Non-equilibrium Quantum Transport and Quantum “Heat Engines”

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Pisa, January 21, 2016

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Key features of non-equilibrium QFT:

- there exist many inequivalent ways to drive a quantum system with infinite degrees of freedom away from equilibrium;
- there exist therefore a large variety of different non-equilibrium configurations;
- lack of a unified framework for all of them;
- several different approaches exist: Keldish perturbation theory, Lindblad operator approach, Landauer-Büttiker scattering formalism,...;
- the art in this context is to construct a non-equilibrium state which provides a realistic description of the physical situation one is dealing with;
- keeping in mind these peculiar features of non-equilibrium QFT, there are two directions among others, which attract recently much attention, triggered by remarkable experiments in condensed matter physics.

Recent progress in non-equilibrium QFT:

(a) quantum transport - systems in non-equilibrium steady state:

- particle and energy (heat) currents;
- current fluctuations and noise (noise specroscopy).

Experiments:
(i) quantum wires, quantum wire junctions and networks (electrons);
(ii) quantum Hall edges (anyons);
(iii) topological superconducting edges (Majorana fermions).

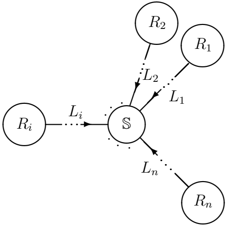
(b) quantum quench - systems in non-equilibrium and non-steady state:

quench at t_0 - Hamiltonian $H(t) = H_0 + \theta(t - t_0)H_1$.

- universal features in the regime of relaxation $(t > t_0)$;
- nature of the final $(t \to \infty)$ equilibrium state.

Experiments:
(i) trapped ultra-cold atomic gases.

Non-equilibrium quantum system with star graph geometry:



Quantum junction with n terminals

- n oriented semi-infinite terminals L_i with coordinates $\{x \geq 0,\ i = 1, \ldots, n\}$;
- n heat reservoirs $R_i = \{\beta_i, \mu_i\}$ attached at infinity;
- large capacity of R_i - $\{\beta_i, \mu_i\}$ remain invariant after particle emission and absorption;
- the vertex of the star graph represents a defect (impurity) characterised by a unitary scattering matrix $\mathbb{S}$;
- the system is away from equilibrium if $\mathbb{S}_{ij} \neq 0$ exists between $(\beta_i, \mu_i) \neq (\beta_j, \mu_j)$.

Goal: Construct a non-equilibrium steady state for this setting and explore its properties.

Plan:

1. General considerations:
   - symmetry content;
   - energy conversion and efficiency;
   - interaction.
2. The Schrödinger junction - definition and basic properties:
   - the non-equilibrium steady state and the second principle of thermodynamics;
   - efficiency of the energy conversion in the Schrödinger junction;
   - role of the statistics.
3. Complete description of the particle transport:
   - exact *n*-point connected current correlation functions and cumulants;
   - reconstruction of the underlying probability distribution - the core of the transport problem;
   - physical interpretation of the distribution.
4. Conclusions and prespectives.

L. Santoni, P. Sorba, M. M., J. Phys. A: Math. Theor. **48** (2015) 055003;
L. Santoni, P. Sorba, M. M., J. Phys. A: Math. Theor. **48** (2015) 285002;
L. Santoni, P. Sorba, M. M., arXiv: 1601.01819.

Assume the following symmetry content:

- particle number conservation: $\partial_t j_t - \partial_x j_x = 0$
- energy conservation: $\partial_t \theta_{tt} - \partial_x \theta_{xt} = 0$
- Kirchhoff rules: $\sum_{i=1}^{n} j_x(t, 0, i) = \sum_{i=1}^{n} \theta_{xt}(t, 0, i) = 0$

The two components of the total energy density:

- chemical potential energy density: $k_t = \mu_i j_t$
- heat energy density: $q_t = \theta_{tt} - \mu_i j_t$
- the associated currents $k_x = \mu_i j_x$ and $q_x = \theta_{xt} - \mu_i j_x$
  satisfy local conservation

$$\partial_t k_t - \partial_x k_x = 0\,, \qquad \partial_t q_t - \partial_x q_x = 0\,,$$

but violate the Kirchhoff rules, if not all of chemical potentials coincide ($\exists\ \mu_i \neq \mu_j$):

$$\sum_{i=1}^{n} q_x(t, 0, i) = -\sum_{i=1}^{n} \mu_i j_x(t, 0, i) \neq 0$$

Simple consequences from the Kirchhoff rule violation:

- the heat and chemical energies are not separately conserved;
- since the total energy is conserved, conversion of heat to chemical energy or vice versa occurs;

Lesson: away from equilibrium the quantum junction operates as energy converter.

- this feature is very general, being based on symmetry considerations only.

Characterising the process of energy conversion:

- define $\dot{Q} = -\sum_{i=1}^{n} q_x(t, 0, i)$;
- let $\Psi \in \mathcal{H}$ be any state of the system;
- with our convention for the lead orientation

$$\begin{aligned} \langle \dot{Q} \rangle_\Psi &< 0\,, \qquad \text{heat energy} \longrightarrow \text{chemical energy}\,, \\ \langle \dot{Q} \rangle_\Psi &> 0\,, \qquad \text{chemical energy} \longrightarrow \text{heat energy}\,. \end{aligned}$$

- efficiency in the sate Ψ: let $\mathcal{K}_+$ ($\mathcal{L}_+$) be the set of positive heat (chemical) currents

$$\begin{aligned} \eta &= \frac{-\langle \dot{Q} \rangle_\Psi}{\sum_{i\in\mathcal{K}_+} \langle q_x \rangle_\Psi}\,, \qquad \langle \dot{Q} \rangle_\Psi < 0\,, \qquad (\text{quantum “heat engine”}); \\ \widetilde{\eta} &= \frac{\langle \dot{Q} \rangle_\Psi}{\sum_{i\in\mathcal{L}_+} \langle \mu_i j_x \rangle_\Psi}\,, \qquad \langle \dot{Q} \rangle_\Psi > 0\,; \end{aligned}$$

- by construction $0 < \eta \leq 1\,, \qquad 0 < \widetilde{\eta} \leq 1$;
- In what follows I will focus on η.

Interaction

- interaction - codified in the unitary scattering matrix $\mathbb{S}$ characterizing the defect;
- since the particle number is conserved,

$$\mathcal{H} = \bigoplus_{m=1}^{\infty} \mathcal{H}^{(m)}\, , \qquad \mathbb{S} = \bigoplus_{m=1}^{\infty} \mathbb{S}^{(m)}\, ,$$

$\mathbb{S}^{(m)}$ - the scattering matrix in the m-particle space $\mathcal{H}^{(m)}$;

- start with the simplest case where the energy transmutation shows up;
- we assume in this talk that

$$\mathbb{S}^{(1)} = \mathbb{S}(k)\, , \qquad \mathbb{S}^{(m)} = \mathbb{I} \qquad \forall\; m \geq 2\, ;$$

- evidence from experiments with quantum wire junctions that the $m \geq 2$-body interactions influence the quantum transport only marginally.

Example - the Schrödinger junction:

- bulk dynamics:

$$\left(\mathrm{i}\partial_t + \frac{1}{2m}\partial_x^2\right)\psi(t,x,i) = 0\,;$$

- boundary condition ($\mathbb{U} \in U(n)$, λ-free parameter):

$$\lim_{x\to 0^-} \sum_{j=1}^{n} \left[\lambda(\mathbb{I}-\mathbb{U})_{ij} + \mathrm{i}(\mathbb{I}+\mathbb{U})_{ij}\partial_x\right]\psi(t,x,j) = 0\,;$$

- this is the most general b.c. ensuring the self-adjointness of the Hamiltonian;
- scattering matrix (Kostrikin-Schrader 2000):

$$\mathbb{S}(k) = -\frac{[\lambda(\mathbb{I}-\mathbb{U}) - k(\mathbb{I}+\mathbb{U})]}{[\lambda(\mathbb{I}-\mathbb{U}) + k(\mathbb{I}+\mathbb{U})]}\,;$$

- scale invariant (critical) elements of this family:

$$\mathbb{S} = U\,\mathbb{S}_{\mathrm{d}}\,U^*\,, \qquad U \in U(n)\,, \qquad \mathbb{S}_{\mathrm{d}} = \mathrm{diag}(\pm 1, \pm 1, ..., \pm 1)$$

The solution:

- the well known expression

$$\psi(t,x,i) = \sum_{j=1}^{n} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \mathrm{e}^{-\mathrm{i}\omega(k)t - \mathrm{i}kx} a_i(k)\,, \qquad \omega(k) = \frac{k^2}{2m}\,.$$

but with a deformed canonical (anti)commutation algebra

$$\mathcal{A}_\pm = \{a_i(k),\, a_i^*(k)\ :\ i = 1, ..., n,\ k \in \mathbb{R}\}\,;$$

$$[a_i(k)\,,\, a_j(p)]_\pm = [a_i^*(k)\,,\, a_j^*(p)]_\pm = 0\,,$$

$$[a_i(k)\,,\, a_j^*(p)]_\pm = 2\pi[\delta_{ij}\delta(k-p) + \mathbb{S}_{ij}(k)\delta(p+k)]\,.$$

- $P(k,p) \equiv 2\pi[\delta_{ij}\delta(k-p) + \mathbb{S}_{ij}(k)\delta(p+k)]$ - integral kernel of a projection operator.

Algebraic construction of the non-equilibrium steady state:

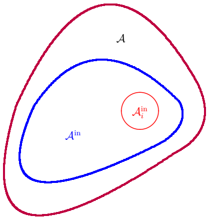
Consider the incoming sub-algebra $\mathcal{A}^{\rm in}_{\pm,i} = \{a_i(k),\, a_i^*(k) \,:\, k > 0\}$ associated to R_i and perform the following three steps:

- take the Gibbs state Ω_{β_i,μ_i} over $\mathcal{A}^{\rm in}_{\pm,i}$;

$$\left(\Omega_{\beta_i,\mu_i}\,,\,\mathcal{O}\,\Omega_{\beta_i,\mu_i}\right) \equiv \langle\mathcal{O}\rangle_{\beta_i,\mu_i} = \frac{1}{Z}\mathrm{Tr}\left[\mathrm{e}^{-K_i}\mathcal{O}\right]\,, \qquad Z = \mathrm{Tr}\left[\mathrm{e}^{-K_i}\right]\,,$$

$$K_i = \beta_i(h_i - \mu_i q_i)\,, \quad h_i = \int_0^\infty \frac{\mathrm{d}k}{2\pi}\omega(k)a_i^*(k)a_i(k)\,, \quad q_i = \int_0^\infty \frac{\mathrm{d}k}{2\pi}a_i^*(k)a_i(k)\,.$$

- perform the tensor product $\Omega^{\rm in}_{\beta,\mu} = \bigotimes_{i=1}^n \Omega_{\beta_i,\mu_i}$;
- extend $\Omega^{\rm in}_{\beta,\mu}$ by linearity to a state $\Omega_{\beta,\mu}$ on the whole algebra $\mathcal{A}_\pm$ using the scattering relations



$$a_i(k) = \sum_{j=1}^n \mathbb{S}_{ij}(k)a_j(-k)\,, \qquad a_i^*(k) = \sum_{j=1}^n a_j^*(-k)\mathbb{S}^*_{ji}(k)\,;$$

Few comments about $\Omega_{\beta,\mu}$:

- $\Omega_{\beta,\mu}$, obtained is this way is called Landauer (1957)-Büttiker (1986) (LB) state.
- adopting a quantum mechanical formalism, LB constructed actually the projection of $\Omega_{\beta,\mu}$ on $\mathcal{H}^{(1)}$ and $\mathcal{H}^{(2)}$, thus determining only the 2-point and 4-point ψ-correlators.
- the above construction allows to derive the n-point non-equilibrium correlators.
- example - the exact two-point function in the state $\Omega_{\beta,\mu}$:

$$\langle \psi^*(t_1,x_1,i)\psi(t_2,x_2,j)\rangle_{\beta,\mu} = \int_0^\infty \frac{\mathrm{d}k}{2\pi}\mathrm{e}^{\mathrm{i}\omega(k)t_{12}}\Big[\delta_{ji}d_i(k)\mathrm{e}^{\mathrm{i}kx_{12}} + d_j^{\pm}(k)\mathbb{S}_{ji}(k)\mathrm{e}^{-\mathrm{i}k\widetilde{x}_{12}} + \mathbb{S}^*_{ji}(k)d_i^{\pm}(k)\mathrm{e}^{\mathrm{i}k\widetilde{x}_{12}} + \sum_{l=1}^n \mathbb{S}^*_{jl}(k)d_l^{\pm}(k)\mathbb{S}_{li}(k)\mathrm{e}^{-\mathrm{i}kx_{12}}\Big],$$

$$d_i^{\pm}(k) = \frac{\mathrm{e}^{-\beta_i[\omega(k)-\mu_i]}}{1\pm \mathrm{e}^{-\beta_i[\omega(k)-\mu_i]}} \qquad \textit{Dirac}(+)/\textit{Bose}(-)\textit{distribution}\,.$$

- In the bosonic case we assume $\mu_i < 0$ for avoiding singularities.

Exploring the properties of $\Omega_{\beta,\mu}$:

- using $\langle \psi^*(t_1, x_1, i)\psi(t_2, x_2, j)\rangle_{\beta,\mu}$, one can compute

$$\langle j_x(t,x,i)\rangle_{\beta,\mu}\,, \qquad \langle \theta_{xt}(t,x,i)\rangle_{\beta,\mu}\,, \qquad \langle q_x(t,x,i)\rangle_{\beta,\mu}\,.$$

and start studying the non-equilibrium features of $\Omega_{\beta,\mu}$.

Here:

- particle current

$$j_x(t,x,i) = \frac{\mathrm{i}}{2m}\left[\psi^*(\partial_x\psi) - (\partial_x\psi^*)\psi\right](t,x,i)\,,$$

- energy current

$$\theta_{xt}(t,x,i) = \frac{1}{4m}[(\partial_t\psi^*)\,(\partial_x\psi) + (\partial_x\psi^*)\,(\partial_t\psi) - (\partial_t\partial_x\psi^*)\,\psi - \psi^*\,(\partial_t\partial_x\psi)](t,x,i)\,.$$

- heat current

$$q_x(t,x,i) = \theta_{xt}(t,x,i) - \mu_i j_x(t,x,i)\,.$$

Fundamental property of $\Omega_{\beta,\mu}$ - nonnegative entropy production:

$$\dot{S} = -\sum_{i=1}^{n} \beta_i q_x(t,x,i)\,,$$

In fact,

$$\langle \dot{S} \rangle_{\beta,\mu} = \sum_{i,j=1}^{n} \int_0^\infty \frac{\mathrm{d}k}{2\pi} \frac{k}{m} |\mathbb{S}_{ij}(k)|^2 \left[\sigma_i(k) - \sigma_j(k)\right] d_j(k) \geq 0\,,$$

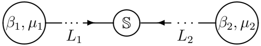
$$\sigma_i(k) = \beta_i \left[\omega(k) - \mu_i\right]\,, \qquad d_j(k) = \frac{1}{\mathrm{e}^{\sigma_j(k)} \pm 1}$$

because the integrand is nonnegative (Nenciu 2007):

- $d_j(k)$ is a strictly decreasing function of σ_j;
- the inequality $F(x) - F(y) \leq (x-y)f(y)$, where F is any primitive of a strictly decreasing function f, holds.

Since $\langle \dot{S} \rangle_{\beta,\mu} \geq 0$ one can expect that η behaves like the efficiency of a "heat engine".

Efficiency η in the two-terminal system in the LB state - fermions:



- without loss of generality one can assume $\beta_2 \geq \beta_1$;
- then the efficiency η of converting heat to chemical energy takes the form

$$\eta = \frac{(\mu_2 - \mu_1)\langle j_x(t,x,1)\rangle_{\beta,\mu}}{\langle q_x(t,x,1)\rangle_{\beta,\mu}}\, ;$$

- in the scale invariant case one gets

$$\eta(\lambda_1, \lambda_2; r) = \frac{(\lambda_1 - r\lambda_2)\left[\ln\left(1+\mathrm{e}^{-\lambda_1}\right) - r\ln\left(1+\mathrm{e}^{-\lambda_2}\right)\right]}{\lambda_1\left[\ln\left(1+\mathrm{e}^{-\lambda_1}\right) - r\ln\left(1+\mathrm{e}^{-\lambda_2}\right)\right] - \left[\mathrm{Li}_2\left(-\mathrm{e}^{-\lambda_1}\right) - r^2\mathrm{Li}_2\left(-\mathrm{e}^{-\lambda_2}\right)\right]}\, ;$$

$$r = \frac{\beta_1}{\beta_2} \in [0,1]\, , \qquad \lambda_i = -\beta_i\mu_i\, , \quad i = 1,2\, , \qquad \text{(dimensionless parameters)}\, .$$

- amount of heat energy converted in chemical energy:

$$-\dot{Q}(\lambda_1, \lambda_2, r) = -\frac{|\mathbb{S}_{12}|^2}{2\pi\beta_1^2}(\lambda_1 - r\lambda_2)\left[r\ln\left(1+\mathrm{e}^{-\lambda_2}\right) - \ln\left(1+\mathrm{e}^{-\lambda_1}\right)\right]\, .$$

Basic properties of η:

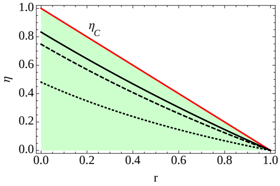
- one easily shows that

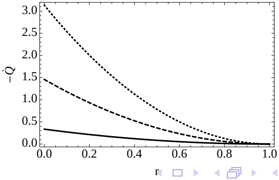
$$\dot{S} \geq 0 \quad \Longrightarrow \quad \eta(\lambda_1, \lambda_2; r) \leq 1 - r \equiv \eta_{\rm C} \ , \quad (\text{Carnot efficiency}) \ ;$$

- the maximal efficiency is obtained in the limit $\lambda_1 = \lambda_2 \to \infty$

$$\eta_{\max}(r) = \lim_{\lambda \to +\infty} \eta(\lambda, \lambda; r) = 1 - r \equiv \eta_{\rm C} \ ;$$

- the efficiency η is decreasing with increasing of the energy conversion $-\dot{Q}$, and vice versa;
- η compared to $\eta_{\rm C}$ (left) and the heat $-\dot{Q}$ converted to chemical energy (right) with $\lambda = 1$ (dotted), $\lambda = 3$ (dashed) and $\lambda = 5$ (continuous) :





Efficiency at optimal energy conversion:

- a simple analysis shows that the function $-\dot{Q}(\lambda_1, \lambda_2, r)$ reaches its maximum at

$$\lambda_1 = \lambda_2 \equiv \lambda^* \, , \qquad \lambda^* - (1 + \mathrm{e}^{\lambda^*}) \ln(1 + \mathrm{e}^{-\lambda^*}) = 0 \, .$$

- the solution is $\lambda^* = 1.14455...$ and the efficiency takes the form

$$\eta^*(r) \equiv \eta(\lambda^*, \lambda^*; r) = \frac{(1-r)\lambda^* \ln\left(1 + \mathrm{e}^{-\lambda^*}\right)}{\lambda^* \ln\left(1 + \mathrm{e}^{-\lambda^*}\right) - (1+r)\mathrm{Li}_2\left(-\mathrm{e}^{-\lambda^*}\right)} \, ;$$

- $\eta^*(r)$ is the counterpart of the concept of efficiency at maximal power η_{max} from heat engines;
- in endoreversible thermodynamics Curzon-Ahlborn (1975) (CA) established the bound

$$\eta_{\mathrm{max}}(r) \leq 1 - \sqrt{r} \, .$$

- the CA bound is satisfied in our case - one can show that $\eta^*(r) \leq 1 - \sqrt{r}$;
- hot topic in the literature: possibility to enhance η^* above the CA bound in the quantum context - couple the electric charge to ambient electromagnetic fields.

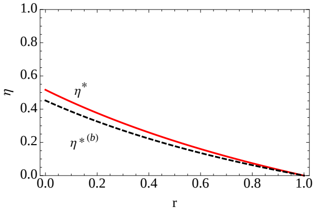
Efficiency η^* in the in the LB state - bosons: at maximal energy conversion one gets

$$\eta_b^*(r) = \frac{(1-r)\lambda_b^* \ln\left(1-\mathrm{e}^{-\lambda_b^*}\right)}{\lambda_b^* \ln\left(1-\mathrm{e}^{-\lambda_b^*}\right) - (1+r)\mathrm{Li}_2\left(+\mathrm{e}^{-\lambda_b^*}\right)} .$$

where

$$\lambda_b^* - (1-\mathrm{e}^{\lambda_b^*})\ln(1-\mathrm{e}^{-\lambda_b^*}) = 0 \quad \Longrightarrow \quad \lambda_b^* = 0.69314\ldots ;$$

- the bosonic junctions are less efficient then the fermionic ones;



N.B.

- up to now we have used only one-point current correlators;
- the transport is fully characterised by the full sequence of n-point correlators;
- e.g. the quantum noise is deduced from the two-point correlators.

The main aspects of the complete picture:

- the complete picture is codified in

$$\{\langle j(t_1, x_1, i) \cdots j(t_n, x_n, i)\rangle^{\rm conn}_{\beta,\mu} \; : \; n = 1, 2, \ldots\} \, .$$

We concentrate on the fermion transport and investigate the full sequence of n-point correlators:

$$\mathcal{C}_n^i(\hat{t}_1, ..., \hat{t}_{n-1}, x_1, ..., x_n) = \langle j(t_1, x_1, i) \cdots j(t_n, x_n, i)\rangle_{\beta,\mu}^{\mathrm{conn}} .$$

$$\{\hat{t}_k \equiv t_k - t_{k+1} \ : \ k = 1, ..., n-1\}$$

- for eliminating the unessential for the transport space-time variables, we consider

$$\mathcal{C}_n^i(x_1, ..., x_n; \nu) = \int_{-\infty}^{\infty} \mathrm{d}\hat{t}_1 \cdots \int_{-\infty}^{\infty} \mathrm{d}\hat{t}_{n-1} \mathrm{e}^{\mathrm{i}\nu(\hat{t}_1 + \cdots \hat{t}_{n-1})} \mathcal{C}_n^i(\hat{t}_1, ..., \hat{t}_{n-1}, x_1, ..., x_n) ,$$

and perform the zero-frequency limit

$$\mathcal{C}_n^i = \lim_{\nu \to 0^+} \mathcal{C}_n^i(x_1, ..., x_n; \nu) .$$

- $\mathcal{C}_n^i$ are x_i-independent and have the following integral representation in the energy ω:

$$\mathcal{C}_n^i = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \, \mathcal{C}_n^i(\omega) ,$$

$$\mathcal{C}_1^i(\omega) = \mathrm{Tr}\left[\mathbb{T}^i \mathbb{D}\right] , \qquad \mathcal{C}_n^i(\omega) = \sum_{\sigma \in \mathcal{P}_{n-1}} \mathrm{Tr}\left[\mathbb{T}^i \mathbb{D}\, \mathbb{C}^i_{\sigma_1\sigma_2} \cdots \mathbb{C}^i_{\sigma_{n-2}\sigma_{n-1}} \mathbb{T}^i (\mathbb{I} - \mathbb{D})\right] , \quad n \geq 2 ,$$

where the sum runs over all permutations $\mathcal{P}_{n-1}$ of $n-1$ elements and

$$\mathbb{C}^i_{\sigma_i\sigma_{i+1}} = \begin{cases} -\mathbb{T}^i\, \mathbb{D} \ , & \sigma_i < \sigma_{i+1} , \\ \ \ \mathbb{T}^i\, (\mathbb{I} - \mathbb{D}) , & \sigma_i > \sigma_{i+1} . \end{cases}$$

$$\mathbb{T}^i_{lm} = \delta_{li}\delta_{mi} - \mathbb{S}_{li}(\sqrt{2m\omega})\, \overline{\mathbb{S}}_{mi}(\sqrt{2m\omega}) , \qquad \mathbb{D} \equiv \mathrm{diag}[d_1(\omega), d_2(\omega), ..., d_n(\omega)]$$

Example: cumulants in the lead L_1 of the two-terminal case:

$$\begin{aligned}
\mathcal{C}_1^1(\omega) &= \tau c_1 \,, \\
\mathcal{C}_2^1(\omega) &= \tau(c_2 - \tau c_1^2) \,, \\
\mathcal{C}_3^1(\omega) &= \tau^2 c_1(1 - 3c_2 + 2\tau c_1^2) \,, \\
\mathcal{C}_4^1(\omega) &= \tau^2[c_2 - 3c_2^2 + 12\tau c_1^2 c_2 - 2\tau c_1^2(2 + 3\tau c_1^2)] \,, \\
\mathcal{C}_5^1(\omega) &= \tau^3 c_1[1 + 30c_2^2 - 15c_2(1 + 4\tau c_1^2) + 4\tau c_1^2(5 + 6\tau c_1^2)] \,, \\
&\dots
\end{aligned}$$

$$\tau(\omega) = |\mathbb{S}_{12}(\sqrt{2m\omega})|^2 \,, \quad c_1(\omega) \equiv d_1(\omega) - d_2(\omega) \,, \quad c_2(\omega) \equiv d_1(\omega) + d_2(\omega) - 2d_1(\omega)d_2(\omega) \,.$$

Understand the mathematical structure and physical meaning of $\mathcal{C}_n^i$

Basic observations: $\{\mathcal{C}_n^1 \, : \, n = 1, 2, \dots\}$ are the cumulants of a probability distribution φ_1;
φ_1 captures the complete information about the particle transport;

Main problem: determine φ_1.

Strategy: determine the moments and solve the moment problem.

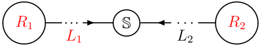
Skipping the details, the probability distribution in lead L_1 is:

$$\varphi_1(\omega;\xi) = p_{12}(\omega)\delta(\xi + e\sqrt{\tau}) + p_{11}(\omega)\delta(\xi) + p_{21}(\omega)\delta(\xi - e\sqrt{\tau})\,,$$

$$p_{12} = \frac{1}{2}(c_2 - c_1\sqrt{\tau})\,, \qquad p_{11} = (1-c_2)\,, \qquad p_{21} = \frac{1}{2}(c_2 + c_1\sqrt{\tau})$$

$$0 \leq p_{ij} \leq 1\,, \qquad p_{12} + p_{11} + p_{21} = 1\,.$$

Physical interpretation: the fundamental elementary processes in L_1 and the probabilities p_{ij}.



Emission of a particle with energy ω:

- from R_1 and absorption by R_2 - probability p_{12};
- from R_1 and reabsorption by R_1 - probability p_{11};
- from R_2 and absorption by R_1 - probability p_{21};
- the δ-functions implement the charge variation in L_1 during these processes;
- $e\sqrt{\tau}$ - effective charge crossing the defect (full transmission $\tau = 1$ - absence of the defect).

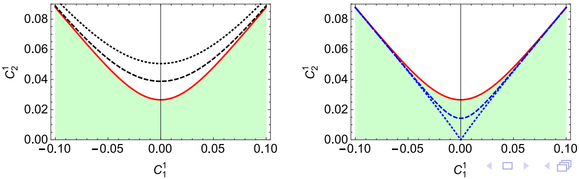
Reconstruction of the cumulants $\mathcal{C}^1_n$ from the distribution φ_1:

The triplet of probabilities $\{p_{12},\, p_{21},\, \tau\}$ fully describes the quantum transport - namely all connected current correlators in the zero frequency limit. In fact:

$$\{p_{12},\, p_{21},\, \tau\} \to \varphi_1(\omega;\xi) \to m^1_n(\omega) = \int_{-\infty}^{\infty} \mathrm{d}\xi\, \xi^n \varphi_1(\omega;\xi) \to \mathcal{C}^1_n(\omega) \to \mathcal{C}^1_n = \int_0^{\infty} \frac{\mathrm{d}\omega}{2\pi}\, \mathcal{C}^1_n(\omega)$$

Example: Practical use of the cumulants - the noise $\mathcal{C}^1_2$ as a function of the current $\mathcal{C}^1_1$: (noise experiments with quantum Hall edges)

- both $\mathcal{C}^1_1$ and $\mathcal{C}^1_2$ depend on $\mu_\pm = (\mu_1 \pm \mu_2)/2$;
- $\mu_+ \neq 0$ parametrises the deviation from the linear regime;
- eliminate μ_- in $\mathcal{C}^1_2$ in favour of $\mathcal{C}^1_1$ and plot for different values of μ_+;
- $\mu_+ \geq 0$ (left - noise enhancement), $\mu_+ \leq 0$ (right - noise reduction), $\mu_+ = 0$ (red line).



Conclusions, further developments and observations:

- the above results are exact - no use of linear response theory (valid only for $\beta_1 \sim \beta_2$ and $\mu_1 \sim \mu_2$);
- a direct extension to multi terminal junctions exists;
- the efficiency $\widetilde{\eta}$ of converting chemical energy to heat can be treated along the same lines;
- we used the LB state $\Omega_{\beta,\mu}$ generated by the Gibbs states of the heat reservoirs;
- the orbit $\{\Omega_{\beta,\mu},\ T\,\Omega_{\beta,\mu},\ P\,\Omega_{\beta,\mu}\ PT\,\Omega_{\beta,\mu}\}$ under parity P and time reversal T provides new physically interesting non equilibrium states, e.g.

$$\eta^*_{PT} = \frac{2(1-r)}{r+3}\,, \qquad \text{(Brownian particles undergoing a Carnot cycle)}$$

- in systems with larger internal symmetry one can use the LB state generated by a generalized Gibbs ensembles;
- test different dynamics and include some $\mathbb{S}^{(m)}$ with $m \geq 2$ (e.g. Tomonaga-Luttinger liquid - P. Sorba, M. M. 2013).

Non-equilibrium QFT is a fascinating subject with many open conceptual problems and concrete physical applications in various fields like:

modern condensed matter,
critical phenomena,
cosmology, ...

Thanks for your attention.