Inclusive Semileptonic Penguin Decays

Subleading power factorisation

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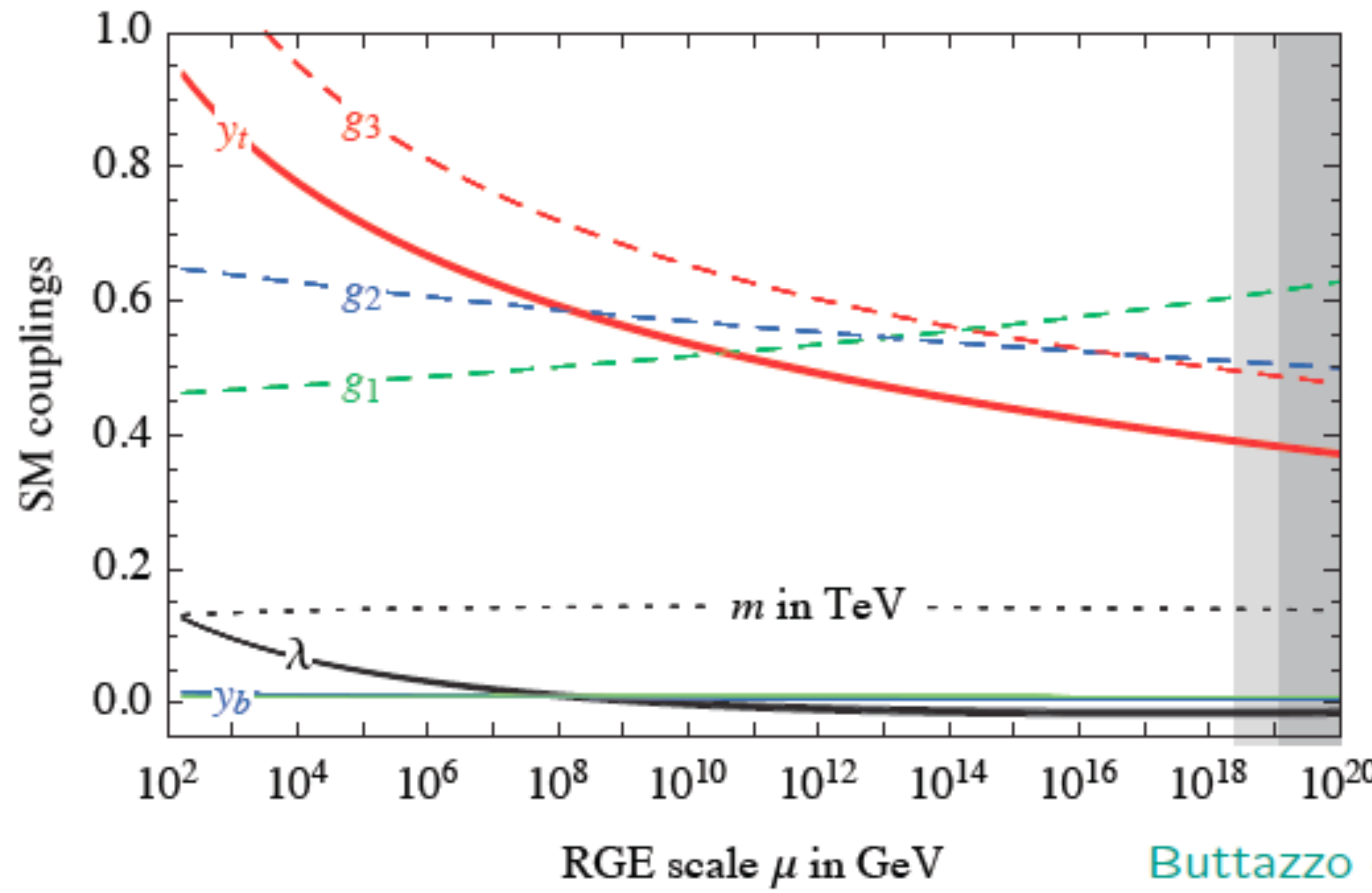
Prologue

Self-consistency of the SM

Do we need new physics beyond the SM ?

- It is possible to extend the validity of the SM up to the M_P as weakly coupled theory.



Buttazzo et al. arXiv:1307.3536

High-energy extrapolation shows that the Yukawa couplings, weak gauge couplings and the Higgs self coupling remain perturbative in the entire energy domain between the electroweak and Planck scale (no Landau poles !).

- Renormalizability implies no constraints on the free parameters of the SM Lagrangian.

Experimental evidence beyond SM

- Dark matter (visible matter accounts for only 4% of the Universe)
- Neutrino masses (Dirac or Majorana masses ?)
- Baryon asymmetry of the Universe (new sources of CP violation needed)

Caveat:

Answers perhaps wait at energy scales which we do not reach with present experiments.

Inclusive Semileptonic Penguin Decays

Based on

Huber,Hurth,Lunghi arXiv:1503.0449

Inclusive $B \to X_s \ell^+ \ell^-$: Complete angular analysis and a thorough study of collinear photons

Benzke,Fickinger,Hurth,Turczyk to appear

Subleading power factorization in $B \to X_s \ell^+ \ell^-$

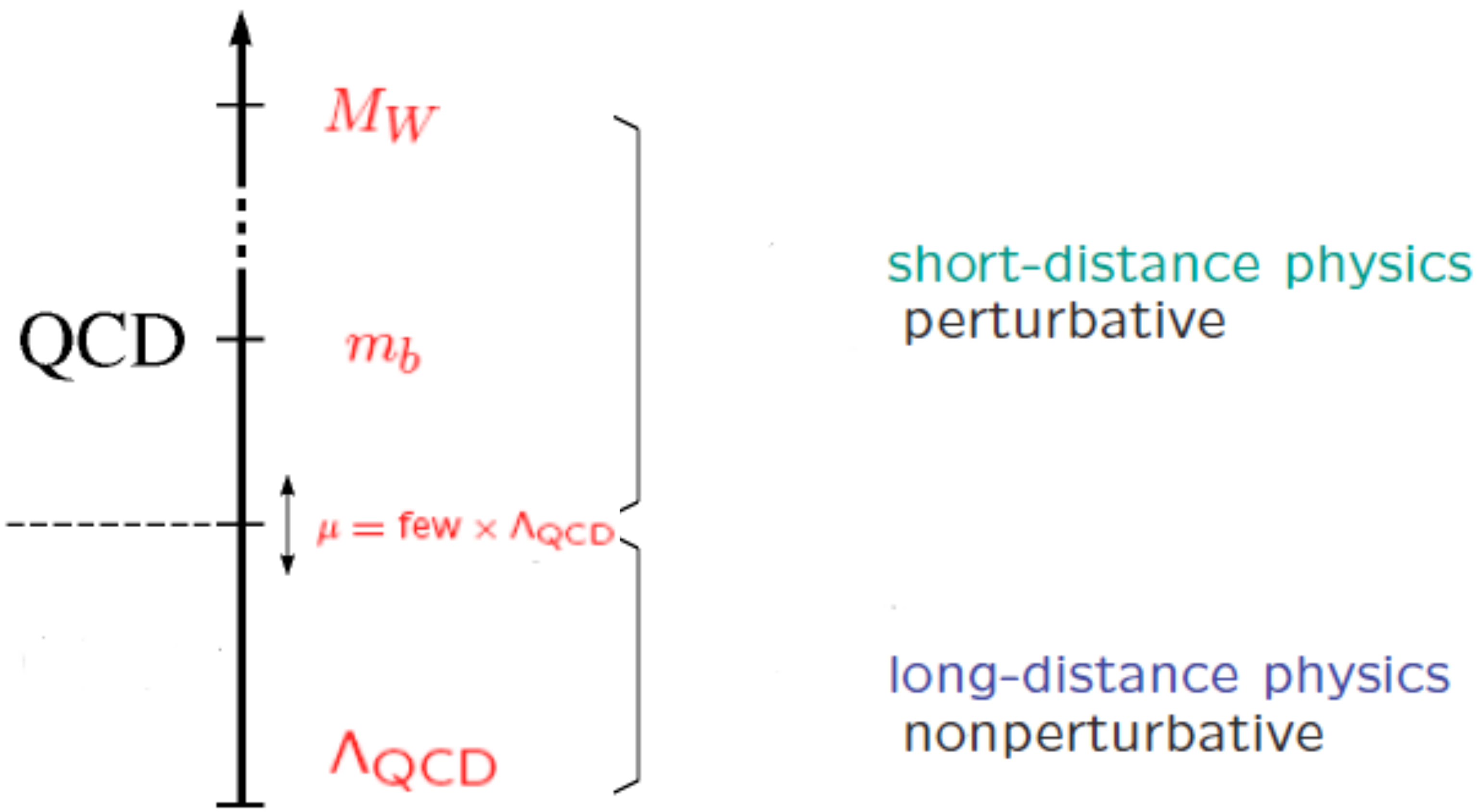
Hurth,Mahmoudi,Neshatpour arXiv:1603.00865

On the anomalies in the latest LHCb data

Motivation

- Radiative and semileptonic rare B decayse are highly sensitive probes for new physics
- Exclusive modes are experimentally easier (LHCb), but have larger theoretical uncertainties (issue of unknown power corrections !)
- Inclusive modes require Belle-II for full exploitation (complete angular analysis) but are theoretically very clean
- Inclusive modes allow for crosschecks of recent LHCb anomalies

Theoretical tools for flavour precision observables



Factorization theorems: separating long- and short-distance physics

- Electroweak effective Hamiltonian: $H_{eff} = -\frac{4G_F}{\sqrt{2}} \sum C_i(\mu, M_{heavy})\ \mathcal{O}_i(\mu)$

- $\mu^2 \approx M_{New}^2 >> M_W^2$: 'new physics' effects: $C_i^{SM}(M_W) + C_i^{New}(M_W)$

How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

Inclusive modes $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$

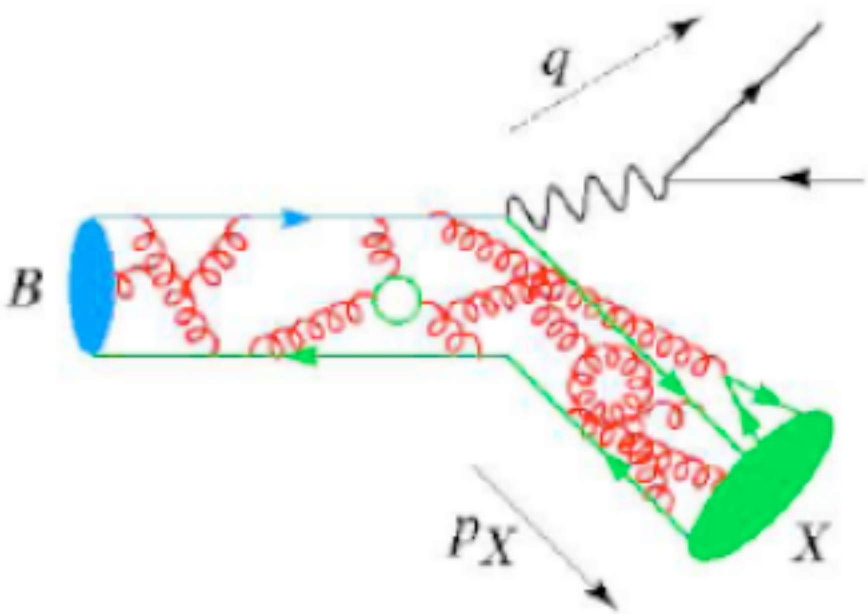
How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

Heavy mass expansion for inclusive modes:

$$\Gamma(\bar{B} \to X_s\gamma) \stackrel{m_b \to \infty}{\longrightarrow} \Gamma(b \to X_s^{parton}\gamma)\,, \quad \Delta^{nonpert.} \sim \Lambda_{QCD}^2/m_b^2$$

No linear term Λ_{QCD}/m_b (perturbative contributions dominant)

Chay, Georgi, Grinstein 1990



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An old story:

- If one goes beyond the leading operator ($\mathcal{O}_7$, $\mathcal{O}_9$):
  breakdown of local expansion

A new dedicated analysis:

naive estimate of non-local matrix elements leads to 5% uncertainty.

Benzke,Lee,Neubert,Paz,arXiv:1003.5012

Analysis in $B \to X_s\ell\ell$ in this talk; Benzke,Fickinger,Hurth,Turczyk

Exclusive modes $B \to K^{(*)}\ell\ell$

QCD-improved factorization: BBNS 1999

$$\mathcal{T}_a^{(i)} = C_a^{(i)}\xi_a + \phi_B \otimes T_a^{(i)} \otimes \phi_{a,K^*} + O(\Lambda/m_b)$$

(Soft-collinear effective theory)

– Separation of perturbative hard kernels from process-independent nonperturbative functions like form factors

– Relations between formfactors in large-energy limit

– Limitation: insufficient information on power-suppressed Λ/m_b terms (breakdown of factorization: 'endpoint divergences')

Difference between exclusive and inclusive $b \to s\gamma, \ell\ell$ modes:

Inclusive

Λ^2/m_b^2 corrections can be calculated for the leading operators in the local OPE .

Λ/m_b corrections to the subleading operators correspond to nonlocal matrix elements and can be estimated !

Exclusive

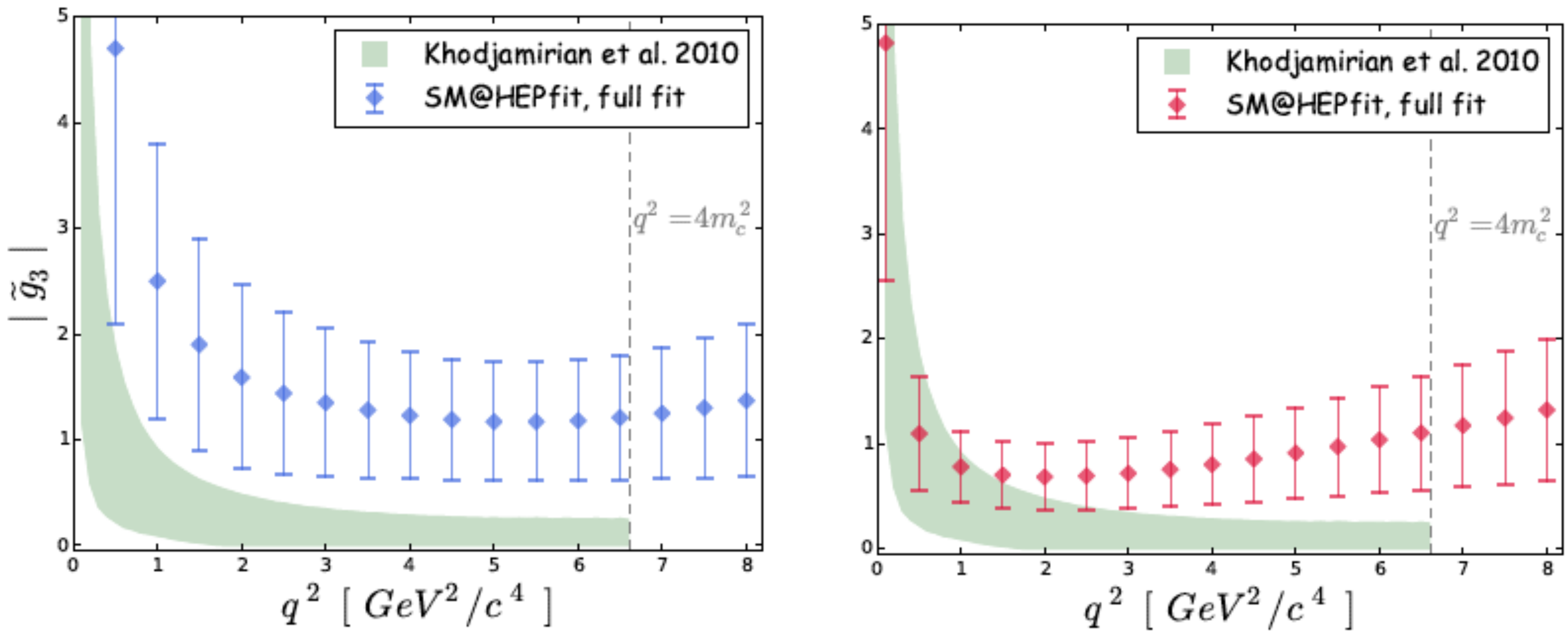
No theory of Λ/mb corrections at all within QCD factorization formula (in the low-q^2 region); these corrections can only be "guesstimated" !

Fit the unknown power corrections to the data

Ciuchini et al. arXiv:1512.07157

Leading SCET amplitude with general ansatz with 18 parameters for power corrections

Fit needs 20 – 50% power corrections (on the observable level)



No sign for q^2 dependence in the theory-independent fit

Significant q^2 dependence if power corrections are fixed at 1GeV via result of LCSR calculation Kjodjamirian et al. arXiv:1211.0234

Significance of the LHCb anomalies depend on the assumptions on the power corrections

Ciuchini et al. (arXiv:1512.07157): Fit produces 20-50% nonfact. power corrections on the observable level in the critical bins.

Hurth et al. (arXiv:1603.00865): Assumption of 60% nonfact. power corrections on the amplitude level lead to 17-20 % on the observable level (S_3, S_4, S_5) only.

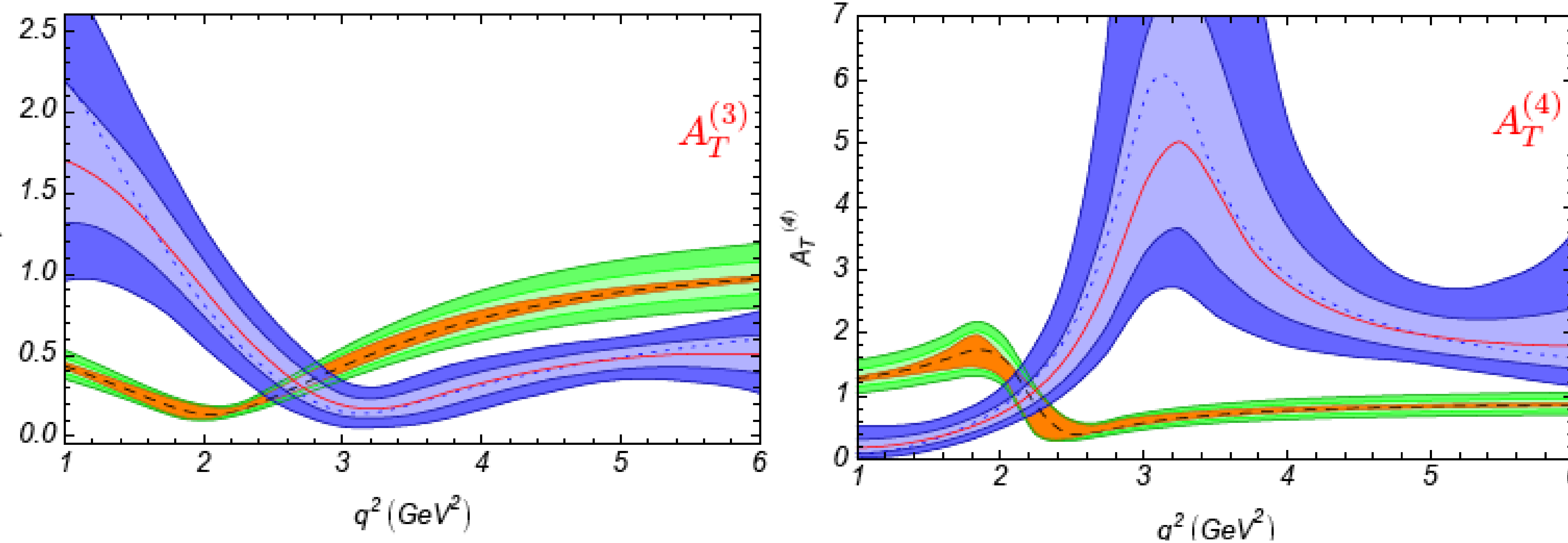
Previous predictions versus LHCb Monte Carlo (10 fb^1)

Egede,Hurth,Matias,Ramon,Reece,arXiv:0807.2589,arXiv:1005.0571

– unknown Λ/m_b power corrections

$A_{\perp,\|,0} = A^0_{\perp,\|,0}\,(1 + c_{\perp,\|,0})$ vary c_i in a range of ±10% and also of ±5%

Guesstimate



The experimental errors assuming SUSY scenario (b) with large-gluino mass and positive mass insertion, is compared to the theoretical errors assuming the SM.

This was the dream in 2008

see also Altmannshofer et al.,arXiv:0811.1214; Bobeth et al.,arXiv:0805.2525

Calculations beyond guessing numbers

Any reasonable calculation is better than a fit!

Methods offered in the analysis of $B \to K\ell^+\ell^-$ to calculate power corrections Kjodjamirian et al. arXIv: 1211.0234, also 1006.4945

Crosschecking errors and correlations of formfactor calculation in Zwicky et al. arXiv: 1503.0553 by independent LCSR analysis

Crosscheck of LHCb anomalies with various ratios $R(e/\mu)$

Hurth,Mahmoudi,Neshatpour arXiv:1603.00865

Altmannshofer,Straub arXiv:1503.06199

R_K is theoretically rather clean compared to LHCb anomalies and its tension with the SM cannot be explained by power corrections.

But both tensions might be healed by new physics in C_9^μ

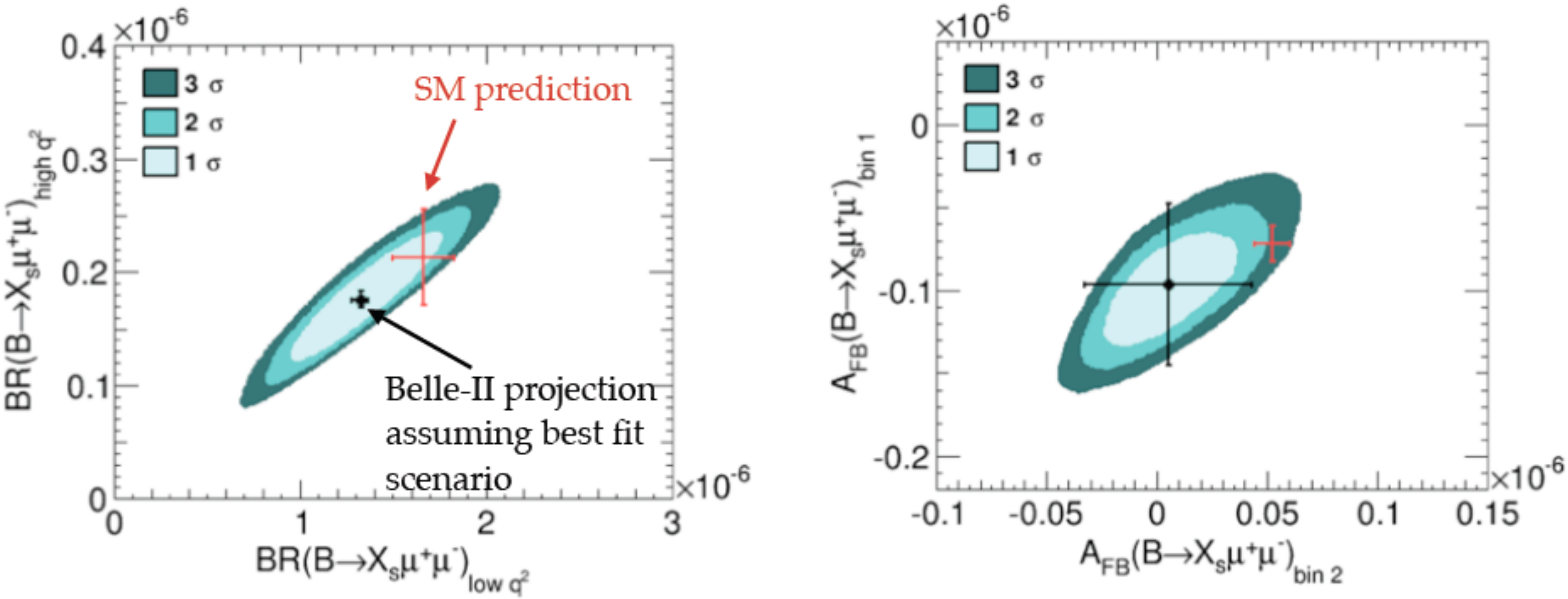
Observable	95% C.L. prediction
$\text{BR}(B \to X_s \mu^+\mu^-)/\text{BR}(B \to X_s e^+e^-)_{q^2\in[1,6](\text{GeV})^2}$	$[0.61, 0.93]$
$\text{BR}(B \to X_s \mu^+\mu^-)/\text{BR}(B \to X_s e^+e^-)_{q^2>14.2(\text{GeV})^2}$	$[0.68, 1.13]$
$\text{BR}(B^0 \to K^{*0}\mu^+\mu^-)/\ \text{BR}(B^0 \to K^{*0}e^+e^-)_{q^2\in[1,6](\text{GeV})^2}$	$[0.65, 0.96]$
$\langle F_L(B^0 \to K^{*0}\mu^+\mu^-)\rangle/\langle F_L(B^0 \to K^{*0}e^+e^-)\rangle_{q^2\in[1,6](\text{GeV})^2}$	$[0.85, 0.96]$
$\langle A_{FB}(B^0 \to K^{*0}\mu^+\mu^-)\rangle/\langle A_{FB}(B^0 \to K^{*0}e^+e^-)\rangle_{q^2\in[4,6](\text{GeV})^2}$	$[-0.21, 0.71]$
$\langle S_5(B^0 \to K^{*0}\mu^+\mu^-)\rangle/\langle S_5(B^0 \to K^{*0}e^+e^-)\rangle_{q^2\in[4,6](\text{GeV})^2}$	$[0.53, 0.92]$
$\text{BR}(B^0 \to K^{*0}\mu^+\mu^-)/\ \text{BR}(B^0 \to K^{*0}e^+e^-)_{q^2\in[15,19](\text{GeV})^2}$	$[0.58, 0.95]$
$\langle F_L(B^0 \to K^{*0}\mu^+\mu^-)\rangle/\langle F_L(B^0 \to K^{*0}e^+e^-)\rangle_{q^2\in[15,19](\text{GeV})^2}$	$[0.998, 0.999]$
$\langle A_{FB}(B^0 \to K^{*0}\mu^+\mu^-)\rangle/\langle A_{FB}(B^0 \to K^{*0}e^+e^-)\rangle_{q^2\in[15,19](\text{GeV})^2}$	$[0.87, 1.01]$
$\langle S_5(B^0 \to K^{*0}\mu^+\mu^-)\rangle/\langle S_5(B^0 \to K^{*0}e^+e^-)\rangle_{q^2\in[15,19](\text{GeV})^2}$	$[0.87, 1.01]$
$\text{BR}(B^+ \to K^+\mu^+\mu^-)/\ \text{BR}(B^+ \to K^+e^+e^-)_{q^2\in[1,6](\text{GeV})^2}$	$[0.58, 0.95]$
$\text{BR}(B^+ \to K^+\mu^+\mu^-)/\ \text{BR}(B^+ \to K^+e^+e^-)_{q^2\in[15,22](\text{GeV})^2}$	$[0.58, 0.95]$

Table 3: Predicted ratios of observables with muons in the final state to electrons in the final state, considering the two operator fit within the $\{C_9^\mu, C_9^e\}$ set.

Crosscheck of LHCb anomalies with inclusive modes

Hurth, Mahmoudi, Neshatpour, arXiv:1410.4545

if SM deviations in R_K and P_5' persist until Belle-II



If NP then the effect of C_9 and C_9' are large enough to be checked at Belle-II with theoretically clean modes.

New physics sensitivity

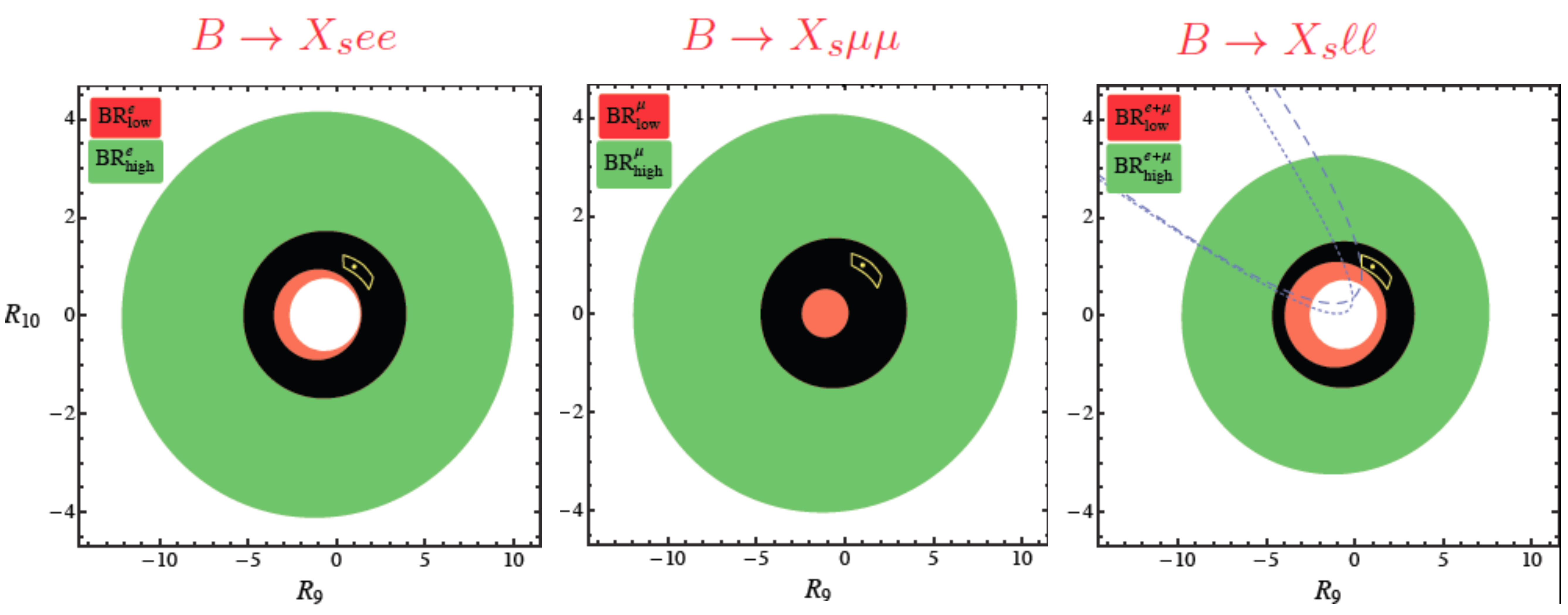
Huber, Hurth, Lunghi, arXiv:1503.04849

Constraints on Wilson coefficients C_9/C_9^{SM} and C_{10}/C_{10}^{SM}

$$R_i = \frac{C_i(\mu_0)}{C_i^{SM}(\mu_0)}$$

that we obtain at 95% C.L. from present experimental data
(red low q^2, green high q^2)

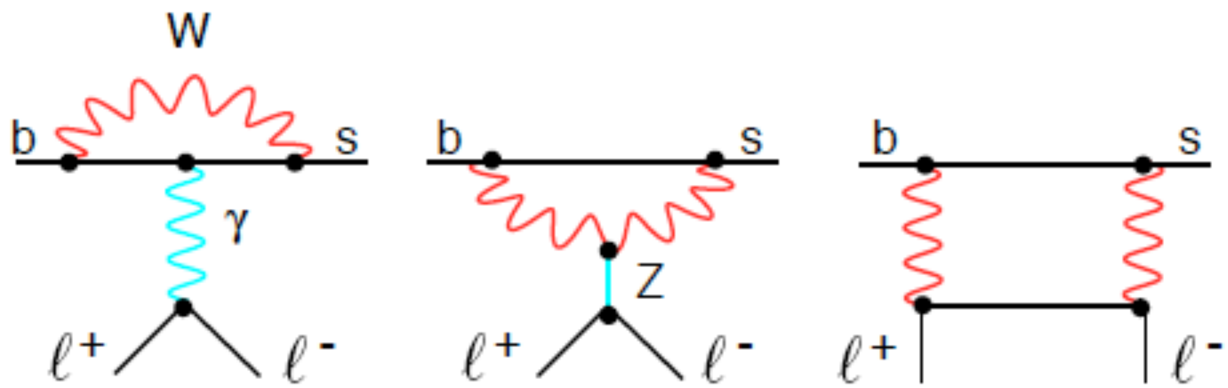
that we will obtain at 95% C.L. from $50ab^{-1}$ data at Belle-II
(yellow)



Complete angular analysis of inclusive $B \to X_s \ell\ell$

Huber,Hurth,Lunghi, arXiv:1503.04849

- "Latest" Belle measurement of branching ratio is based on less than 30% of the total luminosity



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$$\frac{d^2\Gamma}{dq^2\,dz} = \frac{3}{8}\left[(1+z^2)\,H_T(q^2) + 2\,z\,H_A(q^2) + 2\,(1-z^2)\,H_L(q^2)\right] \qquad (z = \cos\theta_\ell)$$

$$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2) \qquad\qquad \frac{dA_{\rm FB}}{dq^2} = 3/4\,H_A(q^2)$$

- Phenomenological analysis to NNLO QCD and NLO QED for all angular observables

- Electromagnetic effects due to energetic photons are large and calculated analytically and crosschecked against Monte Carlo generator events

Large logs $log(mb/m_\ell)$ different for muon and electron !

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Huber,Hurth,Lunghi, arXiv:1503.04849

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  Size of logs depend on experimental set-up $\qquad \alpha_{\rm em}\,\log(m_b^2/m_\ell^2)$

  We assume no photons are included in the definition of q^2 (di-muon channel at Babar/Belle, di-electron at Belle)

  Babar's di-electron channel: Photons that are emitted in a cone of 35 mrad angular opening are included in q^2

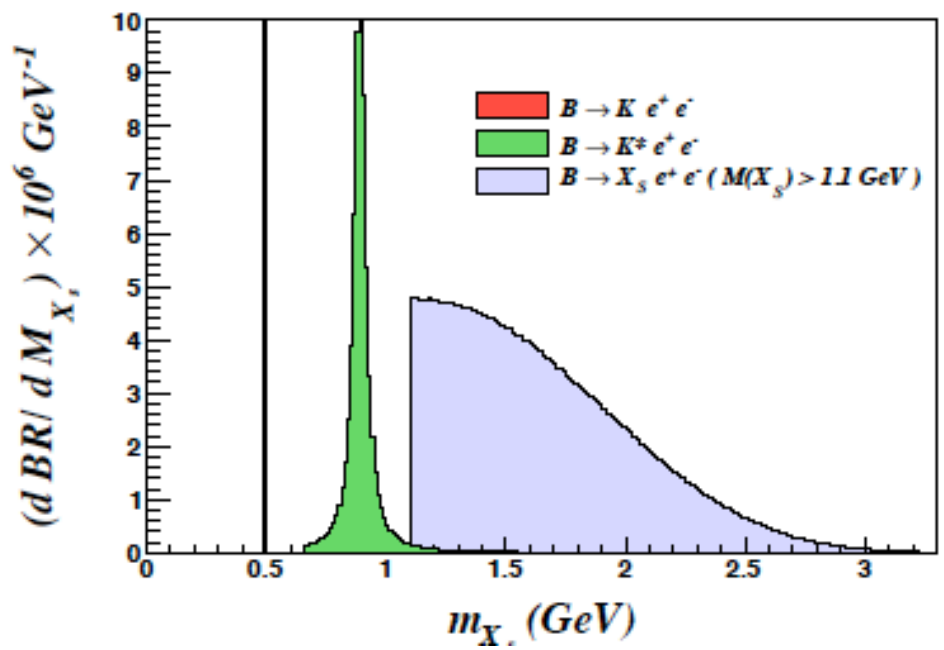
  Monte Carlo techniques needed to estimate this effect !

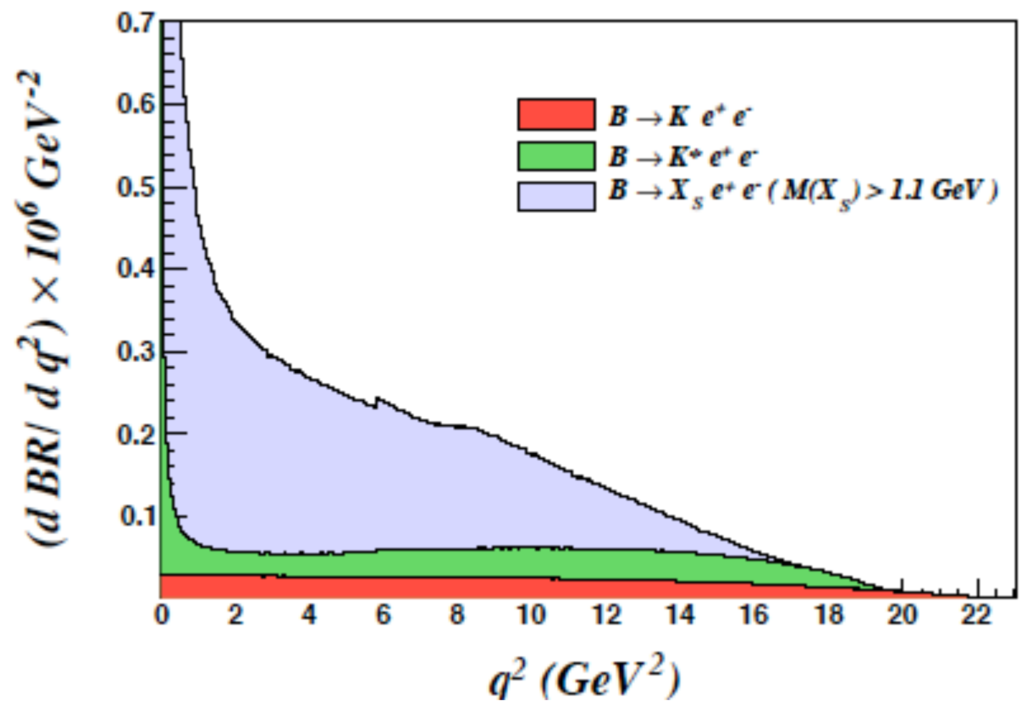
Monte Carlo analysis

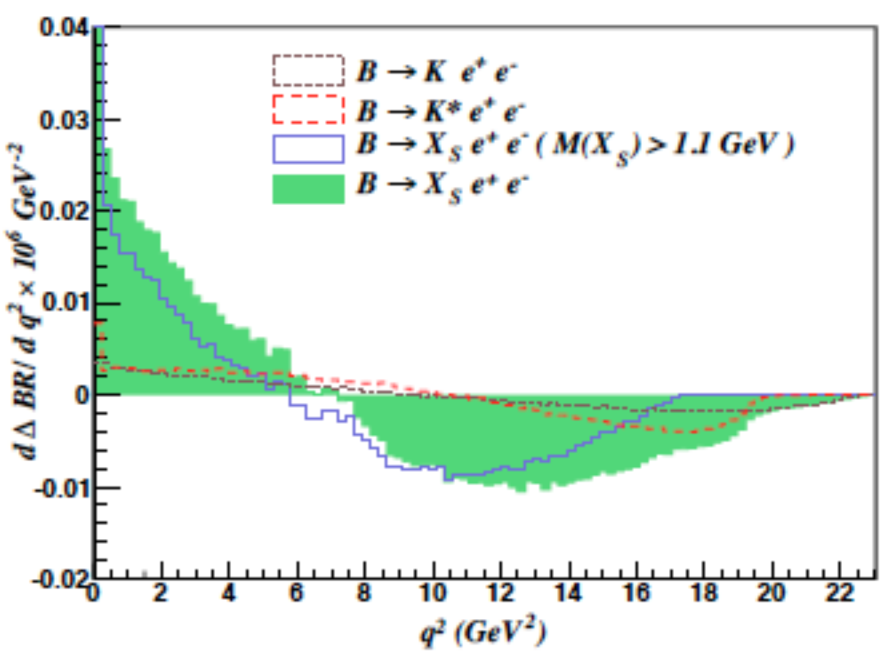
Huber,Hurth,Lunghi, arXiv:1503.04849

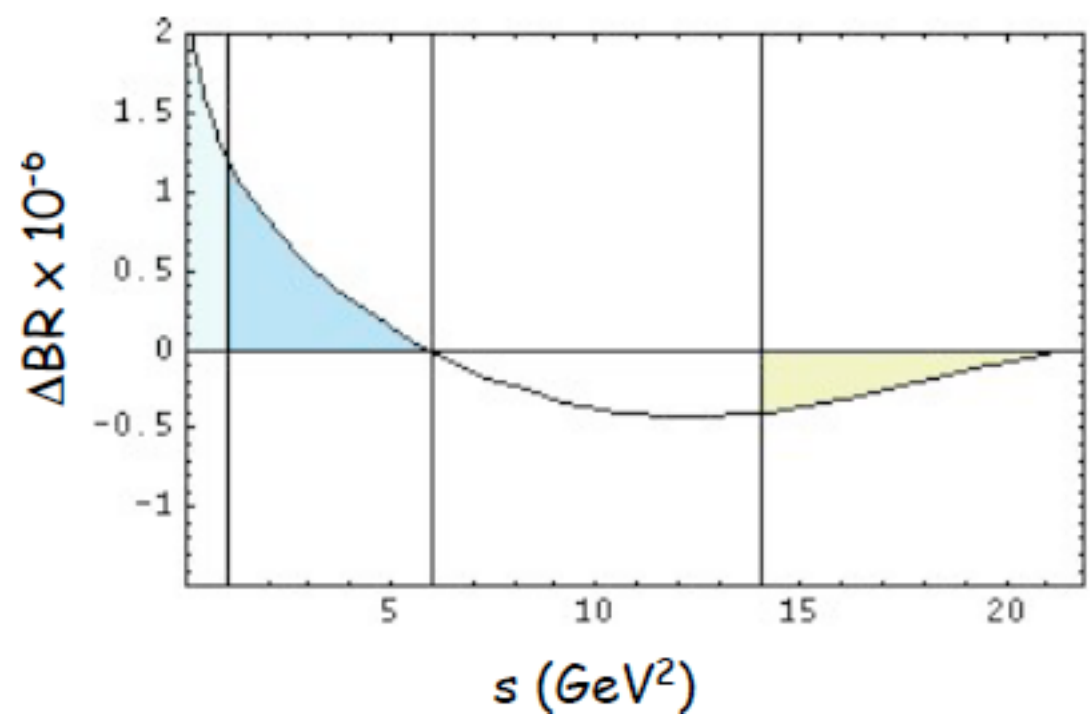
(event generator EVTGEN, hadronization JETSET, EM radiation PHOTOS)

$$\frac{\left[\mathcal{B}_{ee}^{\rm low}\right]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\rm coll}}}}{\left[\mathcal{B}_{ee}^{\rm low}\right]_{q=p_{e^+}+p_{e^-}}} - 1 = 1.65\% \qquad \frac{\left[\mathcal{B}_{ee}^{\rm high}\right]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\rm coll}}}}{\left[\mathcal{B}_{ee}^{\rm high}\right]_{q=p_{e^+}+p_{e^-}}} - 1 = 6.8\%$$





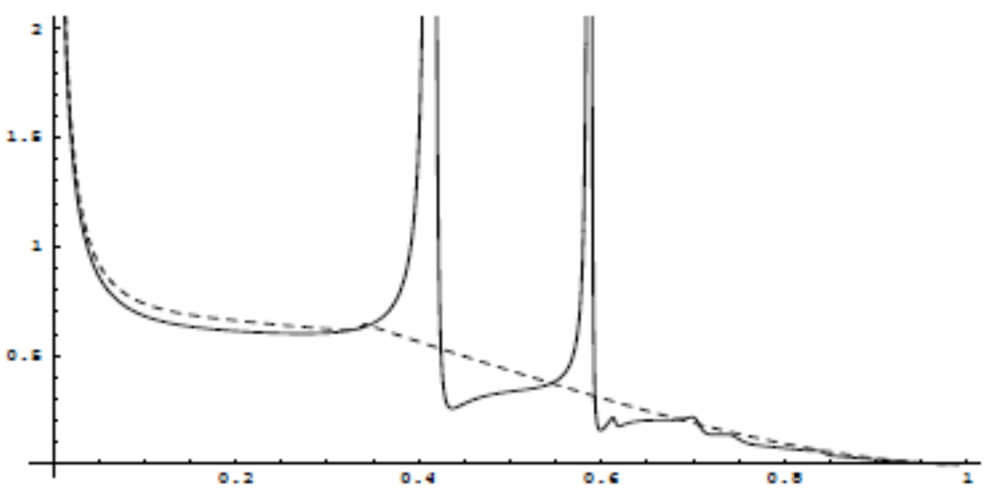


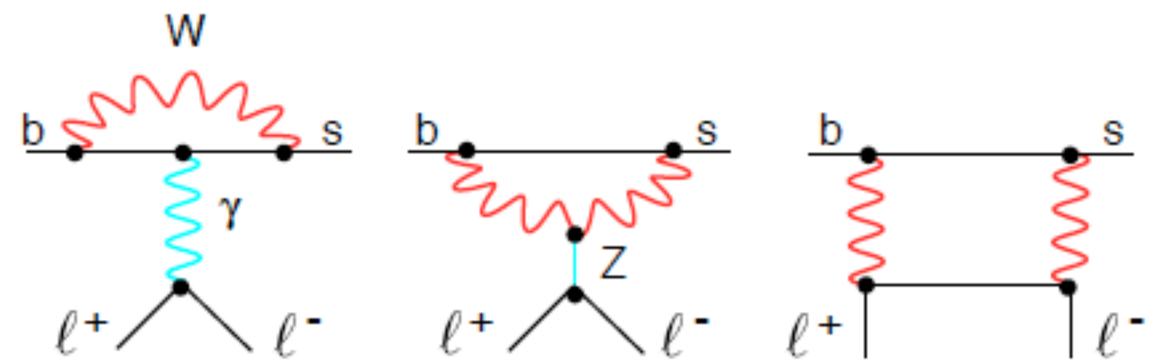


Subleading contributions in $B \to X_s \ell^+ \ell^-$

- On-shell-$c\bar{c}$-resonances $\Rightarrow$ cuts in dlepton mass spectrum necessary :
  1GeV$^2 < q^2 <$ 6GeV^2 and 14.4GeV$^2 < q^2$ $\Rightarrow$ perturbative contributions dominant

$\frac{d}{d\hat{s}} BR(\bar{B} \to X_s l^+ l^-) \times 10^{-5}$





- Again additional subtleties $\Rightarrow$ additional uncertainties

  – Locally: breakdown of OPE in Λ_{QCD}/m_b in the high-s (q^2) endpoint
  Partonic contribution vanishes in the limit $s \to 1$, while the $1/m_b^2$ corrections in $R(s)$ tend towards a nonzero value.

  Theoretically: s-quark propagator in the correlator of OPE:

$$S_s(k) = \frac{\not{k} + i \not{D}}{k^2 + 2ik \cdot D - \not{D} \not{D} + i\varepsilon} \; .$$

  Endpoint region of the q^2 spectrum in $\bar{B} \to X_s l^+ l^-$ different from endpoint region of the photon spectrum of $\bar{B} \to X_s \gamma$:
  $q^2 \approx m_b^2 \approx M_B^2 \;\Rightarrow k \sim \Lambda, \quad k^2 \sim \Lambda^2$ $\Rightarrow$ complete breakdown of OPE
  no partial all-orders resummation possible, shape-function irrelevant
  Buchalla, Isidori

Practically: for integrated high-s (q^2) spectrum one finds an effective expansion ($s_{\rm min} \approx 0.6$): Ghinculov,Hurth,Isidori,Yao hep-ph/0312128

$$\int_{s_{\rm min}}^{1} ds\ R(s) = \left[1 - \frac{1.6\lambda_2}{m_b^2(1-\sqrt{s_{\rm min}})^2} + \frac{1.8\rho_1 + 1.7 f_1}{m_b^3(1-\sqrt{s_{\rm min}})^3}\right] \times \int_{s_{\rm min}}^{1} ds\ \ R(s)|_{m_b\to\infty}$$

– Hadronic invariant-mass cut is imposed in order to eliminate the background like $b \to c\,(\to se^+\nu)e^-\bar{\nu} = b \to se^+e^- +$ missing energy

* Babar,Belle: $m_X < 1.8$ or $2.0 GeV$
* high-q^2 region not affected by this cut
* kinematics: X_s is jetlike and $m_X^2 \leq m_b\Lambda_{QCD}$ $\Rightarrow$ shape function region
* SCET analysis: universality of jet and shape functions found:

  the 10-30% reduction of the dilepton mass spectrum can be accurately computed using the $\bar{B} \to X_s\gamma$ shape function
  5% additional uncertainty for $2.0 GeV$ cut due to subleading shape functions
  Lee,Stewart hep-ph/0511334
  Lee,Ligeti,Stewart,Tackmann hep-ph/0512191
  Lee,Tackmann arXiv:0812.0001 (effect of subleading shape functions)
  Bell,Beneke,Huber,Li arXiv:1007.3758 (NNLO matching QCD $\to$ SCET)

Subleading power factorization in $B \to X_s \ell^+ \ell^-$

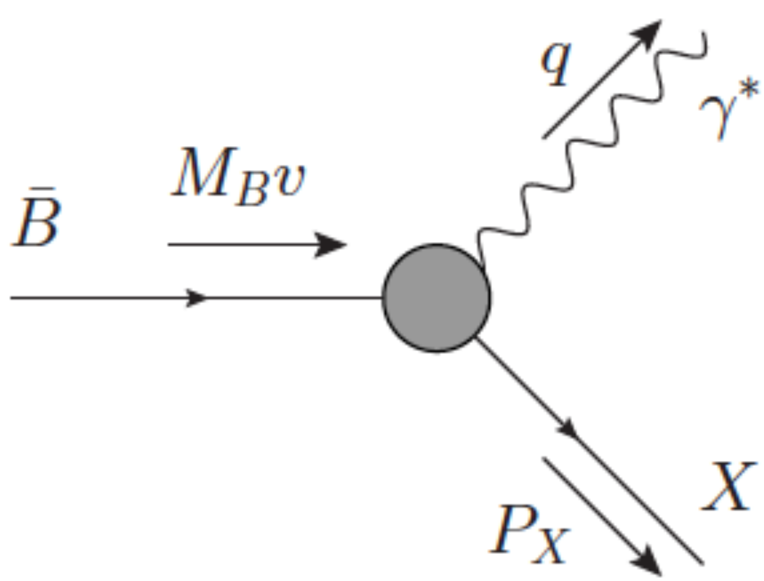
Benzke,Fickinger,Hurth,Turczyk, to appear

Hadronic cut

Additional cut in X_s necessary to reduce background
affects only low-q^2 region.

Hadronic invariant $m_X^2 < 1.8(2.0) GeV^2$, jet-like X_s $E_X \sim \mathcal{O}(m_b)$

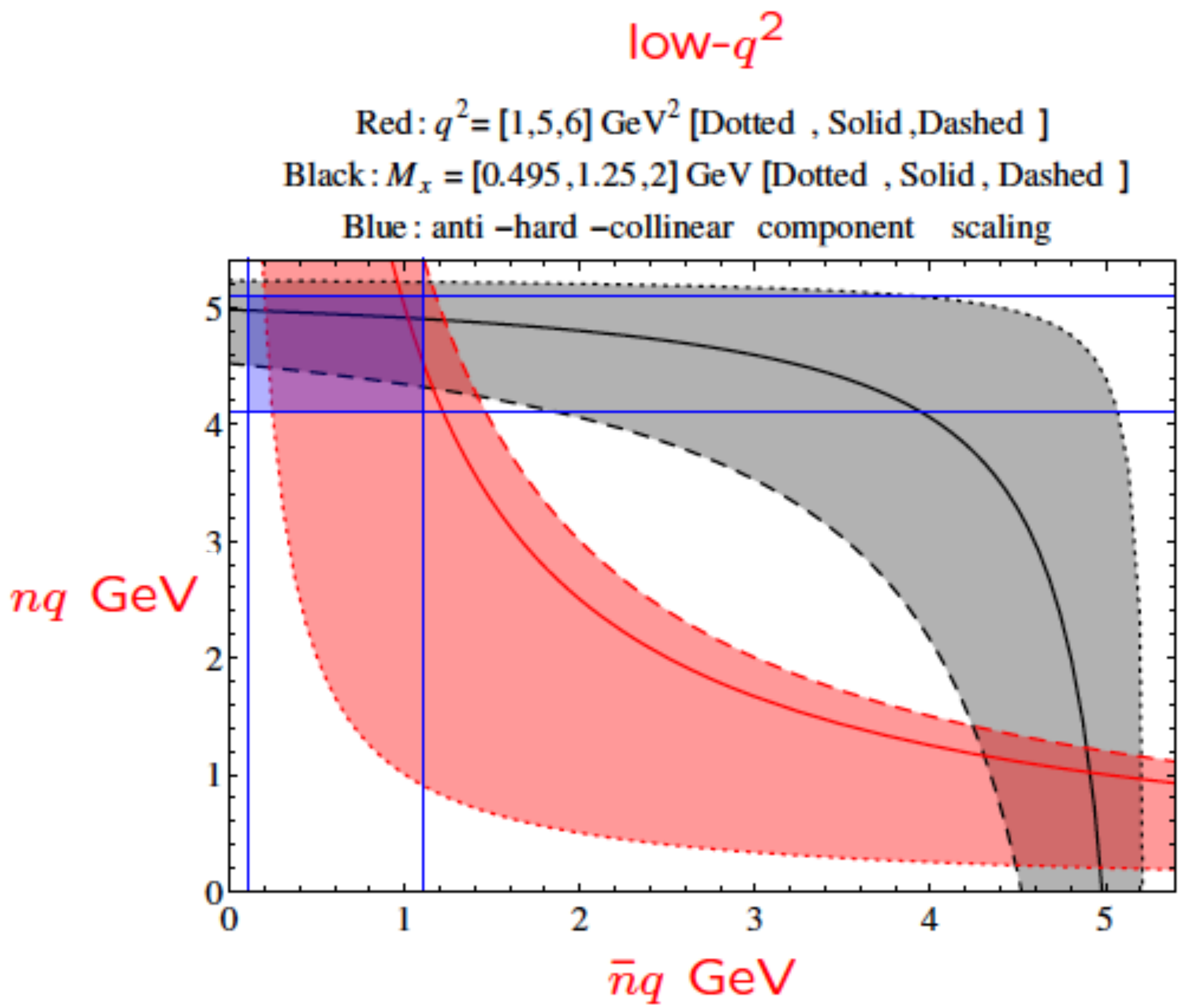
Multiscale problem $\to$ SCET

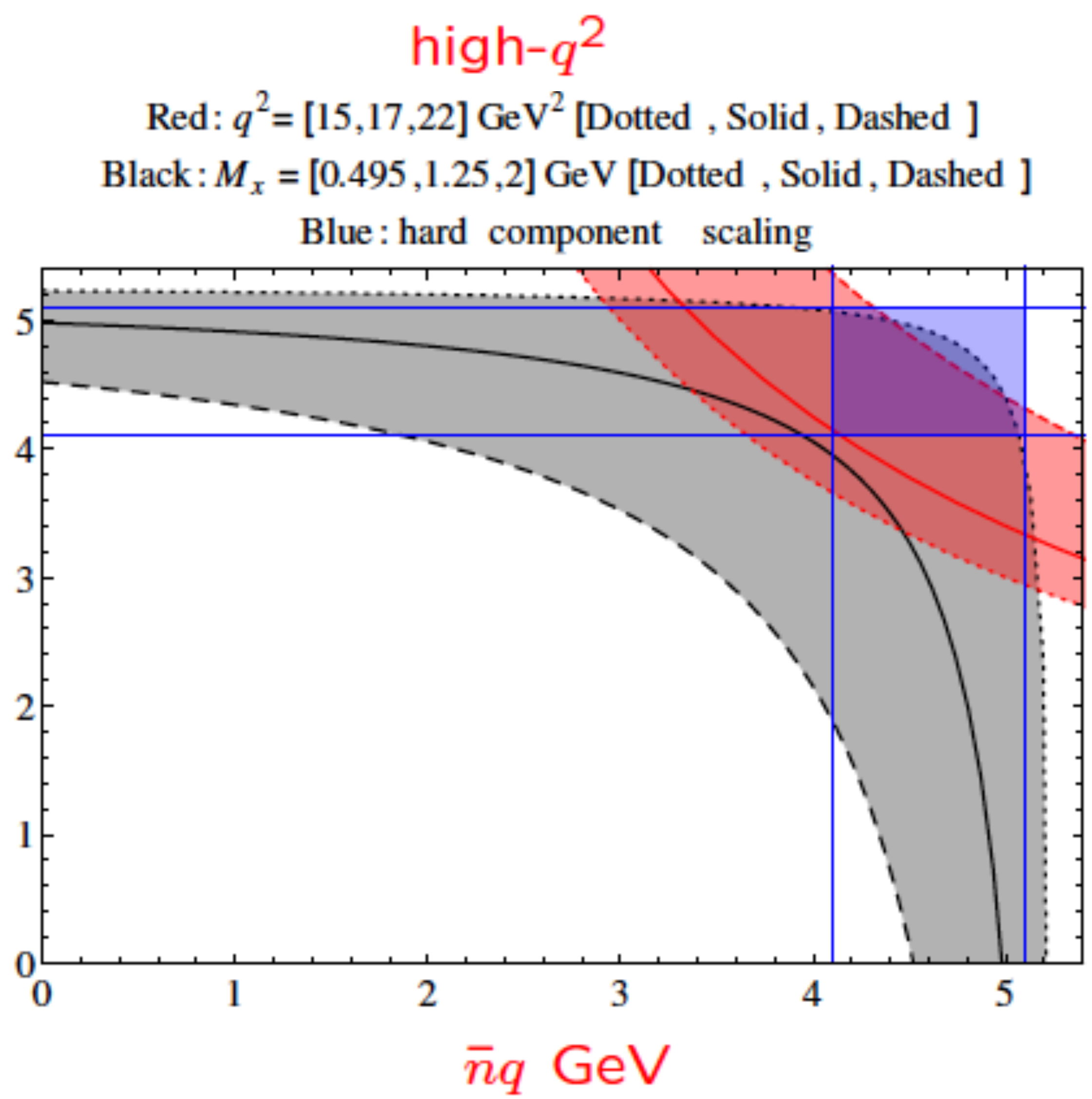


$$m_X^2 = P_X^2 = (M_B - n \cdot q)(M_B - \bar{n} \cdot q)$$

$$M_B^2 \sim m_b^2 \gg m_X^2 \sim \Lambda_{\rm QCD} m_b \gg \Lambda_{\rm QCD}^2$$

Alllowed regions

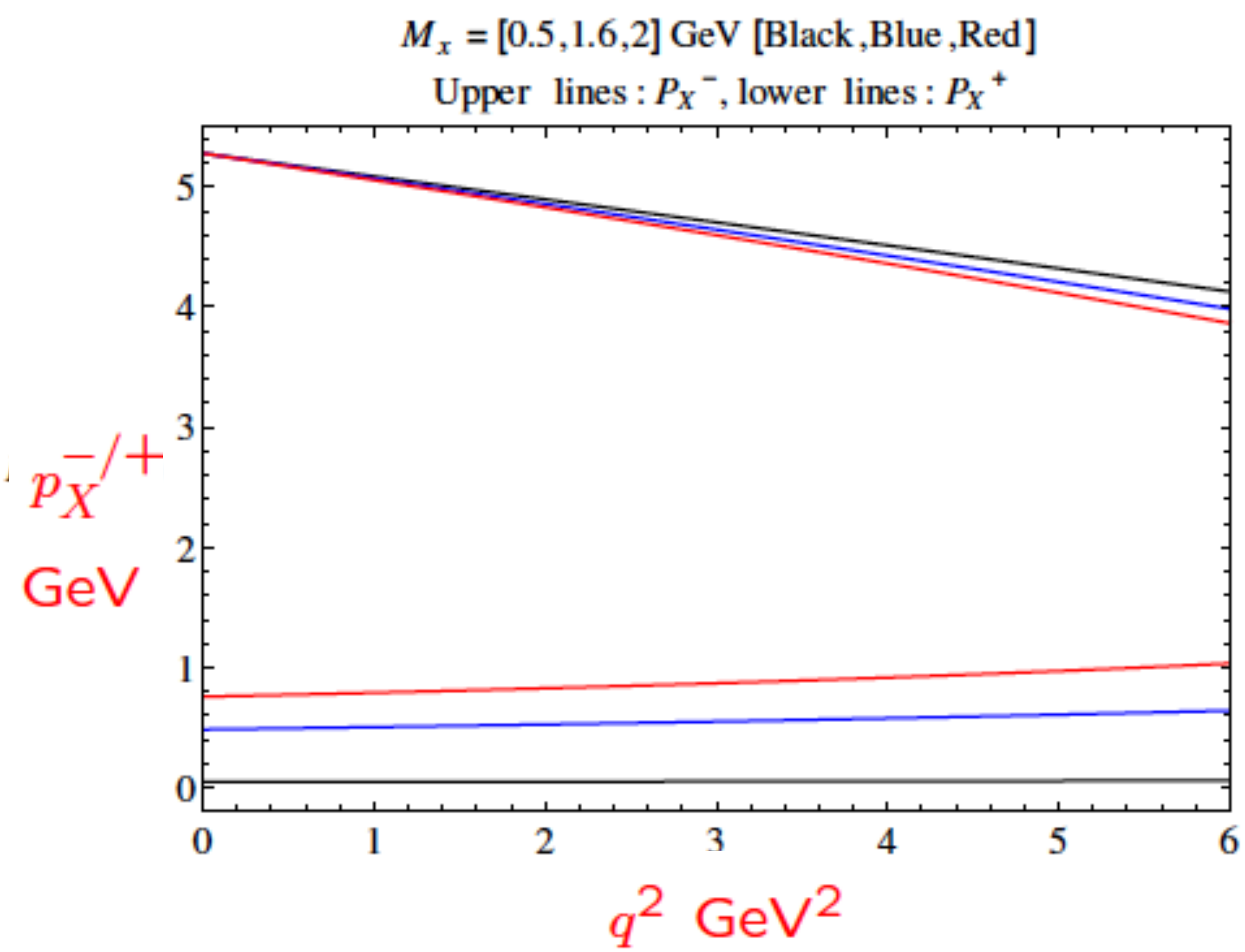


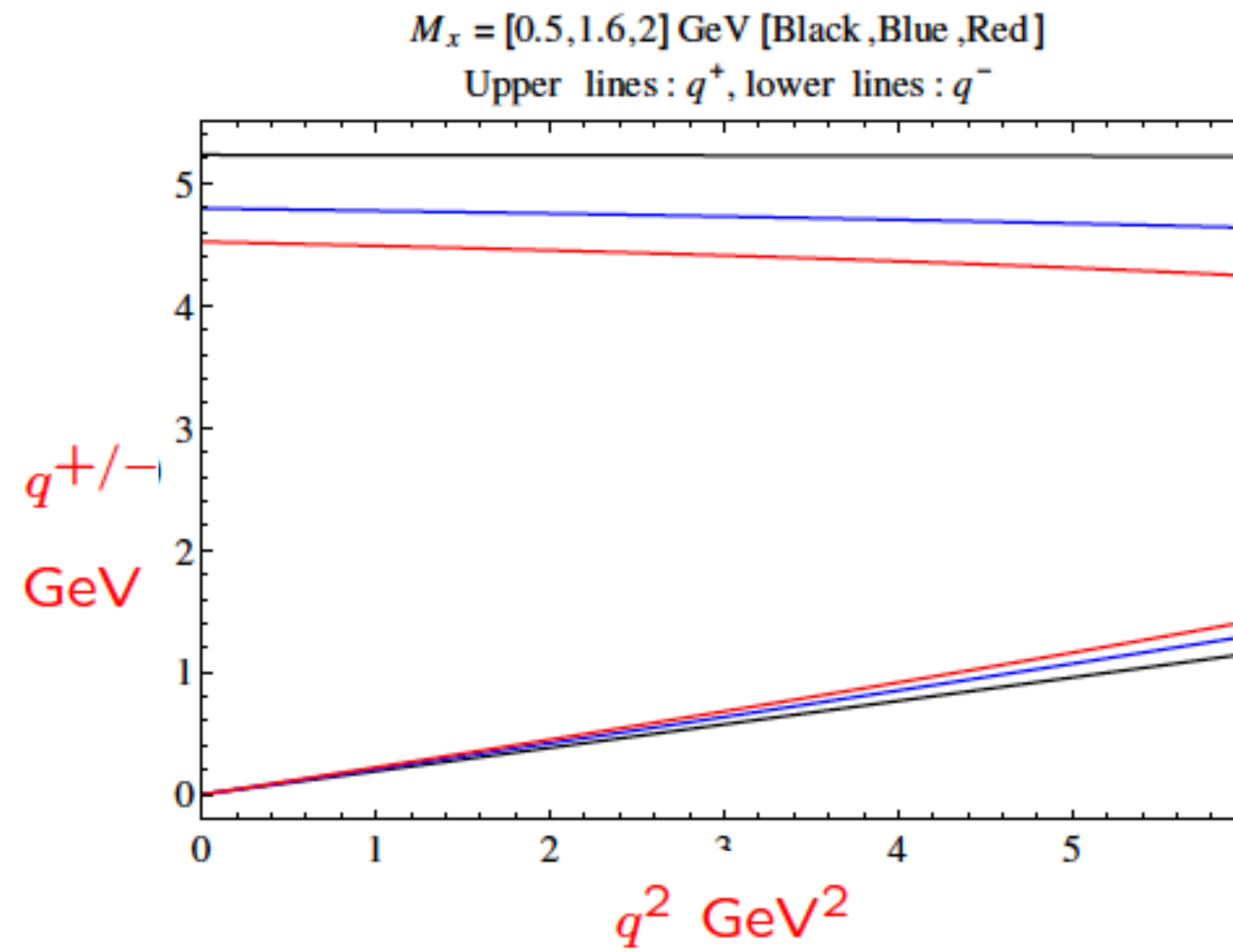


Scaling

$$\lambda = \Lambda_{\mathrm{QCD}}/m_b \qquad m_X^2 \sim \lambda \Rightarrow m_b - n\cdot q \sim \lambda$$

Scaling





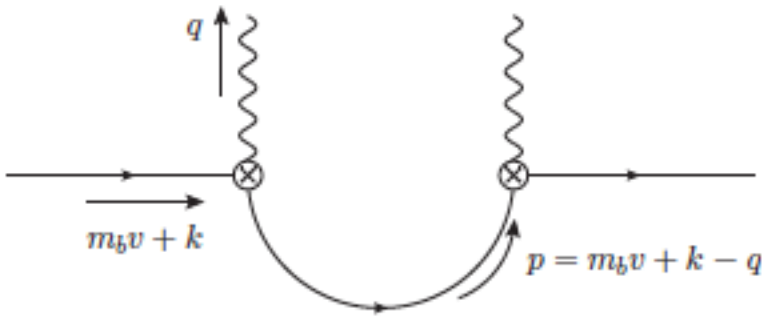
For $q^2 < 6GeV^2$ the scaling of np_X and $\bar{n}p_X$ implies $\bar{n}q$ is of order λ, means q anti-hard-collinear (just kinematics).

Stewart and Lee assume $\bar{n}q$ to be order 1, means q is hard.

This problematic assumption implies a different matching of SCET/QCD.

Shapefunction region

Local OPE breaks down for $m_X^2 \sim \lambda$:



$$\frac{1}{(m_b v+k-q)^2} = \frac{1}{m_b - n\cdot q}\left(1 - \frac{n\cdot k}{m_b - n\cdot q} + \dots\right)\frac{1}{m_b - \bar{n}\cdot q}$$

Resummation of leading contributions into a shape function.

(scaling of $\bar{n}q$ does not matter here; zero in case of $B \to X_s\gamma$)

Factorization theorem $\quad d\Gamma \sim H \cdot J \otimes S$

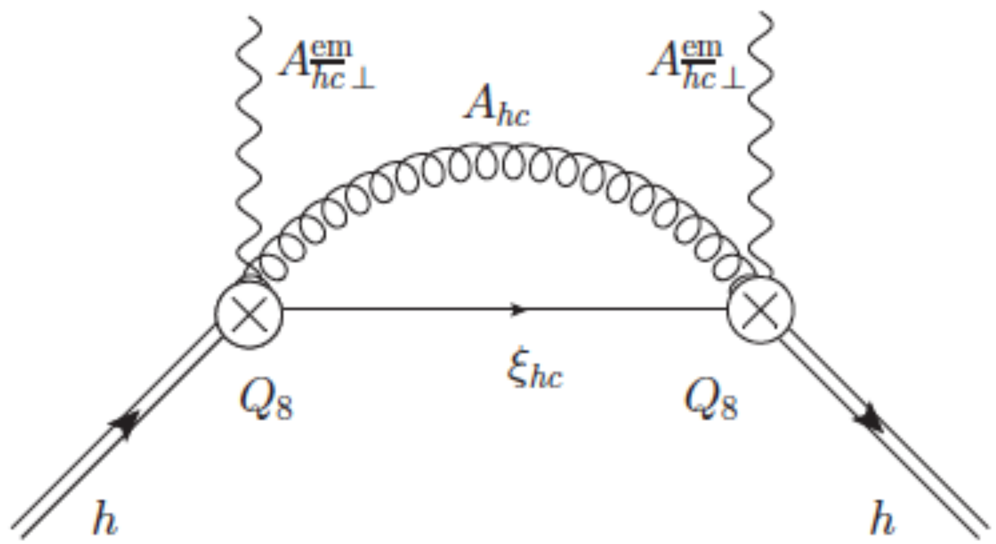
The hard function H and the jet function J are perturbative quantities.

The shape function S is a non-perturbative non-local HQET matrix element.

(universality of the shape function, uncertainties due to subleading shape functions)

Calculation at subleading power

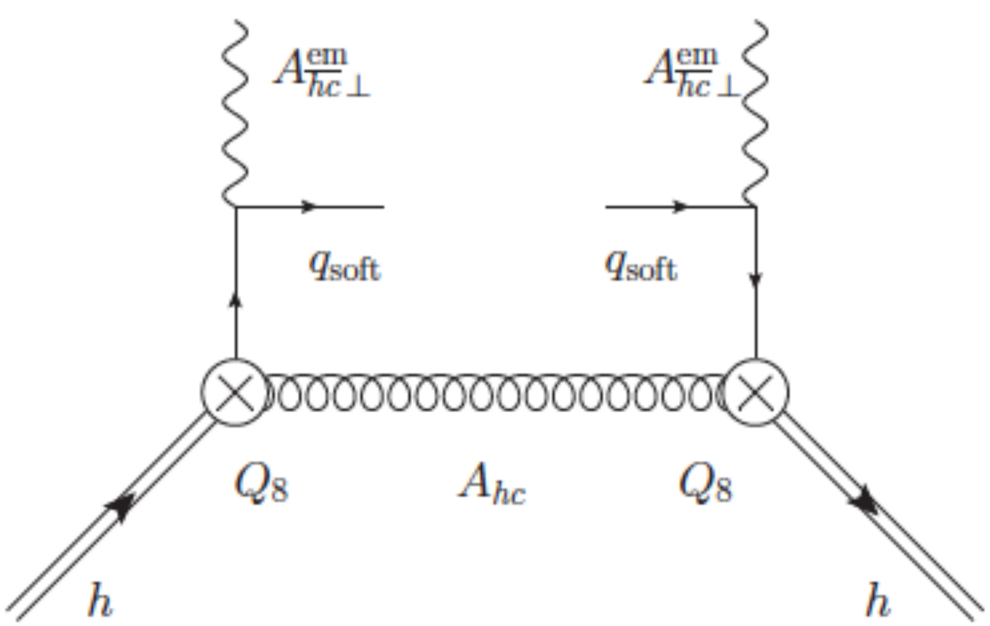
Example of **direct** photon contribution which factorizes $d\Gamma \sim H \cdot j \otimes S$



$\rightarrow \frac{\alpha_s}{m_b}$ in low m_X^2 region

Example of **resolved** photon contribution (double-resolved) which factorizes

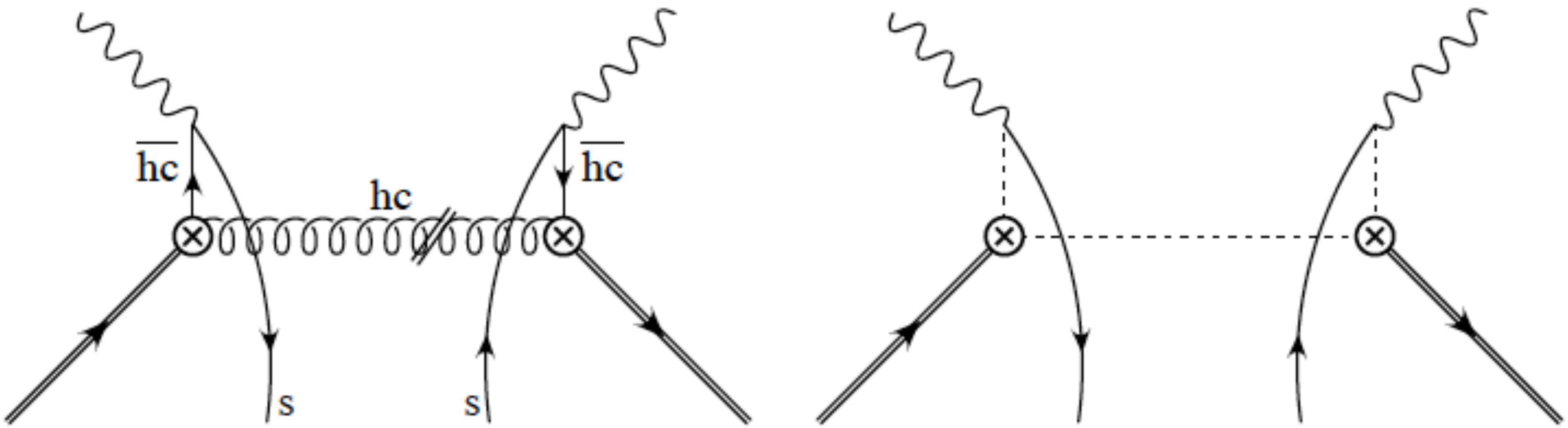
$d\Gamma \sim H \cdot J \otimes s \otimes \bar{J} \otimes \bar{J}$



$\rightarrow \frac{\Lambda}{m_b}$

Shape function is non-local in two light-cone directions.
It survives $M_X \rightarrow 1$ limit (irreducible uncertainty).

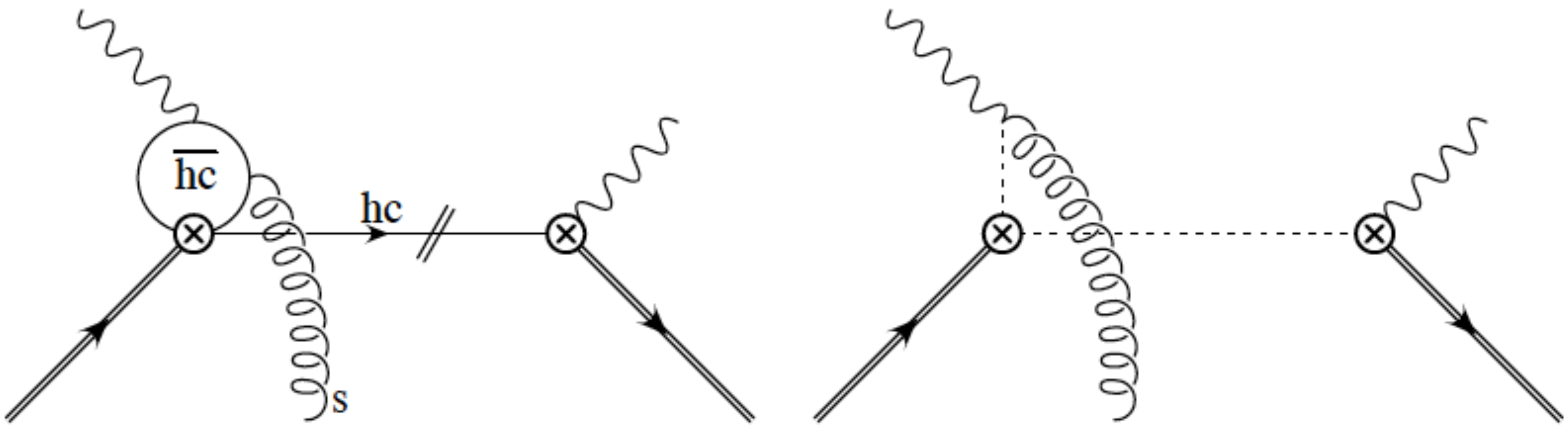
Interference of Q_8 and Q_8



$$\frac{d\Gamma^{\rm res}}{dn\cdot q\, d\bar{n}\cdot q} \sim \frac{e_s^2\alpha_s}{m_b}\int d\omega\, \delta(\omega+p_+)\int \frac{d\omega_1}{\omega_1+\bar{n}\cdot q+i\varepsilon}\int \frac{d\omega_2}{\omega_2+\bar{n}\cdot q-i\varepsilon} g_{88}(\omega,\omega_1,\omega_2)$$

$$g_{88}(\omega,\omega_1,\omega_2) = \frac{1}{M_B}\langle\bar{B}|\bar{h}(\mathbf{tn})\dots s(\mathbf{tn}+\mathbf{u\bar{n}})\bar{s}(\mathbf{r\bar{n}})\dots h(\mathbf{0})|\bar{B}\rangle_{\rm F.T.}$$

Interference of Q_1 and Q_7



$$\frac{d\Gamma^{\rm res}}{dn\cdot q\, d\bar{n}\cdot q} \sim \frac{1}{m_b}\int d\omega\, \delta(\omega+p_+)\int \frac{d\omega_1}{\omega_1+i\varepsilon}$$

$$\frac{1}{\omega_1}\left[\bar{n}\cdot q\left(F\left(\frac{m_c^2}{n\cdot q\bar{n}\cdot q}\right)-1\right)-(\bar{n}\cdot q+\omega_1)\left(F\left(\frac{m_c^2}{n\cdot q(\bar{n}\cdot q+\omega_1)}\right)-1\right)\right.$$

$$\left.+\bar{n}\cdot q\left(G\left(\frac{m_c^2}{n\cdot q\bar{n}\cdot q}\right)-G\left(\frac{m_c^2}{n\cdot q(\bar{n}\cdot q+\omega_1)}\right)\right)\right]g_{17}(\omega,\omega_1)$$

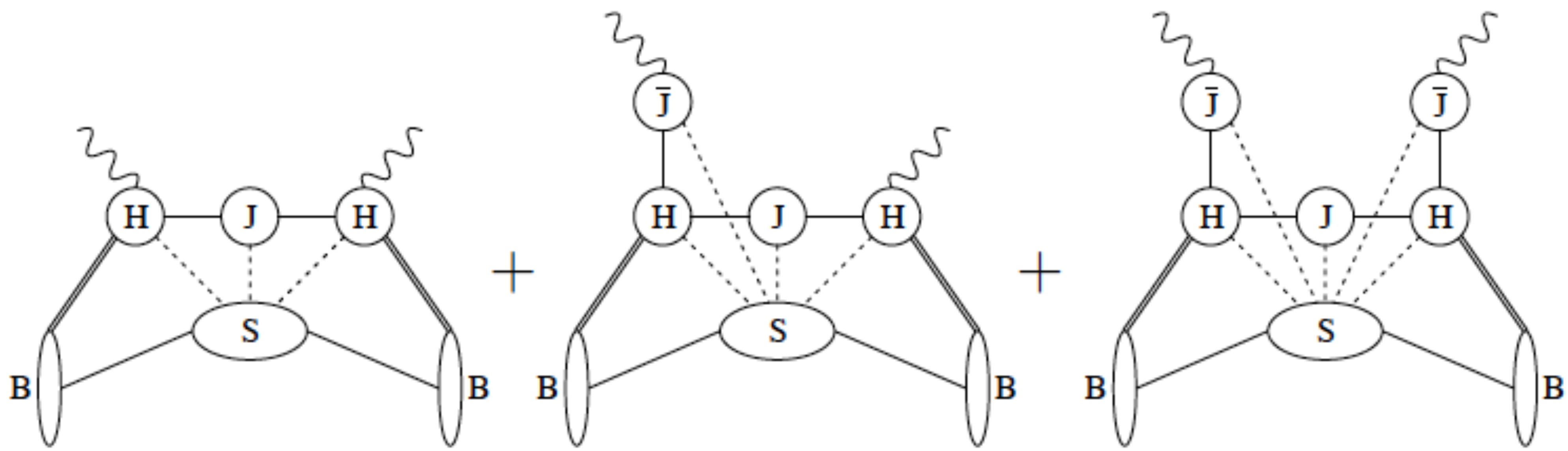
$$g_{17}(\omega,\omega_1)=\int\frac{dr}{2\pi}e^{-i\omega_1 r}\int\frac{dt}{2\pi}e^{-i\omega t}\frac{1}{M_B}\langle\bar{B}|\bar{h}(\mathbf{tn})\ldots G_s^{\alpha\beta}(\mathbf{r\bar{n}})\ldots h(\mathbf{0})|\bar{B}\rangle$$

Expansion for $m_c \sim m_b$ leads to Voloshin term in the total rate $(-\lambda_2/m_c^2)$, the terms stays non-local for $m_c < m_b$.

Factorization formula

In the $m_X^2 \sim \lambda$ and $q^2 \sim \lambda$ region we have the following factorization formula

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum H \cdot j_i \otimes S + \frac{1}{m_b} \sum H \cdot J \otimes s_i + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} \otimes \bar{J} + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$



Numerical evaluation (work in progress)

Similar subleading shape functions as in $B \to X_s \gamma$

Use vacuum insertion approximation, PT invariance,....

Summary

- For q anti-hard-collinear we have identified a new type of subleading power corrections.

- In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.

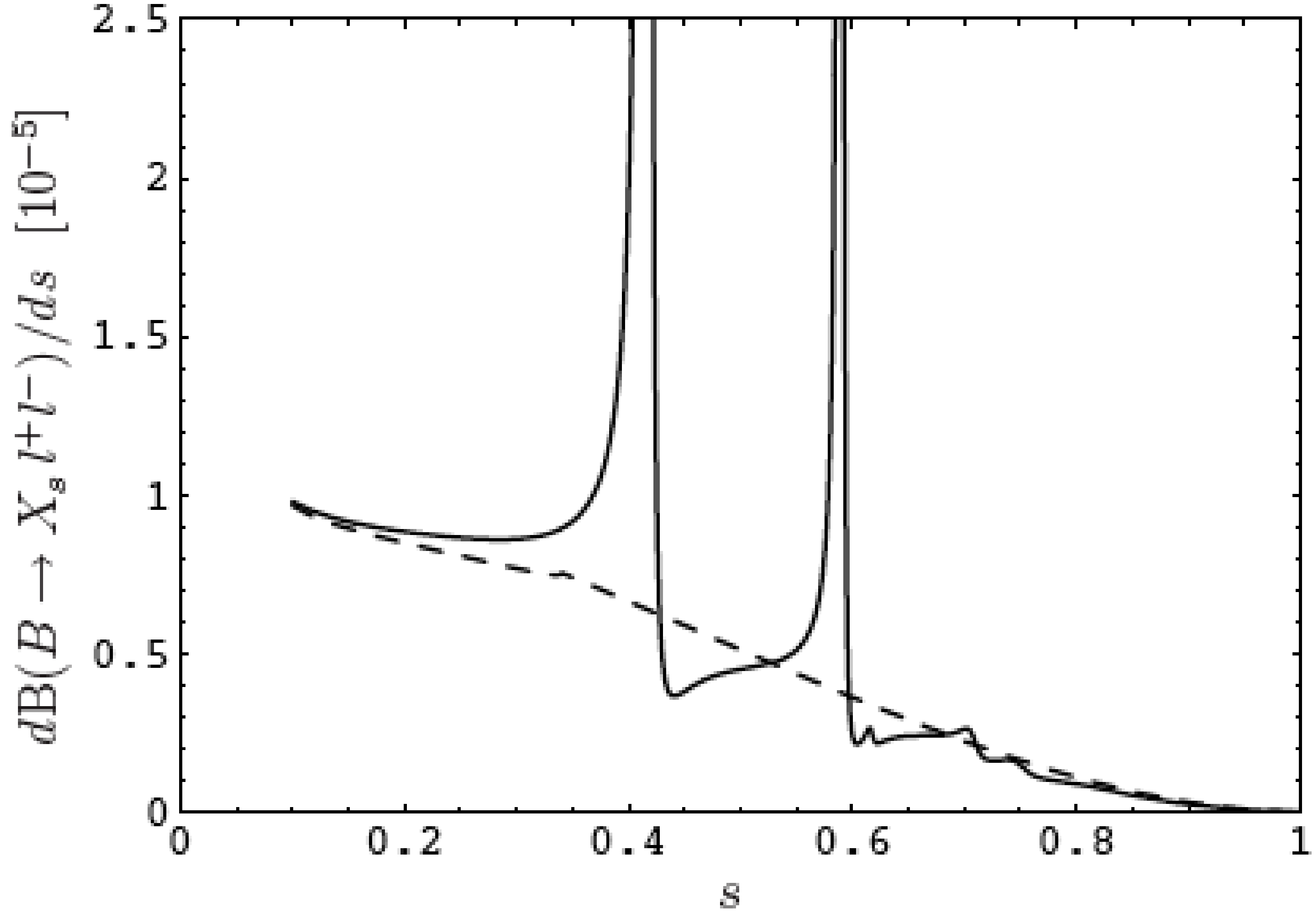
- They constitute an irreducible uncertainty because they survive the $M_X \to 1$ limit.

Extra

Quark-hadron duality violated in $\bar{B} \to X_s \ell^+ \ell^-$?

BBNS, arXiv:0902.4446

Within integrated branching ratio the resonances J/ψ and ψ' exceed the perturbative contributions by two orders of magnitude.



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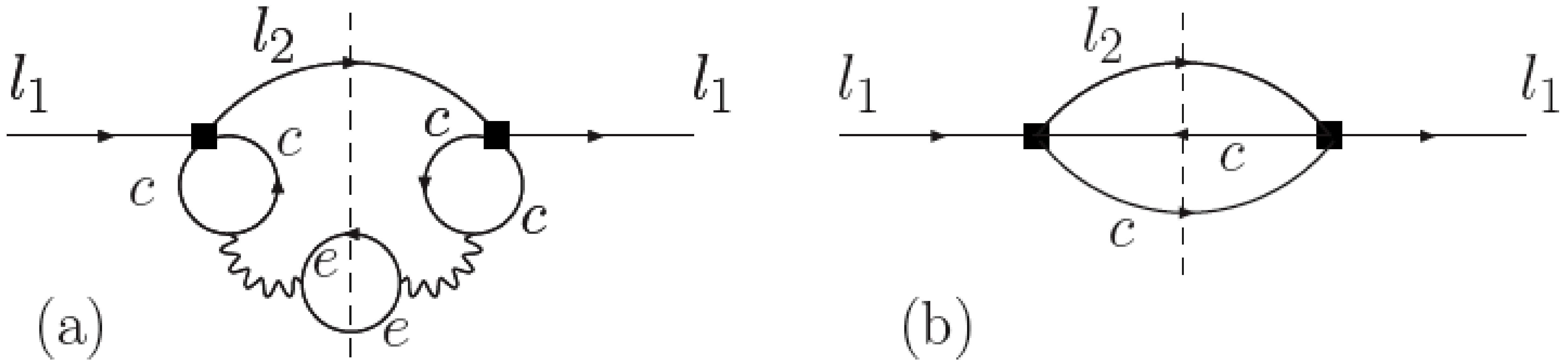


The rate $l_1 \to l_2 e^+ e^-$ (a) is connected to the integral over $|\Pi(q^2)|^2$ for which global duality is NOT expected to hold.

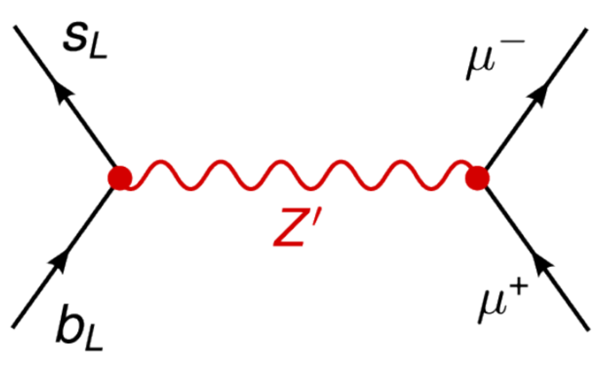
In contrast the inclusive hadronic rate $l_1 \to l_2 X$ (b) corresponds to the imaginary part of the correlator $\Pi(q^2)$.

New physics explanations (1σ solutions)

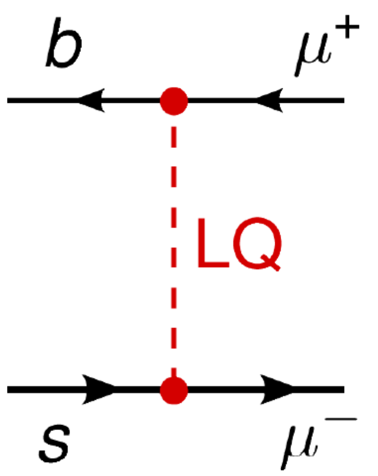
Difficult to generate $\delta C_9 = -1$ at loop level (MSSM with MFV)

Various models under discussion (tree level contributions):

Z' bosons



Leptoquarks



Altmannshofer, Straub arXiv:1308.1501
Gauld, Goertz, Haisch arXiv:1308.1959;1310.1082
Buras,De Fazio,Girrbach arXiv:1311.6729
Altmannshofer,Gori,Pospelov,Yavin arXiv:1403.1269
. . .

Hiller, Schmaltz arXiv:1408.1627
Sahoo, Mohanta arXiv:1501.05193
Becirevic, Fajfer, Kosnik arXiv:1503.09024
Bauer, Neubert arXiv:1511.01900 (loop)
. . .

Bauer, Neubert arXiv:1511.01900

Model explaining all anomalies by one leptoquark

- $$R_{D^{(*)}}^{\tau/l} = \frac{\mathcal{B}(\bar{B} \to D^{(*)}\tau\bar{\nu})/\mathcal{B}(\bar{B} \to D^{(*)}\tau\bar{\nu})_{SM}}{\mathcal{B}(\bar{B} \to D^{(*)}l\bar{\nu})/\mathcal{B}(\bar{B} \to D^{(*)}l\bar{\nu})_{SM}}$$

  3.9σ deviation from $\tau - \mu/e$ universality

- $$R_K^{\mu/e} = \frac{\mathcal{B}(B \to K\mu^+\mu^-)}{\mathcal{B}(B \to Ke^+e^-)} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

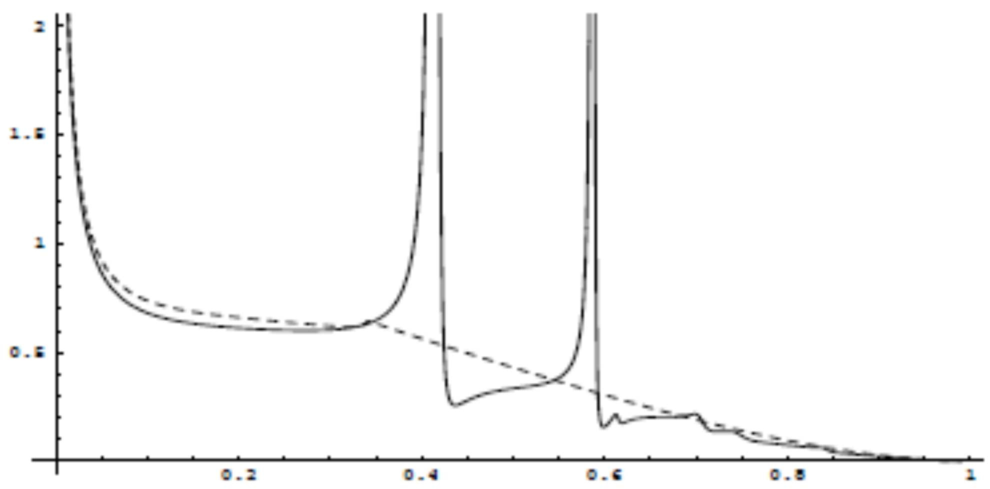
  2.6σ deviation from $\mu - e$ universality

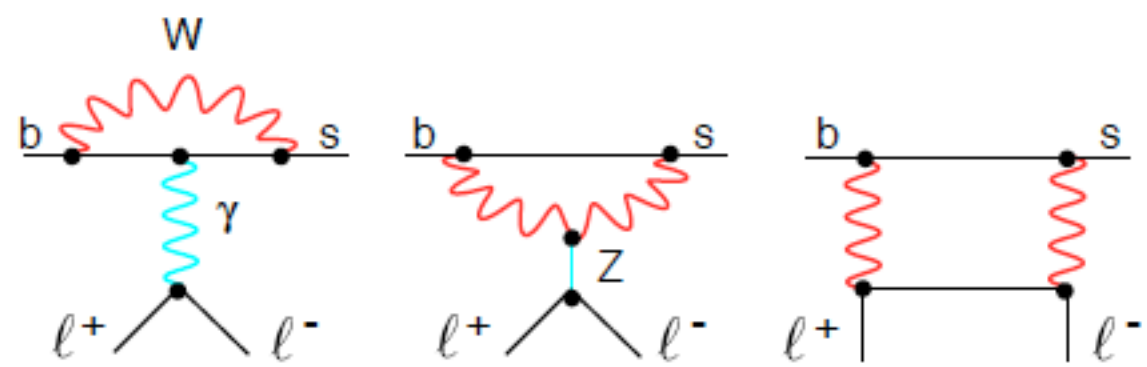
- $(g-2)_\mu$

Review of previous calculations for $B \to X_s \ell\ell$

- On-shell-$c\bar{c}$-resonances $\Rightarrow$ cuts in dlepton mass spectrum necessary :
  1GeV$^2 < q^2 <$ 6GeV^2 and 14.4GeV$^2 < q^2$ $\Rightarrow$ perturbative contributions dominant

$\frac{d}{d\hat{s}} BR(\bar{B} \to X_s l^+ l^-) \times 10^{-5}$





$\hat{s} = q^2/m_b^2$

- NNLL prediction of $\bar{B} \to X_s \ell^+\ell^-$: dilepton mass spectrum
  Asatryan,Asatrian,Greub,Walker, hep-ph/0204341;
  Ghinculov,Hurth,Isidori,Yao hep-ph/0312128:

$$BR(\bar{B} \to X_s \ell^+\ell^-)_{Cut:\ q^2 \in [1GeV^2, 6GeV^2]} = (1.63 \pm 0.20) \times 10^{-6}$$

$$BR(\bar{B} \to X_s l^+ l^-)_{Cut:\ q^2 > 14.4GeV^2} = (4.04 \pm 0.78) \times 10^{-7}$$

NNLL QCD corrections $q^2 \in [1GeV^2, 6GeV^2]$

central value: -14%, perturbative error: $13\% \to 6.5\%$

- Further refinements:
  - Completing NNLL QCD corrections:
    Mixing into $\mathcal{O}_9$ ($+1\%$), NNLL matrixelement of $\mathcal{O}_9$ (-4%)
  - NLL QED two-loop corrections to Wilson coefficients
    -1.5% shift for $\alpha_{em}(\mu = m_b)$, -8.5% for $\alpha_{em}(\mu = m_W)$
    Bobeth,Gambino,Gorbahn,Haisch, hep-ph/0312090
  - QED two-loop corrections to matrix elements
    Large collinear logarithm $Log(m_b/m_\ell)$ which survive intregration if a restricted part of the dilepton mass spectrum is considered
    $+2\%$ effect in the low-q^2 region for muons, for the electrons the effect depends on the experimental cut parameters
    Huber,Lunghi,Misiak,Wyler, hep-ph/0512066

- NNLL prediction of $\bar{B} \to X_s \ell^+ \ell^-$: forward-backward-asymmetry (FBA)
  Asatrian, Bieri, Greub, Hovhannisyan, hep-ph/0209006;
  Ghinculov,Hurth,Isidori,Yao, hep-ph/0208088, hep-ph/0312128:

$$A_{\rm FB} \equiv \frac{1}{\Gamma_{semilep}} \left(\int_0^1 d(\cos\theta) \frac{d^2\Gamma}{dq^2 d\cos\theta} - \int_{-1}^0 d(\cos\theta) \frac{d^2\Gamma}{dq^2 d\cos\theta} \right)$$

(θ angle between ℓ^+ and B momenta in dilepton CMS)

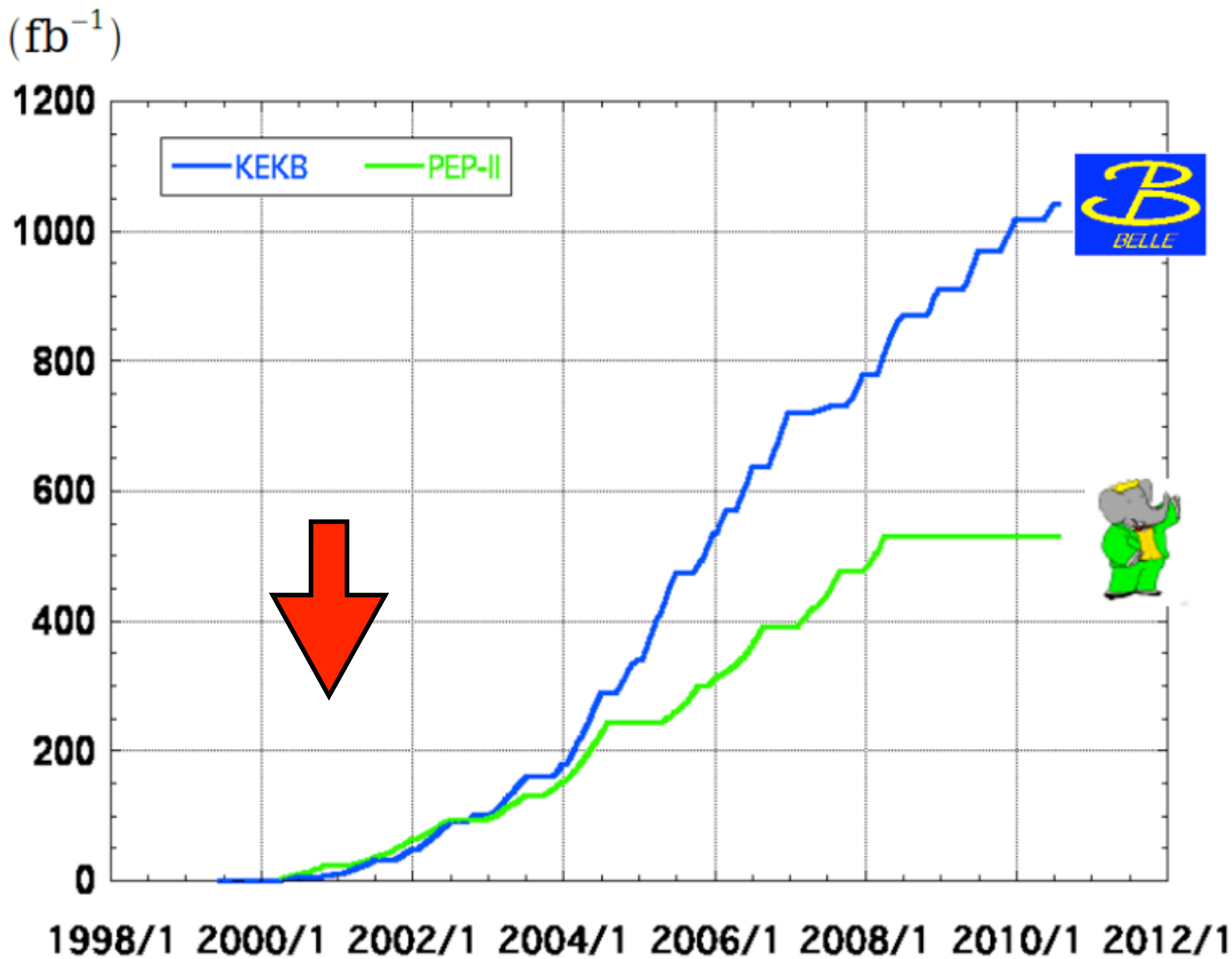
$$A_{FB}(q_0^2) = 0 \quad \text{for} \quad q_0^2 \sim C_7/C_9 \quad q_0^2 = (3.90 \pm 0.25) GeV^2$$

- "Latest" Belle measurement of branching ratio is based on less than 30% of the total luminosity

Belle hep-ex/0503044 (!!!) (based $152 \times 10^6 B\bar{B}$ events)

Babar hep-ex/0404006 (!!!) (based $89 \times 10^6 B\bar{B}$ events)

Integrated luminosity of B factories



> 1 ab^{-1}
On resonance:
$\Upsilon(5S)$: 121 fb^{-1}
$\Upsilon(4S)$: 711 fb^{-1}
$\Upsilon(3S)$: 3 fb^{-1}
$\Upsilon(2S)$: 25 fb^{-1}
$\Upsilon(1S)$: 6 fb^{-1}
Off reson./scan:
$\sim$ 100 fb^{-1}

~ 550 fb^{-1}
On resonance:
$\Upsilon(4S)$: 433 fb^{-1}
$\Upsilon(3S)$: 30 fb^{-1}
$\Upsilon(2S)$: 14 fb^{-1}
Off resonance:
$\sim$ 54 fb^{-1}

Two new analyses from the B factories:

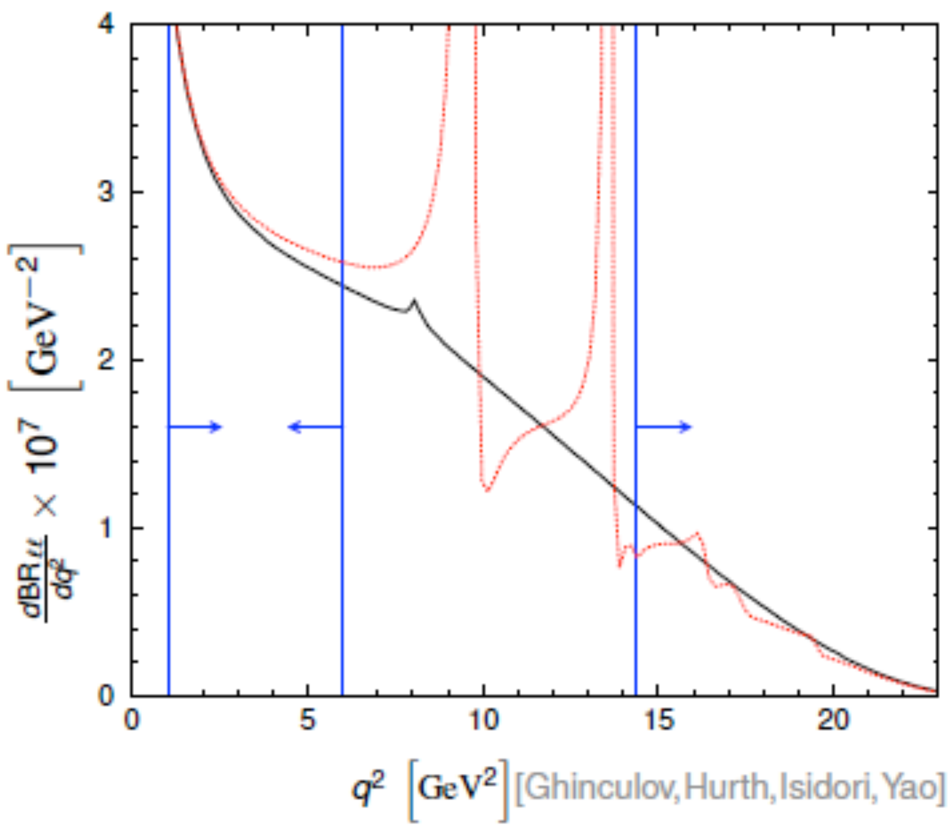
New Babar analysis on dilepton spectrum arXiv:1312.3664

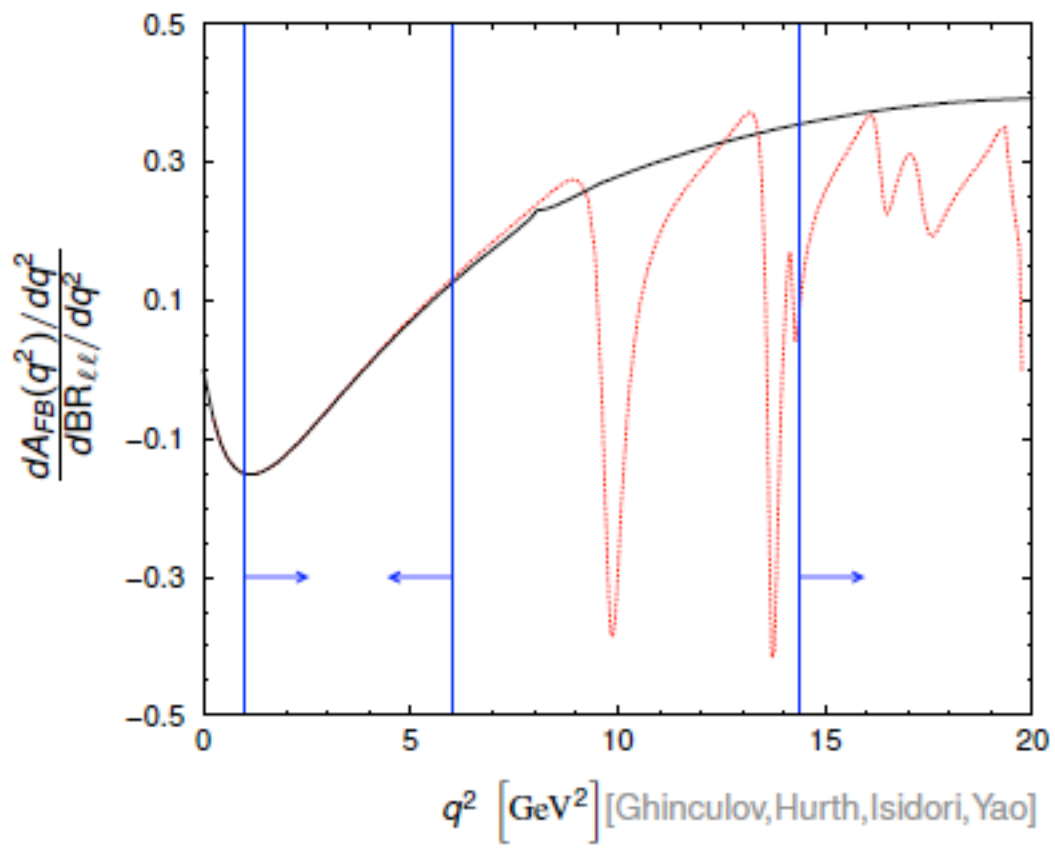
New Belle analysis on AFB arXiv:1402.7134

- Observables

$$\frac{d^2\Gamma}{dq^2\,dz} = \frac{3}{8}\left[(1+z^2)\,H_T(q^2) + 2\,z\,H_A(q^2) + 2\,(1-z^2)\,H_L(q^2)\right] \qquad (z = \cos\theta_\ell)$$

$$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2) \qquad \frac{dA_{\rm FB}}{dq^2} = 3/4\,H_A(q^2)$$





Low-q^2 region: $1\,\mathrm{GeV}^2 < q^2 < 6\,\mathrm{GeV}^2$ High-q^2 region: $q^2 > 14.4\,\mathrm{GeV}^2$

Huber,Hurth,Lunghi, arXiv:1503.04849

- Dependence on Wilson coefficients

$$H_T(q^2) \propto 2s(1-s)^2\left[\left|C_9 + \frac{2}{s}C_7\right|^2 + |C_{10}|^2\right]$$

$$H_A(q^2) \propto -4s(1-s)^2\ \mathrm{Re}\left[C_{10}\left(C_9 + \frac{2}{s}C_7\right)\right]$$

$$H_L(q^2) \propto (1-s)^2\left[\left|C_9 + 2\,C_7\right|^2 + |C_{10}|^2\right]$$

H_T suppressed in low-q^2 window

- Devide low-q^2 bin in two bins (zero of H_A in low-q^2)
  Lee,Ligeti,Stewart, Tackmann hep-ph/0612156
- Most important input parameters

$$m_b^{1S} = (4.691 \pm 0.037)\mathrm{GeV}\,, \qquad \overline{m}_c(\overline{m}_c) = (1.275 \pm 0.025)\mathrm{GeV}$$

$$|V_{ts}^* V_{tb}/V_{cb}|^2 = 0.9621 \pm 0.0027\,, \qquad BR^{exp.}_{b\to c\,e\,\nu} = (10.51 \pm 0.13)\,\%$$

- Perturbative expansion (NNLO QCD + NLO QED) α_s $\kappa = \alpha_{\rm em}/\alpha_s$

$$\begin{aligned} A &= \kappa\left[A_{LO} + \alpha_s A_{NLO} + \alpha_s^2 A_{NNLO} + \mathcal{O}(\alpha_s^3)\right] \\ &+ \kappa^2\left[A_{LO}^{em} + \alpha_s A_{NLO}^{em} + \alpha_s^2 A_{NNLO}^{em} + \mathcal{O}(\alpha_s^3)\right] + \mathcal{O}(\kappa^3) \end{aligned}$$

$$\mathrm{LO} = \alpha_{\rm em}/\alpha_s\,, \qquad \mathrm{NLO} = \alpha_{\rm em}\,, \qquad \mathrm{NNLO} = \alpha_{\rm em}\,\alpha_s$$

- Normalization:

Huber,Hurth,Lunghi, arXiv:1503.04849

$$\frac{d\ BR(\bar{B} \to X_s ll)}{d\ \hat{s}} = BR^{exp.}_{b\to c\,e\,\nu} \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{1}{C} \frac{d\Gamma(\bar{B} \to X_s\, ll)/d\hat{s}}{\Gamma(\bar{B} \to X_u e\bar{\nu})}$$

$$C = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma(\bar{B} \to X_c e\bar{\nu})}{\Gamma(\bar{B} \to X_u e\bar{\nu})} = 0.574 \pm 0.019$$

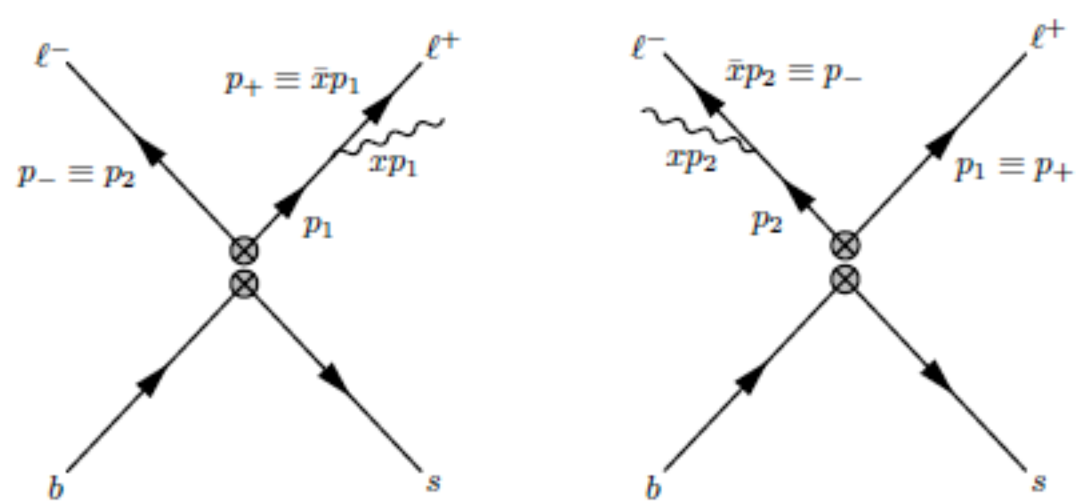
Gambino,Schwanda, arXiv:1307.4551

Huber,Hurth,Lunghi, arXiv:1503.04849

- Collinear Photons give rise to log-enhanced QED corrections $\alpha_{\rm em} \log(m_b^2/m_\ell^2)$
- Higher powers of z in double differential decay width
  - Definition of H_i ? Sensitivity for QED observables ?

We use Legendre poynomials for H_T and H_L and $Sign(z)$ for H_A

We can construct QED sensitive observables (vanish in absence of QED) by Legendre projectors $P_3(z)$ or $P_4(z)$: 10^{-8}



Huber,Hurth,Lunghi, arXiv:1503.04849

- Collinear Photons give rise to log-enhanced QED corrections $\alpha_{\rm em} \log(m_b^2/m_\ell^2)$
- Higher powers of z in double differential decay width
  - Definition of H_i ? Sensitivity for QED observables ?
- Size of logs depend on experimental set-up

$$q^2 = (p_{\ell^+} + p_{\ell^-})^2 \quad \text{vs.} \quad q^2 = (p_{\ell^+} + p_{\ell^-} + p_{\gamma,\rm coll})^2$$

  - We assume no photons are included in the definition of q^2 (di-muon channel at Babar/Belle, di-electron at Belle)
  - Babar's di-electron channel: Photons that are emitted in a cone of 35 mrad angular opening are included in q^2

  Monte Carlo techniques needed to estimate this effect

$$\frac{\left[\mathcal{B}_{ee}^{\rm low}\right]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\rm coll}}}}{\left[\mathcal{B}_{ee}^{\rm low}\right]_{q=p_{e^+}+p_{e^-}}} - 1 = 1.65\% \qquad \frac{\left[\mathcal{B}_{ee}^{\rm high}\right]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\rm coll}}}}{\left[\mathcal{B}_{ee}^{\rm high}\right]_{q=p_{e^+}+p_{e^-}}} - 1 = 6.8\%$$

Huber,Hurth,Lunghi, arXiv:1503.04849

Results

Low-q^2 $(1GeV^2 < q^2 < 6GeV^2)$

$BR(B \to X_s ee) = (1.67 \pm 0.10)\, 10^{-6}$

$BR(B \to X_s \mu\mu) = (1.62 \pm 0.09)\, 10^{-6}$

Babar:$BR(B \to X_s \ell\ell) =$

$= (1.60\ (+0.41 - 0.39)_{stat}(+0.17 - 0.13)_{syst}(\pm 0.18)_{mod})\, 10^{-6}$

good agreement with SM

Results

Huber,Hurth,Lunghi, arXiv:1503.04849

High-q^2, Theory: $q^2 > 14.4 GeV^2$, Babar: $q^2 > 14.2 GeV^2$

$BR(B \to X_s ee) = (0.220 \pm 0.070)\, 10^{-6}$

$BR(B \to X_s \mu\mu) = (0.253 \pm 0.070)\, 10^{-6}$

Babar: $BR(B \to X_s \ell\ell) =$

$(0.57\ (+0.16 - 0.15)_{stat}(+0.03 - 0.02)_{syst})\, 10^{-6}$

2σ higher than SM

Significant higher values predicted in Greub et al. due to missing power and QED corrections and different cut Greub,Pilipp,Schupbach,arXiv:0810.4077

(but perfect agreement if we use their prescriptions)

Further refinement

Huber,Hurth,Lunghi, arXiv:1503.04849

Normalization to semileptonic $B \to X_u \ell \nu$ decay rate with the same cut reduces the impact of $1/m_b$ corrections in the high-q^2 region significantly.

Ligeti,Tackmann arXiv:0707.1694

Theory prediction for ratio

$$R(s_0)_{ee} = (2.25 \pm 0.31)\, 10^{-3}$$

$$R(s_0)_{\mu\mu} = (2.62 \pm 0.30)\, 10^{-3}$$

Largest source of error are CKM elements (V_{ub})

Note: Additional $O(5\%)$ uncertainty due to nonlocal power corrections $O(\alpha_s \Lambda / m_b)$

Huber,Hurth,Lunghi, arXiv:1503.04849

Further results in units of 10^{-6}

$$H_L[1,3.5]_{ee} = 0.64 \pm 0.03 \qquad H_L[1,3.5]_{\mu\mu} = 0.68 \pm 0.04$$

$$H_L[3.5,6]_{ee} = 0.50 \pm 0.03 \qquad H_L[3.5,6]_{\mu\mu} = 0.53 \pm 0.03$$

$$H_L[1,6]_{ee} = 1.13 \pm 0.06 \qquad H_L[1,6]_{\mu\mu} = 1.21 \pm 0.07$$

$$H_T[1,3.5]_{ee} = 0.29 \pm 0.02 \qquad H_T[1,3.5]_{\mu\mu} = 0.21 \pm 0.01$$

$$H_T[3.5,6]_{ee} = 0.24 \pm 0.02 \qquad H_T[3.5,6]_{\mu\mu} = 0.19 \pm 0.02$$

$$H_T[1,6]_{ee} = 0.53 \pm 0.04 \qquad H_T[1,6]_{\mu\mu} = 0.40 \pm 0.03$$

$$H_A[1,3.5]_{ee} = -0.103 \pm 0.005 \qquad H_A[1,3.5]_{\mu\mu} = -0.110 \pm 0.005$$

$$H_A[3.5,6]_{ee} = +0.073 \pm 0.012 \qquad H_A[3.5,6]_{\mu\mu} = +0.067 \pm 0.012$$

$$H_A[1,6]_{ee} = -0.029 \pm 0.016 \qquad H_A[1,6]_{\mu\mu} = -0.042 \pm 0.016$$

Total error $\mathcal{O}(5-8\%)$. Still dominated by scale uncertainty.