Power corrections in radiative leptonic B decay

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Why power corrections?

- Understanding the general properties power expansion in EFTs (HQET, SCET, NRQCD).

- Interesting to understand the strong interaction dynamics of heavy quark decays.
  - Factorization properties of the subleading-power amplitudes.
  - Renormalization and asymptotic properties of higher-twist B-meson DA.
  - Interplay of different QCD techniques.

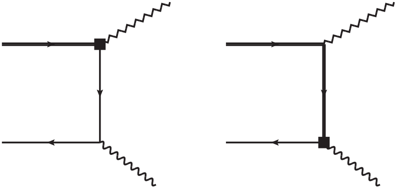
- Precision determinations of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$.
  Power corrections, QED corrections, BSM physics.

- Crucial to understand the CP violation in B-meson decays.
  Strong phase of $\mathscr{A}(B \to M_1 M_2)$ @ m_b scale in the leading power.

- Indispensable for understanding the flavour puzzles.
  - P_5' anomaly in $B \to K^* \ell^+ \ell^-$.
  - Color suppressed hadronic B-meson decays.
  - Polarization fractions of penguin dominated $B_{(s)} \to VV$ decays.

General aspects of $B \to \gamma \ell \nu$

- Tree diagrams:



Kinematics:

$$p_B \equiv p + q = m_B \, v \,, \qquad p = \frac{n \cdot p}{2} \, \bar{n} \,, \qquad q = \frac{n \cdot q}{2} \, \bar{n} + \frac{\bar{n} \cdot q}{2} \, n \,.$$

- Decay amplitude:

$$\mathcal{M}(B^- \to \gamma \ell \, \nu) = \frac{G_F \, V_{ub}}{\sqrt{2}} \, (i \, g_{em} \, \varepsilon_\nu^*) \left\{ T^{\nu\mu}(p,q) \, \bar{\ell} \, \gamma_\mu \, (1-\gamma_5) \nu + Q_\ell \, f_B \, \bar{\ell} \, \gamma^\nu \, (1-\gamma_5) \nu \right\} .$$

Hadronic tensor:

$$\begin{aligned} T_{\nu\mu}(p,q) &\equiv \int d^4x \, e^{ip\cdot x} \langle 0 | \mathrm{T}\{ j_{\nu,em}(x), \left[\bar{u} \gamma_\mu (1-\gamma_5) b \right] (0) \} | B^-(p+q) \rangle \,, \\ &= v \cdot p \left[-i \, \varepsilon_{\mu\nu\rho\sigma} \, n^\rho \, v^\sigma \, F_V(n \cdot p) + g_{\mu\nu} \, \hat{F}_A(n \cdot p) \right] + v_\nu \, p_\mu \, F_1(n \cdot p) \\ &\quad + v_\mu \, p_\nu \, F_2(n \cdot p) + v \cdot p \, v_\mu \, v_\nu \, F_3(n \cdot p) + \frac{p_\mu \, p_\nu}{v \cdot p} \, F_4(n \cdot p) \,. \end{aligned}$$

General aspects of $B \to \gamma \ell\, \nu$

- Ward identity [Grinstein and Pirjol, 2000; Khodjamirian and Wyler, 2001]:

$$p_\nu\, T^{\nu\mu}(p,q) = -(Q_b - Q_u) f_B\, p_B^\mu \,.$$

$$\Downarrow$$

$$\hat{F}_A(v\cdot p) = -F_1(v\cdot p)\,, \qquad F_3(v\cdot p) = -\frac{(Q_b - Q_u) f_B\, m_B}{(v\cdot p)^2}\,.$$

- Reduced parametrization:

$$T_{\nu\mu}(p,q) = -i\, v\cdot p\, \varepsilon_{\mu\nu\rho\sigma}\, n^\rho\, v^\sigma\, F_V(n\cdot p) + \left[g_{\mu\nu}\, v\cdot p - v_\nu\, p_\mu\right] \hat{F}_A(n\cdot p) \underbrace{- \frac{(Q_b - Q_u) f_B\, m_B}{v\cdot p}\, v_\mu\, v_\nu}_{\text{contact term}}\,.$$

- Absorb the photon emission off the lepton [Beneke and Rohrwild, 2011]:

$$\left[g_{\mu\nu}\, v\cdot p - v_\nu\, p_\mu\right] \hat{F}_A(n\cdot p) = -Q_\ell f_B\, g_{\mu\nu} + \left[g_{\mu\nu}\, v\cdot p - v_\nu\, p_\mu\right] \underbrace{\left[\hat{F}_A(n\cdot p) + \frac{Q_\ell f_B}{v\cdot p}\right]}_{F_A(n\cdot p)} \underbrace{+ \frac{v_\nu p_\mu}{v\cdot p}\, Q_\ell f_B}_{\text{irrelevant after the contraction with } \varepsilon_\nu^*}\,.$$

Current status of $B \to \gamma \ell \nu$

- Factorization properties at leading power [Korchemsky, Pirjol and Yan, 2000; Descotes-Genon and Sachrajda, 2002; Lunghi, Pirjol and Wyler, 2003; Bosch, Hill, Lange and Neubert, 2003].

- Leading power contributions at NLL and subleading power corrections at tree level [Beneke and Rohrwild, 2011].

- Subleading power soft two-particle correction at tree level [Braun and Khodjamirian, 2013].

- Subleading power soft two-particle correction at one loop [this talk!].

- Three-particle B-meson DA's contribution at tree level [this talk!].

Dispersion approach

- Basic idea [Khodjamirian, 1999]:

$$\tilde{T}_{\nu\mu}(p,q) \equiv \int d^4x\, e^{ip\cdot x} \langle 0|\mathrm{T}\{j_{\nu,em}(x), [\bar{u}\gamma_\mu(1-\gamma_5)b](0)\}|B^-(p+q)\rangle\big|_{p^2<0},$$

$$= v\cdot p\left[-i\,\varepsilon_{\mu\nu\rho\sigma}\,n^\rho\,v^\sigma\,F_V^{B\to\gamma^*}(n\cdot p,\bar{n}\cdot p) + g_{\mu\nu}^{\perp}\,\hat{F}_A^{B\to\gamma^*}(n\cdot p,\bar{n}\cdot p)\right] + \ldots$$

- Power counting: $n\cdot p \sim \mathcal{O}(m_b), \qquad \bar{n}\cdot p \sim \mathcal{O}(\Lambda)$.
- Dispersion relations:

$$F_V^{B\to\gamma^*}(n\cdot p,\bar{n}\cdot p) = \frac{2}{3}\,\frac{f_\rho\,m_\rho}{m_\rho^2-p^2-i0}\,\frac{2\,m_B}{m_B+m_\rho}\,V(q^2) + \frac{1}{\pi}\int_{\omega_s}^{\infty} d\omega'\,\frac{\mathrm{Im}_{\omega'}\,F_V^{B\to\gamma^*}(n\cdot p,\omega')}{\omega'-\bar{n}\cdot p-i0},$$

$$\hat{F}_A^{B\to\gamma^*}(n\cdot p,\bar{n}\cdot p) = \frac{2}{3}\,\frac{f_\rho\,m_\rho}{m_\rho^2-p^2-i0}\,\frac{2\,(m_B+m_\rho)}{n\cdot p}\,A_1(q^2) + \frac{1}{\pi}\int_{\omega_s}^{\infty} d\omega'\,\frac{\mathrm{Im}_{\omega'}\,\hat{F}_A^{B\to\gamma^*}(n\cdot p,\omega')}{\omega'-\bar{n}\cdot p-i0}.$$

- LCSR for the $B\to\rho$ form factors:

$$\frac{2}{3}\,\frac{f_\rho\,m_\rho}{n\cdot p}\,\mathrm{Exp}\left[-\frac{m_\rho^2}{n\cdot p\,\omega_M}\right]\frac{2\,m_B}{m_B+m_\rho}\,V(q^2) = \frac{1}{\pi}\int_0^{\omega_s} d\omega'\,e^{-\omega'/\omega_M}\left[\mathrm{Im}_{\omega'}\,F_V^{B\to\gamma^*}(n\cdot p,\omega')\right],$$

$$\frac{2}{3}\,\frac{f_\rho\,m_\rho}{n\cdot p}\,\mathrm{Exp}\left[-\frac{m_\rho^2}{n\cdot p\,\omega_M}\right]\frac{2\,(m_B+m_\rho)}{n\cdot p}\,A_1(q^2) = \frac{1}{\pi}\int_0^{\omega_s} d\omega'\,e^{-\omega'/\omega_M}\left[\mathrm{Im}_{\omega'}\,\hat{F}_A^{B\to\gamma^*}(n\cdot p,\omega')\right].$$

Dispersion approach

- Improved dispersion relations (setting $\bar{n}\cdot p = 0$) [Master formula I]:

$$F_V(n\cdot p) = \underbrace{\frac{1}{\pi}\int_0^{\omega_s} d\omega' \, \frac{n\cdot p}{m_\rho^2}\, \mathrm{Exp}\left[\frac{m_\rho^2 - \omega'\, n\cdot p}{n\cdot p\, \omega_M}\right] \left[\mathrm{Im}_{\omega'}\, F_V^{B\to\gamma^*}(n\cdot p, \omega')\right]}_{\text{nonperturbative modification}},$$

$$+\frac{1}{\pi}\int_{\omega_s}^{\infty} d\omega' \, \frac{1}{\omega'} \left[\mathrm{Im}_{\omega'}\, F_V^{B\to\gamma^*}(n\cdot p, \omega')\right].$$

- Comparison with the HQE result [Master formula II]:

$$F_V(n\cdot p) = \underbrace{\frac{1}{\pi}\int_0^{\infty} d\omega' \, \frac{1}{\omega'} \left[\mathrm{Im}_{\omega'}\, F_V^{B\to\gamma^*}(n\cdot p, \omega')\right]}_{\text{HQE expression, not always well defined}}$$

$$+\underbrace{\frac{1}{\pi}\int_0^{\omega_s} d\omega' \left\{\frac{n\cdot p}{m_\rho^2}\, \mathrm{Exp}\left[\frac{m_\rho^2 - \omega'\, n\cdot p}{n\cdot p\, \omega_M}\right] - \frac{1}{\omega'}\right\}}_{\mathcal{O}(1)} \left[\mathrm{Im}_{\omega'}\, F_V^{B\to\gamma^*}(n\cdot p, \omega')\right].$$

- Spectral density at tree level:

$$\frac{1}{\pi}\,\mathrm{Im}_{\omega'}\, F_V^{B\to\gamma^*}(n\cdot p, \omega') = \frac{Q_u f_B m_B}{n\cdot p}\, \underbrace{\phi_B^+(\omega', \mu)}_{\text{of } \mathcal{O}(1/\Lambda)[\mathcal{O}(1/m_b)] \text{ for } \omega' \sim \mathcal{O}(\Lambda)[\omega' \sim \mathcal{O}(\Lambda^2/m_b)]} + \mathcal{O}(\alpha_s, \Lambda/m_b).$$

Power suppressed soft contribution!

Power suppressed soft contributions at one loop

- Perturbative QCD corrections to $\tilde{T}_{\nu\mu}(p,q)$:

$$F_{V,2P}^{B\to\gamma^*}(n\cdot p,\bar{n}\cdot p) = \hat{F}_{A,2P}^{B\to\gamma^*}(n\cdot p,\bar{n}\cdot p)$$
$$= \frac{Q_u \tilde{f}_B(\mu)\, m_B}{n\cdot p}\, C_\perp(n\cdot p,\mu) \int_0^\infty d\omega\, \frac{\phi_B^+(\omega,\mu)}{\omega - \bar{n}\cdot p}\, J_\perp(n\cdot p,\bar{n}\cdot p,\omega,\mu)\,.$$

- B-meson light-cone distribution amplitude [Grozin and Neubert, 1997; Beneke and Feldmann, 2001]:

$$i F_{\rm stat}(\mu)\, \phi_B^+(\omega,\mu) = \frac{1}{2\pi} \int dt\, e^{i\omega t} \langle 0|(\bar{q}_s\, Y_s)(t\bar{n})\, \not{\bar{n}}\, \gamma_5\, (Y_s^\dagger\, b_v)(0)|B(v)\rangle\,.$$

  - One-loop renormalization of $\phi_B^+(\omega,\mu)$ [Lange and Neubert, 2003].
  - Renormalization of $[\bar{q}_s(t\bar{n})\,\Gamma\, b_v(0)]$ does not commute with the shot-distance expansion [Braun, Ivanov and Korchemsky, 2004].

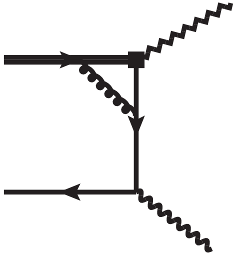
$$[(\bar{q}_s\, Y_s)(t\bar{n})\not{\bar{n}}\,\Gamma\,(Y_s^\dagger\, b_v)(0)]_R \neq \sum_{p=0} \frac{t^p}{p!} \left[\bar{q}_s(0)\,(n\cdot \overleftarrow{D})^p\, \not{\bar{n}}\,\Gamma\, b_v(0)\right]_R\,.$$

  - Eigenfunctions of the Lange-Neubert renormalization kernel [Bell, Feldmann, YMW and Yip, 2013].

- Evaluating the hard and jet functions using the method of regions [Beneke and Smirnov, 1997].

Power suppressed soft contributions at one loop

- Weak vertex correction:



- Hard function only from the weak vertex diagram and the wavefunction renormalization of the external b-quark field:

$$C_\perp(n\cdot p,\mu) = 1 - \frac{\alpha_s(\mu)\,C_F}{4\pi}\left[2\ln^2\frac{\mu}{n\cdot p} + 5\ln\frac{\mu}{m_b} - 2\,\mathrm{Li}_2\left(1-\frac{1}{r}\right) - \ln^2 r + \frac{3r-2}{1-r}\ln r + \frac{\pi^2}{12} + 6\right].$$

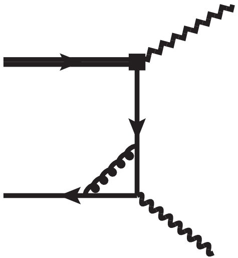
- Hard-collinear contributions obtained from the NLO corrections to the vacuum-to-B-meson correlation function for $B\to\pi$ form factors [or replacing $\bar{n}\cdot k \to \bar{n}\cdot(k-p)$ in $B\to\gamma\ell\nu$].

- Expansion by regions:

$$\begin{aligned}
\tilde{T}^{weak}_{\nu\mu}(p,q) \;&=\; -ig_s^2\,C_F\,\mu^{2\epsilon}\int\frac{d^D l}{(2\pi)^D}\,\frac{1}{[(p_b+l)^2-m_b^2+i0][(p-k+l)^2+i0][l^2+i0]} \\
&\quad\times\left\{n\cdot l\,[(D-2)\,\bar{n}\cdot l+2\,m_b]+2\,n\cdot p\,(\bar{n}\cdot l+m_b)+(D-4)\,l_\perp^2\right\}\,\tilde{T}^{tree}_{\nu\mu}(p,q)\,, \\
&\overset{\text{hc}}{\Rightarrow}\; -ig_s^2\,C_F\,\mu^{2\epsilon}\int\frac{d^D l}{(2\pi)^D}\,\frac{2\,m_b\,n\cdot(p+l)\,\tilde{T}^{tree}_{\nu\mu}(p,q)}{[m_b\,n\cdot l+i0][n\cdot(p+l)\,\bar{n}\cdot(p-k+l)+l_\perp^2+i0][l^2+i0]}\,.
\end{aligned}$$

Power suppressed soft contributions at one loop

- Electromagnetic vertex correction:



- Only contributes to the jet function.
- Partonic amplitude:

$$\tilde{T}^{em}_{\nu\mu}(p,q) = \frac{Q_u\, g_s^2\, C_F}{n\cdot p\, \bar{n}\cdot(k-p)}\, \mu^{2\epsilon} \int \frac{d^D l}{(2\pi)^D} \frac{1}{[l^2+i0][(p-l)^2+i0][(l-k)^2+i0]}$$
$$\bar{u}(k)\, \gamma_\rho \not{l}\, \gamma_\nu^\perp\, (\not{p}-\not{l})\, \gamma^\rho\, (\not{p}-\not{k})\, \gamma_\mu^\perp\, (1-\gamma_5)\, b(p_b)\,.$$

- The scaling behavior of the scalar integral is m_b/Λ.
  The Dirac algebra must induce a power-suppression factor.

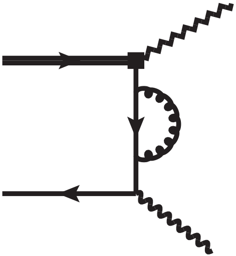
- The resulting jet function:

$$J_{\perp,em} = \frac{\alpha_s(\mu)\, C_F}{4\pi} \left\{ \frac{\ln(1+\eta)}{\eta} \left[2\ln\frac{\mu^2}{-p^2} - \ln(1+\eta) + 3 \right] \underbrace{- \ln\frac{\mu^2}{n\cdot p\,\bar{n}\cdot(k-p)} - 4}_{\text{consistent with } B\to\gamma\ell\nu} \right\},$$
$$\eta = -\bar{n}\cdot k/\bar{n}\cdot p$$

- Can be also obtained in SCET, but more complicated [Pirjol and Wyler, 2003].

Power suppressed soft contributions at one loop

- Renormalization of the internal quark propagator:

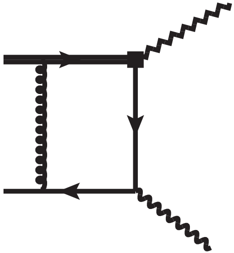


- Only contributes to the jet function:

$$J_{\perp,wfc} = \frac{\alpha_s(\mu)\, C_F}{4\,\pi} \left[\ln \frac{\mu^2}{n\cdot p\, \bar{n}\cdot(k-p)} + 1 \right] .$$

Free of soft and collinear divergences.

- Box diagram:



No leading-power hard-collinear contribution:

$$\tilde{T}^{box}_{\nu\mu}(p,q) = -Q_u\, g_s^2\, C_F\, \mu^{2\varepsilon} \int \frac{d^D l}{(2\,\pi)^D} \frac{1}{[(p_b+l)^2 - m_b^2][(p-k+l)^2][(k-l)^2][l^2]}$$
$$\bar{u}(k)\, \gamma_\rho\, (\not{k} - \not{l})\, \gamma_\nu^\perp\, (\not{p} - \not{k} + \not{l})\, \gamma_\mu^\perp\, (1-\gamma_5)\, (\not{p}_b + \not{l} + m_b)\, \gamma^\rho\, b(p_b)\, .$$

Different from the vacuum-to-B-meson correlator for $B \to \pi$ form factors [YMW and Shen, 2015].

Power suppressed soft contributions at one loop

- Jet function at one loop:

$$J_\perp(n\cdot p,\bar{n}\cdot p,\omega,\mu) = 1 - \frac{\alpha_s(\mu)\,C_F}{4\pi}\left\{\ln^2\frac{\mu^2}{n\cdot p\,(\omega-\bar{n}\cdot p)} - \frac{\pi^2}{6} - 1 - \frac{\bar{n}\cdot p}{\omega}\ln\frac{\bar{n}\cdot p-\omega}{\bar{n}\cdot p}\left[\ln\frac{\mu^2}{-p^2} + \ln\frac{\mu^2}{n\cdot p\,(\omega-\bar{n}\cdot p)} + 3\right]\right\}.$$

- Factorization-scale independence:

$$\frac{d}{d\ln\mu}F_{V,2P}^{B\to\gamma^*}(n\cdot p,\bar{n}\cdot p) = \mathcal{O}(\alpha_s^2)\,.$$

- Static B-meson decay constant:

$$\tilde{f}_B(\mu) = f_B\,K^{-1}(\mu,m_b)\,,\qquad K(\mu,m_b) = 1 + \frac{\alpha_s(\mu)\,C_F}{4\pi}\left[3\ln\frac{m_b}{\mu} - 2\right].$$

- Resummation for the hard functions at NLL:

$$\frac{d}{d\ln\mu}C_\perp(n\cdot p,\mu) = \Big[-\underbrace{\Gamma_{\rm cusp}(\mu)}_{\text{three loops}}\ln\frac{\mu}{n\cdot p} + \underbrace{\gamma_h(\mu)}_{\text{two loops}}\Big]\,C_\perp(n\cdot p,\mu)\,,$$

$$\frac{d}{d\ln\mu}\tilde{f}_B(\mu) = \underbrace{\tilde{\gamma}(\mu)}_{\text{two loops}}\tilde{f}_B(\mu)\,.$$

Leading-power contributions at one loop

- Leading-power HQE expression [Beneke and Rohrwild, 2011]:

$$F_{V,2P}^{\mathrm{LP},\mathrm{NLL}}(n\cdot p) = \frac{Q_u f_B m_B}{n\cdot p} C_{\perp,\mathrm{RG}}(n\cdot p,\mu)\, K_{\mathrm{RG}}^{-1}(\mu,m_b) \times \underbrace{\int_0^\infty d\omega \frac{\phi_B^+(\omega,\mu)}{\omega} J_\perp(n\cdot p,\bar{n}\cdot p=0,\omega,\mu)}\,.$$

$$= \lambda_B^{-1}(\mu)\left\{1+\frac{\alpha_s(\mu)\,C_F}{4\pi}\left[\ln^2\frac{\mu^2}{n\cdot p\,\mu_0}+2\ln\frac{\mu^2}{n\cdot p\,\mu_0}\sigma_1(\mu)+\sigma_2(\mu)-\frac{\pi^2}{6}-1\right]\right\}$$

- Inverse-logarithmic moments:

$$\lambda_B^{-1}(\mu)=\int_0^\infty \frac{d\omega}{\omega}\,\phi_B^+(\omega,\mu)\,,\qquad \sigma_B^{(n)}(\mu)=\lambda_B(\mu)\int_0^\infty\frac{d\omega}{\omega}\ln^n\frac{\mu_0}{\omega}\,\phi_B^+(\omega,\mu)\,.$$

$$\frac{\lambda_B(\mu_0)}{\lambda_B(\mu)}=1+\frac{\alpha_s(\mu_0)\,C_F}{4\pi}\ln\frac{\mu}{\mu_0}\left[2-2\ln\frac{\mu}{\mu_0}-4\,\sigma_B^{(1)}(\mu_0)\right]+\mathcal{O}(\alpha_s^2)\,.$$

$$\frac{d}{d\ln\mu}\sigma_B^{(n)}(\mu)=\mathcal{O}(\alpha_s)\Rightarrow \text{No } [\alpha_s(\mu)]^0\ \ln(\mu/\mu_0) \text{ due to the evolution}\,.$$

- Not aiming at resummation of $\ln^k(\mu/\mu_0)$.
  Resummation in the "dual" momentum space [Bell, Feldmann, YMW and Yip, 2013].

Power suppressed soft contributions at one loop

- Soft contributions at one loop [Master formula II]:

$$F_{V,2P}^{\rm NLP,NLL}(n\cdot p) = \frac{Q_u f_B m_B}{n\cdot p} C_{\perp,\rm RG}(n\cdot p,\mu)\, K_{\rm RG}^{-1}(\mu,m_b) \times \int_0^{\omega_s} d\omega' \left\{ \frac{n\cdot p}{m_\rho^2} {\rm Exp}\left[\frac{m_\rho^2-\omega' n\cdot p}{n\cdot p\,\omega_M}\right] - \frac{1}{\omega'} \right\} \phi_{B,\rm eff}^+(\omega',\mu)\,.$$

- Hard-collinear corrections absorbed into $\phi_{B,\rm eff}^+(\omega',\mu)$:

$$\begin{aligned}\phi_{B,\rm eff}^+(\omega',\mu) &= \phi_B^+(\omega',\mu) + \frac{\alpha_s(\mu)C_F}{4\pi}\left\{ \int_0^{\omega'} d\omega \left[\frac{2}{\omega-\omega'} \ln^2 \frac{\mu^2}{n\cdot p\,(\omega'-\omega)}\right]_+ \phi_B^+(\omega,\mu) \right.\\ &-\omega' \int_0^{\omega'} d\omega \left[\frac{1}{\omega-\omega'} \ln \frac{\omega'-\omega}{\omega}\right]_+ \frac{\phi_B^+(\omega,\mu)}{\omega} \\ &+\frac{\omega'}{2}\int_0^\infty d\omega \ln^2\left|\frac{\omega-\omega'}{\omega'}\right| \frac{d}{d\omega}\left[\frac{\phi_B^+(\omega,\mu)}{\omega}\right] \\ &-\int_{\omega'}^\infty d\omega \left[\ln^2\frac{\mu^2}{n\cdot p\,\omega'} - \frac{\pi^2}{2} - 1\right]\frac{d}{d\omega}\phi_B^+(\omega,\mu) \\ &+\omega'\int_{\omega'}^\infty d\omega \left[\ln\frac{\mu^2}{n\cdot p\,\omega'} \ln\frac{\omega-\omega'}{\omega'} - \frac{1}{2}\ln^2\frac{\mu^2}{n\cdot p\,(\omega-\omega')} + \frac{1}{2}\ln^2\frac{\mu^2}{n\cdot p\,\omega'} \right.\\ &\left.\left. +3\ln\frac{\omega-\omega'}{\omega'} - \frac{2\pi^2}{3}\right]\frac{d}{d\omega}\left[\frac{\phi_B^+(\omega,\mu)}{\omega}\right] \right\}.\end{aligned}$$

End-point divergences in QCD factorization.

Sub-leading power corrections in QCD factorization

- Power corrections from the hard-collinear light-quark and the b-quark propagators:

$$F^{\rm HQE}_{V,NLP}(n\cdot p) = F^{\rm NLC}_{V,NLP}(n\cdot p) + F^{\rm LC}_{V,NLP}(n\cdot p)\,,$$
$$F^{\rm HQE}_{A,NLP}(n\cdot p) = F^{\rm NLC}_{V,NLP}(n\cdot p) + F^{\rm LC}_{A,NLP}(n\cdot p)\,.$$

- Subleading-power local contribution [Beneke and Rohrwild, 2011]:

$$F^{\rm LC}_{V,NLP}(n\cdot p) = \frac{Q_u f_B m_B}{(n\cdot p)^2} + \frac{Q_b f_B m_B}{n\cdot p\, m_b}\,,$$
$$F^{\rm LC}_{A,NLP}(n\cdot p) = -\left[\frac{Q_u f_B m_B}{(n\cdot p)^2} + \frac{Q_b f_B m_B}{n\cdot p\, m_b}\right] + \frac{2\,Q_l f_B}{n\cdot p}\,.$$

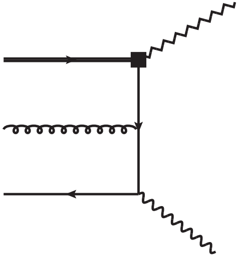
- Subleading-power non-local contribution:

$$F^{\rm NLC}_{V,NLP}(n\cdot p) = \xi(v\cdot p) \stackrel{?}{=} -\frac{i Q_u}{(n\cdot p)^2}\,\overline{C}_{\perp,NLP}(n\cdot p,\mu)$$
$$\times \int ds \left\langle 0 \left| [\bar{u}\,Y_s](s\bar{n})\,\frac{i n\cdot \overleftarrow{\partial}}{i\bar{n}\cdot \overleftarrow{\partial}}\,\not{n}\,(1-\gamma_5)\,[Y_s^\dagger\, b_v](0)\right| B^-(v)\right\rangle \tilde{J}_{NLP}\left(\frac{\mu^2\, s}{v\cdot p}\right).$$

Can $\xi(v\cdot p)$ be matched onto $F^{\rm NLP}_{V,2P}$ [Braun and Khodjamirian, 2013]?

Three-particle B-meson DA's contributions

- Subleading power corrections from higher Fock states:



- Quark propagator in the background gluon field [Balitsky and Braun, 1988]:

$$\langle 0|T\{q(x)\,,\bar{q}(0)\}|0\rangle|_G \supset -\frac{i}{16\pi^2}\frac{1}{x^2}\int_0^1 du\,\left[\not{x}\,\sigma_{\alpha\beta} - 4\,i\,u\,x_\alpha\,\gamma_\beta\right] \times \underbrace{G^{\alpha\beta}(u\,x)}_{\equiv g_s T^a G^a_{\mu\nu}}\,.$$

- Three-particle B-meson DA's contributions [Khodjamirian, Mannel and Offen, 2007]:

$$\langle 0|\bar{u}_\alpha(x)\,G_{\lambda\rho}(u\,x)\,b_v(0)|B^-(v)\rangle\Big|_{x^2=0} = \frac{F_{\rm stat}(\mu)}{4}\int_0^\infty d\omega\int_0^\infty d\xi\,e^{-i(\omega+u\,\xi)\,v\cdot x}\Bigg[(1+\not{v})\Bigg\{(v_\lambda\,\gamma_\rho - v_\rho\,\gamma_\lambda)\left[\Psi_A(\omega,\xi)-\Psi_V(\omega,\xi)\right] - i\,\sigma_{\lambda\rho}\,\Psi_V(\omega,\xi) - \frac{x_\lambda\,v_\rho - x_\rho\,v_\lambda}{v\cdot x}\,X_A(\omega,\xi) + \frac{x_\lambda\,\gamma_\rho - x_\rho\,\gamma_\lambda}{v\cdot x}\,Y_A(\omega,\xi)\Bigg\}\gamma_5\Bigg]\,.$$

See also [Kawamura, Kodaira, Qiao and Tanaka, 2001; Geye and Witzel, 2013].

- Work in the coordinate space, compute the $\int d^4x\,e^{ip\cdot x}$ integral, and do the power counting.

Three-particle B-meson DA's contributions

- Three-particle contributions at tree level:

$$F_{V,3P}^{B\to\gamma^*}(n\cdot p,\bar{n}\cdot p) = \hat{F}_{A,3P}^{B\to\gamma^*}(n\cdot p,\bar{n}\cdot p)$$
$$= -\frac{Q_u\tilde{f}_B(\mu)\,m_B}{(n\cdot p)^2}\int_0^\infty d\omega\int_0^\infty d\xi\int_0^1 du\left\{\frac{\rho_{3P}^{(2)}(u,\omega,\xi)}{[\bar{n}\cdot p-\omega-u\,\xi]^2}+\frac{\rho_{3P}^{(3)}(u,\omega,\xi)}{[\bar{n}\cdot p-\omega-u\,\xi]^3}\right\},$$
$$\rho_{3P}^{(2)}(u,\omega,\xi) = \Psi_V(\omega,\xi)+(1+2u)\,\Psi_A(\omega,\xi)\,,\qquad \rho_{3P}^{(3)}(u,\omega,\xi) = -2\,(1+2u)\,\bar{X}_A(\omega,\xi)\,,$$
$$\bar{X}_A(\omega,\xi) = \int_0^\omega d\eta\,X_A(\eta,\xi)\,,\qquad \bar{Y}_A(\omega,\xi) = \int_0^\omega d\eta\,Y_A(\eta,\xi)\,.$$

- Small-momenta behaviours ($\omega\to 0\,,\xi\to 0$) [Khodjamirian, Mannel and Offen, 2007]:

$$\Psi_V(\omega,\xi)\sim\Psi_A(\omega,\xi)\sim\xi^2\,,\qquad \bar{X}_A(\omega,\xi)\sim\omega\,\xi^2\,,\qquad \bar{Y}_A(\omega,\xi)\sim\omega\,\xi\,.$$

  LO QCD sum rule analysis, assume the "true" behaviors reproduced by the perturbative analysis.

- HQE result for the three-particle DA's effect:

$$F_{V,3P}^{\rm HQE}(n\cdot p) = -\frac{Q_u\tilde{f}_B(\mu)\,m_B}{(n\cdot p)^2}\int_0^\infty d\omega\int_0^\infty d\xi\left\{\frac{1}{\omega(\omega+\xi)}\,\Psi_V(\omega,\xi)\right.$$
$$\left.+\left[\frac{1}{\omega(\omega+\xi)}-\frac{2}{\xi\,(\omega+\xi)}+\frac{2}{\xi^2}\ln\frac{\omega+\xi}{\omega}\right]\Psi_A(\omega,\xi)+\frac{4\,\omega+\xi}{\omega^2\,(\omega+\xi)^2}\,\bar{X}_A(\omega,\xi)\right\}.$$

  End-point divergences in QCD factorization (speculated in [Braun and Khodjamirian, 2013])!
  $\Rightarrow$ Three-particle contributions cannot be written as a "pure" HQE expression plus a correction.

Three-particle B-meson DA's contributions

- Nonperturbative modification [Master formula I]:

$$F_{V,3P}(n\cdot p) = \underbrace{F_{3P,soft}(n\cdot p)}_{\text{IR cutoff}:\ \omega_M.} + F_{3P,hard}(n\cdot p),$$

$$F_{3P,soft}(n\cdot p) = \frac{1}{\pi}\int_0^{\omega_s} d\omega' \, \frac{n\cdot p}{m_\rho^2}\, \mathrm{Exp}\left[\frac{m_\rho^2 - \omega'\, n\cdot p}{n\cdot p\, \omega_M}\right] \left[\mathrm{Im}_{\omega'}\, F^{B\to\gamma^*}_{V,3P}(n\cdot p, \omega')\right],$$

$$F_{3P,hard}(n\cdot p) = \frac{1}{\pi}\int_{\omega_s}^{\infty} d\omega' \, \frac{1}{\omega'} \left[\mathrm{Im}_{\omega'}\, F^{B\to\gamma^*}_{V,3P}(n\cdot p, \omega')\right].$$

- Power counting.
  Leading power contributions [Beneke and Feldmann, 2003]:

$$F^{\mathrm{HQE}}_{V,LP}(n\cdot p) \sim \langle \gamma(p)|\bar{q}_s \not{A}_{\perp(\gamma)} \frac{1}{i\bar{n}\cdot \overleftarrow{D}_s} \frac{\not{n}}{2}\, \Gamma\, b_v\, |B(p+q)\rangle \sim \left(\frac{\Lambda}{m_b}\right)^{1/2},$$

  Three-particle contributions:

$$\omega_s \sim \omega_M \sim \Lambda^2/m_b \;\Rightarrow\; F_{3P,soft}(n\cdot p) \sim F_{3P,hard}(n\cdot p) \sim \left(\frac{\Lambda}{m_b}\right)^{3/2}.$$

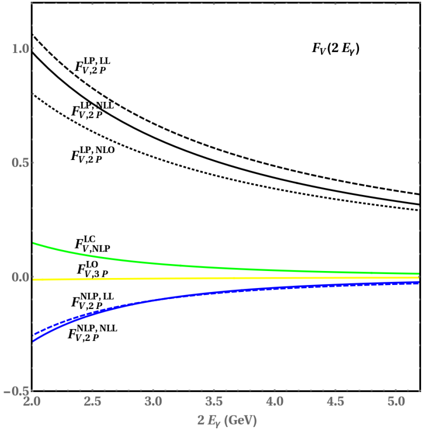
  - Three-particle contributions power suppressed.
  - But both the "soft" and "hard" effects from the three-particle DA are of the same power.
  - Can be speculated from the rapidity divergences of the HQE result.

Final expressions of the $B \to \gamma$ form factors

- Adding up the leading and subleading power contributions:

$$F_V(n\cdot p) = F_{V,2P}^{\rm LP,NLL}(n\cdot p) + F_{V,2P}^{\rm NLP,NLL}(n\cdot p) + F_{V,3P}(n\cdot p) + F_{V,NLP}^{\rm LC}(n\cdot p)\,,$$
$$F_A(n\cdot p) = \hat{F}_{A,2P}^{\rm LP,NLL}(n\cdot p) + \hat{F}_{A,2P}^{\rm NLP,NLL}(n\cdot p) + \hat{F}_{A,3P}(n\cdot p) + F_{A,NLP}^{\rm LC}(n\cdot p)\,.$$

- Breakdown of various contributions [$\lambda_B = 354\,\mathrm{MeV}$]:



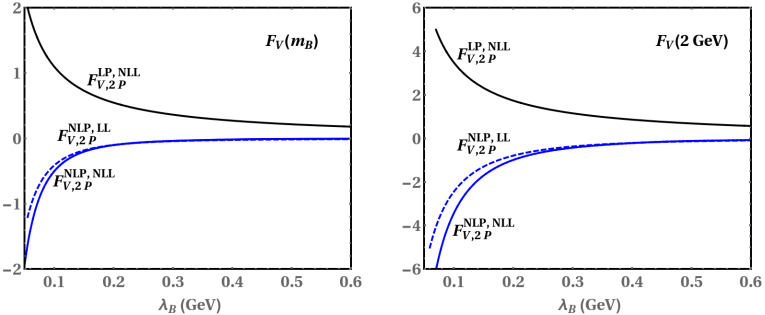
Numerics with central inputs:

$$\begin{aligned}
F_{V,2P}^{\rm LP,LL}(m_B) &= 0.35\,, \\
F_{V,2P}^{\rm LP,NLO}(m_B) &= 0.29\,, \\
F_{V,2P}^{\rm LP,NLL}(m_B) &= 0.31\,, \\
F_{V,2P}^{\rm NLP,LL}(m_B) &= -0.027\,, \\
F_{V,2P}^{\rm NLP,NLL}(m_B) &= -0.022\,, \\
F_{V,NLP}^{\rm LC}(m_B) &= 0.013\,, \\
F_{A,NLP}^{\rm LC}(m_B) &= -0.093\,, \\
F_{V,3P}(m_B) &= -0.0031\,.
\end{aligned}$$

Fixed-order corrections dominant. Resummation effect comparable to the power corrections.

λ_B dependence of the $B \to \gamma$ form factors

- Sizeable soft two-particle correction at small λ_B:



  - For $\lambda_B(\mu_0) = 100\,{\rm MeV}$, $\mathcal{O}\,(45\ \%)$ [$\mathcal{O}\,(100\ \%)$] correction at $n \cdot p = m_B$ [2 GeV].
  - NLL correction to the soft two-particle correction around $\mathcal{O}\,(20 \sim 40)\ \%$.

- Power counting analysis:

$$F_{V,2P}^{\rm LP} \sim F_{V,2P}^{\rm NLP} \sim \left(\frac{m_b}{\Lambda}\right)^{1/2}, \qquad \text{for} \qquad \lambda_B(\mu_0) \sim \Lambda^2/m_b\,,$$

$$F_{V,2P}^{\rm LP} \sim \left(\frac{\Lambda}{m_b}\right)^{1/2},\ F_{V,2P}^{\rm NLP} \sim \left(\frac{\Lambda}{m_b}\right)^{3/2}, \qquad \text{for} \qquad \lambda_B(\mu_0) \sim \Lambda\,.$$

Only consider $\lambda_B(\mu_0) \geq 200\,{\rm MeV}$.

$\phi_B^+(\omega,\mu)$ dependence of the $B \to \gamma$ form factors

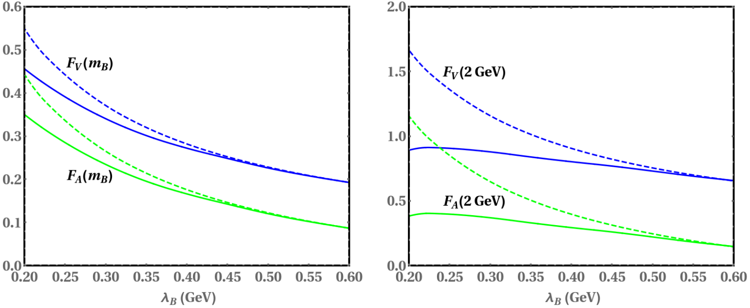
- Two models of $\phi_B^+(\omega,\mu)$ from QCD sum rules:

$$\phi_{B,\mathrm{I}}^+(\omega,\mu_0) = \frac{\omega}{\omega_0^2}\, e^{-\omega/\omega_0}\,,$$

$$\phi_{B,\mathrm{II}}^+(\omega,\mu_0) = \frac{1}{4\pi\,\omega_0}\,\frac{k}{k^2+1}\left[\frac{1}{k^2+1} - \frac{2(\sigma_1(\mu_0)-1)}{\pi^2}\,\ln k\right], \qquad k = \frac{\omega}{1\,\mathrm{GeV}}\,.$$

Constructing the models of $\phi_B^+(\omega,\mu)$ with OPE constraints [Feldmann, Lange and YMW, 2014].

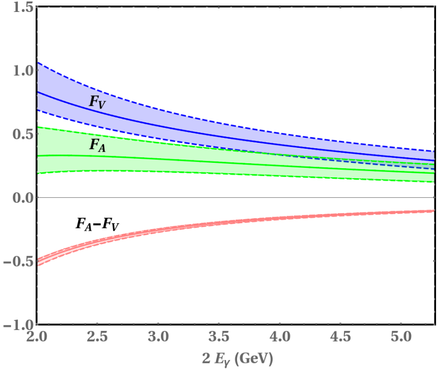
- Numerics of the $B \to \gamma$ form factors:



Solid curves for $\phi_{B,\mathrm{I}}^+(\omega,\mu_0)$, dashed curves for $\phi_{B,\mathrm{II}}^+(\omega,\mu_0)$.

Photon-energy dependence of the $B \to \gamma$ form factors

Including power suppressed two-particle and three-particle corrections:



- Dominant uncertainties from $\lambda_B(\mu_0)$, $\sigma_1(\mu_0)$, $\sigma_2(\mu_0)$ and μ.
- $F_A - F_V$ depends only on f_B:

$$F_A(n \cdot p) - F_V(n \cdot p) = \frac{2 f_B}{n \cdot p} \left[Q_\ell - \frac{Q_u\, m_B}{n \cdot p} - \frac{Q_b\, m_B}{m_b} \right].$$

  Only local symmetry-breaking effect!

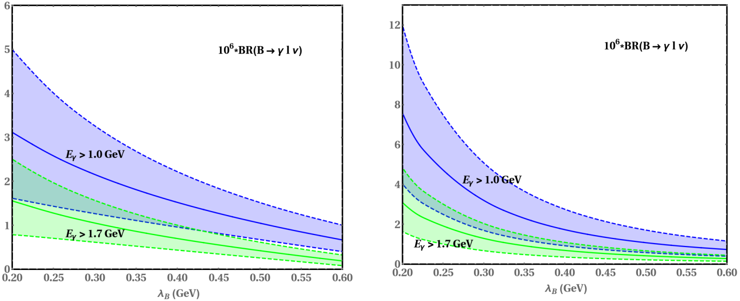
- Faster growing F_V than F_A with the decrease of E_γ.

Partial branching fractions of $B \to \gamma \ell \nu$

- Integrated decay rate $\Delta BR(E_{\rm cut})$:

$$\Delta BR(E_{\rm cut}) = \tau_B \int_{E_{\rm cut}}^{m_B/2} d E_\gamma \, \frac{d\Gamma}{d E_\gamma} \, (B \to \gamma \ell \nu) \, .$$

- $\lambda_B(\mu_0)$ dependence of $\Delta BR(E_{\rm cut})$:

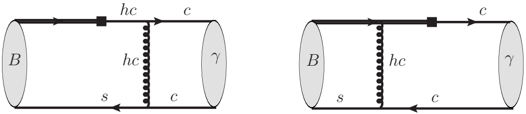


Belle 2015 data: $\Delta BR(1\,{\rm GeV}) < 3.5 \times 10^{-6} \Rightarrow$

- No interesting bound on $\lambda_B(\mu_0)$ for the model $\phi^+_{B,{\rm I}}(\omega, \mu_0)$.
- $\lambda_B(\mu_0) > 214\,{\rm MeV}$ for the model $\phi^+_{B,{\rm II}}(\omega, \mu_0)$.

Photon emission at large distance

- Long-distance photon contribution:



  - “Hadronic" photon effect power suppressed [Beneke and Feldmann, 2003].
  - Can be described by the photon DA [Ball, Braun and Kivel, 2002]:

$$\langle 0|\bar{q}(z)\,\sigma_{\alpha\beta}\,q(-z)|\gamma(q)\rangle = Q_q\, g_{em}\, \underbrace{\chi(\mu)}\, \langle \bar{q} q\rangle \left[q_\beta\,\varepsilon_\alpha - q_\alpha\,\varepsilon_\beta\right] \int_0^1 du\, e^{i(2u-1)\,q\cdot z}\, \underbrace{\phi_\gamma(u,\mu)}\,.$$

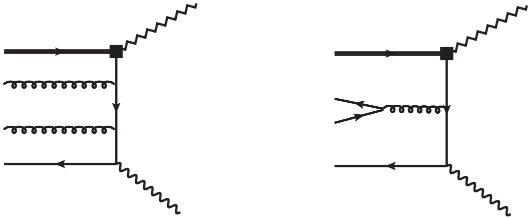
$$\text{magnetic susceptibility, } \chi_{\rm VMD} \approx 2/m_\rho^2\,, \qquad \phi_\gamma^{asy}(u,\mu) = 6\,u\,(1-u)\,.$$

  - Long-distance photon contribution divergent for $\gamma^*\pi \to \gamma$ in QCD factorization. “Hadronic" photon effect in $B \to \gamma\ell\nu$ calculable in HQE?
  - Double counting when adding the “hadronic" photon effect and $F_{2P,soft}$ [$F_{3P,soft}$] together?

- “Hadronic" photon effect from LCSR with photon DAs and j_B [Ball and Kou, 2003].
  - Twist-2 photon DA's contribution: $F_{V,\gamma}^{\rm twist-2}(n\cdot p) = \hat{F}_{A,\gamma}^{\rm twist-2}(n\cdot p)$.
  - Higher-twist phton DA's contribution: $F_{V,\gamma}^{\rm higher-twist}(n\cdot p) \neq \hat{F}_{A,\gamma}^{\rm higher-twist}(n\cdot p)$.
  - $F_{V,\gamma}(m_B) = 0.09 \pm 0.02\,,\ \hat{F}_{A,\gamma}(m_B) = 0.07 \pm 0.02.$ $\Rightarrow$ 30 % hadronic photon corrections.

Future Work

- Yet higher-twist contributions:



Two-gluon-field-strength terms and the covariant derivative of $G^{\mu\nu}$ terms [Balitsky and Braun, 1988]:

$$D_\mu \, G^{\mu\nu}(x) = -g_s \sum_q \bar{q}(x) \gamma^\nu \, T^a \, q(x) \, .$$

Can factorization of four-particle DA be trusted?

- Why is it interesting?
  Lesson from the pion-photon form factor [Agaev, Braun, Offen and Porkert, 2011]:
  Soft dominant ⇒ Correspondence between power expansion and twist counting lost.
  ⇒ Contributions of *all* higher twists yield the power corrections suppressed by *one* power of Q^2.
  True for the B-meson-to-photon form factor [Braun and Khodjamirian, 2013]?

Concluding Remarks

- Understanding power corrections in $B \to \gamma \ell \nu$ important for precision flavour physics.

- Power suppressed contributions in the dispersion approach.
  - NLL two-particle soft correction approximately $(10 \sim 30)\,\%$ at $\lambda_B = 354\,\mathrm{MeV}$.
  - NLO correction to the soft two-particle contribution around $(10 \sim 20)\,\%$.
  - Soft two-particle correction grows rapidly with the decrease of $\lambda_B(\mu_0)$.
  - Three-particle contributions (soft $\oplus$ hard) of order $\mathcal{O}(1)\%$.
  - “Soft" and “hard" three-particle contributions are of the same power.
  - Rapidity divergences of three-particle contributions in QCD factorization.

- The inverse moment $\lambda_B(\mu_0)$ not sufficient to describe $B \to \gamma \ell \nu$ in general.

- Can the power suppressed soft contributions be identified as $\xi(E_\gamma)$ in QCD factorization? Non-local subleading power corrections in SCET [Beneke, Kirilin and Rohrwild].

- Understanding the symmetry breaking effects due to the yet higher-twist photon DA. Mismatch of the power expansion and the twist expansion.

- Perturbative corrections to the three-particle contribution in the dispersion approach. Renormalization properties of the three-particle B-meson DAs.