A bottom up determination of lepton mass matrices

Andrea Romanino, SISSA

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The topic of this talk

- A bottom-up approach to lepton masses and mixings
- Based on a simple "stability" principle
- Leading to a model-independent determination of *lepton mass matrices*

Are lepton mass matrices physical?

- In the SM only the eigenvalues and the PMNS matrix are physical

$$M_\nu = U_\nu^T M_\nu^{\text{diag}} U_\nu \qquad M_\nu^{\text{diag}},\ M_E^{\text{diag}}$$
$$M_E = U_{e^c}^T M_E^{\text{diag}} U_e \qquad U = U_e U_\nu^\dagger \quad \text{(up to phases)}$$

- In a theory of flavour they typically are

The "stability" principle

- Physical quantities (e.g. m_e/m_τ, m_μ/m_τ, $|\Delta m^2_{12}/\Delta m^2_{23}|$) should be stable wrt variations of individual matrix entries

- Motivation: an understanding of their smallness requires the smallness is not accidental, i.e. its stability wrt variations of independent, fundamental parameters

- Underlying assumption: matrix elements correspond to *independent* fundamental parameters

- Caveat: correlations might arise e.g. because of non-abelian symmetries

Caveat

$$m_\nu = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

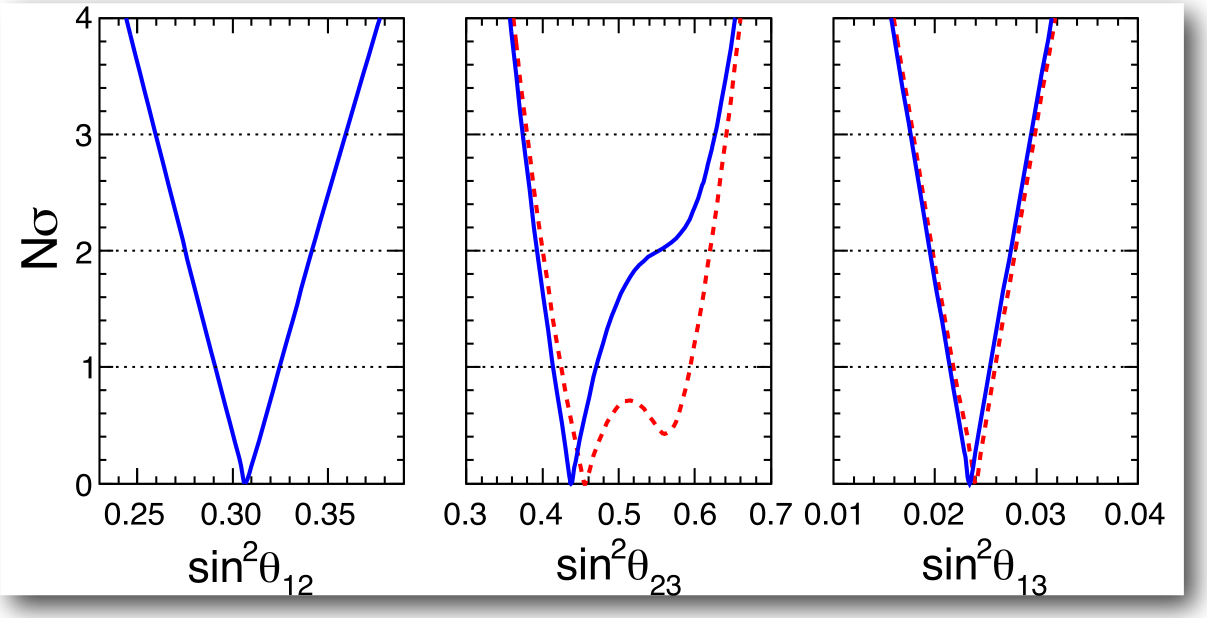
$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

3 independent correlations

$m_{12} = m_{13} \quad m_{22} = m_{33}$

$m_{11} + m_{12} = m_{22} + m_{23}$

(non trivial model-building: alignment, charged fermions, selective flavour breaking)



The “stability” principle: mathematical formulation

- Charged leptons: $$\frac{\Delta m_k}{m_k} \lesssim \left| \frac{\Delta M^E_{ij}}{M^E_{ij}} \right| \qquad \text{if } |\Delta M^E_{ij}| \ll |M^E_{ij}|$$

- Neutrinos: $$\frac{\Delta(\Delta m^2_{12})}{\Delta m^2_{12}} \lesssim \left| \frac{\Delta M^\nu_{ij}}{M^\nu_{ij}} \right| \qquad \text{if } |\Delta M^\nu_{ij}| \ll |M^\nu_{ij}|$$

M_ν stable textures: $\Delta m^2_{12} \to 0$ limit (Majorana)

- Technical details

  - define $\Delta \equiv (\Delta m^2_{12} \Delta m^2_{23} \Delta m^2_{13})^2$ (polynomial in the matrix entries)

  - simple algebraic condition: $|M_{ij}| \; \Delta_{M_{ij} \to M_{ij} + \Delta M_{ij}} = 0$
    as a polynomial in ΔM_{ij}

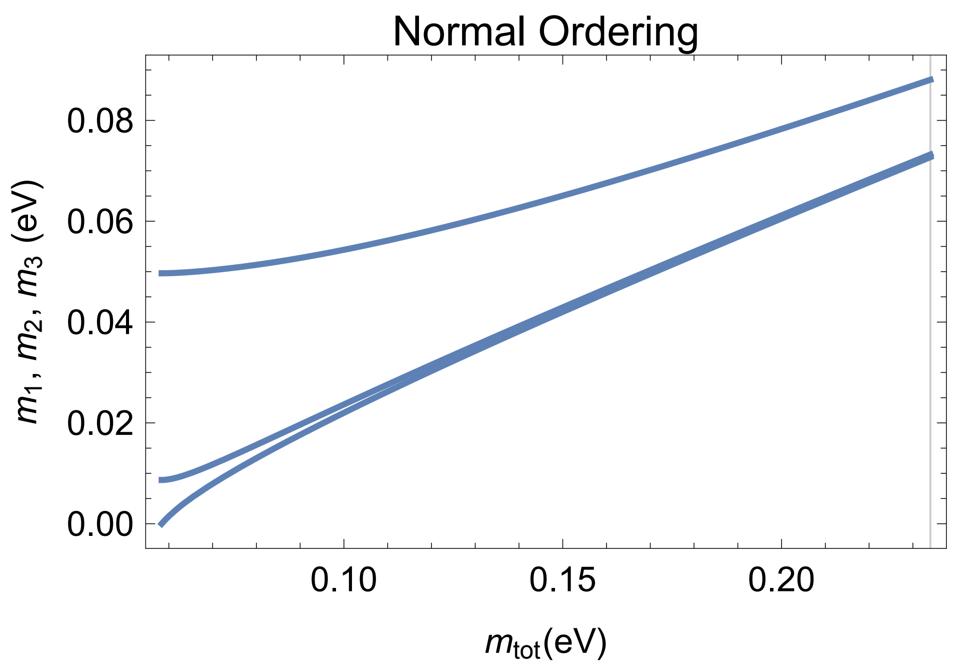
M_ν stable textures: $\Delta m^2_{12} \to 0$ limit (Majorana)

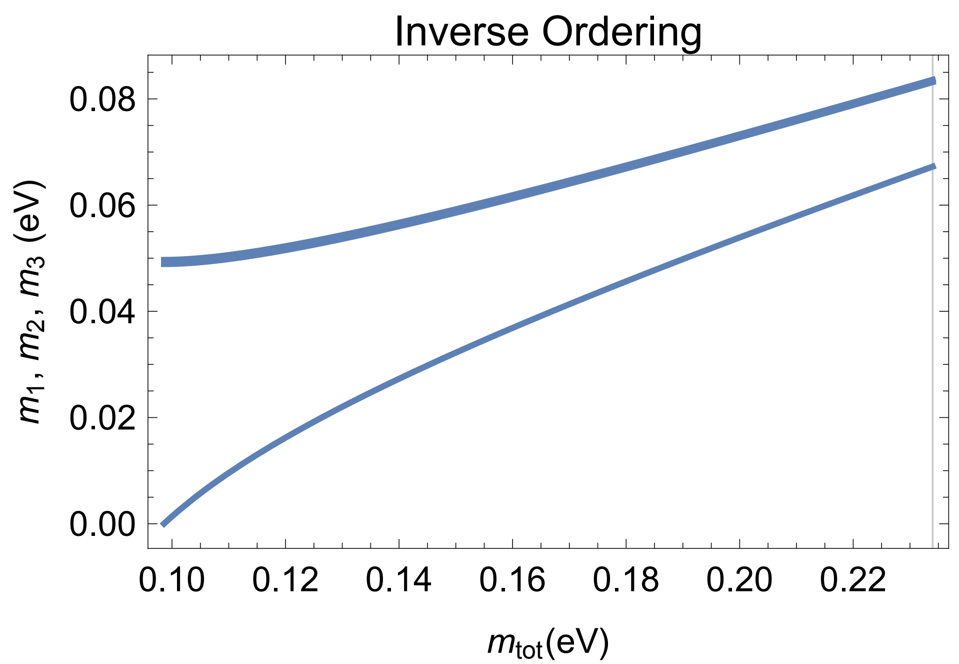
	$\begin{pmatrix} & X & \\ X & & \\ & & X \end{pmatrix}$	$\begin{pmatrix} & X & X \\ X & & \\ X & & \end{pmatrix}$	$\begin{pmatrix} & X & \\ X & & \\ & & \end{pmatrix}$	$\begin{pmatrix} & & \\ & & \\ & & X \end{pmatrix}$
IH	✓	✓	✓	X
NH	✓	X	X	✓
semi-deg.	✓	X	X	X

experiments can determine the leading order form of M_ν

Example: semi-degeneracy

- m_{tot} close to the exp limit ⇒ semi-degeneracy





$$M_\nu = \begin{pmatrix} & m & \\ m & & \\ & & m_3 \end{pmatrix} + \text{small}$$

Example: semi-degeneracy

$$M_\nu \sim \begin{pmatrix} \epsilon^2 m & m & \epsilon\, m \\ m & \epsilon^2 m & \epsilon\, m \\ \epsilon\, m & \epsilon\, m & m_3 \end{pmatrix} \qquad \epsilon^2 = \frac{\Delta m^2_{12}}{2m^2}$$

near the exp bound $m_{tot} < 0.23$ eV:

	ϵ	ϵ^2
NH	0.07	0.005
IH	0.08	0.007

$$\boxed{\theta^\nu_{12} = \frac{\pi}{4} \pm 0.005} \qquad \theta_{12} \approx \frac{\pi}{4} - 0.2$$

M_E stable textures: $m_e\, m_\mu \to 0$ limit

$$
\begin{pmatrix} & & \\ & & \\ & & X \end{pmatrix}
\quad
\begin{pmatrix} & & \\ & & \\ & X & X \end{pmatrix}
\quad
\begin{pmatrix} & & \\ & & \\ X & X & X \end{pmatrix}
$$

$$
\begin{pmatrix} & & \\ & & X \\ & & X \end{pmatrix}
\quad
\begin{pmatrix} & & X \\ & & X \\ & & X \end{pmatrix}
$$

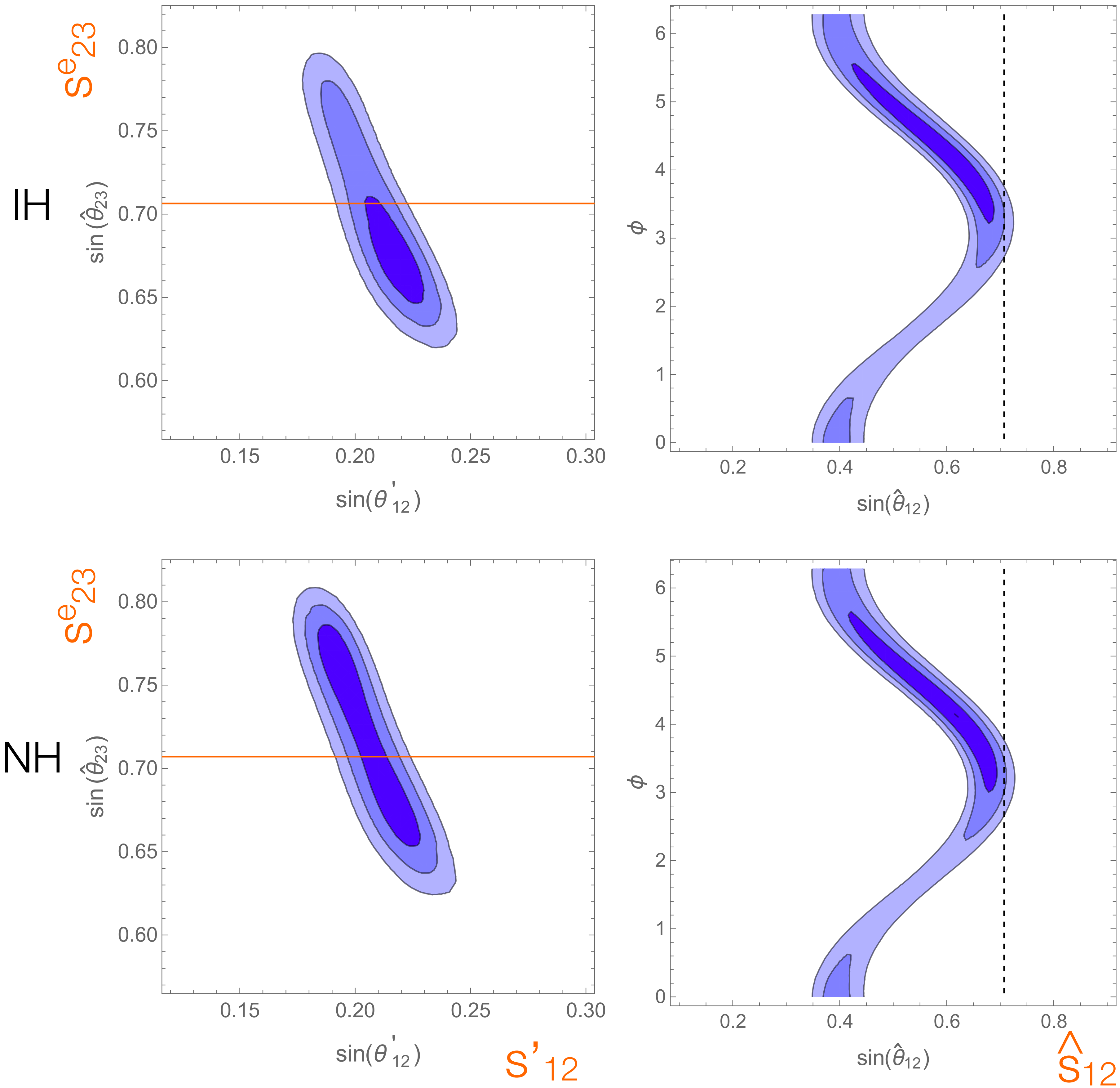
M_E (with neutrino semi-degeneracy)

$$\boxed{U_e = U U_\nu}$$

$$M_E = U_{e^c}^T M_E^{\text{diag}} U_e = \begin{pmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ U_{31}^e & U_{32}^e & U_{33}^e \end{pmatrix} m_\tau + \mathcal{O}(0.003)$$

Experimental determination of the last row in the stability hypothesis
(in the parameterisation below)

$$|M_E| = \begin{pmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ s_{12}^e s_{23}^e & c_{12}^e s_{23}^e & c_{23}^e \end{pmatrix} m_\tau$$



$$\hat{\theta}_{12} = \frac{\pi}{4} - \theta^e_{12}$$

$$\downarrow$$

$$\hat{\theta}_{12} = \left| \frac{\pi}{4} - \theta^e_{12} e^{i\alpha} \right|$$

$$\frac{\pi}{4} - \hat{\theta}_{12} < \theta^e_{12} < \frac{\pi}{4} + \hat{\theta}_{12}$$

$\theta^e_{12} \neq 0$ at 2σ C.L.

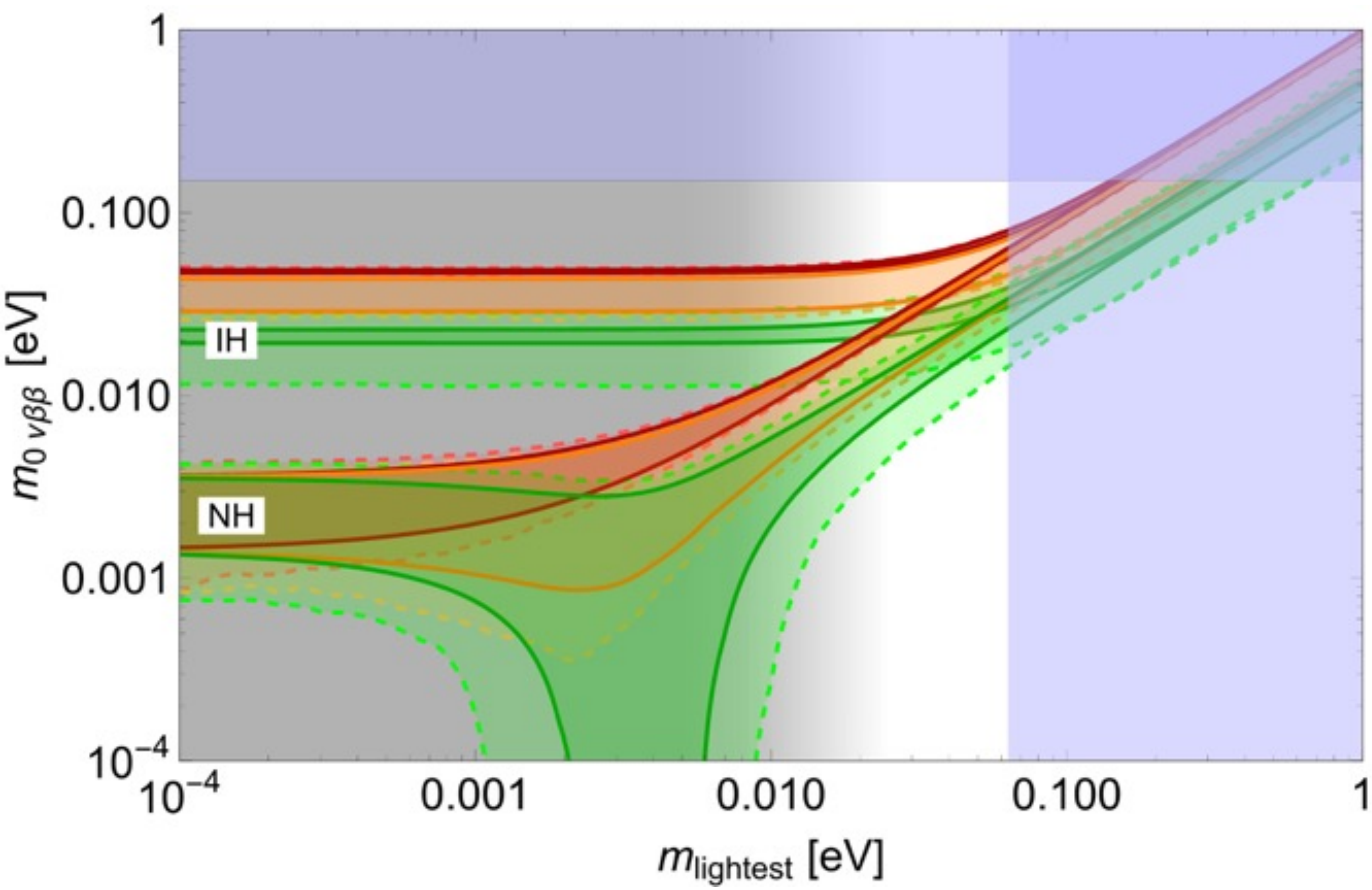
$\theta^e_{12} \sim 0.13$ no phases

$\theta^e_{12} \gtrsim 0.13$ with phases

$\theta^e_{12} \sim 0.13$ no canc.

α ~ π/2

$$m_{0\nu\beta\beta} \approx m\,|\cos^2\theta_{12} + e^{2i\hat{\alpha}}\sin^2\theta_{12}|$$



α = 0

α = π/4

α = π/2

Structure of the lepton matrices

$$|M_\nu| \sim \begin{pmatrix} \epsilon^2 m & m & \epsilon\, m \\ m & \epsilon^2 m & \epsilon\, m \\ \epsilon\, m & \epsilon\, m & m_3 \end{pmatrix}$$

$$|M_E| \sim \begin{pmatrix} m_e & m_e/\lambda & m_e/\lambda \\ \lambda m_\mu & m_\mu & m_\mu \\ \lambda m_\tau & m_\tau & m_\tau \end{pmatrix}$$

Summary

- Bottom-up determination of lepton mass matrices
- Within assumptions, the experiment can identify the LO form of the neutrino mass matrix
- In the (experimentally relevant) example of semi-degenerate neutrinos, the LO form of the lepton mass matrices, and the size of the corrections, are determined by experimental data + the stability condition
- Bottom-up identification of symmetry principle?