Many-body forces in magnetic neutron stars

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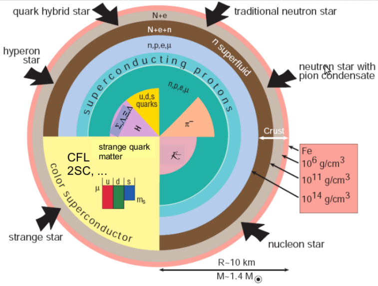
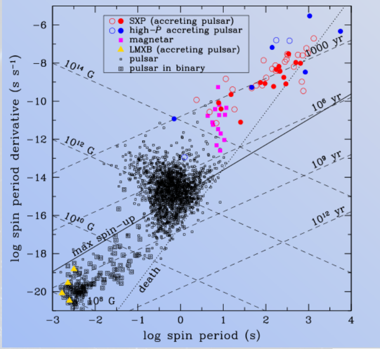
CSQCD V - L'aquila, Italy (2016)

Motivation



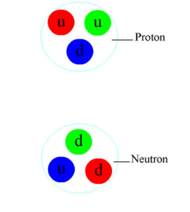
Nuclear EoS



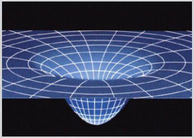
Magnetic NS

Modeling neutron stars

- matter inside the star: *MBF formalism* $\rightarrow$ **equation of state (EoS)**;



- stellar structure: *Einstein equations* $\rightarrow$ **hydrostatic equilibrium (TOV/LORENE).**



Many-body forces formalism

MBF formalism

$$
\begin{aligned}
\mathcal{L} = & \sum_b \overline{\psi}_b \left[\gamma_\mu \left(i\partial^\mu - g^*_{\omega b\xi}\omega^\mu - g^*_{\phi b\kappa}\phi^\mu - \frac{1}{2} g^*_{\varrho b\eta} \boldsymbol{\tau}.\boldsymbol{\varrho}^{\boldsymbol{\mu}} \right) - m^*_{b\zeta} \right] \psi_b \\
& + \left(\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - m^2_\sigma\sigma^2 \right) + \left(\frac{1}{2}\partial_\mu\sigma^*\partial^\mu\sigma^* - m^2_{\sigma^*}\sigma^{*2} \right) \\
& + \frac{1}{2}\left(-\frac{1}{2}\omega_{\mu\nu}\omega^{\mu\nu} + m^2_\omega\omega_\mu\omega^\mu \right) + \frac{1}{2}\left(-\frac{1}{2}\phi_{\mu\nu}\phi^{\mu\nu} + m^2_\phi\phi_\mu\phi^\mu \right) \\
& + \frac{1}{2}\left(-\frac{1}{2}\boldsymbol{\varrho}_{\boldsymbol{\mu\nu}}.\boldsymbol{\varrho}^{\boldsymbol{\mu\nu}} + m^2_\varrho\boldsymbol{\varrho}_{\boldsymbol{\mu}}.\boldsymbol{\varrho}^{\boldsymbol{\mu}} \right) + \left(\frac{1}{2}\partial_\mu\boldsymbol{\delta}.\partial^\mu\boldsymbol{\delta} - m^2_\delta\boldsymbol{\delta}^2 \right) \\
& + \sum_l \overline{\psi}_l\gamma_\mu \left(i\partial^\mu - m_l \right) \psi_l .
\end{aligned}
$$

R.O. Gomes, V. Dexheimer, S. Schramm, C.A.Z. Vasconcellos, The Astrophysical Journal 808 (2015) 1

MBF formalism

$$\mathcal{L} = \sum_b \overline{\psi}_b \left[\gamma_\mu \left(i\partial^\mu - g^*_{\omega b\xi}\omega^\mu - g^*_{\phi b\kappa}\phi^\mu - \frac{1}{2}g^*_{\varrho b\eta}\boldsymbol{\tau}.\boldsymbol{\varrho}^\mu\right) - m^*_{b\zeta}\right]\psi_b$$
$$+ \left(\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - m^2_\sigma\sigma^2\right) + \left(\frac{1}{2}\partial_\mu\sigma^*\partial^\mu\sigma^* - m^2_{\sigma^*}\sigma^{*2}\right)$$
$$+ \frac{1}{2}\left(-\frac{1}{2}\omega_{\mu\nu}\omega^{\mu\nu} + m^2_\omega\omega_\mu\omega^\mu\right) + \frac{1}{2}\left(-\frac{1}{2}\phi_{\mu\nu}\phi^{\mu\nu} + m^2_\phi\phi_\mu\phi^\mu\right)$$
$$+ \frac{1}{2}\left(-\frac{1}{2}\boldsymbol{\varrho}_{\mu\nu}.\boldsymbol{\varrho}^{\mu\nu} + m^2_\varrho\boldsymbol{\varrho}_\mu.\boldsymbol{\varrho}^\mu\right) + \left(\frac{1}{2}\partial_\mu\boldsymbol{\delta}.\partial^\mu\boldsymbol{\delta} - m^2_\delta\boldsymbol{\delta}^2\right)$$
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$$+ \left(\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - m^2_\sigma\sigma^2\right) + \left(\frac{1}{2}\partial_\mu\sigma^*\partial^\mu\sigma^* - m^2_{\sigma^*}\sigma^{*2}\right)$$

$$+ \frac{1}{2}\left(-\frac{1}{2}\omega_{\mu\nu}\omega^{\mu\nu} + m^2_\omega\omega_\mu\omega^\mu\right) + \frac{1}{2}\left(-\frac{1}{2}\phi_{\mu\nu}\phi^{\mu\nu} + m^2_\phi\phi_\mu\phi^\mu\right)$$

$$+ \frac{1}{2}\left(-\frac{1}{2}\boldsymbol{\varrho}_{\mu\nu}.\boldsymbol{\varrho}^{\mu\nu} + m^2_\varrho\boldsymbol{\varrho}_\mu.\boldsymbol{\varrho}^\mu\right) + \left(\frac{1}{2}\partial_\mu\boldsymbol{\delta}.\partial^\mu\boldsymbol{\delta} - m^2_\delta\boldsymbol{\delta}^2\right)$$

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& + \sum_l \overline{\psi}_l \gamma_\mu \left(i\partial^\mu - m_l \right) \psi_l .
\end{aligned}
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The Astrophysical Journal 808 (2015) 1

MBF formalism

$$\textbf{Coupling: } g^*_{ib\lambda} = m^\star_{\lambda b}\, g_{ib} = \left(1 + \frac{g_{\sigma b}\sigma + g_{\sigma^* b}\sigma^* + \frac{1}{2} g_{\delta b}\delta.\tau}{\lambda\, m_b}\right)^{-\lambda} g_{ib}$$

$i = \sigma, \omega, \varrho, \delta, \sigma^*, \phi$ (mesons) $\lambda = \zeta, \xi, \eta, \zeta, \zeta, \kappa$ (parameters)

- **Effective mass:** $m^*_b = m_b - m^\star_{\zeta b}(g_{\sigma b}\sigma + g_{\sigma^* b}\sigma^* + \frac{1}{2} g_{\delta b}\delta.\tau)$
- **Many-body forces contribution:**
- **Scalar version:**

$$g^*_{\sigma b} = m^\star_{\zeta b}\, g_{\sigma b}, \quad g^*_{\delta b} = m^\star_{\zeta b}\, g_{\delta b}, \quad g^*_{\sigma^* b} = m^\star_{\zeta b}\, g_{\sigma^* b}$$

$$g^*_{\omega b} = g_{\omega b}, \quad g^*_{\varrho b} = g_{\varrho b}, \quad g^*_{\phi b} = g_{\phi b}.$$

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- **Many-body forces contribution:**

$$m^\star_{\lambda b} = 1 - \left(\frac{g_{\sigma b}\sigma + g_{\sigma^* b}\sigma^* + \frac{1}{2} g_{\delta b}\delta \cdot \tau}{m_b}\right) + \frac{(\lambda+1)}{2!\lambda}\left(\frac{g_{\sigma b}\sigma + g_{\sigma^* b}\sigma^* + \frac{1}{2} g_{\delta b}\delta \cdot \tau}{m_b}\right)^2$$

$$-\frac{(\lambda^2 + 3\lambda + 2)}{3!\lambda^2}\left(\frac{g_{\sigma b}\sigma + g_{\sigma^* b}\sigma^* + \frac{1}{2} g_{\delta b}\delta \cdot \tau}{m_b}\right)^3 + \dots$$

- **Scalar version:**

$g^*_{\sigma b} = m^\star_{\zeta b}\, g_{\sigma b}$, $\quad g^*_{\delta b} = m^\star_{\zeta b}\, g_{\delta b}$, $\quad g^*_{\sigma^* b} = m^\star_{\zeta b}\, g_{\sigma^* b}$

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MBF formalism

Coupling: $g^{*}_{ib\lambda} = m^{\star}_{\lambda b}\, g_{ib} = \left(1 + \frac{g_{\sigma b}\sigma + g_{\sigma^* b}\sigma^* + \frac{1}{2} g_{\delta b}\delta.\tau}{\lambda\, m_b}\right)^{-\lambda} g_{ib}$

$i = \sigma,\ \omega,\ \varrho,\ \delta,\ \sigma^*,\ \phi$ (mesons) $\lambda = \zeta,\ \xi,\ \eta,\ \zeta,\ \zeta,\ \kappa$ (parameters)

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$$m^{\star}_{\lambda b} = 1 - \left(\frac{g_{\sigma b}\sigma + g_{\sigma^* b}\sigma^* + \frac{1}{2} g_{\delta b}\delta \cdot \tau}{m_b}\right) + \frac{(\lambda+1)}{2!\lambda}\left(\frac{g_{\sigma b}\sigma + g_{\sigma^* b}\sigma^* + \frac{1}{2} g_{\delta b}\delta \cdot \tau}{m_b}\right)^2 - \frac{(\lambda^2+3\lambda+2)}{3!\lambda^2}\left(\frac{g_{\sigma b}\sigma + g_{\sigma^* b}\sigma^* + \frac{1}{2} g_{\delta b}\delta \cdot \tau}{m_b}\right)^3 + \dots$$

- **Scalar version:**

$$g^{*}_{\sigma b} = m^{\star}_{\zeta b}\, g_{\sigma b}, \quad g^{*}_{\delta b} = m^{\star}_{\zeta b}\, g_{\delta b}, \quad g^{*}_{\sigma^* b} = m^{\star}_{\zeta b}\, g_{\sigma^* b}$$
$$g^{*}_{\omega b} = g_{\omega b}, \quad g^{*}_{\varrho b} = g_{\varrho b}, \quad g^{*}_{\phi b} = g_{\phi b}.$$

MBF formalism

$$\sigma_0 = \frac{1}{m_\sigma^2}\sum_b \Big[g_{\sigma b}\left(m^\star_{\zeta b}\right) - \frac{g_{\sigma b}}{m_b}\left(m^\star_{\zeta b}\right)^{\frac{\zeta+1}{\zeta}}\left(g_{\sigma b}\sigma_0 + g_{\delta b}\delta_0^3\, I^{3b} + g_{\sigma_0^* b}\sigma^*\right)\Big]\rho_{sb},$$

$$\delta_0^3 = \frac{1}{m_\delta^2}\sum_b \Big[g_{\delta b}\left(m^\star_{\zeta b}\right) - \frac{g_{\delta b}}{m_b}\left(m^\star_{\zeta b}\right)^{\frac{\zeta+1}{\zeta}}\left(g_{\sigma b}\sigma_0 + g_{\delta b}\delta_0^3\, I^{3b} + g_{\sigma * b}\sigma_0^*\right)\Big] I^{3b}\,\rho_{sb},$$

$$\sigma_0^* = \frac{1}{m_{\sigma *}^2}\sum_b \Big[g_{\sigma * b}\left(m^\star_{\zeta b}\right) - \frac{g_{\sigma * b}}{m_b}\left(m^\star_{\zeta b}\right)^{\frac{\zeta+1}{\zeta}}\left(g_{\sigma b}\sigma_0 + g_{\delta b}\delta_0^3\, I^{3b} + g_{\sigma * b}\sigma_0^*\right)\Big]\rho_{sb},$$

$$\omega_0 = \frac{1}{m_\omega^2}\sum_b g_{\omega b}\rho_b,$$

$$\varrho_0^3 = \frac{1}{m_\varrho^2}\sum_B g_{\varrho b}\, I^{3b}\,\rho_b,$$

$$\phi_0 = \frac{1}{m_\phi^2}\sum_b g_{\phi b}\rho_b.$$

MBF formalism

$$\sigma_0 = \frac{1}{m_\sigma^2}\sum_b \left[g_{\sigma b}\left(m^\star_{\zeta b}\right) - \frac{g_{\sigma b}}{m_b}\left(m^\star_{\zeta b}\right)^{\frac{\zeta+1}{\zeta}}\left(g_{\sigma b}\sigma_0 + g_{\delta b}\delta_0^3\, I^{3b} + g_{\sigma_0^* b}\sigma^*\right)\right]\rho_{sb},$$

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$$\sigma_0^* = \frac{1}{m_{\sigma *}^2}\sum_b \left[g_{\sigma * b}\left(m^\star_{\zeta b}\right) - \frac{g_{\sigma * b}}{m_b}\left(m^\star_{\zeta b}\right)^{\frac{\zeta+1}{\zeta}}\left(g_{\sigma b}\sigma_0 + g_{\delta b}\delta_0^3\, I^{3b} + g_{\sigma * b}\sigma_0^*\right)\right]\rho_{sb},$$

$$\omega_0 = \frac{1}{m_\omega^2}\sum_b g_{\omega b}\rho_b,$$

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$$\phi_0 = \frac{1}{m_\phi^2}\sum_b g_{\phi b}\rho_b.$$

MBF formalism

$$\sigma_0 = \frac{1}{m_\sigma^2}\sum_b \Big[g_{\sigma b}\left(m^\star_{\zeta b}\right) - \frac{g_{\sigma b}}{m_b}\left(m^\star_{\zeta b}\right)^{\frac{\zeta+1}{\zeta}}\left(g_{\sigma b}\sigma_0 + g_{\delta b}\delta_0^3\, I^{3b} + g_{\sigma_0^* b}\sigma^*\right)\Big]\rho_{sb},$$

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MBF formalism

$$\sigma_0 = \frac{1}{m_\sigma^2}\sum_b \Big[g_{\sigma b}\left(m^\star_{\zeta b}\right) - \frac{g_{\sigma b}}{m_b}\left(m^\star_{\zeta b}\right)^{\frac{\zeta+1}{\zeta}}\left(g_{\sigma b}\sigma_0 + g_{\delta b}\delta_0^3\, I^{3b} + g_{\sigma_0^* b}\sigma^*\right)\Big]\rho_{sb},$$

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$$\phi_0 = \frac{1}{m_\phi^2}\sum_b g_{\phi b}\rho_b.$$

$$\sum_b \Big[i\gamma_\mu\partial^\mu - g_{\omega b}\gamma_0\omega_0 - g_{\phi b}\gamma_0\phi_0 - \frac{1}{2}g_{\varrho b}\gamma_0\tau^3\varrho_{03} - m_b^* \Big]\psi_b = 0$$

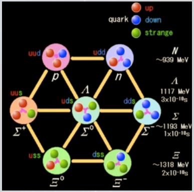
Particle Population

Chemical Potential

Interacting particles:

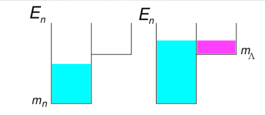
$$\mu_b = \sqrt{k_{F_b}^2 + \left(m_b^*\right)^2} + g_{\omega b}\omega_0 + g_{\phi b}\phi_0 + g_{\varrho b}\varrho_{03} I^{3b}$$

Free particles: $\mu_l = \sqrt{k_{F_l}^2 + m_l^2}$



Chemical Equilibrium

$$\mu_i = q_{b_i}\,\mu_n - q_{e_i}\,\mu_e$$



Conservation laws

Electrical charge:

$$\rho_p + \rho_{\Sigma^+} = \rho_{\Sigma^-} + \rho_{\Xi^-} + \rho_e + \rho_\mu$$

Baryon number:

$$\rho_{bT} = \rho_p + \rho_n + \rho_\Lambda + \rho_{\Sigma^+} + \rho_{\Sigma^0} + \rho_{\Sigma^-} + \rho_{\Xi^-} + \rho_{\Xi^0}$$

Hyperon-nucleon interaction

$$\frac{1}{3}g_{\omega N} = \frac{1}{2}g_{\omega\Lambda} = \frac{1}{2}g_{\omega\Sigma} = g_{\omega\Xi},$$

$$g_{\varrho N} = \frac{1}{2}g_{\varrho\Sigma} = g_{\varrho\Xi},$$

$$2g_{\phi\Lambda} = 2g_{\phi\Sigma} = g_{\phi\Xi} = -\frac{2\sqrt{2}}{3}g_{\omega N},$$

$$g_{\delta N} = \frac{1}{2}g_{\delta\Sigma} = g_{\delta\Xi},$$

$$g_{\varrho\Lambda} = g_{\delta\Lambda} = 0.$$

C. Dover and A. Gal, Prog. Part. Nucl. Phys. 12, 171 (1985); J. Schaffner et al, Annals Phys. 235, 35 (1994).

$$U_Y^N = g_{\omega Y}\omega_0(\rho_0) - g_{\sigma Y}\sigma_0(\rho_0)$$

$$U_\Lambda^N = -28\,MeV,$$

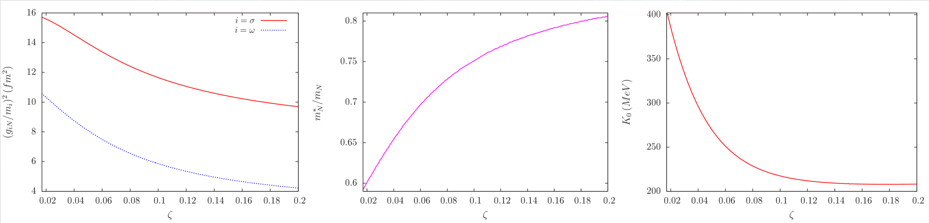
$$U_\Sigma^N = +30\,MeV,$$

$$U_\Xi^N = -18\,MeV.$$

Results: nuclear saturation and hyperon stars

Results: saturation

ζ	m^*_n/m_n	K_0 (MeV)	$(g_{\sigma N}/m_\sigma)^2$	$(g_{\omega N}/m_\omega)^2$
0.040	0.66	297	14.51	8.74
0.049	0.68	272	13.99	8.14
0.059	0.70	253	13.44	7.55
0.071	0.72	237	12.82	6.94
0.085	0.74	225	12.21	6.37
0.104	0.76	216	11.53	5.75
0.129	0.78	211	10.84	5.16



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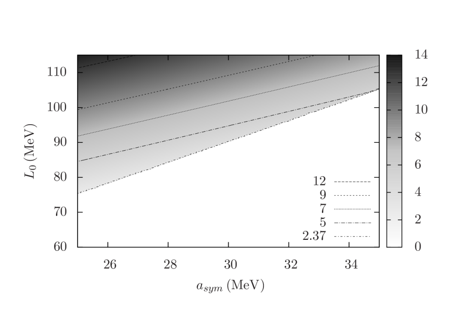
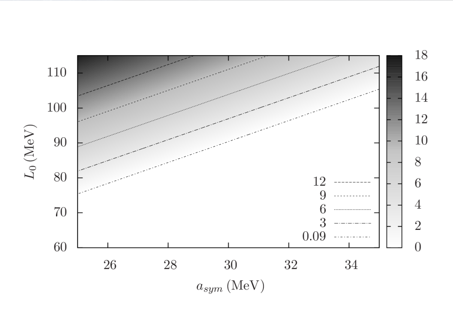
Results: saturation

$\zeta = 0.040$





$(g_\varrho/m_\varrho)^2$ $(g_\delta/m_\delta)^2$

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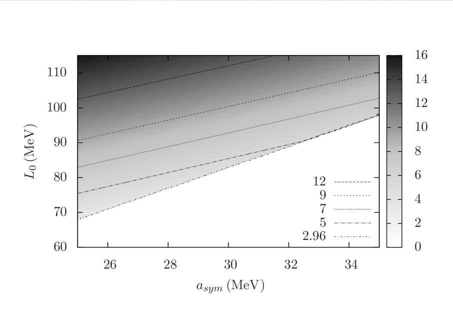
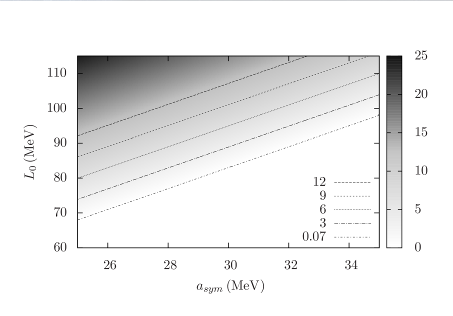
Results: saturation

$\zeta = 0.129$





$(g_\varrho/m_\varrho)^2$ $(g_\delta/m_\delta)^2$

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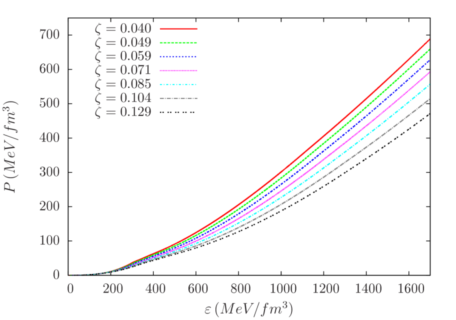
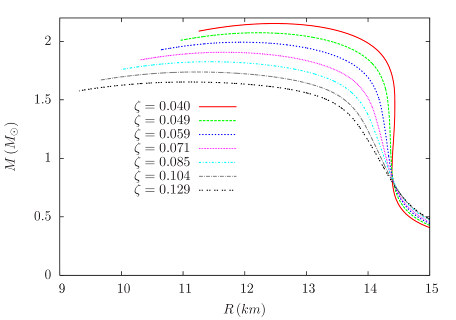
Results: hyperon stars



EoS



TOV

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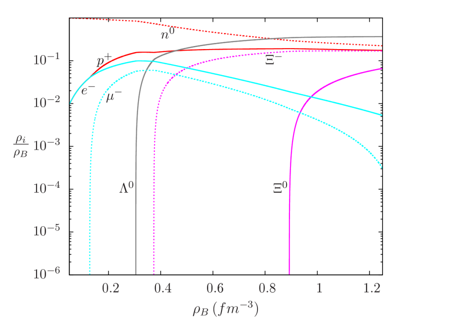
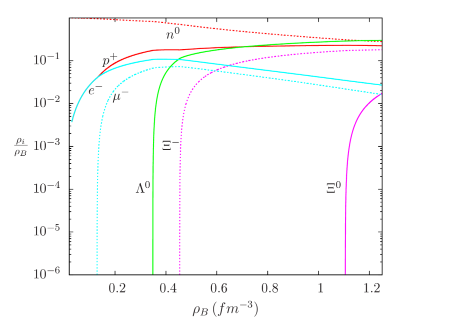
Results: population





$\zeta = 0.040$ $\zeta = 0.129$

R.O. Gomes, V. Dexheimer, S. Schramm, C.A.Z. Vasconcellos, The Astrophysical Journal 808 (2015) 1

Magnetic case: effects on the EoS and the structure

MBF formalism with magnetic fields

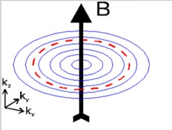
$$
\begin{aligned}
\mathcal{L} = &\sum_b \overline{\psi}_b \left[\gamma_\mu \left(i\partial^\mu - g^*_{\omega b\xi}\omega^\mu - g^*_{\phi b\kappa}\phi^\mu - \frac{1}{2}g^*_{\varrho b\eta}\boldsymbol{\tau}.\boldsymbol{\varrho}^\mu\right) - q_e\gamma_\mu A^\mu - m^*_{b\zeta}\right]\psi_b \\
&+ \sum_l \overline{\psi}_l \gamma_\mu \left(i\partial^\mu - q_e\gamma_\mu A^\mu - m_l\right)\psi_l - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \left(\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) \\
&+ \frac{1}{2}\left(-\frac{1}{2}\omega_{\mu\nu}\omega^{\mu\nu} + m_\omega^2\omega_\mu\omega^\mu\right) + \frac{1}{2}\left(-\frac{1}{2}\phi_{\mu\nu}\phi^{\mu\nu} + m_\phi^2\phi_\mu\phi^\mu\right) \\
&+ \frac{1}{2}\left(-\frac{1}{2}\boldsymbol{\varrho}_{\mu\nu}.\boldsymbol{\varrho}^{\mu\nu} + m_\varrho^2\boldsymbol{\varrho}_\mu.\boldsymbol{\varrho}^\mu\right) + \left(\frac{1}{2}\partial_\mu\boldsymbol{\delta}.\partial^\mu\boldsymbol{\delta} - m_\delta^2\boldsymbol{\delta}^2\right).
\end{aligned}
$$

R.O. Gomes, V.Dexheimer, C.A.Z. Vasconcellos, Astron.Nachr. 335 (2014) 666

Landau Quantization

Energy levels for interacting particles

$$E(k) = g_{\omega b}\omega_0 + g_{\phi b}\phi_0 + g_{\varrho b} I^{3b} \varrho_{03} \pm E_0^*$$



$$E_0^*(\nu) = \sqrt{k_{z_i}^2 + m_i^2 + k_\perp^2}$$

$$k_\perp = \begin{cases} \sqrt{2|q_e|B\nu}, & \nu = l + \frac{1}{2} - \frac{s}{2}\frac{q_e}{|q_e|}, \quad (q_e \neq 0), \\ \sqrt{k_x^2 + k_y^2}, & (q_e = 0). \end{cases}$$

A. Broderick, M. Prakash, J.M. Lattimer, ApJ, 537, 351. (2000);
A. Broderick, M. Prakash, J.M. Lattimer, Physics Letters B, V. 531, Issues 3-4, 11, 167 (2002)

Maximum Landau level $(k_F^2 > 0)$**:**

$$\nu_{max} < \frac{\left((\mu_b^*)^2 + s\kappa_b B\right)^2 - (m_b^*)^2}{2|q_e|B}, \qquad \nu_{max} < \frac{\left(\mu_l^2 + s\kappa_l B\right)^2 - (m_l)^2}{2|q_e|B}$$

Equation of state

Energy-momentum tensor:

$$T^{\mu\nu} = T^{\mu\nu}_{matter} + T^{\mu\nu}_{field}$$

Energy density:

$$\boxed{\varepsilon = T^{00}}$$

Pressure:

$$\boxed{P_{\parallel} = T^{zz}, \qquad P_{\perp} = \frac{1}{2}\left(T^{xx} + T^{yy}\right)}$$

$$\varepsilon = \frac{1}{2}\sum_{\kappa=\sigma,\,\omega,\,\varrho,\,\delta,\,\phi,\,\sigma^*} m_\kappa^2 \kappa_0^2 + \sum_{b,l} \varepsilon^{mag} + \frac{B^2}{2},$$

$$P_{\parallel} = -\frac{1}{2}\sum_{\kappa=\sigma,\,\delta,\,\sigma^*} m_\kappa^2 \kappa_0^2 + \frac{1}{2}\sum_{\kappa=\omega,\,\varrho,\,\phi} m_\kappa^2 \kappa_0^2 + \sum_{b,l} P_{\parallel}^{mag} - \frac{B^2}{2},$$

$$P_{\perp} = -\frac{1}{2}\sum_{\kappa=\sigma,\,\delta,\,\sigma^*} m_\kappa^2 \kappa_0^2 + \frac{1}{2}\sum_{\kappa=\omega,\,\varrho,\,\phi} m_\kappa^2 \kappa_0^2 + \sum_{b,l} P_{\perp}^{mag} + \frac{B^2}{2},$$

Magnetic fields on the structure of neutron stars

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY
MNRAS **447**, 3785–3796 (2015) doi:10.1093/mnras/stu2706

Consistent neutron star models with magnetic-field-dependent equations of state

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ABSTRACT
We present a self-consistent model for the study of the structure of a neutron star in strong magnetic fields. Starting from a microscopic Lagrangian, this model includes the effect of the magnetic field on the equation of state (EoS), the interaction of the electromagnetic field with matter (magnetization), and anisotropies in the energy–momentum tensor, as well as general relativistic aspects. We build numerical axisymmetric stationary models and show the applicability of the approach with one example quark matter EoS often employed in the recent literature for studies of strongly magnetized neutron stars. For this EoS, the effect of inclusion of magnetic field dependence or the magnetization do not increase the maximum mass significantly in contrast to what has been claimed by previous studies.

Astron. Astrophys. 278, 421–443 (1993)

**Axisymmetric rotating relativistic bodies
a new numerical approach for "exact" s**

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Received January 14, accepted April 2, 1993

Abstract. A new set of equations and a new numerical method for computing stationary axisymmetric rapidly
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Astron. Astrophys. 301, 757–775 (1995)

Rotating neutron star models with a m

M. Bocquet[1,2], S. Bonazzola[1], E. Gourgoulhon[1], and J. Novak[1,2]
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Received 20 October 1994 / Accepted 2 March 1995

Abstract. We present the first numerical solutions of the coupled Einstein-Maxwell equations describing rapidly rotating neutron stars endowed with a magnetic field. These solutions are fully relativistic and self-consistent, all the effects of the electromagnetic field on the star's equilibrium (Lorentz force
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Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY
MNRAS **456,** 2937–2945 (2016) doi:10.1093/mnras/stv2606

A self-consistent study of magnetic field effects on hybrid stars

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ABSTRACT
In this work we study the effects of strong magnetic fields on hybrid stars by using a full general-relativity approach, solving the coupled Maxwell–Einstein equation in a self-consistent way. The magnetic field is assumed to be axisymmetric and poloidal. We take into consideration the anisotropy of the energy–momentum tensor due to the magnetic field, magnetic field effects on equation of state (EoS), the interaction between matter and the magnetic field (magnetization), and the anomalous magnetic moment of the hadrons. The EoS used is an extended hadronic and quark SU(3) non-linear realization of the sigma model that describes magnetized hybrid stars containing nucleons, hyperons, and quarks. According to our results, the effects of the magnetization and the magnetic field on the EoS do not play an important role on global properties of these stars. On the other hand, the magnetic field causes the central density in these objects to be reduced, inducing major changes in the populated degrees of freedom and,

LORENE in a nutshell:

Input: $\Omega = 0$, j_0

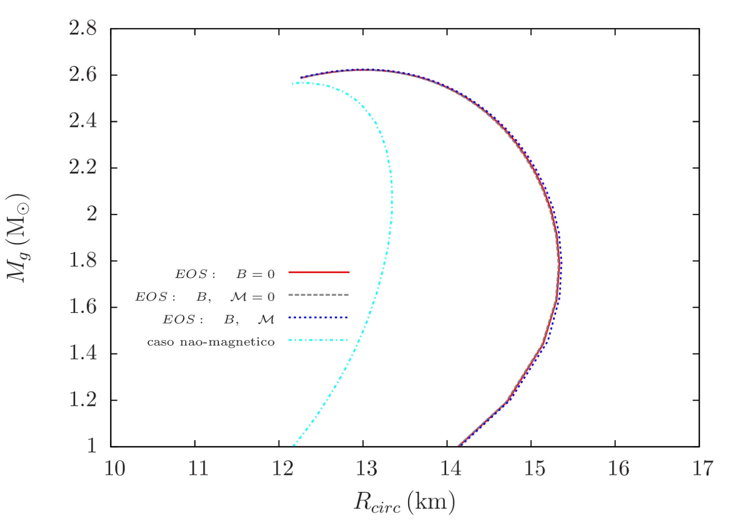
- Axisymmetric metric: deformed stars,
- Poloidal distribution,
- EoS ($h(\rho_b, B)$),
- **Einstein-Maxwell solution,**
- **Magnetostatic equilibrium.**

Output: M_g, $\mathcal{A}$, R_{circ}, ...

S. Bonazzola, E. Gourgoulhon, M. Salgado, J. A. Marck, Astronomy and Astrophysics, 278, 2, 421

Results: magnetic stars

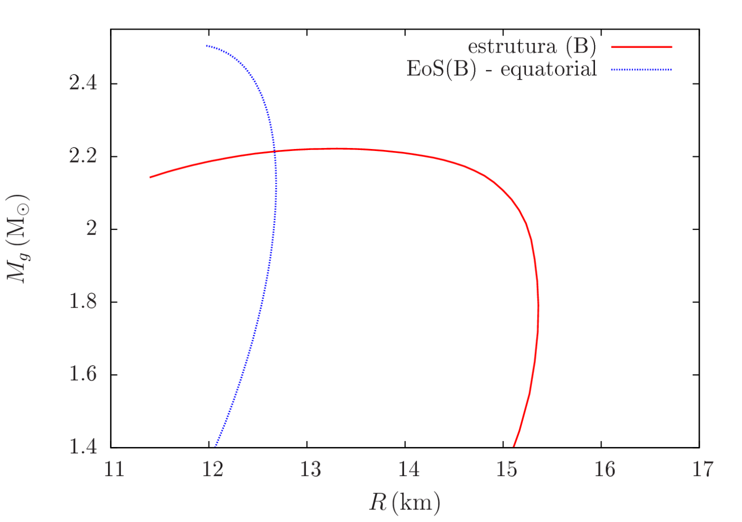
effects of B on the EoS:



80 ANOS 1934|2014 UFRGS UNIVERSIDADE FEDERAL DO RIO GRANDE DO SUL

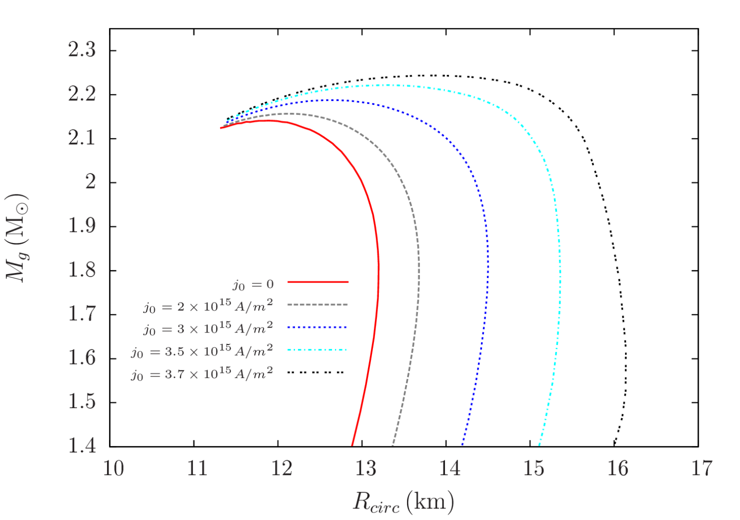
Results: magnetic stars

Comparison TOV and LORENE:



Results: magnetic stars

Global effects:



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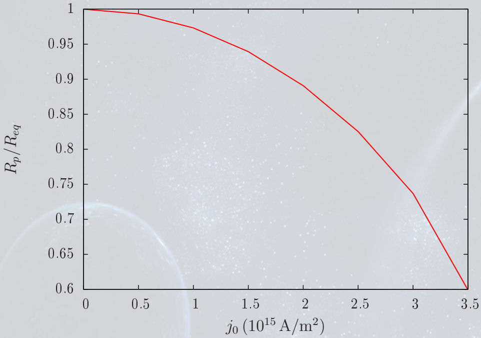

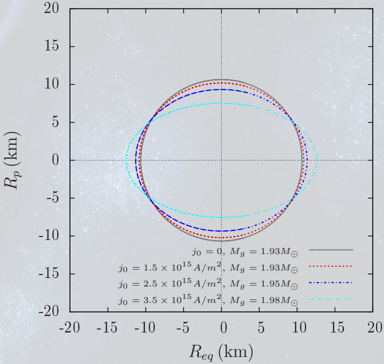

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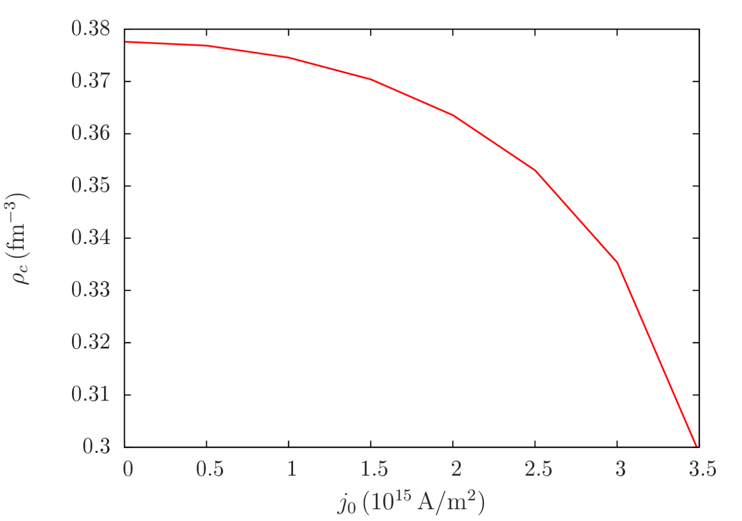
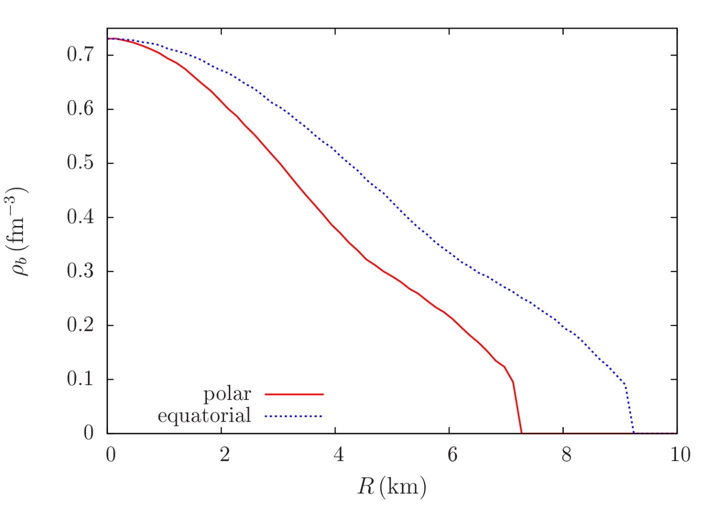
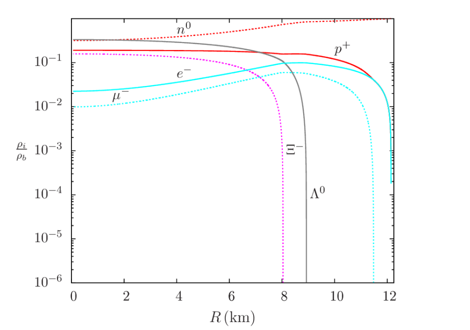


Results: maximum mass star

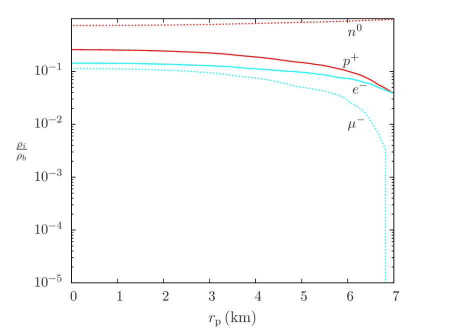


$\zeta = 0.040$, $M_g = 2.22\,M_\odot$, $M_b = 2.56\,M_\odot$, $j_0 = 3.5 \times 10^{15} a/m^2$, $B_s = 3.8 \times 10^{17}\,G$, $B_c \sim 1.2 \times 10^{18}\,G$

Results: Population



$\zeta = 0.040$, $M_g = 2.15\, M_\odot$,
$\rho_c = 0.86\, fm^{-}3$



$\zeta = 0.040$, $M_g = 2.22\, M_\odot$,
$\rho_c = 0.7\, fm^{-3}$, $B_c \sim 1.2 \times 10^{18}\, G$

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- Applying the EoS to compact stars, we describe massive hyperon stars. In particular, the astrophysical data narrows the values of the parameter of the model to a range of $0.040 \leq \zeta \leq 0.065$, being $2.1 M_\odot$ the maximum achieved by the model to describe stars with hyperon content.

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Summary

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- We conclude that the inclusion of magnetic effects on the EoS of the MBF formalism does not present significant changes on the global properties of the stars;
- The error of including the magnetic effects only on the EoS (by solving TOV equations) is estimated in comparison to the LORENE solution, from which we conclude an overestimation of 13% in the maximum mass and an underestimation of $\sim 25\%$ in the radius;
- The Lorentz force acts against gravity, increasing the radius and mass of the stars and, in particular, we verify these effects on the global and internal properties of neutrons stars for the MBF formalism;
- The Landau quantization, together with the reduction of the central density due to magnetic fields, make the hyperon population vanish inside strongly magnetized stars

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Thank you.

This work is a collaboration with:

B. Franzon (Frankfurt Intitute for Advanced Studies, Germany)

V. Dexheimer (Kent State University, USA)

S. Schramm (Frankfurt Intitute for Advanced Studies, Germany)

C. A. Z. Vasconcellos (Universidade Federal do Rio Grande do Sul, Brazil)

Outlook

- Extend the many-body contributions to the vector mesons (vector versions of the model);
- Verify the effects of temperature and thermal evolution of the hyper stars described by different parameterizations of the model.
- Investigate magnetic field effects on other populations (kaon condensates, Δ ressonances, etc..);