

Notes on soft theorems

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Abstract

This lecture is thought to provide a pedagogical introduction to the subject of scattering amplitudes, with focus on the recent developments about soft theorems.

The lecture will be structured in two parts, the first of which will concern fundamental concepts such as unitarity and analyticity, as well as the singularity structure of the amplitudes and their physical importance.

As far as the second part is concerned, we will review the so-called “soft theorems”, their origin and the renewed interest in their structure. The relation with asymptotic symmetries will be highlighted, the attempt being to emphasize what is known and what is new.

More in detail, in the first part will touch the following topics:

- Generalities: Definition and the Lorentz little group;
- Analyticity, unitarity and locality;
- BCFW method and the singularity structure of perturbative field theories;
- Soft limits.

As far as the second part is concerned:

- Statement and history of soft theorems [1], [2] ; historical importance: QCD [3], double scalars as probes of $\mathcal{N} = 8$ moduli space [4]. Sketch of a derivation [5].
- Relation of soft theorems to asymptotic symmetries of the theory [6]; definition of BMS symmetry for gravitational scattering and its relevance.¹
- Soft theorems as Ward identities, soft bosons as Goldstone bosons [8]

References:

- *Reviews:*
 - H. Elvang, Y-t. Huang, "Scattering amplitudes", arXiv:1308.1697 [hep-th]
 - P. Benincasa, "New structures in scattering amplitudes: a review", arXiv:1312.5583 [hep-th], Int.J.Mod.Phys. A29 (2014) 5, 1430005
 - J.M. Henn, J.C. Plefka, "Scattering amplitudes in gauge theories", Lect.Notes Phys. 883 (2014) 1-195
- *BCFW method and the singularity structure of perturbative field theories:*
 - R. Britto, F. Cachazo, B. Feng, E. Witten, "Direct proof of tree-level recursion relation in Yang-Mills theory", arXiv:0501052 [hep-th], Phys. Rev. Lett. 94 (2005).
 - P. Benincasa, C. Boucher-Veronneau, and F. Cachazo, "Taming tree amplitudes in general relativity", JHEP 11 (2007) 057, arXiv:hep-th/0702032.
 - T. Cohen, H. Elvang, and M. Kiermaier, "On-shell constructibility of tree amplitudes in general field theories", JHEP 1104 (2011) 053, arXiv:1010.0257 [hep-th].
 - C. Cheung, C.-H. Shen, and J. Trnka, "Simple recursion relations for general field theories," arXiv:1502.05057 [hep-th]
 - N. Arkani-Hamed, F. Cachazo, and J. Kaplan, "What is the simplest quantum field theory?", JHEP 1009 (2010) 016, arXiv:0808.1446 [hep-th].

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¹A partial list of papers dealing with the topic is [7,8,2,9], but of course there's no time to cover all these

- *Generalised unitarity and the leading singularities:*
 - Z. Bern, L. J. Dixon, D. C. Dunbar and D. A. Kosower, "One-loop n-point gauge theory amplitudes, unitarity and collinear limits", Nucl.Phys. B425, 217 (1994), arXiv:hep-ph/9403226 [hep-ph].
 - Z. Bern, L. J. Dixon, D. C. Dunbar and D. A. Kosower, "Fusing gauge theory tree amplitudes into loop amplitudes", Nucl.Phys. B435, 59 (1995), arXiv:hep-ph/9409265 [hep-ph].
 - Z. Bern, L. J. Dixon, D. C. Dunbar and D. A. Kosower, "One-loop self-dual and N=4 Super Yang-Mills", Phys.Lett. B394, 105 (1997), arXiv:hep-th/9611127 [hep-th].
 - R. Britto, F. Cachazo and B. Feng, "Generalized unitarity and one-loop amplitudes in N=4 super-Yang-Mills", Nucl.Phys. B725 (2005) 275-305, hep-th/0412103.
 - F. Cachazo, "Sharpening the leading singularity", arXiv:0803.1988 [hep-th].
- *Original papers on soft theorems and earlier applications:*
 - S. Weinberg, "Infrared photons and gravitons", Phys.Rev. 140 (1965) B516-B524.
 - F. Low, "Bremsstrahlung of very low-energy quanta in elementary particle collisions", Phys.Rev. 110 (1958) 974-977.
 - T. Burnett and N. M. Kroll, "Extension of the low soft photon theorem", Phys.Rev.Lett. 20 (1968) 86.
 - V. Del Duca, "High-energy Bremsstrahlung Theorems for Soft Photons", Nucl.Phys. B345 (1990) 369-388
 - N. Arkani-Hamed, F. Cachazo, and J. Kaplan, "What is the Simplest Quantum Field Theory?", JHEP 1009 (2010) 016, arXiv:0808.1446 [hep-th]
- *Soft theorems and asymptotic symmetries:*
 - A. Strominger, "On BMS Invariance of Gravitational Scattering", arXiv:1312.2229 [hep-th].
 - A. Strominger, "Asymptotic Symmetries of Yang-Mills Theory", arXiv:1308.0589 [hep-th].
 - F. Cachazo and A. Strominger, "Evidence for a New Soft Graviton Theorem", arXiv:1404.4091 [hep-th].
 - T. He, V. Lysov, P. Mitra, and A. Strominger, "BMS supertranslations and Weinberg's soft graviton theorem", arXiv:1401.7026 [hep-th]
 - A not too complete but way longer set of references is found in the bibliography, which covers different flavours of the topic; the ones above is thought to cover (at least partially) the core of the ideas. Most likely, there are important papers missing, and we apologise in advance for any reference missing.

References

- [1] S. Weinberg, “Infrared photons and gravitons,” *Phys.Rev.* **140** (1965) B516–B524. • F. Low, “Bremsstrahlung of very low-energy quanta in elementary particle collisions,” *Phys.Rev.* **110** (1958) 974–977. • T. Burnett and N. M. Kroll, “Extension of the low soft photon theorem,” *Phys.Rev.Lett.* **20** (1968) 86.
- [2] F. Cachazo and A. Strominger, “Evidence for a New Soft Graviton Theorem,” [arXiv:1404.4091](#) [hep-th].
- [3] V. Del Duca, “High-energy Bremsstrahlung Theorems for Soft Photons,” *Nucl.Phys.* **B345** (1990) 369–388.
- [4] N. Arkani-Hamed, F. Cachazo, and J. Kaplan, “What is the Simplest Quantum Field Theory?,” *JHEP* **1009** (2010) 016, [arXiv:0808.1446](#) [hep-th].
- [5] E. Casali, “Soft sub-leading divergences in Yang-Mills amplitudes,” [arXiv:1404.5551](#) [hep-th]. • B. U. W. Schwab and A. Volovich, “Subleading soft theorem in arbitrary dimension from scattering equations,” [arXiv:1404.7749](#) [hep-th].
- [6] A. Strominger, “On BMS Invariance of Gravitational Scattering,” [arXiv:1312.2229](#) [hep-th]. • A. Strominger, “Asymptotic Symmetries of Yang-Mills Theory,” [arXiv:1308.0589](#) [hep-th].
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- [8] T. He, V. Lysov, P. Mitra, and A. Strominger, “BMS supertranslations and Weinberg’s soft graviton theorem,” [arXiv:1401.7026](#) [hep-th].
- [9] S. Pasterski, A. Strominger, and A. Zhiboedov, “New Gravitational Memories,” [arXiv:1502.06120](#) [hep-th]. • D. Kapec, M. Pate, and A. Strominger, “New Symmetries of QED,” [arXiv:1506.02906](#) [hep-th]. • D. Kapec, V. Lysov, S. Pasterski, and A. Strominger, “Higher-Dimensional Supertranslations and Weinberg’s Soft Graviton Theorem,” [arXiv:1502.07644](#) [gr-qc]. • T. He, P. Mitra, and A. Strominger, “2D Kac-Moody Symmetry of 4D Yang-Mills Theory,” [arXiv:1503.02663](#) [hep-th].