Knowns and unknowns in the prediction of $B \to K^*\mu\mu$

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- Known knowns
- Known unknowns
  or perhaps unknown knowns…
- Using data

Sixth Workshop on Theory, Phenomenology and Experiments in Flavour Physics - FPCapri2016

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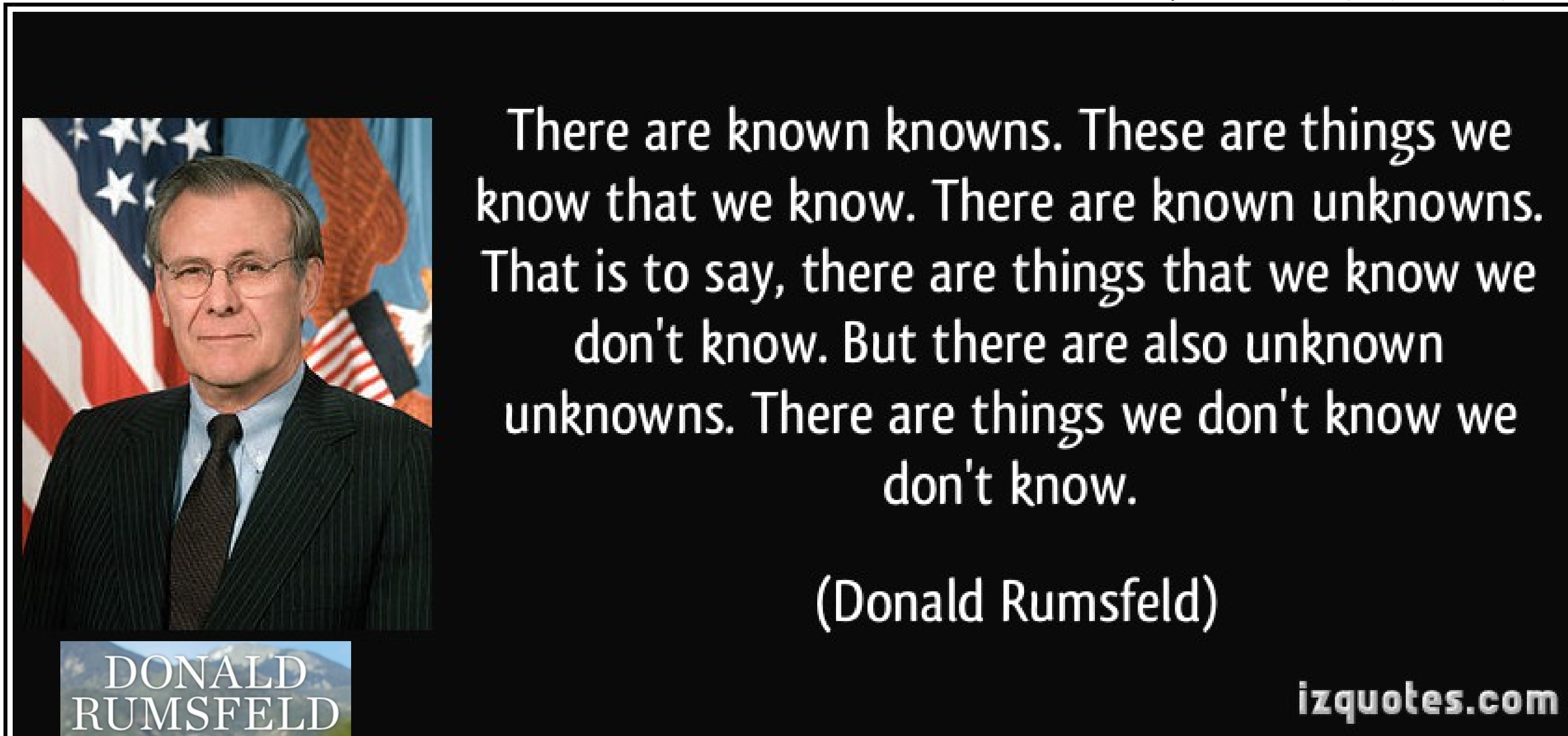
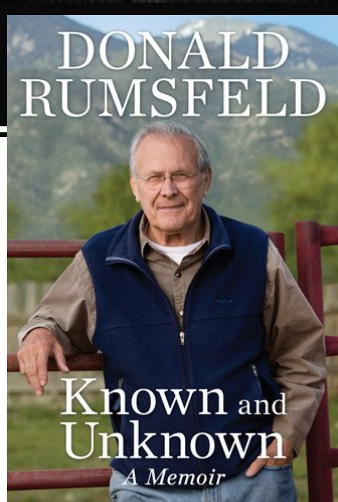
Knowns and Unknowns

2003 Foot in Mouth Award





The concept of unknown knowns (thing we know but we prefer to ignore), introduced by later thinkers, may also be relevant for the present discussion

Known knowns in B → K*μμ (i)

$$\frac{d^{(4)}\Gamma}{dq^2\, d(\cos\theta_l) d(\cos\theta_k) d\phi} = \frac{9}{32\pi} \Big(I_1^s \sin^2\theta_k + I_1^c \cos^2\theta_k + (I_2^s \sin^2\theta_k + I_2^c \cos^2\theta_k)\cos 2\theta_l + I_3 \sin^2\theta_k \sin^2\theta_l \cos 2\phi + I_4 \sin 2\theta_k \sin 2\theta_l \cos\phi + I_5 \sin 2\theta_k \sin\theta_l \cos\phi + (I_6^s \sin^2\theta_k + I_6^c \cos^2\theta_K)\cos\theta_l + I_7 \sin 2\theta_k \sin\theta_l \sin\phi + I_8 \sin 2\theta_k \sin 2\theta_l \sin\phi + I_9 \sin^2\theta_k \sin^2\theta_l \sin 2\phi \Big)$$

angular analysis

$$S_i = \left(I_i^{(s,c)} + \bar{I}_i^{(s,c)} \right) / \Gamma'$$

$$\left(2\,\Gamma' \equiv d\Gamma/dq^2 + d\bar{\Gamma}/dq^2 \right)$$

8 CP-AVERAGED OBSERVABLES

$$F_L, A_{FB}, S_{3,4,5,7,8,9}$$

In the helicity amplitude formalism: $(m_\ell \sim 0)$

$$I_1^c = -I_2^c = \frac{F}{2}\left(|H_V^0|^2 + |H_A^0|^2\right),$$

$$I_1^s = 3I_2^s = \frac{3}{8} F\left(|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2\right),$$

$$I_3 = -\frac{F}{2}\mathrm{Re}\left[H_V^+(H_V^-)^* + H_A^+(H_A^-)^*\right],$$

$$I_4 = \frac{F}{4}\mathrm{Re}\left[(H_V^+ + H_V^-)(H_V^0)^* + (H_A^+ + H_A^-)(H_A^0)^*\right],$$

$$I_5 = \frac{F}{4}\mathrm{Re}\left[(H_V^- - H_V^+)(H_A^0)^* + (H_A^- - H_A^+)(H_V^0)^*\right],$$

$$I_6^s = F\mathrm{Re}\left[H_V^-(H_A^-)^* - H_V^+(H_A^+)^*\right],$$

$$I_6^c = 0,$$

$$I_7 = \frac{F}{2}\mathrm{Im}\left[(H_A^+ + H_A^-)(H_V^0)^* + (H_V^+ + H_V^-)(H_A^0)^*\right],$$

$$I_8 = \frac{F}{4}\mathrm{Im}\left[(H_V^- - H_V^+)(H_V^0)^* + (H_A^- - H_A^+)(H_A^0)^*\right],$$

$$I_9 = \frac{F}{4}\mathrm{Im}\left[H_V^+(H_V^-)^* + H_A^+(H_A^-)^*\right].$$

We need to compute few helicity amplitudes:

$$H_{V,A}^{\lambda} \qquad \lambda = 0, \pm$$

Known knowns in B → K*μμ (ii)

$$H_V{}^\lambda = \frac{4iG_F m_B}{\sqrt{2}} \frac{e^2}{16\pi^2} \lambda_t \left\{ C_9^{\text{eff}} \tilde{V}_{L\lambda} + \frac{m_B^2}{q^2} \left[\frac{2m_b}{m_B} C_7^{\text{eff}} \tilde{T}_{L\lambda} - 16\pi^2 h_\lambda \right] \right\},$$

$$H_A{}^\lambda = \frac{4iG_F m_B}{\sqrt{2}} \frac{e^2}{16\pi^2} \lambda_t C_{10} \tilde{V}_{L\lambda}.$$

NNLO Wilson coefficients from the ΔB=1 effective Hamiltonian:

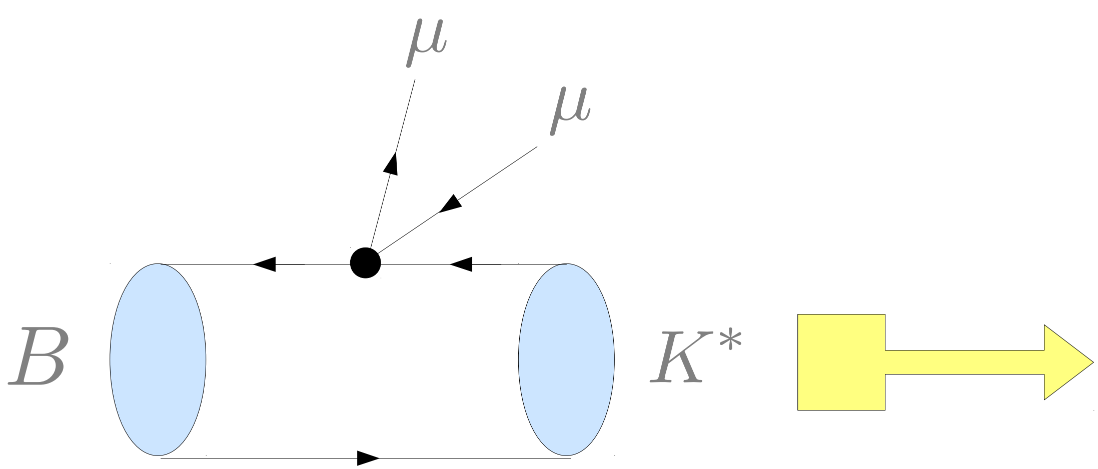
$$\mathcal{H}_{\text{eff}}^{\Delta B=1} = \mathcal{H}_{\text{eff}}^{sl+\gamma} + \mathcal{H}_{\text{eff}}^{had}$$

$$\mathcal{H}_{\text{eff}}^{sl+\gamma} = -\frac{4G_F}{\sqrt{2}} \lambda_t \left(C_7 Q_{7\gamma} + C_9 Q_{9V} + C_{10} Q_{10A} \right)$$

$$Q_{7\gamma} = \frac{e}{16\pi^2} m_b \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R\,,$$

$$Q_{9V} = \frac{\alpha_e}{4\pi} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \ell)\,,$$

$$Q_{10A} = \frac{\alpha_e}{4\pi} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \gamma^5 \ell)\,.$$



Hadronic matrix elements of quark currents: **FORM FACTORS**

Known knowns in B → K*μμ (iii)

Seven form factors are computed in QCD using:

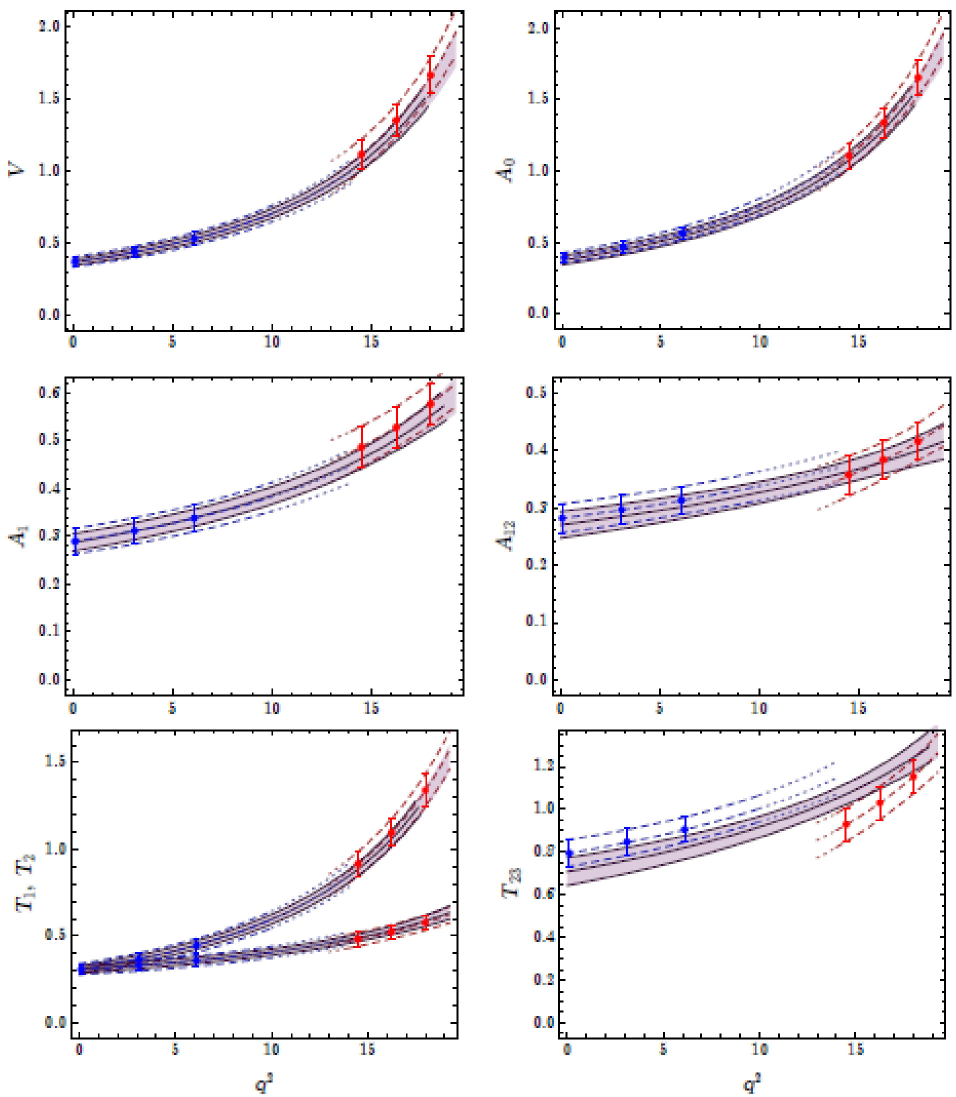
- lattice in the low recoil region
- light-cone sum rules in the large recoil region

$$F^{(i)}(q^2) = \sum_k \alpha_k^{(i)} \frac{\left(z(q^2) - z(0)\right)^k}{1 - q^2/m_{R,i}^2}$$

Results given in terms of the coefficients of the z-expansion with the full correlation matrix

The FF results in QCD with full correlations make the debate about the infinite mass limit with soft functions, power corrections, optimization, etc. rather obsolete

Bharucha, Straub, Zwicky, 1503.05534



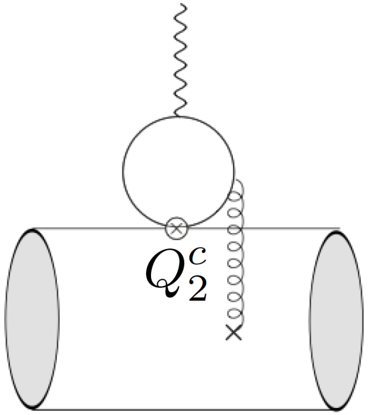
LQCD results from Horgan et al., 1310.3722

Known unknowns in B → K*μμ

$$H_V{}^{\lambda} = \frac{4iG_F m_B}{\sqrt{2}} \frac{e^2}{16\pi^2} \lambda_t \left\{ C_9^{\text{eff}} \tilde{V}_{L\lambda} + \frac{m_B^2}{q^2} \left[\frac{2m_b}{m_B} C_7^{\text{eff}} \tilde{T}_{L\lambda} - 16\pi^2 h_\lambda \right] \right\}$$

$$h_\lambda(q^2) = \frac{\epsilon_\mu^*(\lambda)}{m_B^2} \int d^4x e^{iqx} \langle \bar{K}^* | T\{ j^\mu_{\text{em}}(x) \mathcal{H}^{\text{had}}_{\text{eff}}(0) \} | \bar{B} \rangle$$

Non-factorizable power-suppressed contributions of 4-quark operators to the matrix element



- dominated by

$$Q_1^c = (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L),$$
$$Q_2^c = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L),$$

the charm pair can be close to the resonant region

Do we know how to compute them?

In general, no!

Making the unknowns known…

An exploratory study in 2 steps:

KMPW, arXiv:1006.4945

1. at $q^2 \ll 4m_c^2$ the c loop is dominated by light-cone dynamics. One can write $\left[\mathcal{H}_\mu^{(B\to K^{(*)})}(p,q)\right]_{nonfact} = 2C_1\langle K^{(*)}(p)|\widetilde{\mathcal{O}}_\mu(q)|B(p+q)\rangle$, where $\widetilde{\mathcal{O}}_\mu(q) = \int d\omega\, I_{\mu\rho\alpha\beta}(q,\omega)\bar{s}_L\gamma^\rho\delta[\omega - \frac{(in_+\mathcal{D})}{2}]\widetilde{G}_{\alpha\beta}b_L$ is a non-local operator representing the first subleading term of an expansion in $\Lambda^2/(4m_c^2-q^2)$, (single soft gluon approximation), whose ME is computed using LCSR

   1 ➤ estimate the hadronic contribution for $q^2 < 1$ GeV^2

   but no hard gluons, no phases, large errors (100%?), …?

2. extend the previous result to all q^2 using a dispersion relation, where the spectral functions are given by the 2 physical poles plus additional effective poles that parametrize the integral over the cut at $q^2 > 4m_D^2$

2 ➤

$$\Delta C_{9,i}^{(c\bar{c})}(q^2) = \frac{r_{1,i}\left(1-\frac{\bar{q}^2}{q^2}\right) + \Delta C_{9,i}^{(c\bar{c})}(\bar{q}^2)\frac{\bar{q}^2}{q^2}}{1 + r_{2,i}\frac{\bar{q}^2-q^2}{m_{J/\psi}^2}}$$

$r_{1,i}$	$r_{2,i}$
$0.10^{+0.02}_{-0.00}$	$1.13^{+0.00}_{-0.01}$
$0.09^{+0.01}_{-0.00}$	$1.12^{+0.00}_{-0.01}$
$0.06^{+0.04}_{-0.10}$	$1.05^{+0.05}_{-0.04}$

but model dependence(?), no pert. gluons and phases, ρ positivity…?

Knowing unknowns using data

MC, Fedele, Franco, Mishima, Paul, Silvestrini, Valli, 1512.07157

1. adopt the following parametrization

$$h_\lambda(q^2) = h_\lambda^{(0)} + q^2 h_\lambda^{(1)} + q^4 h_\lambda^{(2)}$$

$$\mathrm{abs}(h_\lambda^{(i)}) \in [0,\, 2\cdot 10^{-3}]$$
$$\arg(h_\lambda^{(i)}) \in [0,\, 2\pi)$$

2. enforce the helicity amplitude suppression on h_+ Jaeger&Camalich, 1212.2263
3. assume there are no contribution beyond the SM
4. include/exclude the effect of th. constraints on $h_\lambda^{(i)}$:
   - KMPW: LCSR results at $q^2 \leq 1$ GeV^2 Khodjamirian, Mannel, Pivovarov, Wang, 1006.4945
   - fullKMPW: LCSR+dispersion relation on the whole q^2 range
   - noKMPW: no KMPW constraints on $h_\lambda^{(i)}$
5. fit the $h_\lambda^{(i)}$ from the LHCb binned angular variables (including correlations) and BRs of $B \to K^*\mu\mu$, $B \to K^*ee$, $B \to K^*\gamma$

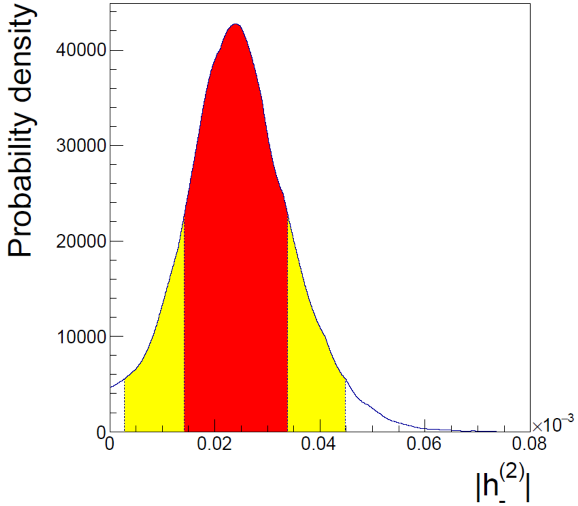
(using HEPfit – https://hepfit.roma1.infn.it)

Results for the h_λ's

KD

Parameter	Absolute value	Phase (rad)
$h_0^{(0)}$	$(5.7 \pm 2.0) \cdot 10^{-4}$	3.57 ± 0.55
$h_0^{(1)}$	$(2.3 \pm 1.6) \cdot 10^{-4}$	0.1 ± 1.1
$h_0^{(2)}$	$(2.8 \pm 2.1) \cdot 10^{-5}$	-0.2 ± 1.7
$h_+^{(0)}$	$(7.9 \pm 6.9) \cdot 10^{-6}$	0.1 ± 1.7
$h_+^{(1)}$	$(3.8 \pm 2.8) \cdot 10^{-5}$	-0.7 ± 1.9
$h_+^{(2)}$	$(1.4 \pm 1.0) \cdot 10^{-5}$	3.5 ± 1.6
$h_-^{(0)}$	$(5.4 \pm 2.2) \cdot 10^{-5}$	3.2 ± 1.4
$h_-^{(1)}$	$(5.2 \pm 3.8) \cdot 10^{-5}$	0.0 ± 1.7
$h_-^{(2)}$	$(2.5 \pm 1.0) \cdot 10^{-5}$	0.09 ± 0.77

Hint of a large charm contribution even beyond the SM



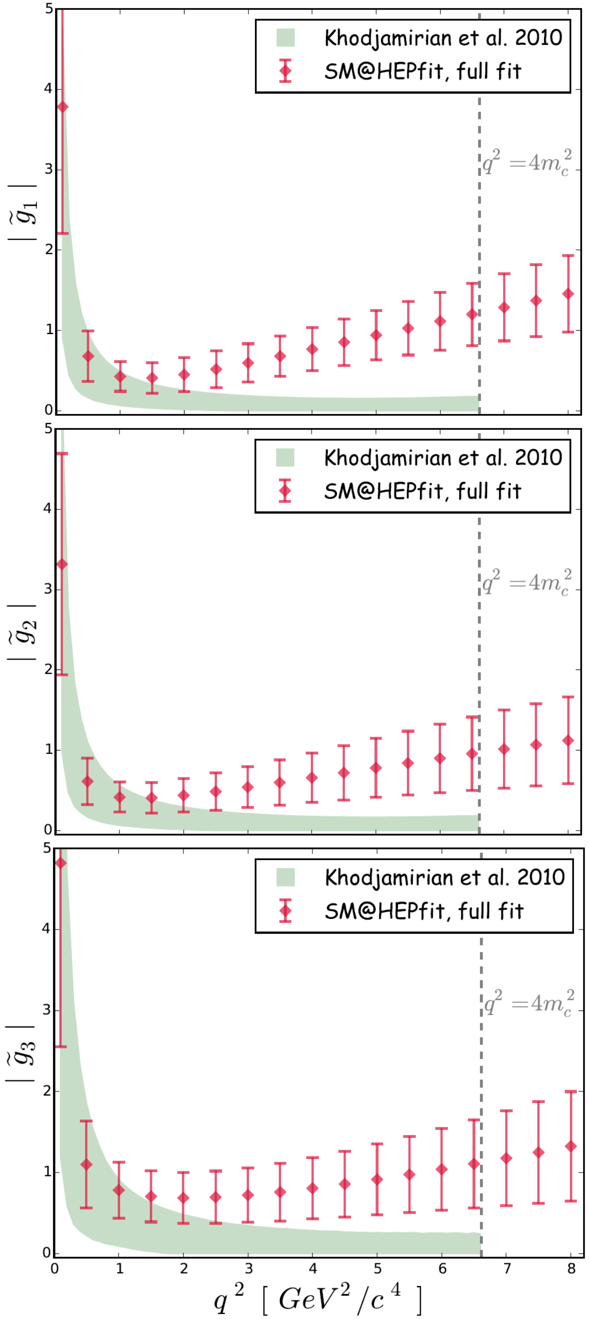
noKD

Parameter	Absolute value	Phase (rad)
$h_0^{(0)}$	$(5.8 \pm 2.1) \cdot 10^{-4}$	3.54 ± 0.56
$h_0^{(1)}$	$(2.9 \pm 2.1) \cdot 10^{-4}$	0.2 ± 1.1
$h_0^{(2)}$	$(3.4 \pm 2.8) \cdot 10^{-5}$	-0.4 ± 1.7
$h_+^{(0)}$	$(4.0 \pm 4.0) \cdot 10^{-5}$	0.2 ± 1.5
$h_+^{(1)}$	$(1.4 \pm 1.1) \cdot 10^{-4}$	0.1 ± 1.7
$h_+^{(2)}$	$(2.6 \pm 2.0) \cdot 10^{-5}$	3.8 ± 1.3
$h_-^{(0)}$	$(2.5 \pm 1.5) \cdot 10^{-4}$	$1.85 \pm 0.45 \cup 4.75 \pm 0.75$
$h_-^{(1)}$	$(1.2 \pm 0.9) \cdot 10^{-4}$	$-0.90 \pm 0.70 \cup 0.80 \pm 0.80$
$h_-^{(2)}$	$(2.2 \pm 1.4) \cdot 10^{-5}$	0.0 ± 1.2

No firm conclusion, but...

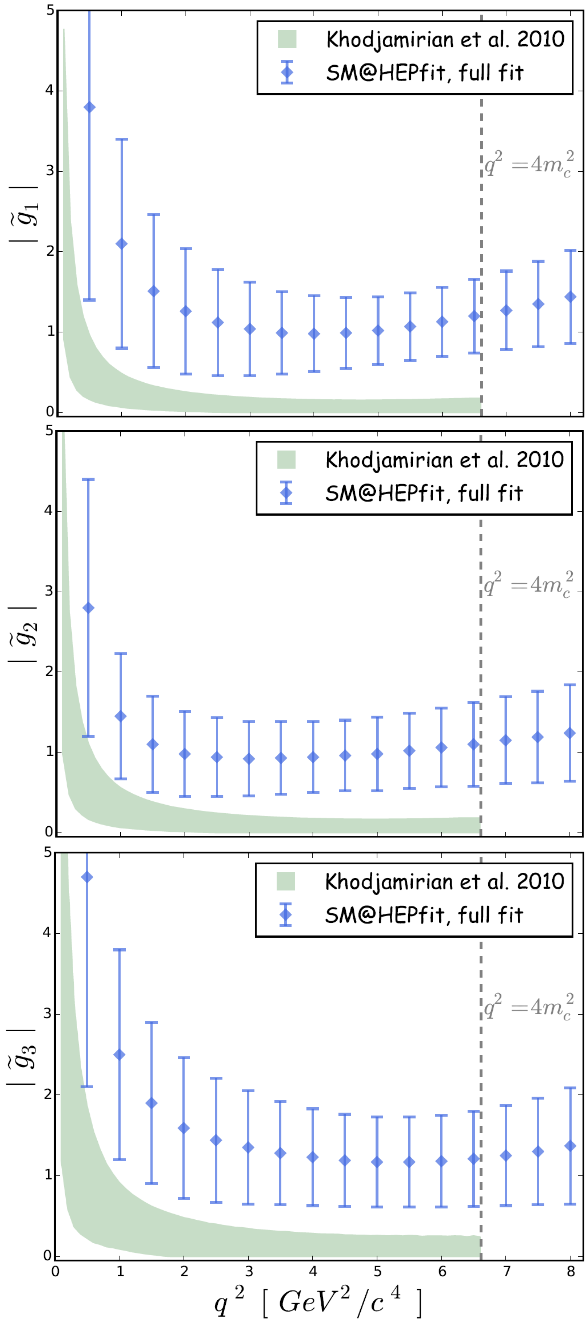
Comparing with KMPW

KD

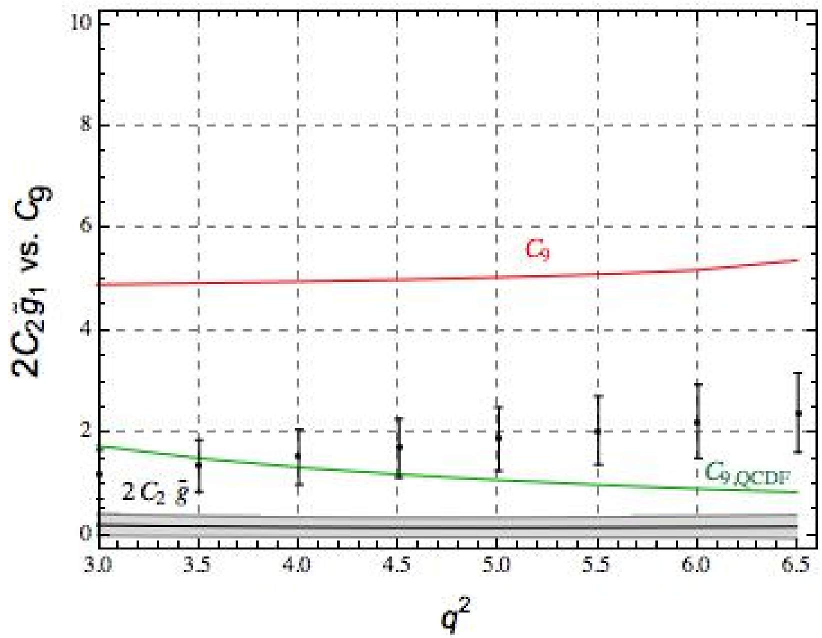


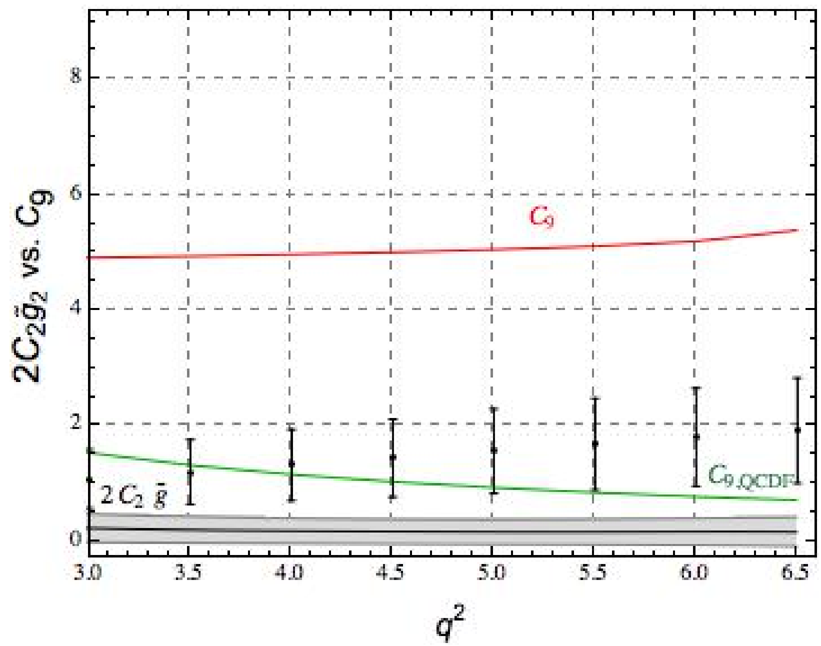
$$\Delta C_{9,i}^{\mathrm{had}} = 2C_2\tilde{g}_i$$

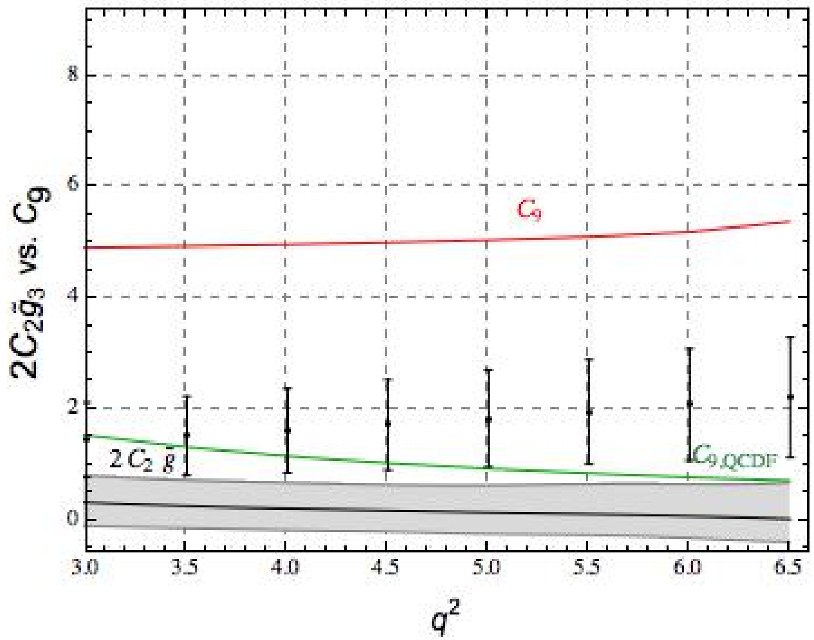
noKD



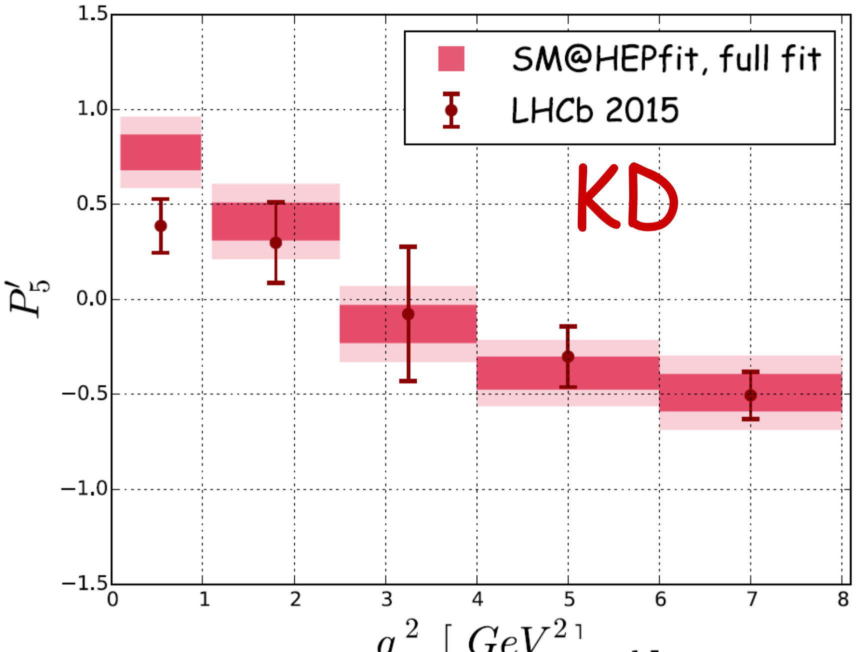
How large is the correction?

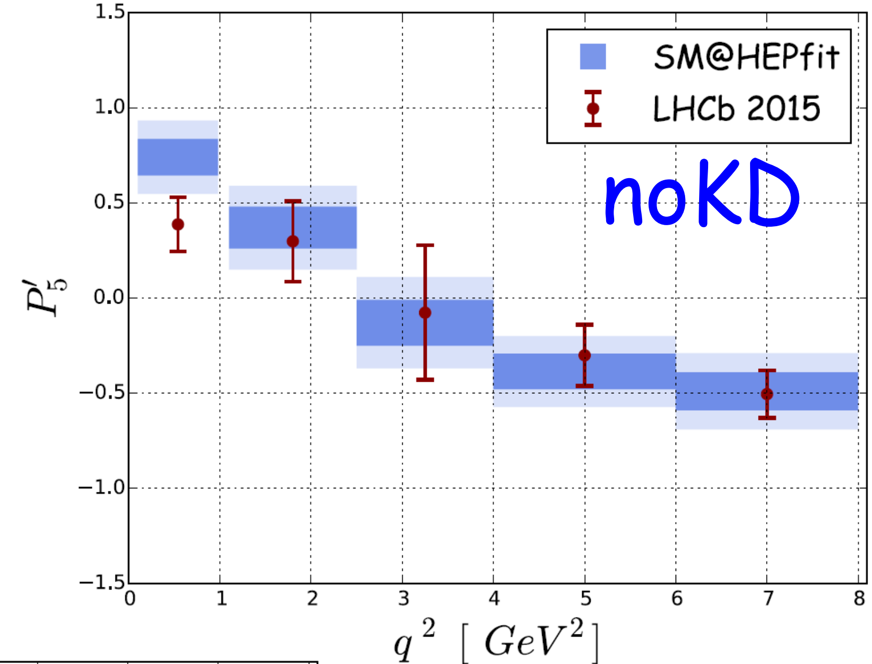


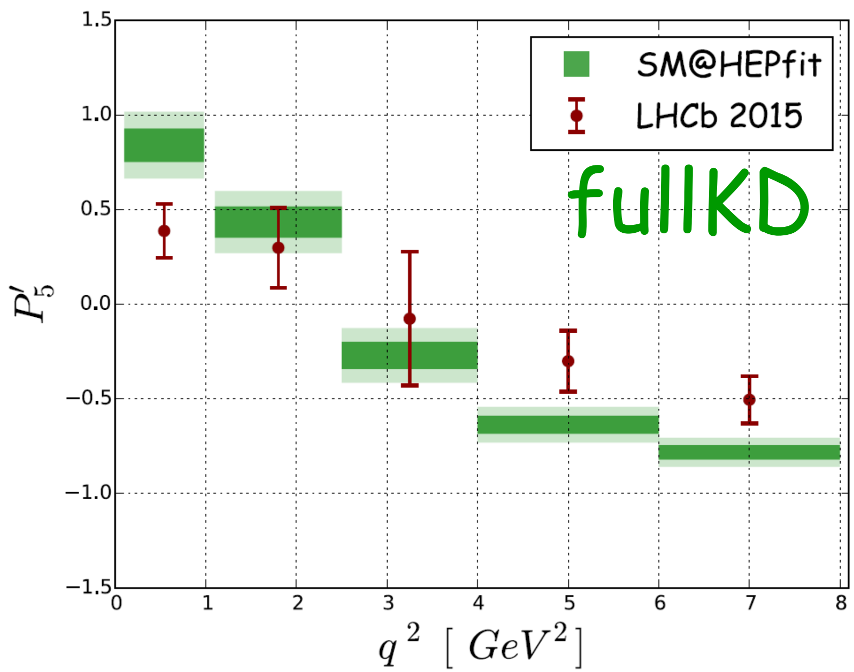




The fate of the anomaly







Comparison of the fits

The Bayes factor cannot be computed from the MCMC

Information Criterion

T. Ando, Am. J. Math. and Manag. Sc. 31 (2011), no. 1-2 13

$$\mathrm{IC} = -2\overline{\log L} + 4\sigma^2_{\log L}$$

Results

noKMPW:	IC=72
KMPW:	IC=78
no q^4 terms:	IC=81
fullKMPW:	IC=111

Summary

The factorizable part of the amplitude seems nowadays under control in QCD - *known knows*

The non-factorizable, power suppressed charm contribution is poorly known - *known unknowns*

Anomalies appear and disappear depending on the assumptions on the charm contribution, yet no violation of the naive power counting expectation is required to describe the $B \to K^*\mu\mu$ data

precise SM prediction of $B \to K^*\mu\mu \leftrightarrow$ control on h_λ's

NP contributions in C_7 and C_9 cannot be disentangled in $B \to K^*\mu\mu$ decays unless they violate symmetries conserved by the charm contribution (CP, LF, LFU,…)

- Towards a calculations...

"Any reasonable calculation is better than a fit!" – T. Hurth

from Nazila's talk

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from Nazila's talk

- Towards a calculations...

"Any ~~reasonable~~ calculation is better than a fit!" – T. Hurth

solid

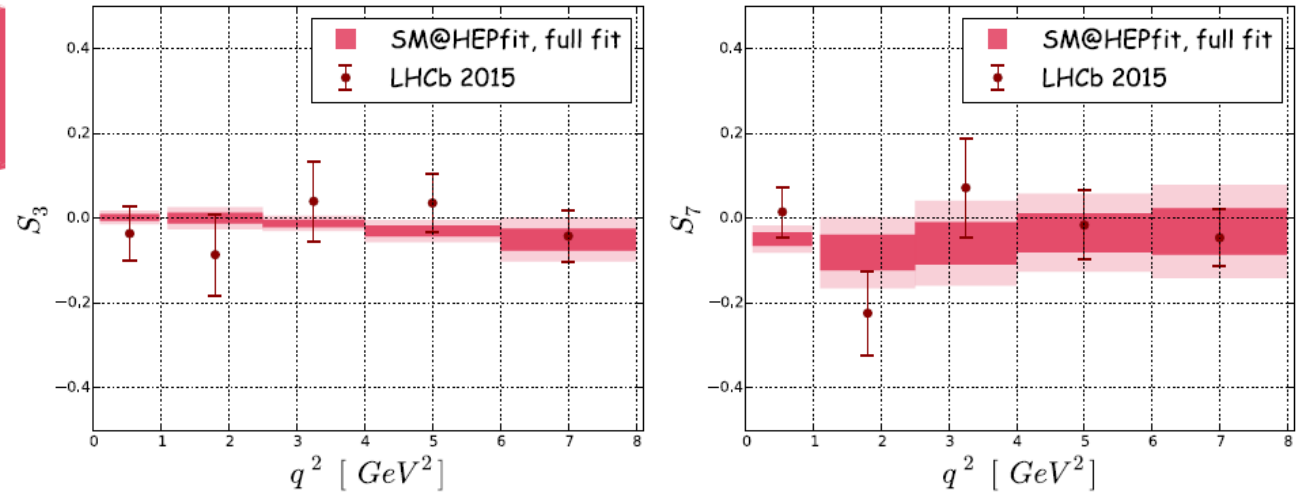
from Nazila's talk

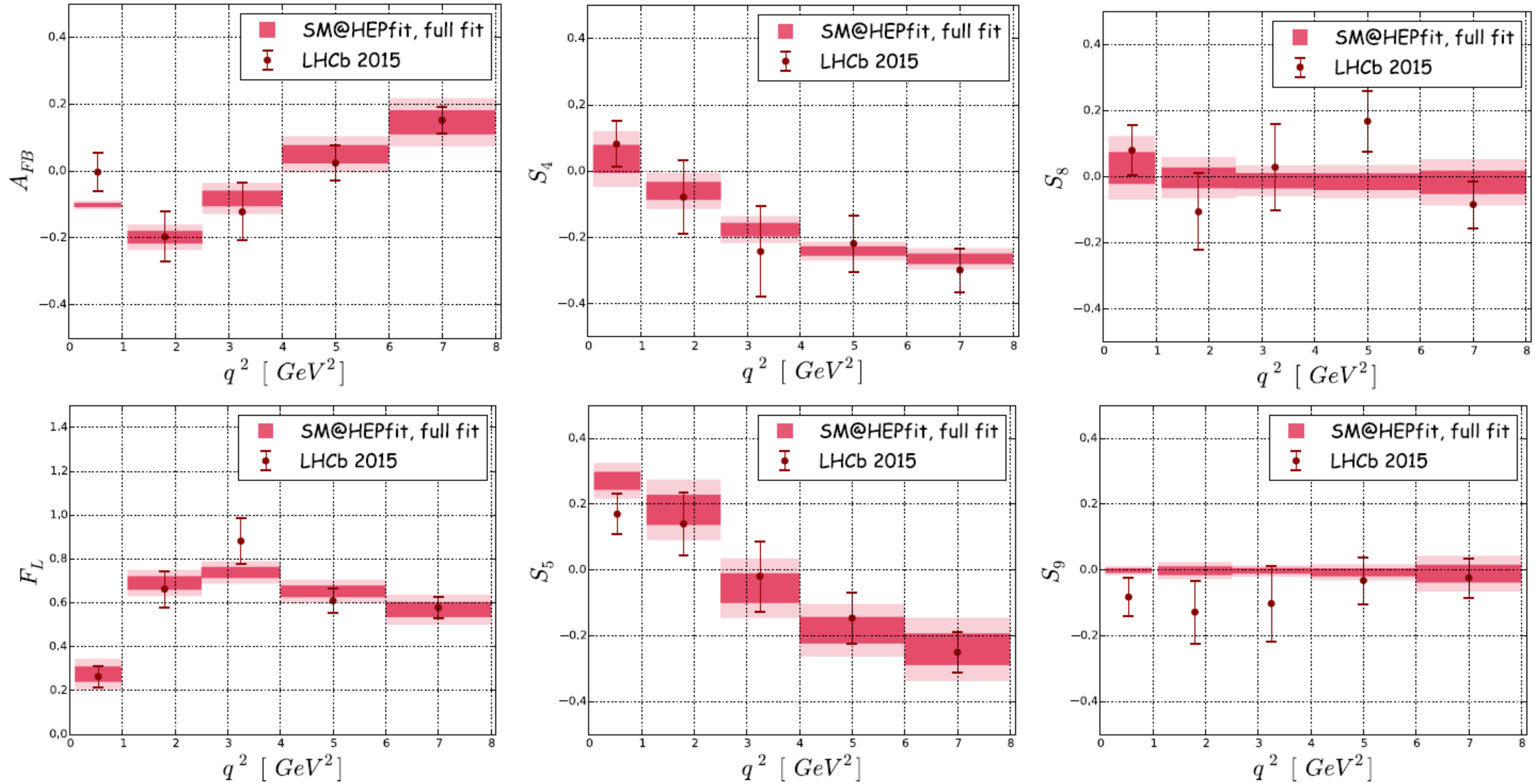
NP CLAIMS

- M. Ciuchini

Backup

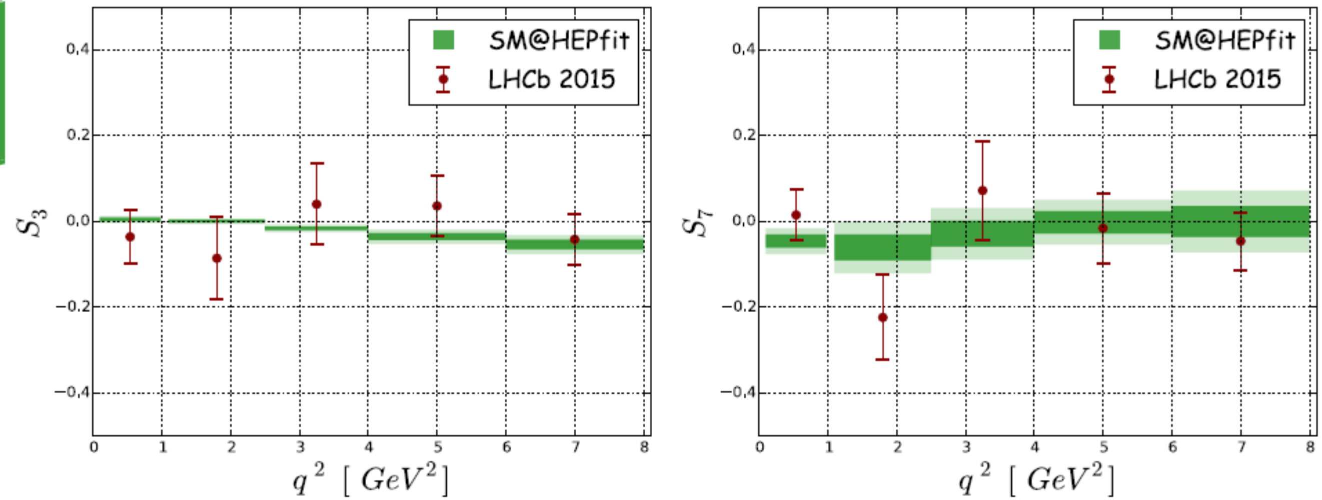
full fit

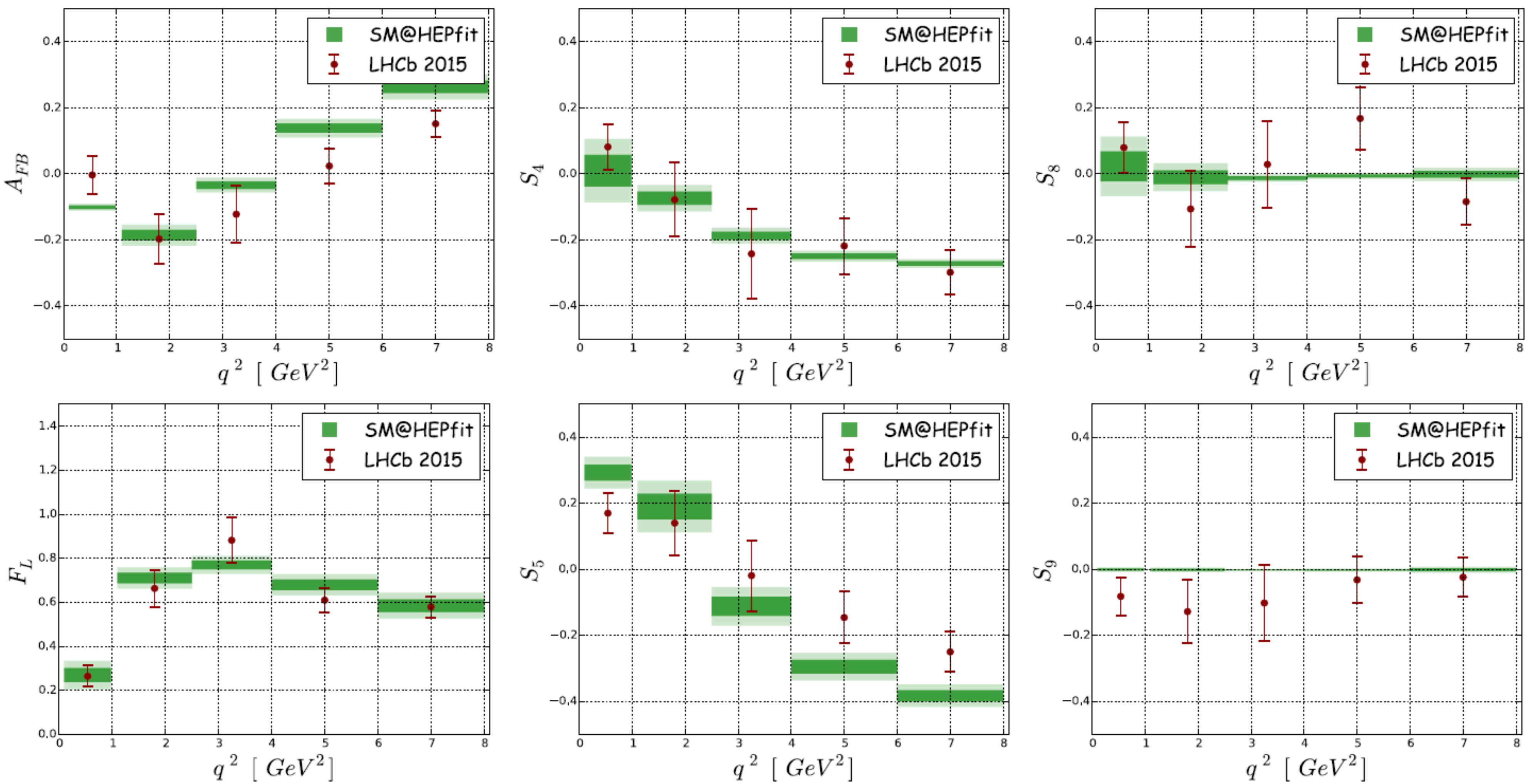

SM@HEPfit, full fit
LHCb 2015
S_3
S_7
q^2 [GeV^2]


SM@HEPfit, full fit
LHCb 2015
A_{FB}
S_4
S_8
F_L
S_5
S_9
q^2 [GeV^2]

HEPfit

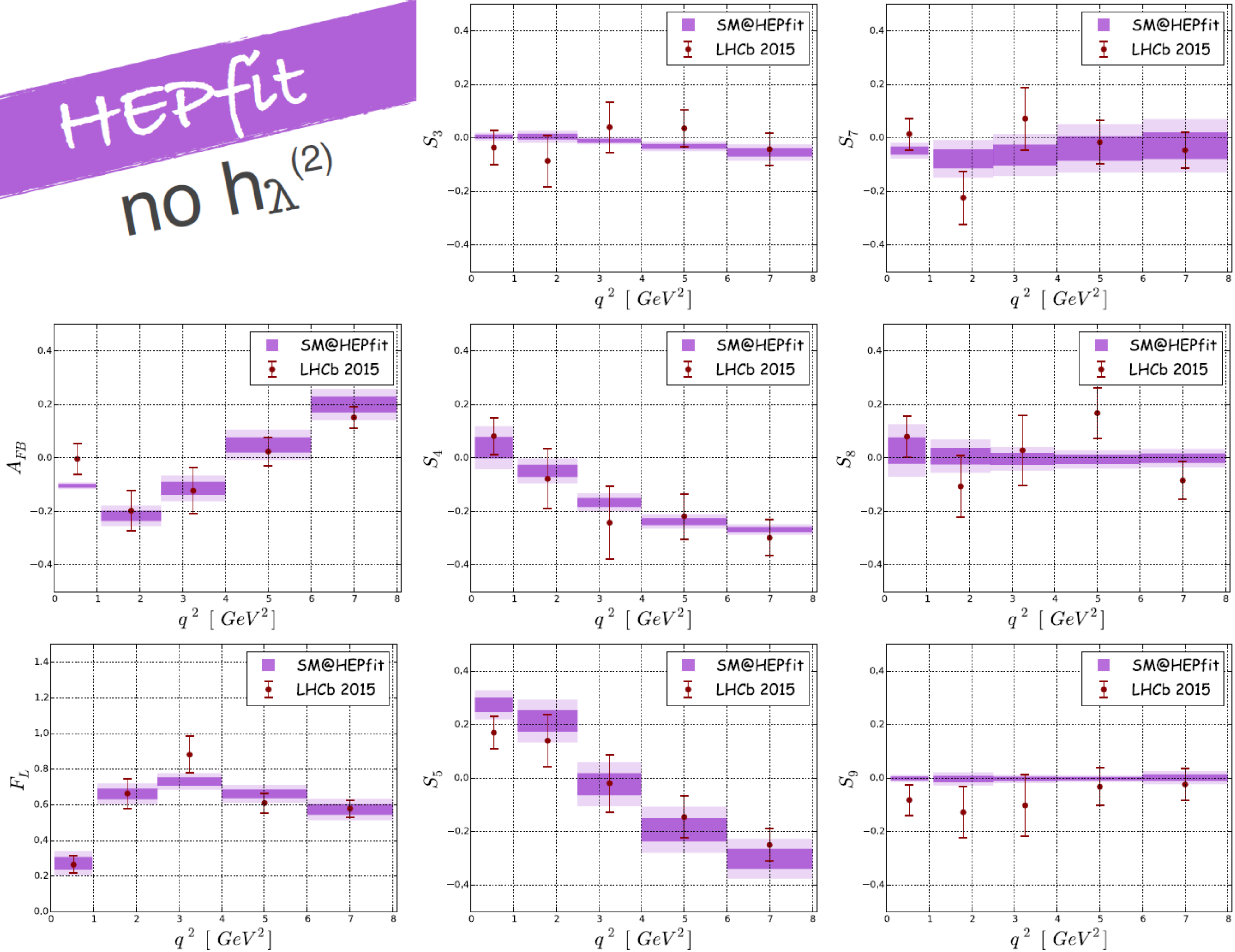
“Full” Khodj.

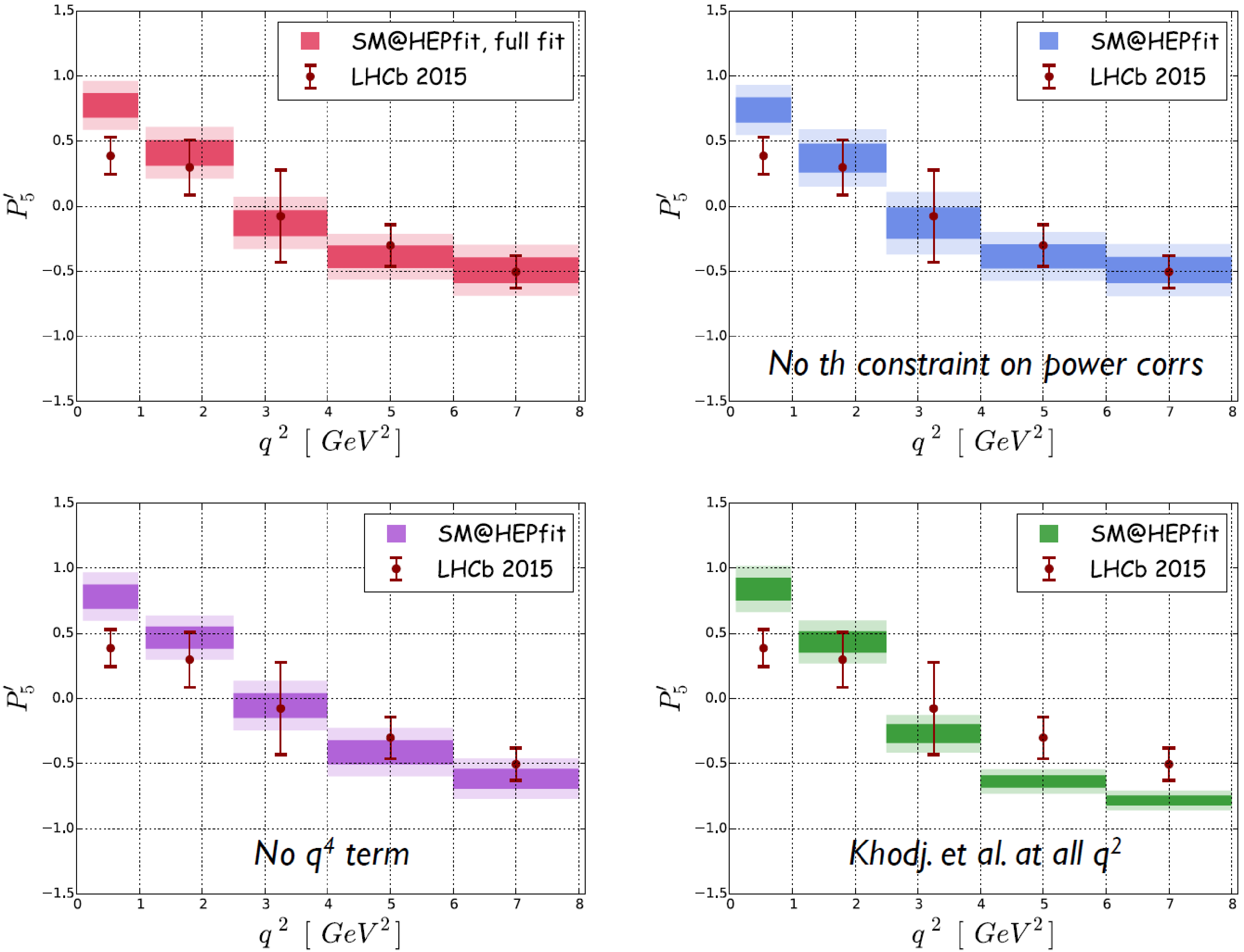




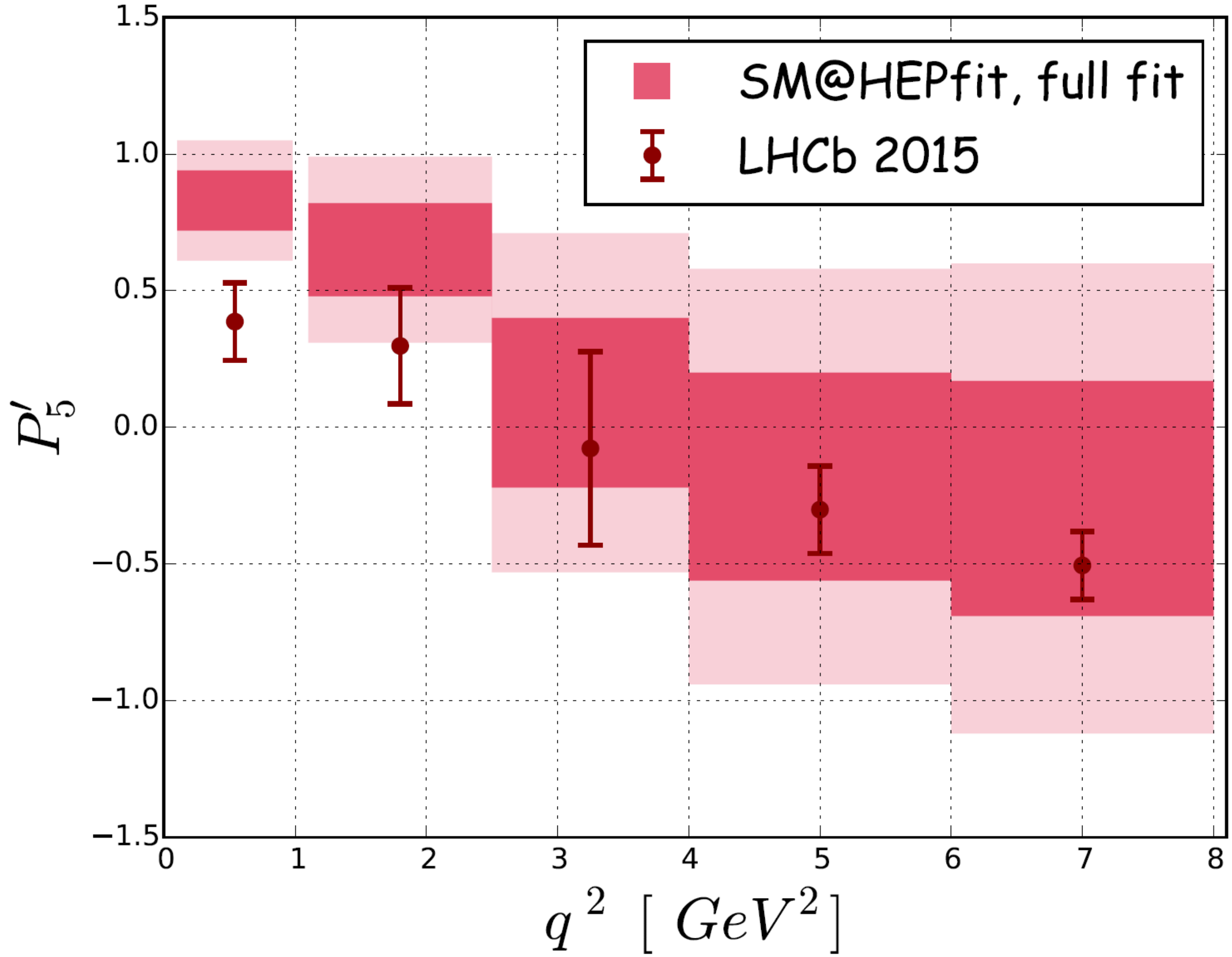
HEPfit

no $h_\lambda^{(2)}$




SM@HEPfit, full fit
LHCb 2015
SM@HEPfit
LHCb 2015
No th constraint on power corrs
SM@HEPfit
LHCb 2015
No q⁴ term
SM@HEPfit
LHCb 2015
Khodj. et al. at all q²
P_5'
q^2 [GeV^2]

Prediction?



SM@HEPfit, full fit
LHCb 2015
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
1
2
3
4
5
6
7
8
P'_5
q^2 [GeV^2]