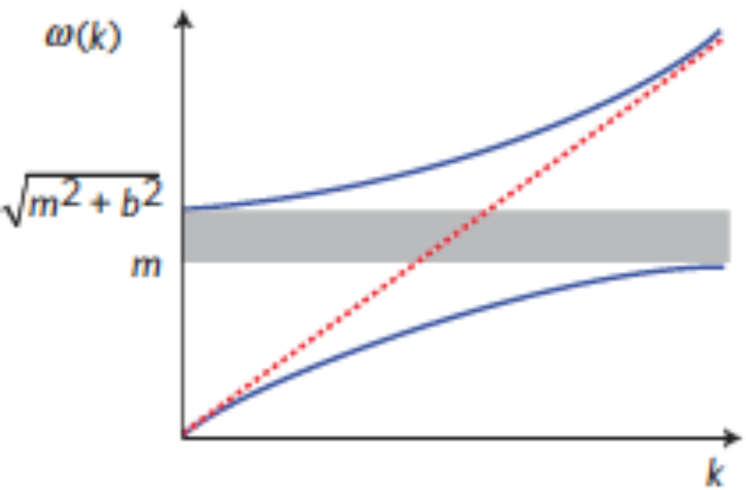
Happens in TI with long-range antiferromagnetic order where the fluctuation of the axion field couples linearly to E:

$$\frac{\alpha}{4\pi^2}\delta\theta E\cdot B_0$$

$$\frac{\partial^2}{\partial t^2}\mathbf{E}-c'^2\nabla^2\mathbf{E}+\frac{\alpha \mathbf{B}_0}{\pi\epsilon}\frac{\partial^2}{\partial t^2}\delta\theta=0$$

$$\frac{\partial^2}{\partial t^2}\delta\theta-v^2\nabla^2\delta\theta+m_0^2\delta\theta-\frac{\alpha \mathbf{B}_0}{8\pi^2 g^2 J}\mathbf{E}=0$$

Axion Polariton: coupled mode of light and axion field



$$\omega_\pm^2(k)=\frac{1}{2}\Big[(c'^2k^2+m^2+b^2)\pm\sqrt{(c'^2k^2+m^2+b^2)^2-4c'^2k^2m^2}\Big]$$

$$b^2=\alpha^2 \mathrm{B}_0{}^2/8\pi^3\epsilon g^2 J$$

Same linear coupling of the axion field to E happens in the DCDW quark system in a magnetic field!

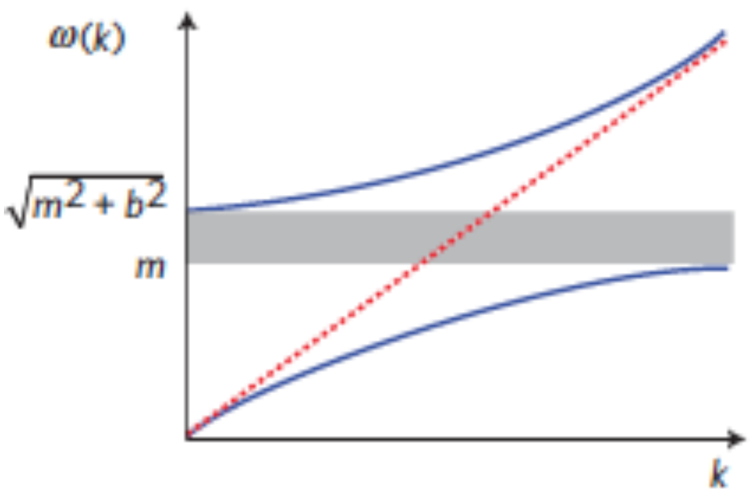
Axion Polariton in DCDW in B

Ferrer & VI, 1512.03972 [nucl-th]

A linear coupling of the axion fluctuation field to E also happens in the DCDW quark system in a magnetic field: $\frac{\kappa}{4\pi}\delta\theta E \cdot B_0$

$$\frac{\partial^2}{\partial t^2}\mathbf{E} - c'^2\nabla^2\mathbf{E} + \frac{\kappa B_0}{\epsilon}\frac{\partial^2}{\partial t^2}\delta\theta = 0$$

$$\frac{\partial^2}{\partial t^2}\delta\theta - v^2\nabla^2\delta\theta + m_0^2\delta\theta - \frac{\kappa B_0}{8\pi}\cdot\mathbf{E} = 0$$

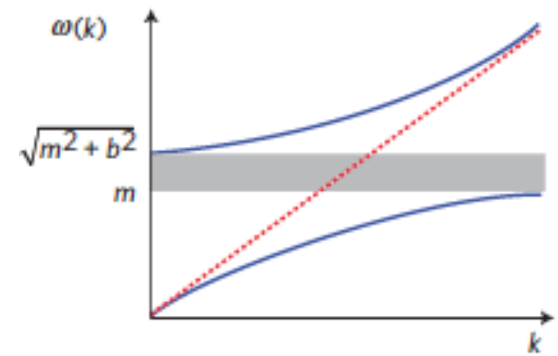


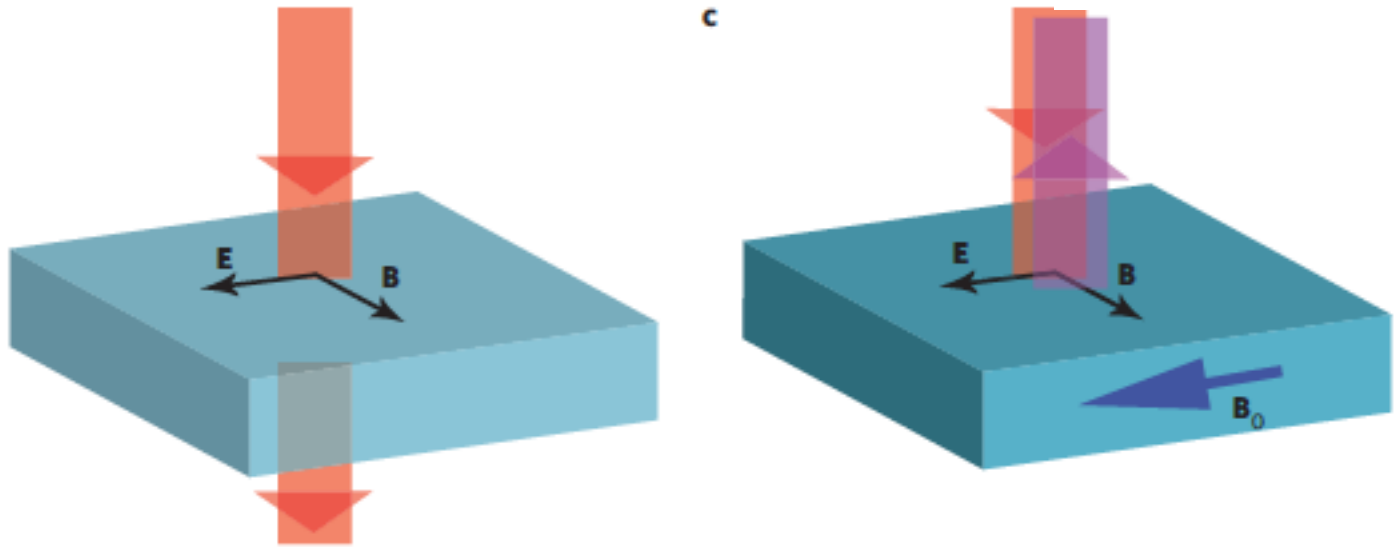
$$\omega_\pm^2(k) = \frac{1}{2}\Big[(c'^2k^2 + m^2 + b^2) \pm \sqrt{(c'^2k^2 + m^2 + b^2)^2 - 4c'^2k^2m^2}\Big]$$

$$b^2 = \frac{\kappa^2 B_0^2}{8\pi\epsilon}$$

Wave Attenuation

If we shine linearly polarized light with $E \| B_0$ and frequency within the gap of the axion polariton, there will be a noticeable wave attenuation





The gap depends on B, so the attenuated frequencies will too. Potentially observable effects in HIC and NS

Magnetoelectricity

$$\nabla \cdot \mathbf{D} = J_0 \quad \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}_V$$

$$\nabla \cdot \mathbf{B} = 0 \quad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

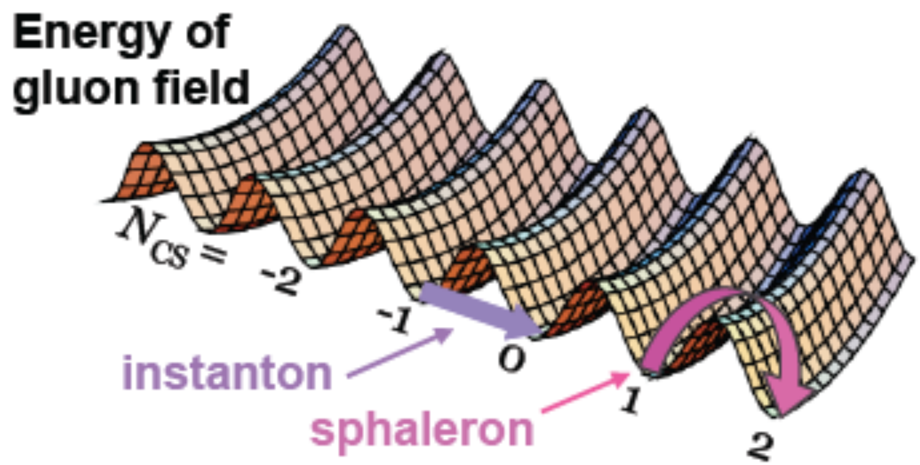
$$\mathbf{H} = \mathbf{B} - \kappa\theta\mathbf{E} \qquad \mathbf{D} = \mathbf{E} + \kappa\theta\mathbf{B}$$

Anomalous magnetization

Anomalous polarization

Different Roles of B at High T and High Density

At high T: B is a probe for the CME, but the topology is produced by the sphaleron transitions



$$\lambda_{n=0,p,\epsilon} = \epsilon\sqrt{m^2 + k_3^2} + q/2$$

$$L_{ax} \sim qzF^{\mu\nu}\tilde{F}_{\mu\nu}$$

At high density: B is needed together with the DCDW, to produce the LLL spectral asymmetry and hence the nontrivial topology

Topology Influences IR Physics!

Topology comes from spectral asymmetry of the LLL. It is an UV phenomenon

$$\eta_H = \lim_{s \to 0} \sum_k sgn(\lambda_k) |\lambda_k|^{-s}$$

$$Q_B^{anom} = \frac{e^2}{4\pi^2} qB$$

The topology affects the gap equation through the anomalous term, so it influences the value of q, which comes from the pairing dynamics, hence IR

$$\frac{\partial \Omega}{\partial q} = \boxed{\frac{\partial \Omega_{anom}}{\partial q}} + \frac{\partial \Omega_{vac}}{\partial q} + \frac{\partial \Omega_{med}}{\partial q} = 0$$

Outlook

- We should exploit the analogy with TI and other topological materials to design potential countertop experiments to get insight of the physics of neutron stars and HIC in a magnetic field.
- Quantitative studies are needed to identify measurable observables that will allow to probe the presence of the DCDW phase at high baryon density at NICA.