The determination of α_s by ALPHA Collaboration

Rainer Sommer @ FPCapri2016

based on work by

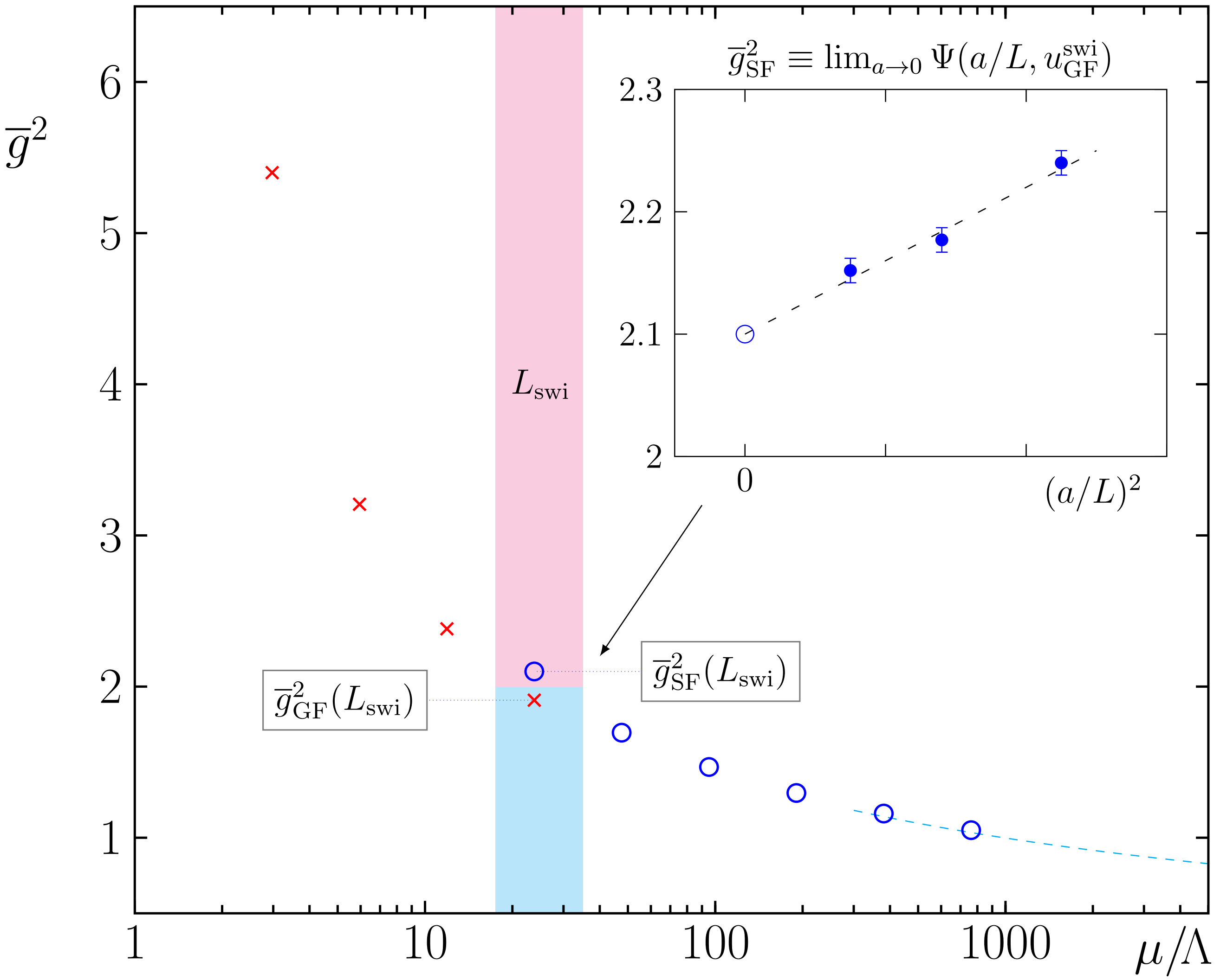
Mattia Bruno, Mattia Dalla Brida, Patrick Fritzsch,
Tomasz Korzec, Alberto Ramos, Stefan Schaefer,
Hubert Simma, Stefan Sint, RS

and simulations by

CLS

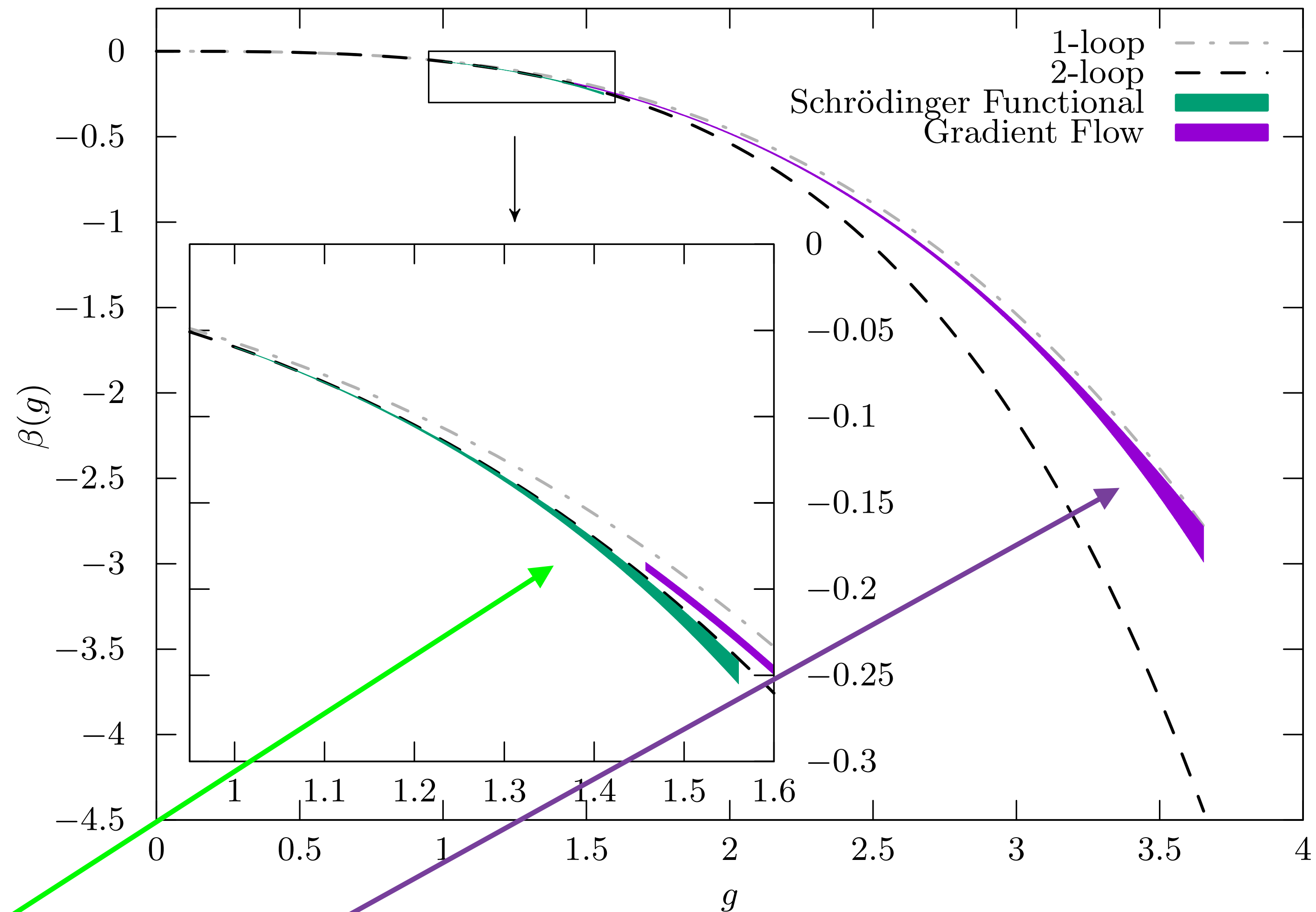
Summary

- Connect low energy f_π, f_K to $g^2(\mu$=100GeV) non-perturbatively
- extract $\Lambda^{(3)}$
- connect to $\Lambda^{(5)}$ by PT $\Longrightarrow \alpha(m_Z)$
- using two different intermediate renormalization schemes
  - SF scheme
  - GF scheme



Summary

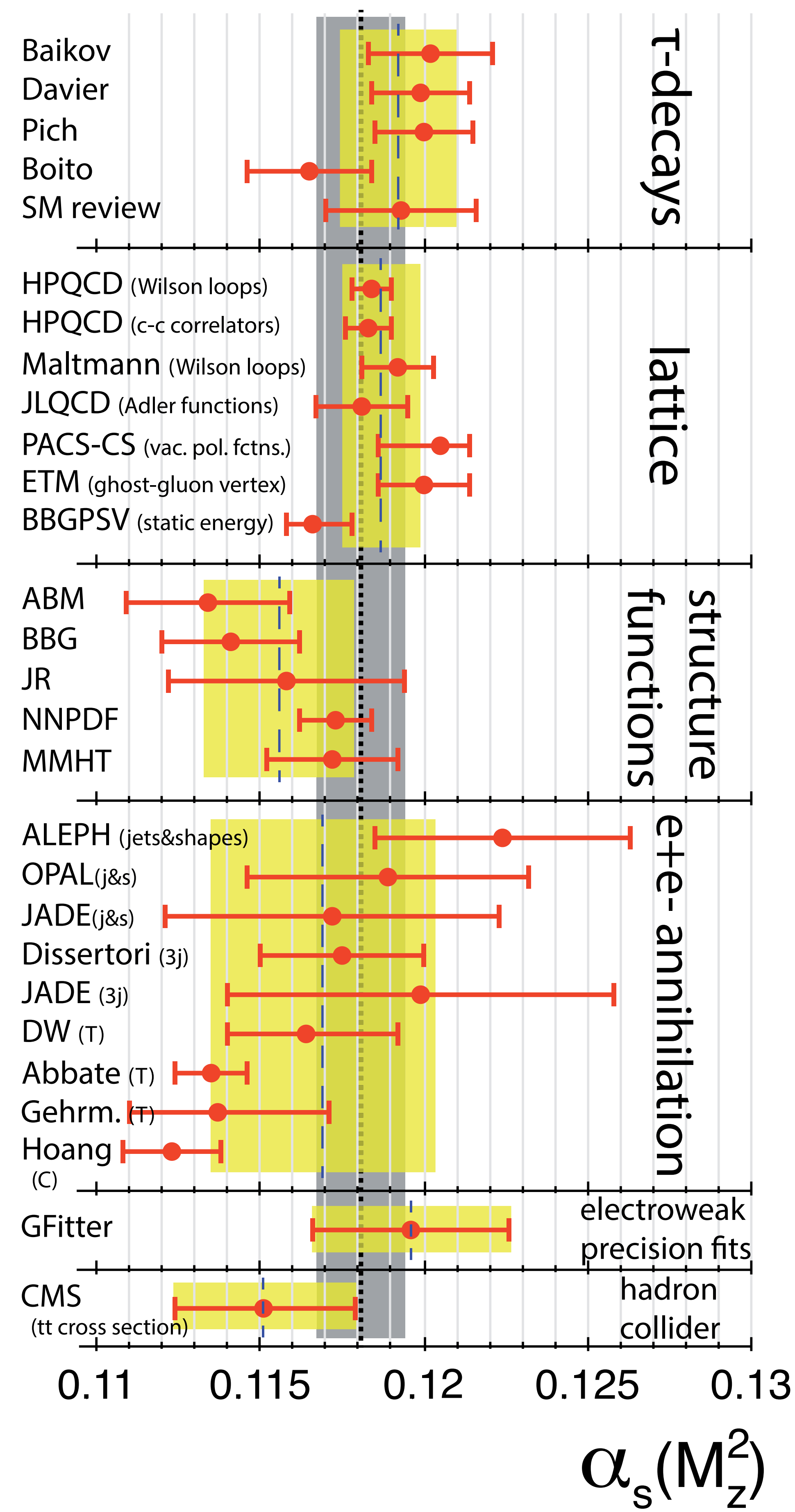
- Connect low energy f_π, f_K to $g^2(\mu=100\text{GeV})$ non-perturbatively
- extract $\Lambda^{(3)}$
- connect to $\Lambda^{(5)}$ by PT $\Longrightarrow \alpha(m_Z)$
- using two different intermediate renormalization schemes
  - SF scheme
  - GF scheme



- NP **β**- functions

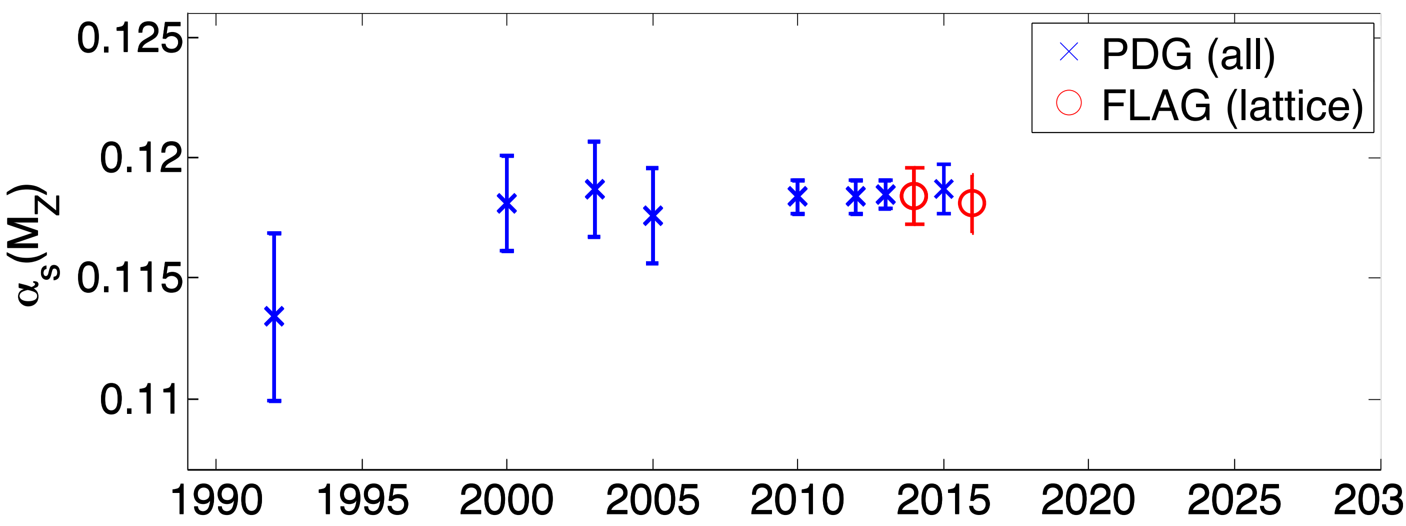
PDG 2016

- Individual determinations differ beyond estimated errors
- lattice most precise
- Here: new very controlled determination



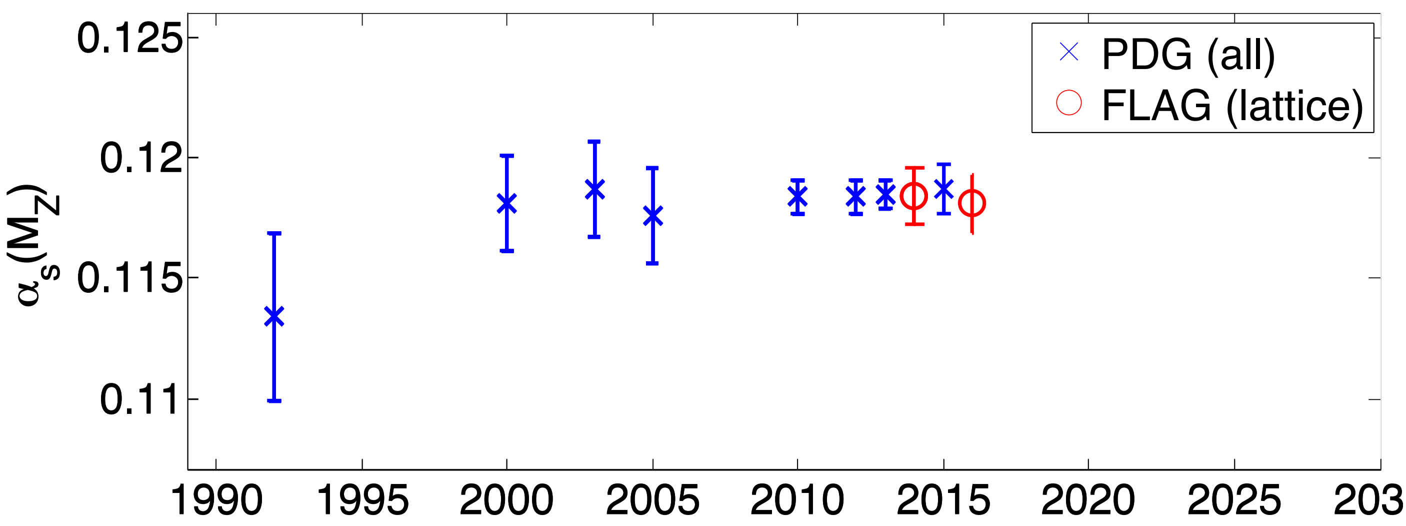
PDG 2016

- Individual determinations differ beyond estimated errors
- lattice most precise
- Here: new very controlled determination



PDG 2016

- Individual determinations differ beyond estimated errors
- lattice most precise
- Here: new very controlled determination



$$\text{PDG, lattice: } 0.1187\binom{10}{11}$$

$$\text{PDG, pheno: } 0.1175(17)$$

$$\text{FLAG2: } 0.1184(12)$$

$$\text{FLAG3: } 0.1181(12) \quad \text{to be confirmed}$$

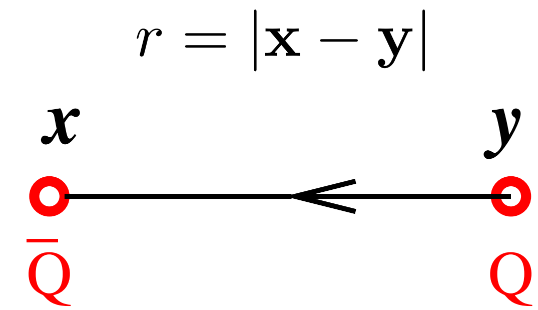
before we compute the QCD coupling

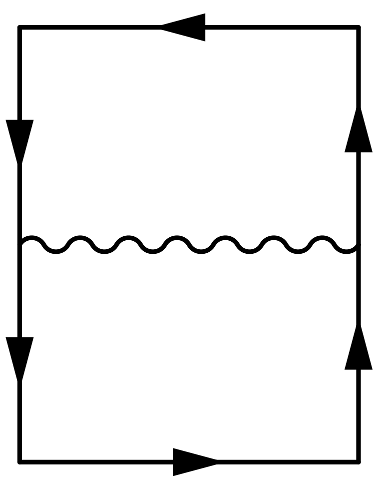
we should define it

… at least when we talk about a non-perturbative computation

Definition of QCD coupling (an example)

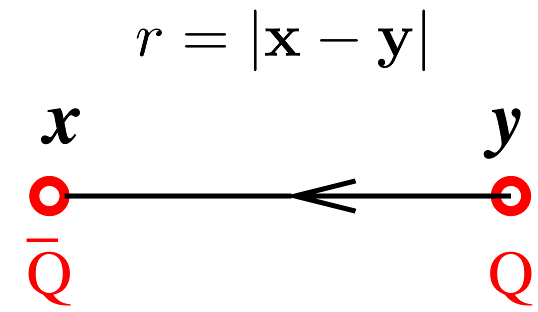
$$\alpha_{qq}(\mu) \equiv \frac{3r^2}{4} F_{Q\bar{Q}}(r)\,, \quad \mu = \frac{1}{r}$$

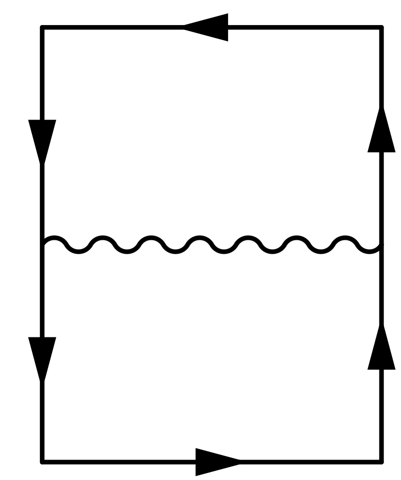




Definition of QCD coupling (an example)

$$\alpha_{\rm qq}(\mu) \equiv \frac{3r^2}{4} F_{Q\bar{Q}}(r)\,, \quad \mu = \frac{1}{r}$$





then

$$\alpha_{\rm qq}(\mu) = \alpha_{\overline{\rm MS}}(\mu) + c_1 \alpha^2_{\overline{\rm MS}}(\mu) + \ldots$$

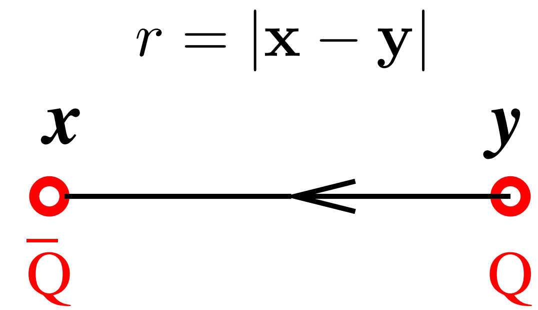
always
(non-perturbatively)
defined
physics!

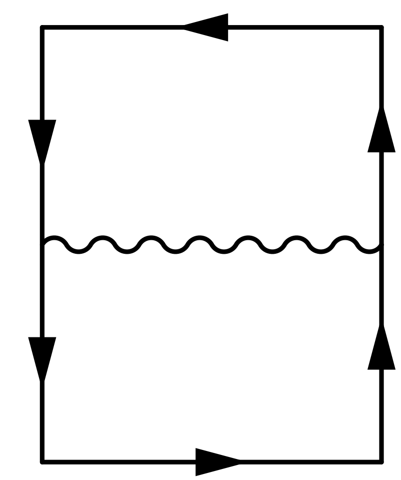
perturbatively defined
by such relations

makes sense for $\alpha \ll 1$

Definition of QCD coupling (an example)

$$\alpha_{\mathrm{qq}}(\mu) \equiv \frac{3r^2}{4} F_{Q\bar{Q}}(r)\,, \quad \mu = \frac{1}{r}$$





then

$$\alpha_{\mathrm{qq}}(\mu) = \alpha_{\overline{\mathrm{MS}}}(\mu) + c_1 \alpha^2_{\overline{\mathrm{MS}}}(\mu) + \ldots$$

always (non-perturbatively) defined physics!

perturbatively defined by such relations

makes sense for $\alpha \ll 1$

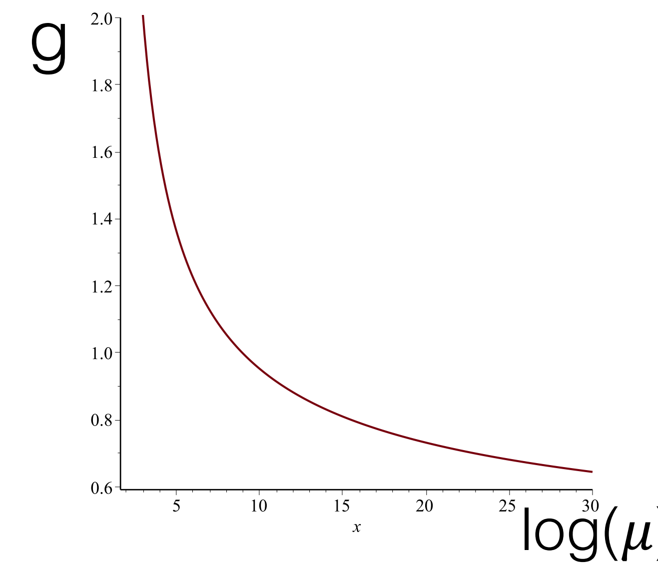
There are many definitions. Equivalent at small α.

QCD coupling, energy dependence

$$\text{RGE:} \quad \mu \frac{\partial \bar{g}}{\partial \mu} = \beta(\bar{g}) \qquad \bar{g}(\mu)^2 = 4\pi\alpha(\mu)$$

$$\beta(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^3 \left\{ b_0 + b_1\bar{g}^2 + b_2\bar{g}^4 + \ldots \right\}$$

$$b_0 = \frac{1}{(4\pi)^2}\left(11 - \frac{2}{3}N_\mathrm{f}\right)$$



Asymptotic freedom

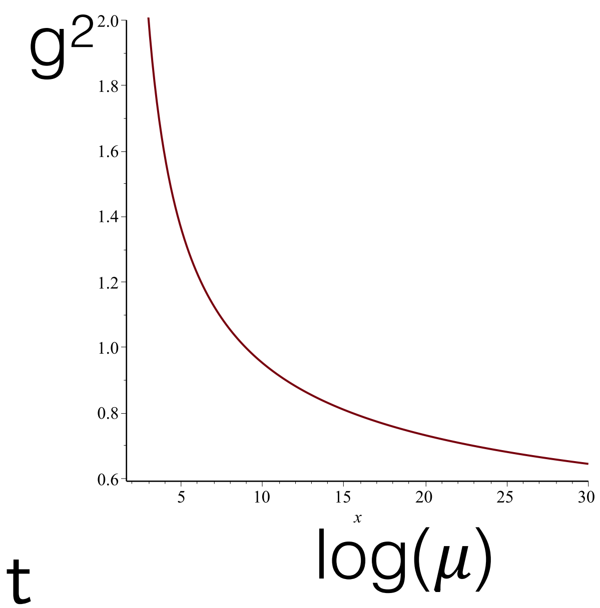
μ = energy = physical

QCD coupling, energy dependence

$$\text{RGE:} \quad \mu \frac{\partial \bar{g}}{\partial \mu} = \beta(\bar{g}) \qquad \bar{g}(\mu)^2 = 4\pi\alpha(\mu)$$

$$\beta(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^3 \left\{ b_0 + b_1 \bar{g}^2 + b_2 \bar{g}^4 + \ldots \right\}$$

$$b_0 = \frac{1}{(4\pi)^2} \left(11 - \frac{2}{3} N_\mathrm{f} \right)$$

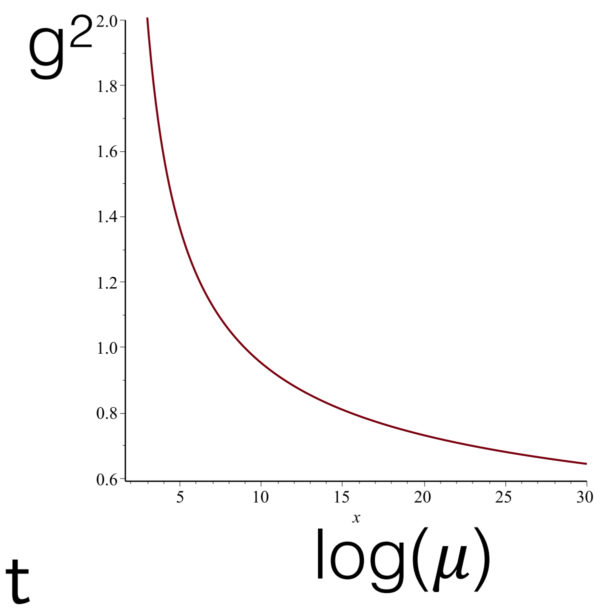


Λ-parameter $(\bar{g} \equiv \bar{g}(\mu))$ = Renormalization Group Invariant
= intrinsic scale of QCD = integration constant of RGE

$$\Lambda = \mu \left(b_0 \bar{g}^2 \right)^{-b_1/2b_0^2} \mathrm{e}^{-1/2b_0\bar{g}^2} \exp\left\{ - \int_0^{\bar{g}} \mathrm{d}g \left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g} \right] \right\}$$

$$\bar{g} \equiv \bar{g}(\mu)$$

QCD coupling, energy dependence



$$\text{RGE:} \quad \mu \frac{\partial \bar{g}}{\partial \mu} = \beta(\bar{g}) \qquad \bar{g}(\mu)^2 = 4\pi\alpha(\mu)$$

$$\beta(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^3 \{ b_0 + b_1 \bar{g}^2 + b_2 \bar{g}^4 + \ldots \}$$

$$b_0 = \frac{1}{(4\pi)^2} \left(11 - \frac{2}{3} N_\mathrm{f}\right)$$

Λ-parameter $(\bar{g} \equiv \bar{g}(\mu))$ = Renormalization Group Invariant
= intrinsic scale of QCD = integration constant of RGE

$$\Lambda = \mu \, (b_0 \bar{g}^2)^{-b_1/2b_0^2} \mathrm{e}^{-1/2b_0\bar{g}^2} \exp\left\{ -\int_0^{\bar{g}} \mathrm{d}g \left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g} \right] \right\}$$

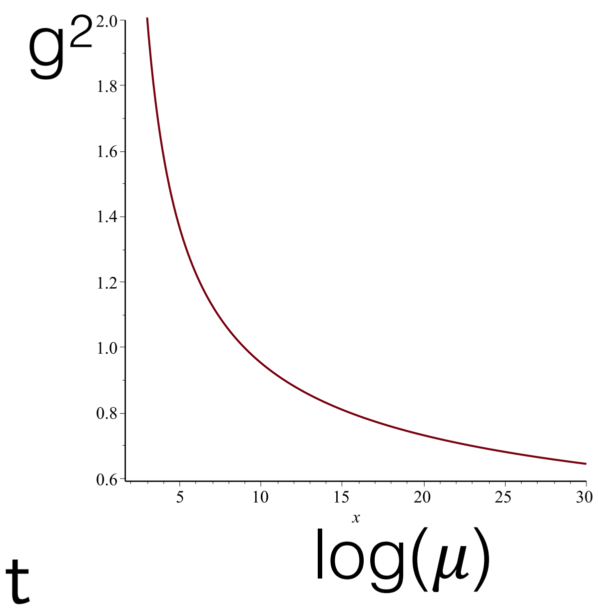
singular behavior

QCD coupling, energy dependence

RGE: $\mu \frac{\partial \bar{g}}{\partial \mu} = \beta(\bar{g})$ $\qquad \bar{g}(\mu)^2 = 4\pi\alpha(\mu)$

$$\beta(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^3 \left\{ b_0 + b_1 \bar{g}^2 + b_2 \bar{g}^4 + \ldots \right\}$$

$$b_0 = \frac{1}{(4\pi)^2} \left(11 - \frac{2}{3} N_{\mathrm{f}}\right)$$



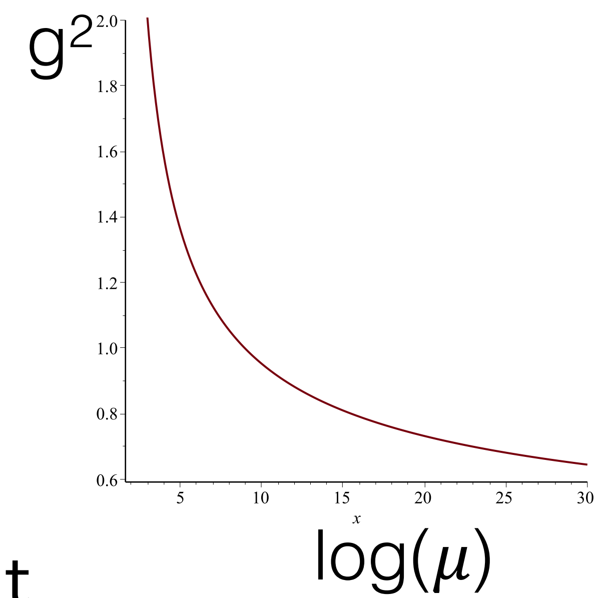
Λ-parameter $(\bar{g} \equiv \bar{g}(\mu))$ = Renormalization Group Invariant
= intrinsic scale of QCD = integration constant of RGE

$$\Lambda = \mu \left(b_0 \bar{g}^2\right)^{-b_1/2b_0^2} \mathrm{e}^{-1/2b_0\bar{g}^2} \exp\left\{ -\int_0^{\bar{g}} \mathrm{d}g \left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g} \right] \right\}$$

singular behavior

convergent for g -> 0

QCD coupling, energy dependence



$$\text{RGE:} \quad \mu \frac{\partial \bar{g}}{\partial \mu} = \beta(\bar{g}) \qquad \bar{g}(\mu)^2 = 4\pi\alpha(\mu)$$

$$\beta(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^3\left\{b_0 + b_1\bar{g}^2 + b_2\bar{g}^4 + \ldots\right\}$$

$$b_0 = \frac{1}{(4\pi)^2}\left(11 - \frac{2}{3}N_\mathrm{f}\right)$$

Λ-parameter $(\bar{g} \equiv \bar{g}(\mu)) =$ Renormalization Group Invariant
$=$ intrinsic scale of QCD = integration constant of RGE

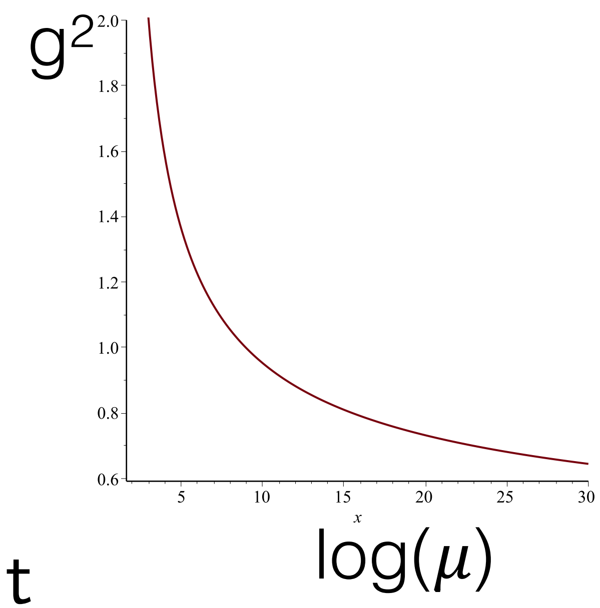
$$\Lambda = \mu\,(b_0\bar{g}^2)^{-b_1/2b_0^2}\,\mathrm{e}^{-1/2b_0\bar{g}^2}\,\exp\left\{-\int_0^{\bar{g}} \mathrm{d}g\left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right]\right\}$$

$$\bar{g} = \bar{g}_{\overline{\mathrm{MS}}} \to \Lambda = \Lambda_{\overline{\mathrm{MS}}}\,, \quad \bar{g} = \bar{g}_{\mathrm{qq}} \to \Lambda = \Lambda_{\mathrm{qq}}$$

$$\Lambda_{\overline{\mathrm{MS}}}/\Lambda_{\mathrm{qq}} = \exp\left(c_1/(2b_0)\right)$$

$$\alpha_{\mathrm{qq}}(\mu) = \alpha_{\overline{\mathrm{MS}}}(\mu) + c_1\alpha^2_{\overline{\mathrm{MS}}}(\mu) + \ldots$$

QCD coupling, energy dependence



$$\text{RGE:} \quad \mu\frac{\partial\bar{g}}{\partial\mu} = \beta(\bar{g}) \qquad \bar{g}(\mu)^2 = 4\pi\alpha(\mu)$$

$$\beta(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^3\left\{b_0 + b_1\bar{g}^2 + b_2\bar{g}^4 + \ldots\right\}$$

$$b_0 = \frac{1}{(4\pi)^2}\left(11 - \frac{2}{3}N_\mathrm{f}\right)$$

Λ-parameter $(\bar{g} \equiv \bar{g}(\mu))$ = Renormalization Group Invariant
= intrinsic scale of QCD = integration constant of RGE

$$\Lambda = \mu\,(b_0\bar{g}^2)^{-b_1/2b_0^2}\mathrm{e}^{-1/2b_0\bar{g}^2}\,\exp\left\{-\int_0^{\bar{g}}\mathrm{d}g\left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right]\right\}$$

$$\bar{g} = \bar{g}_{\overline{\mathrm{MS}}} \to \Lambda = \Lambda_{\overline{\mathrm{MS}}}\,, \qquad \bar{g} = \bar{g}_{\mathrm{qq}} \to \Lambda = \Lambda_{\mathrm{qq}}$$

$$\Lambda_{\overline{\mathrm{MS}}}/\Lambda_{\mathrm{qq}} = \exp\left(c_1/(2b_0)\right)$$

$$\alpha_{\mathrm{qq}}(\mu) = \alpha_{\overline{\mathrm{MS}}}(\mu) + c_1\alpha^2_{\overline{\mathrm{MS}}}(\mu) + \ldots$$

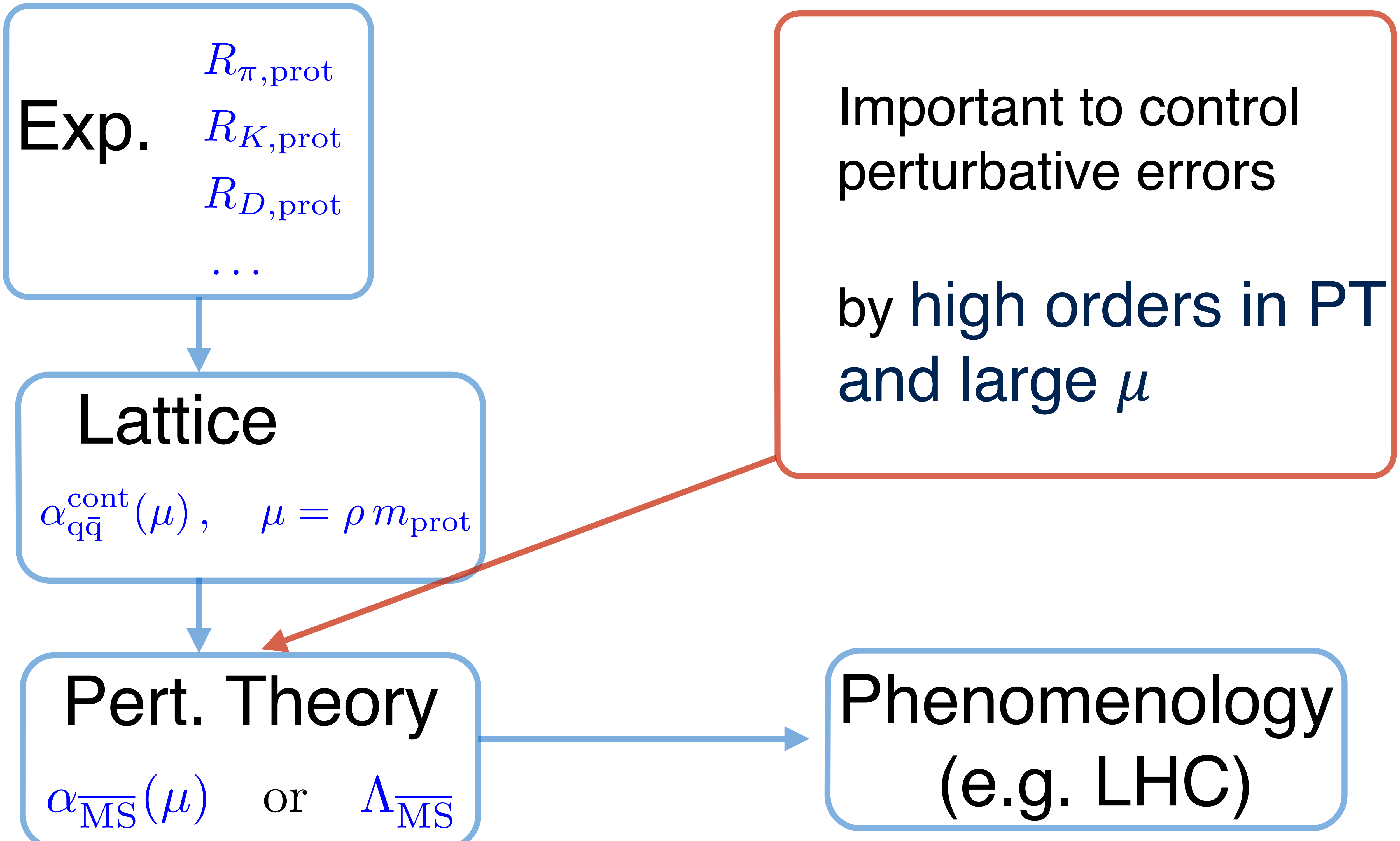
Λ is the goal, relative uncertainty: $k\alpha^n$ for n+1 - loop $\beta(g)$

An aside:

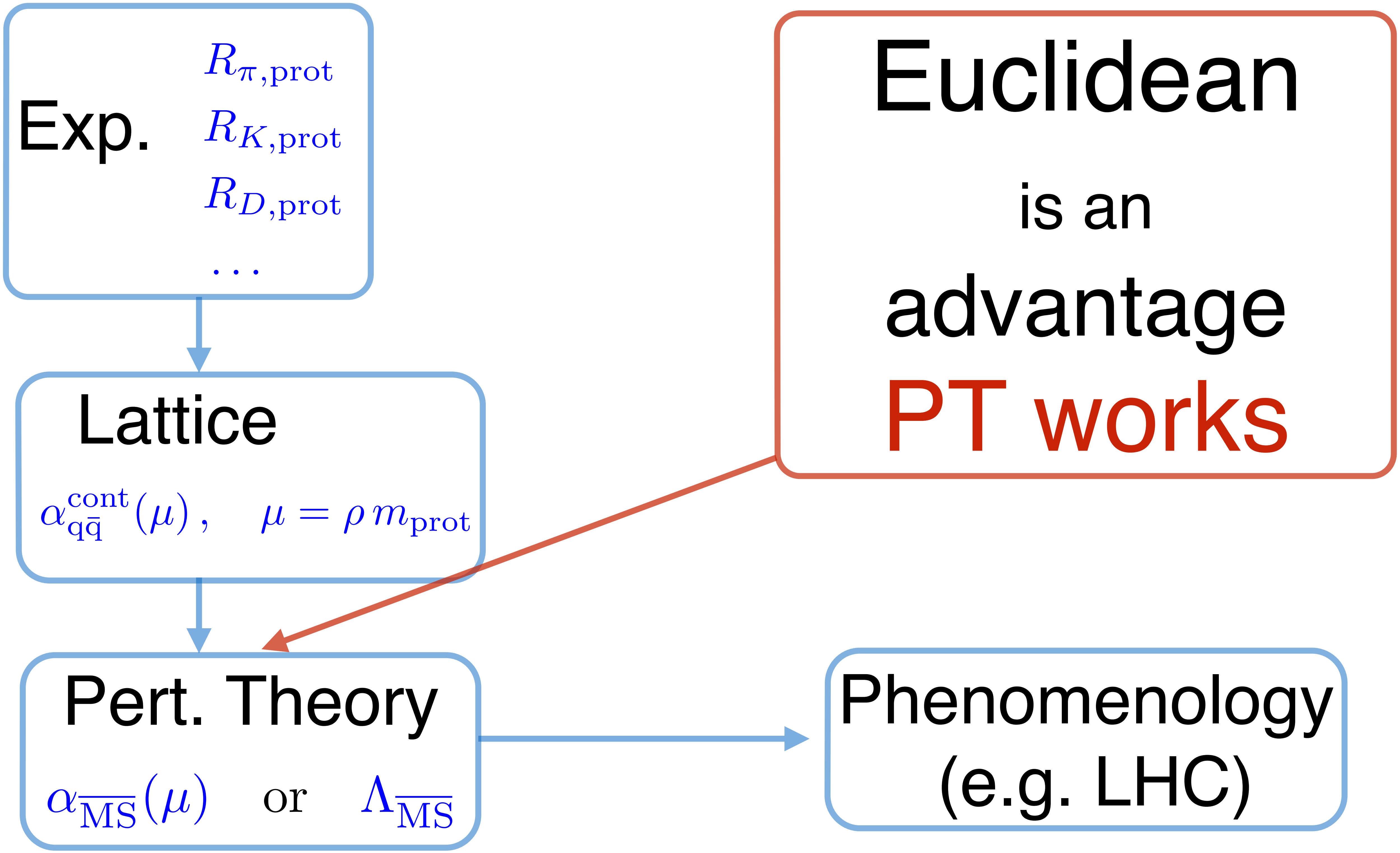
Perturbative but not short distance?

Perturbative = short distance!

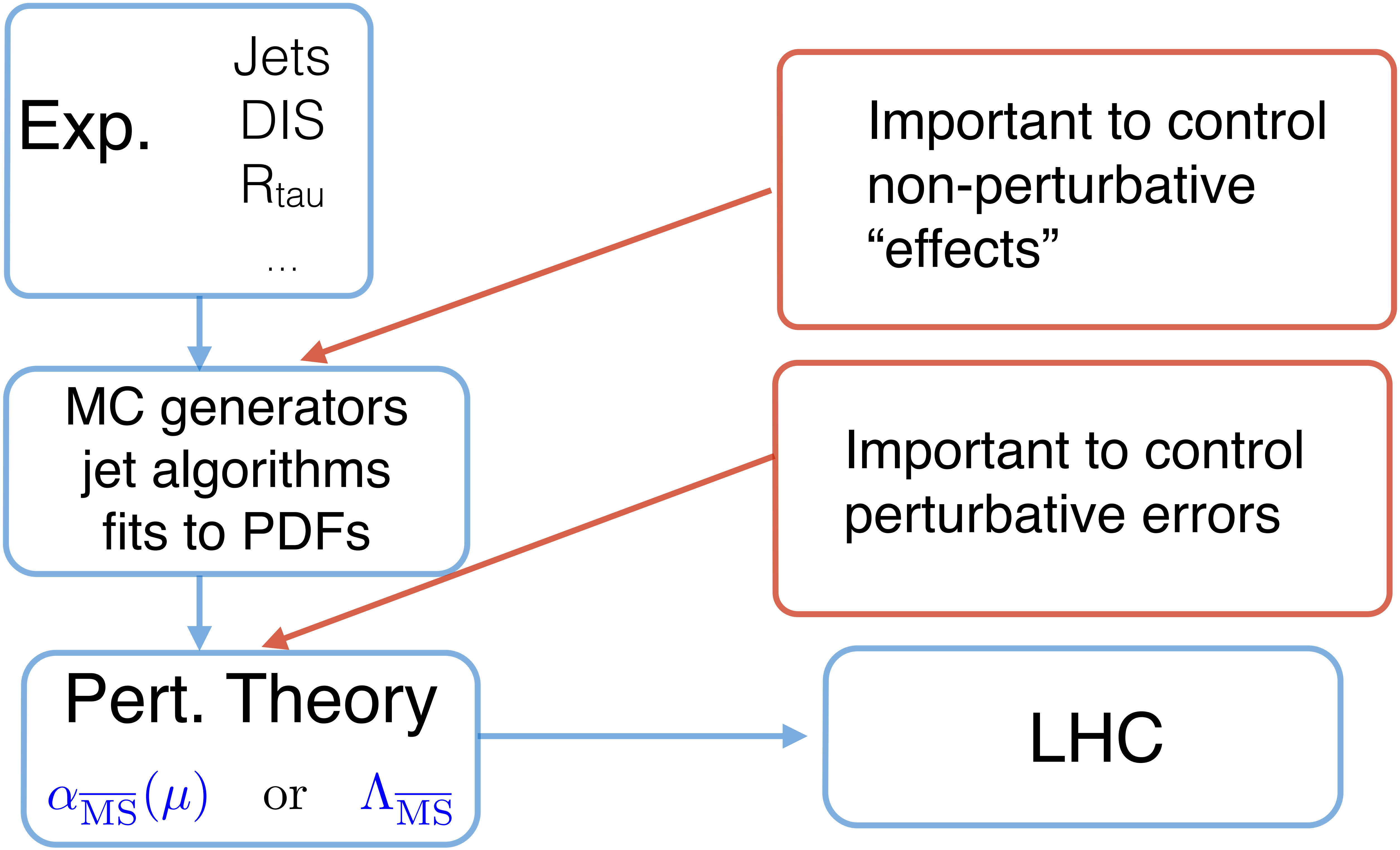
Summary of the principle



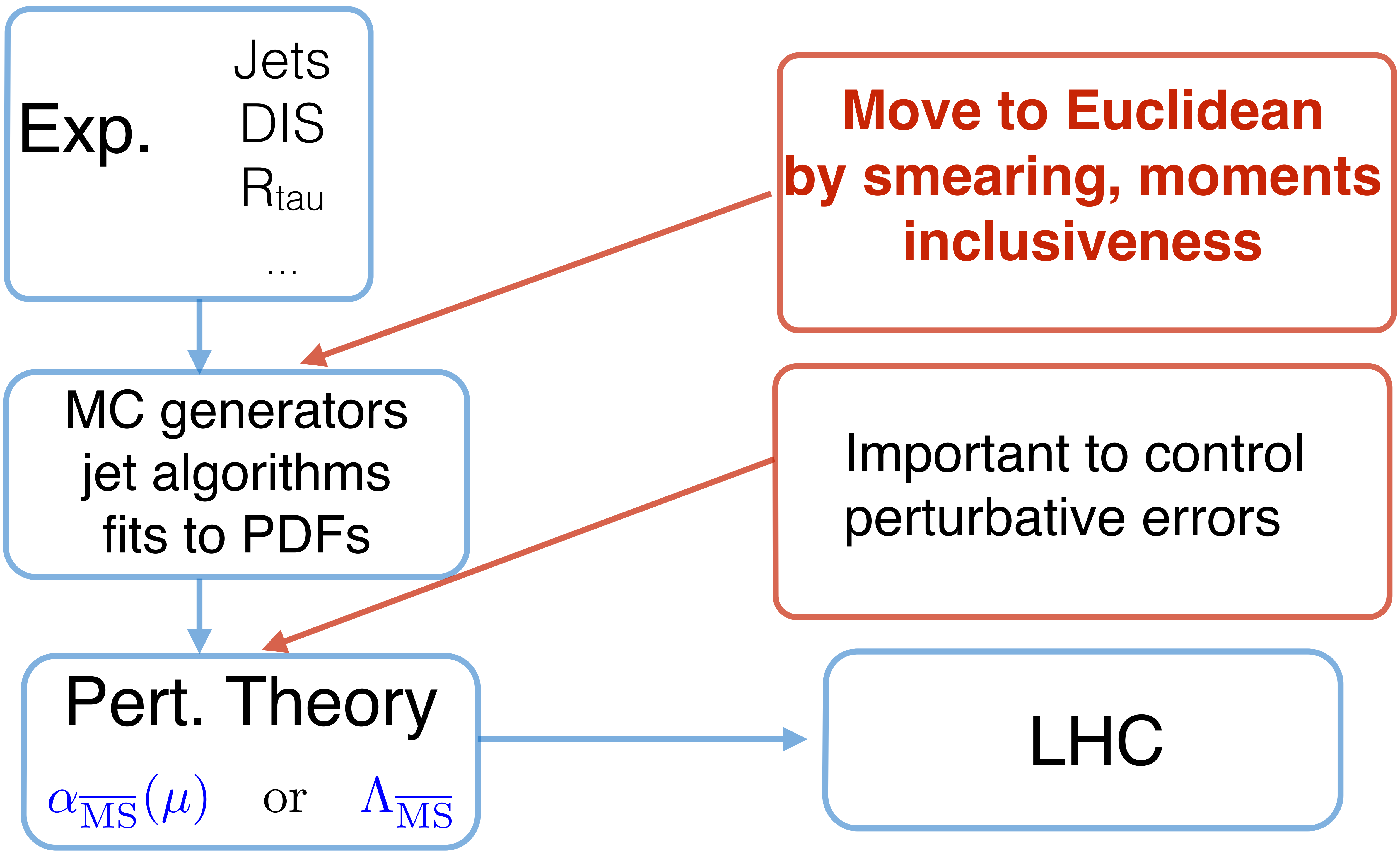
Summary of the principle



Compare to phenomenology



Compare to phenomenology



Limitations of lattice computations

FLAG 2013

- Observable with energy/momentum scale μ

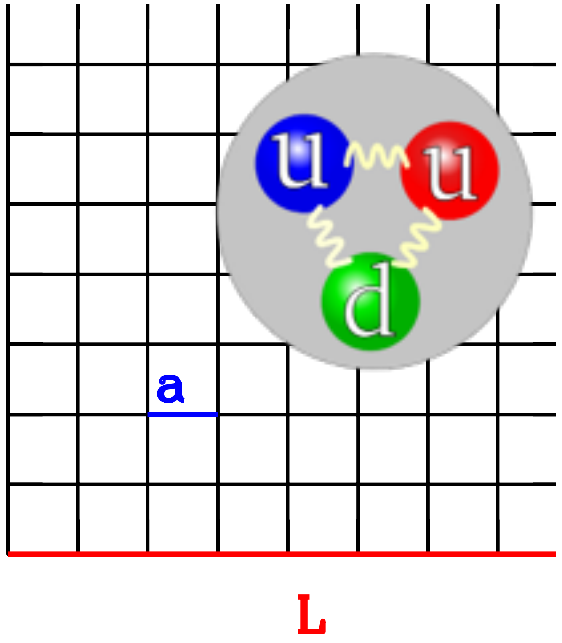
$$\mathcal{O}(\mu) \equiv \lim_{a\to 0} \mathcal{O}_{\rm lat}(a,\mu) \text{ with } \mu \text{ fixed}$$

- avoid finite size and discretization effects

$$L \gg \text{hadron size} \sim \Lambda_{\rm QCD}^{-1} \quad \text{and} \quad 1/a \gg \mu$$

or:

$$L/a \ggg \mu/\Lambda_{\rm QCD}$$



$$\mu \lll L/a \times \Lambda_{\rm QCD} \sim 5 - 20\,\mathrm{GeV}$$

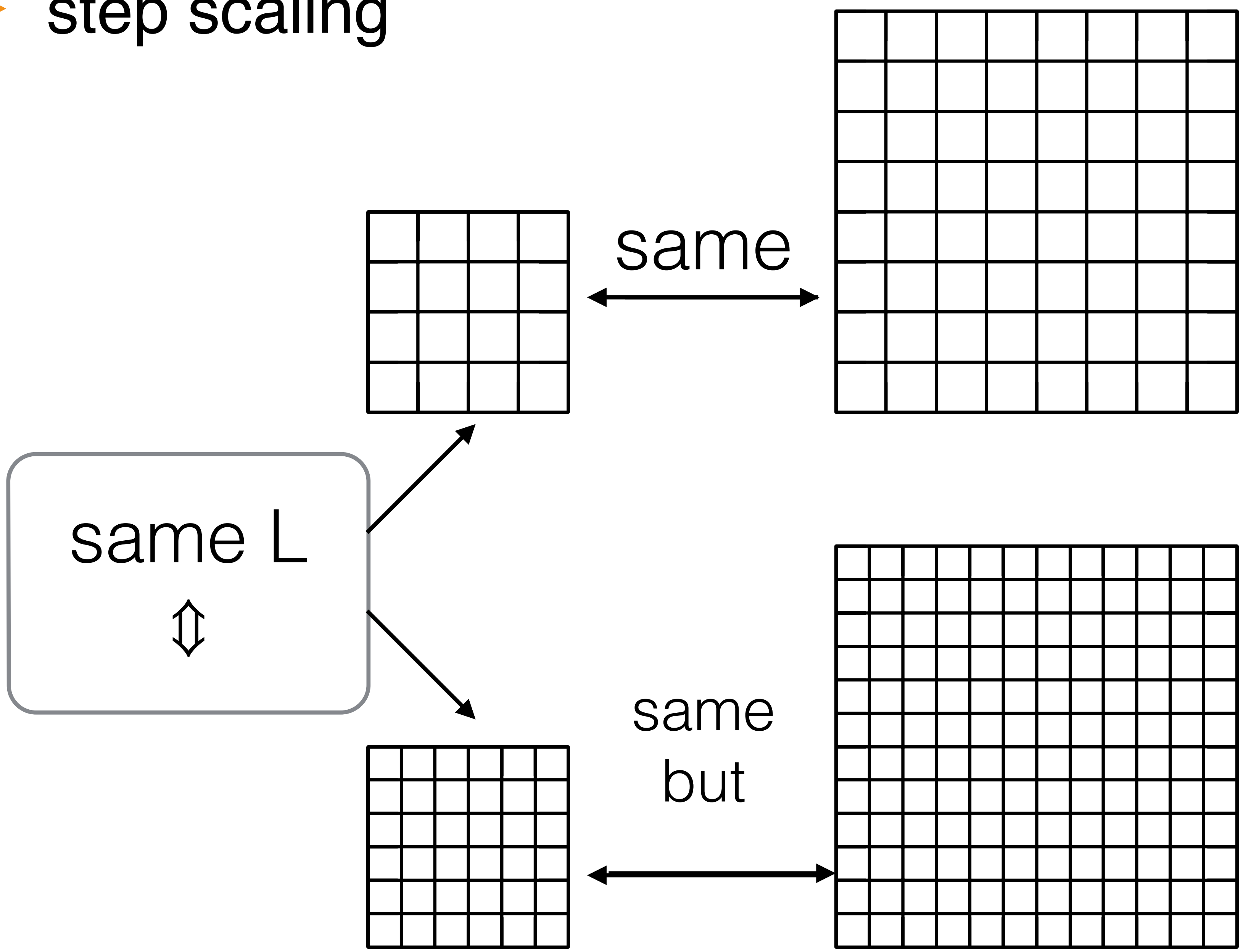
$1-3\,\mathrm{GeV}$ at most, in conflict with $\frac{\Delta\Lambda}{\Lambda} \sim \{\alpha(\mu)\}^n$

a challenge!

Our Strategy to meet the Challenge

ALPHA Collaboration

- finite volume: $\mu=1/L$, with $L/a \gg 1$ get $\mu^2 a^2 \ll 1$ for any μ
- step scaling



$g^2(2L,a/L)$

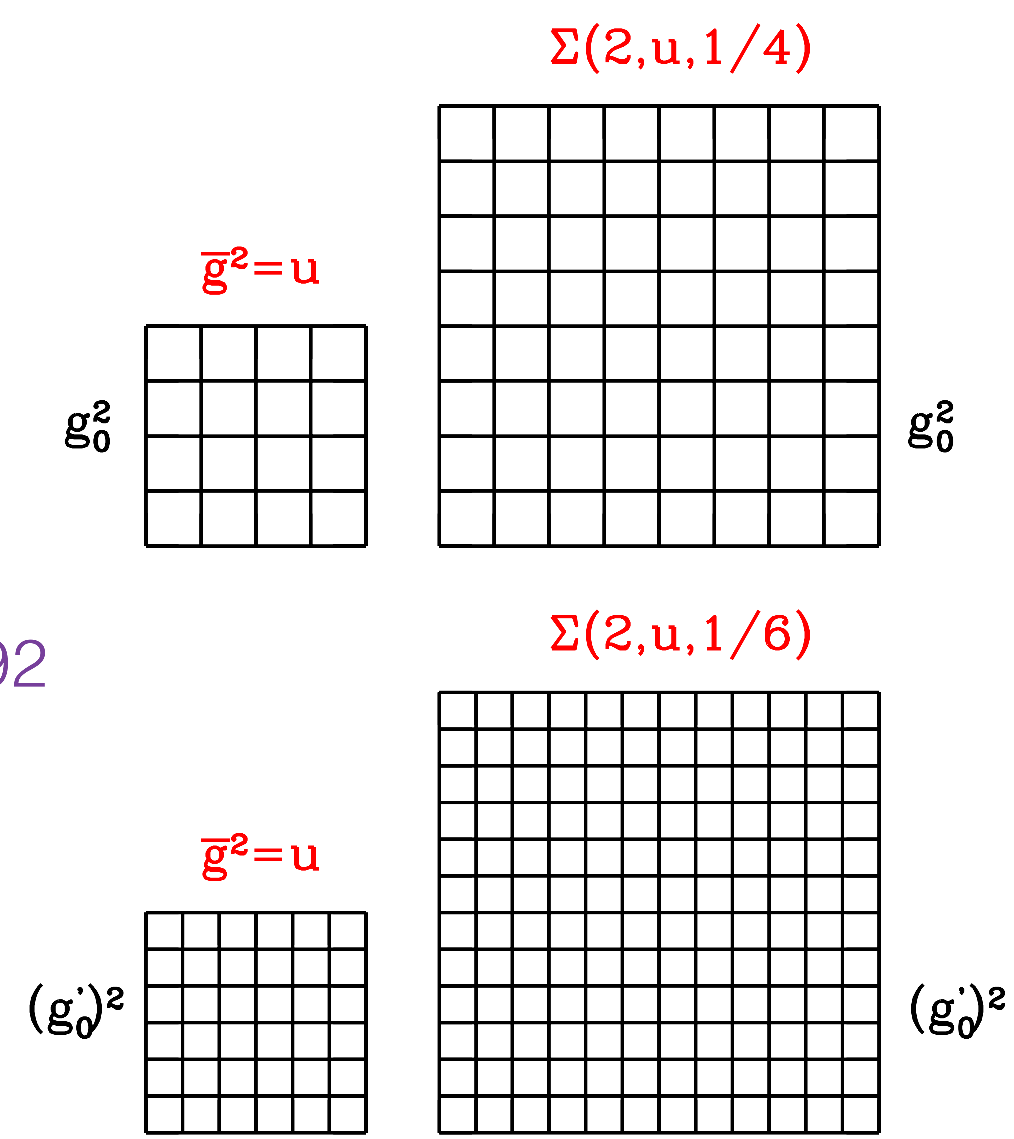
$g^2(2L,a'/L) =$
$g^2(2L,1/6)$

⇓ extrapolate

- needs $L/a \gg 1$, not more: $g^2(2L,0)$ continuum

Our Strategy

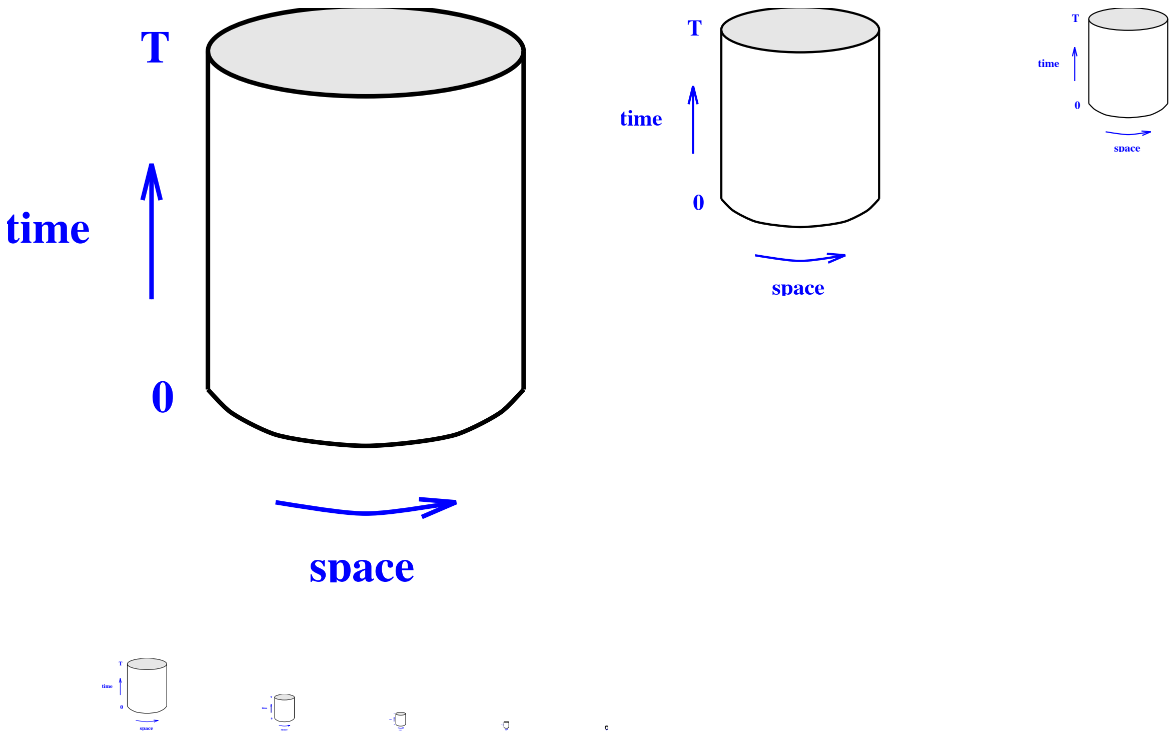
- finite volume: μ=1/L, L/a ≫ 1 at any μ
- step scaling function: $\bar{g}^2(2L) = \sigma(\,\bar{g}^2(L)\,) = \lim_{a/L\to 0} \Sigma(2,u,a/L)$



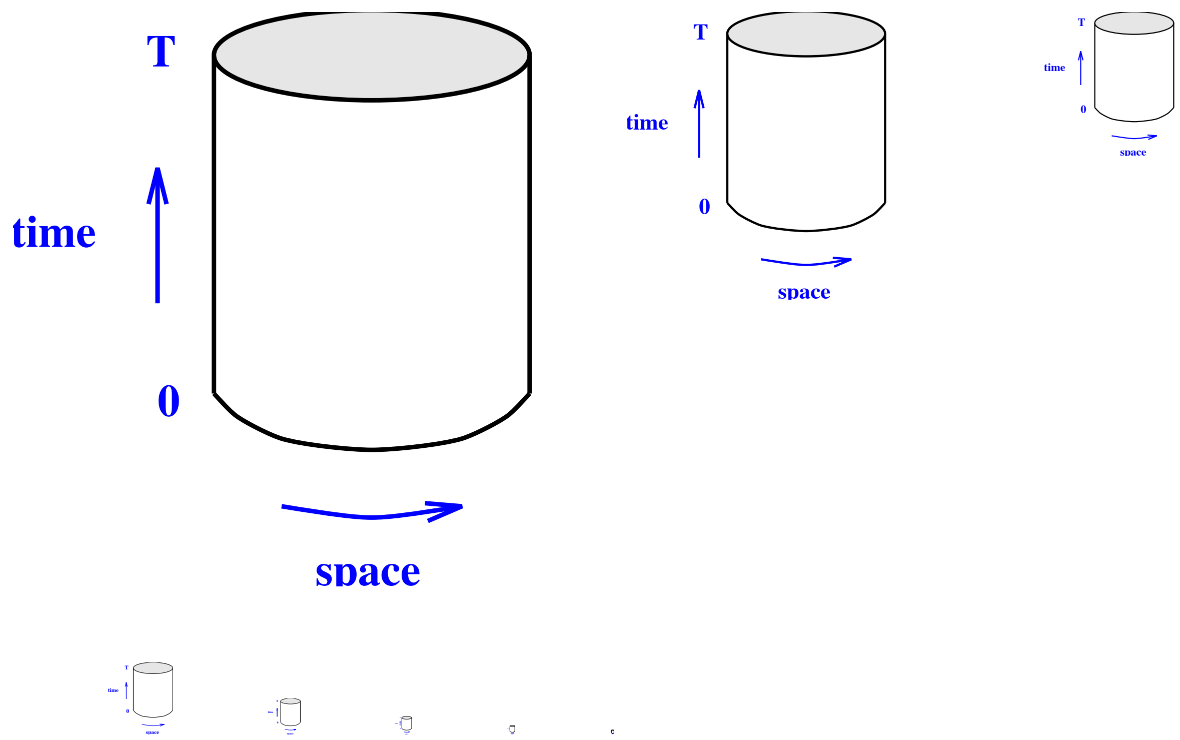
Lüscher, Weisz, Wolff, '91
Lüscher, Narayanan, Weisz, Wolff, '92
Lüscher, Sommer, Weisz, Wolff, '94

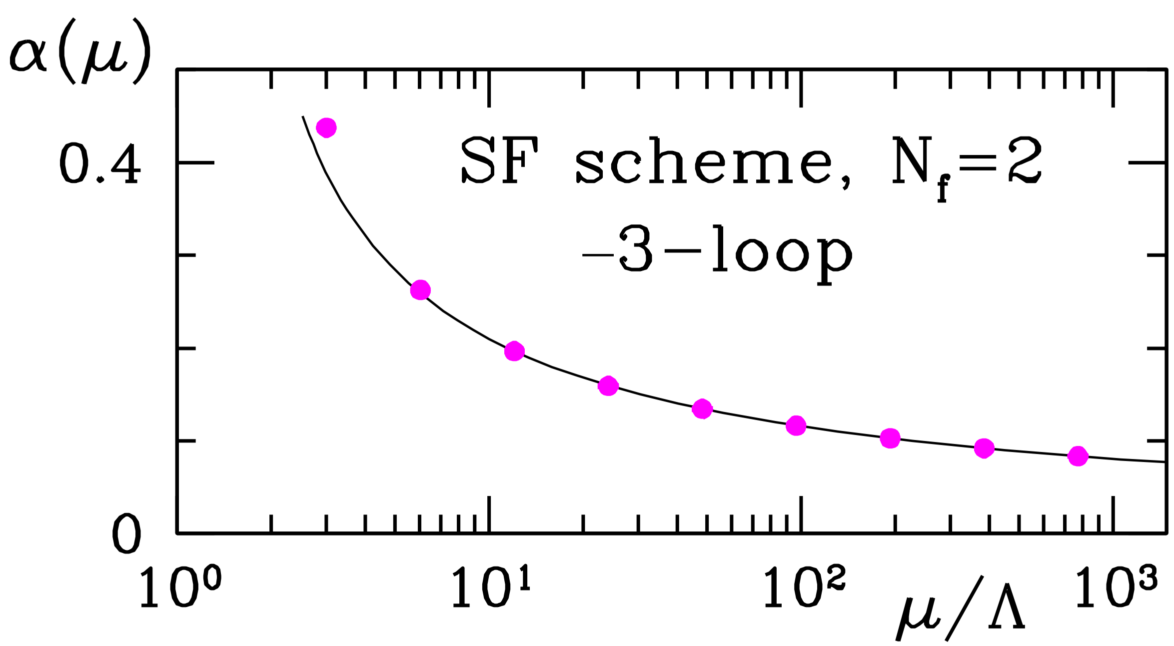
Running to (almost) any scale non-perturbatively

Running to (almost) any scale non-perturbatively

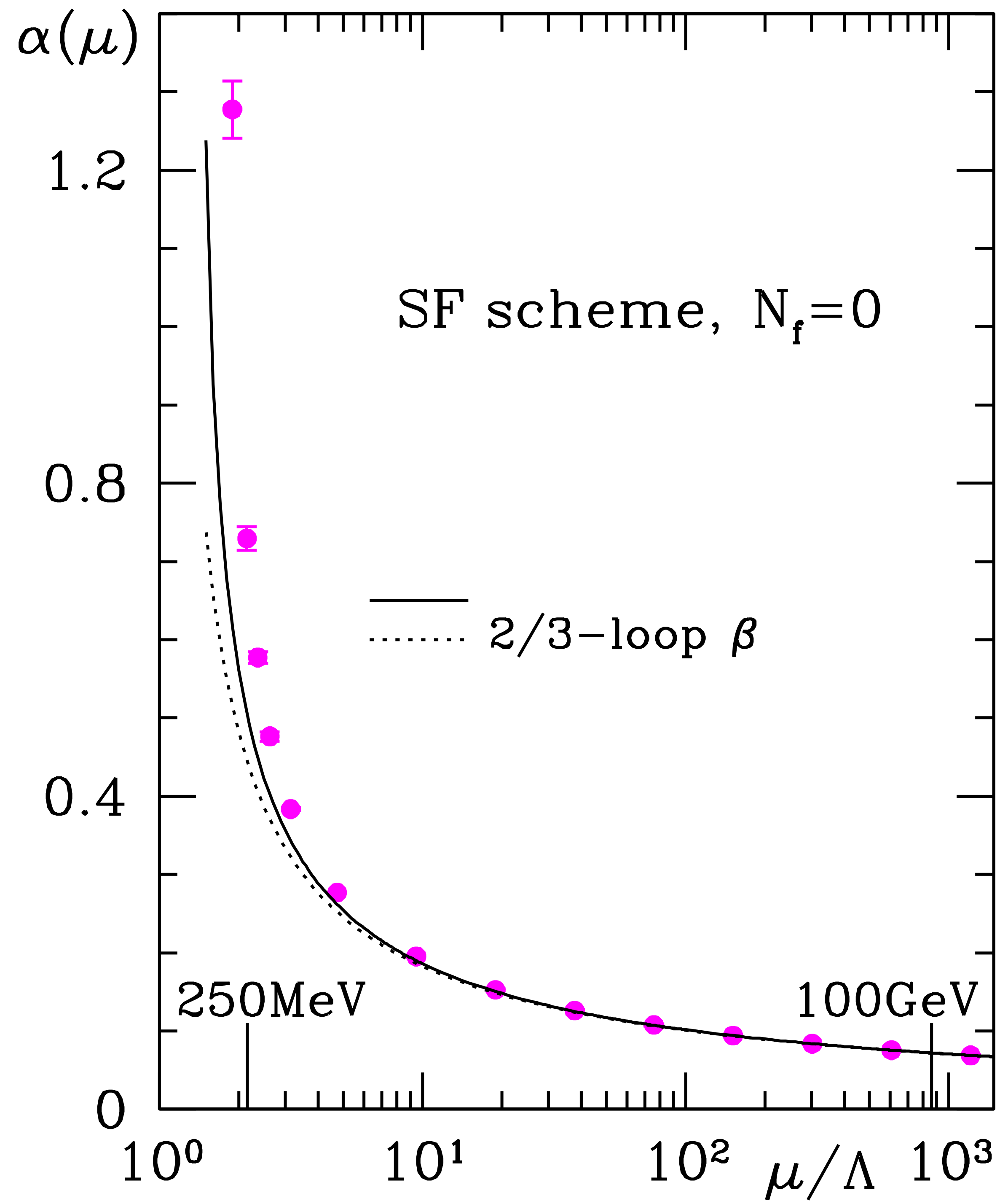


Running to (almost) any scale non-perturbatively





[ALPHA Collaboration, 2005]



[ALPHA Collaboration, 2001]

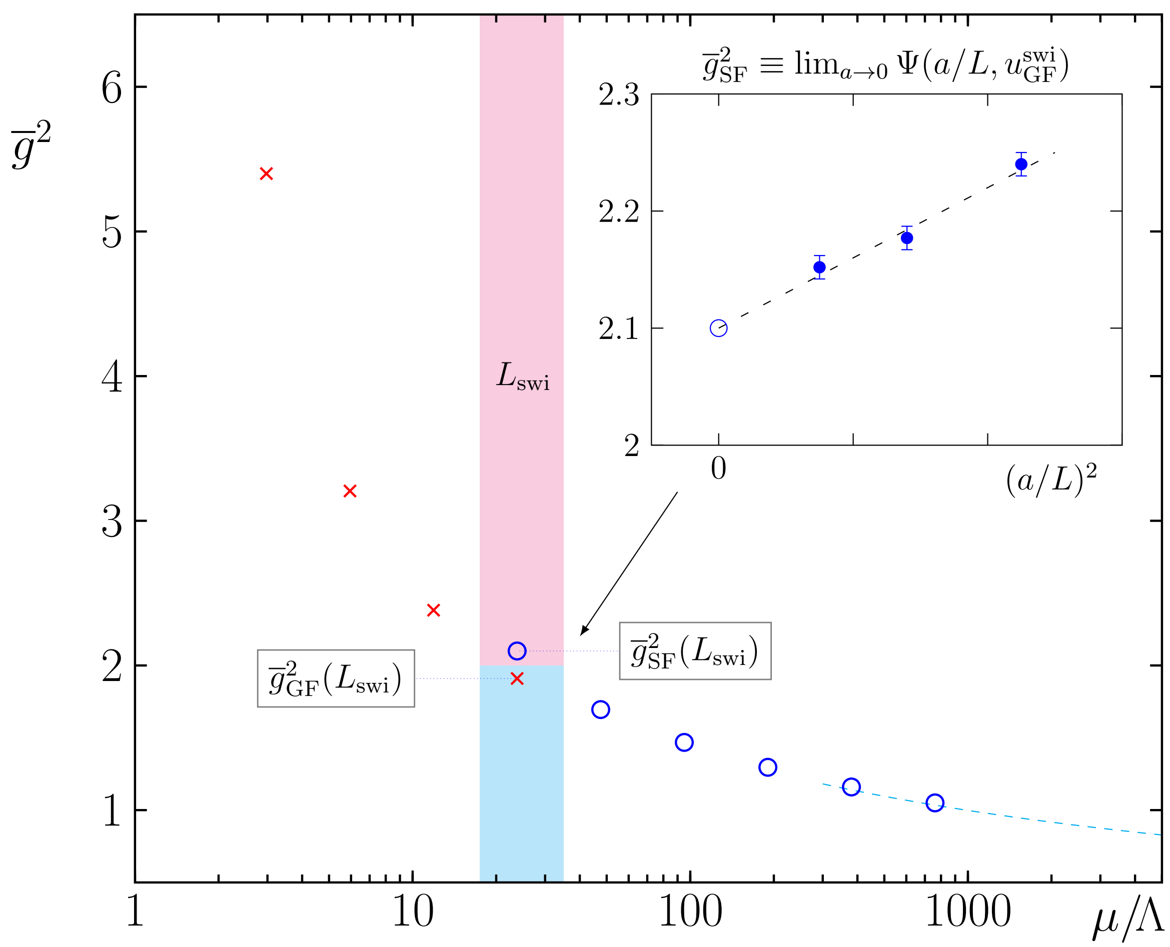
Now: N_f=3 (hadronic world and running)

with up, down, strange; others decoupled

two different schemes

Gradient flow
200 MeV ← 8 GeV

Schrödinger functional
4 GeV ← 200 GeV



Now: N_f=3 (hadronic world and running)

with up, down, strange; others decoupled

ALPHA Collaboration

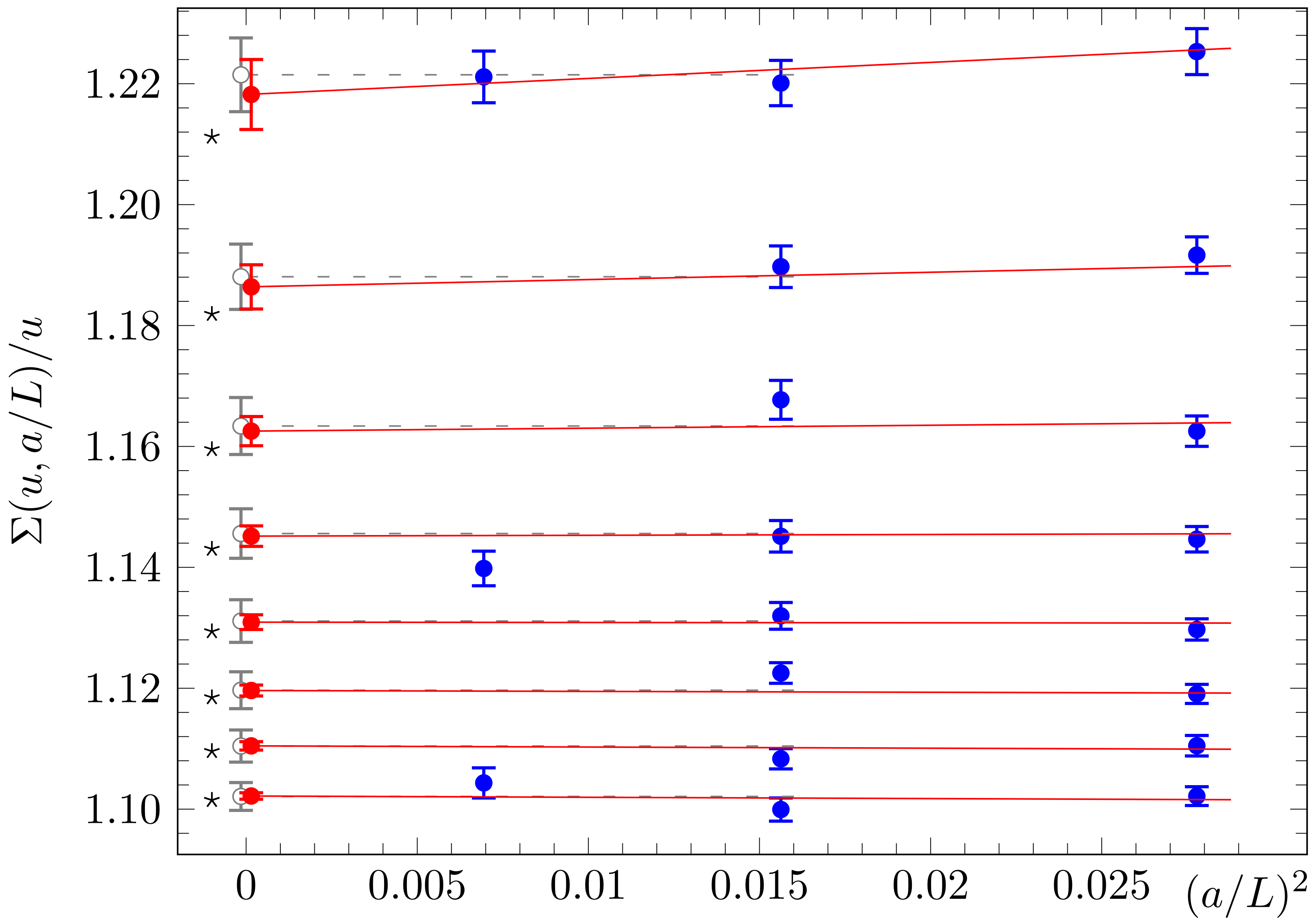
two different schemes

Gradient flow	Schrödinger functional
200 MeV ← 8 GeV	4 GeV ← 200 GeV
high precision in MC	high precision at small g
significant a^2 effects	small a-effects
PT not yet known	3-loop β-function
	2-loop a-effects

Lüscher, 2010
Lüscher, Weisz, 2011
Fritzsch, Ramos, 2013

Lüscher, Weisz, Wolff, '91
Lüscher, Narayanan, Weisz, Wolff, '92
Lüscher, Sommer, Weisz, Wolff, '93
Sint '93

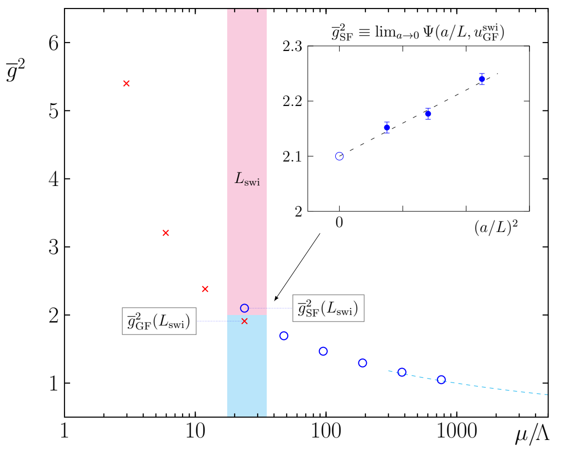
Continuum limit $\sigma(g^2) = \Sigma(g^2,0)$ in small g^2 region



- χ^2 of global fits is good - continuum limit is precise

- constant continuum extrapolation has larger errors
  due to propagation of boundary improvement error

Determination of Λ L_0



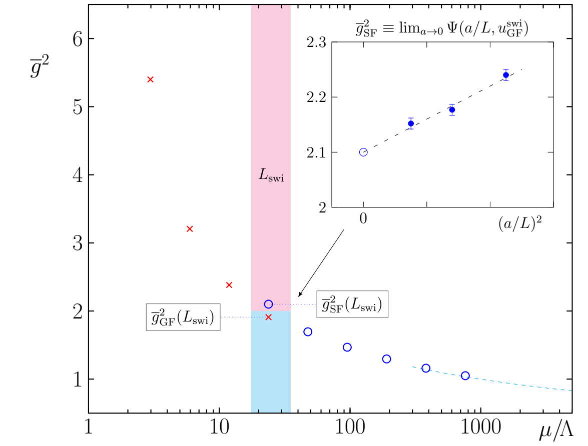
- step scaling (from u_0=2.012)

$$\bar{g}^2(1/L_0) = u_0\,, \quad u_k = \sigma(u_{k+1}), \quad \text{non-pert}$$

$$L_0\Lambda = 2^n \varphi^{\rm pert}(\sqrt{u_n})$$

$$\varphi_s(\bar{g}_s) = (b_0\bar{g}_s^2)^{-b_1/(2b_0^2)} \mathrm{e}^{-1/(2b_0\bar{g}_s^2)} \times \exp\left\{-\int_0^{\bar{g}_s} \mathrm{d}x \left[\frac{1}{\beta_s(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x}\right]\right\}$$

Determination of ΛL_0



- step scaling (from u_0=2.012)

$$\bar{g}^2(1/L_0) = u_0\,, \quad u_k = \sigma(u_{k+1}), \quad \text{non-pert}$$

$$L_0\Lambda = 2^n \varphi^{\rm pert}(\sqrt{u_n})$$

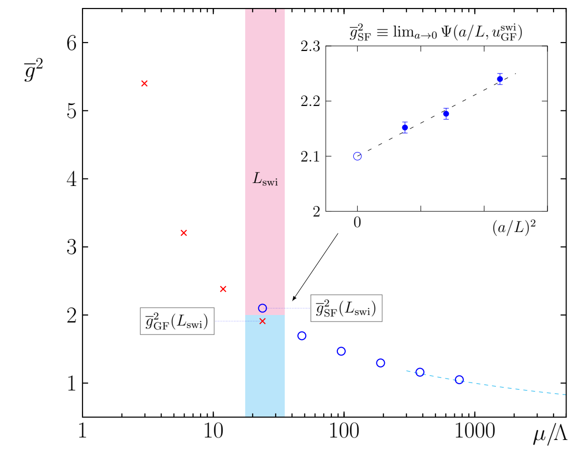
$$\varphi_s(\bar{g}_s) = (b_0\bar{g}_s^2)^{-b_1/(2b_0^2)} \mathrm{e}^{-1/(2b_0\bar{g}_s^2)} \times \exp\left\{-\int_0^{\bar{g}_s} \mathrm{d}x \left[\frac{1}{\beta_s(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x}\right]\right\}$$

- relation $\dfrac{1}{\bar{g}_\nu^2} = \dfrac{1}{\bar{g}^2} - \nu\,\bar{v}$ through $\Omega(u, a/L) = \bar{v}|_{\bar{g}^2(L)=u}$ $\quad \omega(u) = \Omega(u,0)$

$$\omega(2.012) = 0.1199(10)$$

- repeat step scaling for different $\boldsymbol{\nu}$ ⇔ different schemes

Determination of Λ L_0



- step scaling (from u_0=2.012)

$$\bar{g}^2(1/L_0) = u_0\,, \quad u_k = \sigma(u_{k+1}), \quad \text{non-pert}$$

$$L_0\Lambda = 2^n \varphi^{\rm pert}(\sqrt{u_n})$$

$$\varphi_s(\bar{g}_s) = (b_0\bar{g}_s^2)^{-b_1/(2b_0^2)} \mathrm{e}^{-1/(2b_0\bar{g}_s^2)} \times \exp\left\{-\int_0^{\bar{g}_s} \mathrm{d}x \left[\frac{1}{\beta_s(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x}\right]\right\}$$

- relation $\frac{1}{\bar{g}_\nu^2} = \frac{1}{\bar{g}^2} - \nu\,\bar{v}$ through $\Omega(u, a/L) = \bar{v}|_{\bar{g}^2(L)=u}$ $\quad \omega(u) = \Omega(u, 0)$

$$\omega(2.012) = 0.1199(10)$$

- repeat step scaling for different $\boldsymbol{\nu}$ ⇔ different schemes

- use $r_\nu = \Lambda/\Lambda_\nu = \mathrm{e}^{-\nu\times 1.25516}$ exact

- Λ independent of $\boldsymbol{\nu}$, n ? ⟸ excellent check of accuracy of PT

Results for Λ L_0

fit	u_n	i	$\frac{L}{a}\big\rvert_{\min}$	$n_\rho^{(i)}$	n_c	$L_0\Lambda$ $\times 100$	b_3^{eff} $\times(4\pi)^4$	χ^2	d.o.f.
A	1.193(4)	0	6	2	1	3.04(8)		14.7	16
B	1.194(4)	1	6	2	1	3.07(8)		14.2	16
C	1.193(5)	2	6	2	1	3.03(8)		14.5	16
D	1.192(7)	2	6	2	2	3.03(13)		14.5	15
E		2	6	2	1	3.00(11)	4(3)	14.6	16
F		2	8	1	1	3.01(11)	4(3)	12.7	9
G	1.191(11)	2	8	0	2	3.02(20)		13.0	9
H		1	6	2	1	3.04(10)	3(3)	14.1	16

fit	ν	i	$\frac{L}{a}\big\rvert_{\min}$	$n_\rho^{(i)}$	n_c	$L_0\Lambda$ $\times 100$	$b_{3,\nu}^{\mathrm{eff}}$ $\times(4\pi)^4$	χ^2	d.o.f
H	−0.5	1	6	2	1	3.03(15)	11(5)	10.4	16
H	0.3	1	6	2	1	3.04(10)	0(3)	20.0	16

TABLE I. Results for $\nu = 0$ in the upper part.

- all results agree when PT is used at $\alpha = 0.1$
- what happens at larger α ?

$$\varphi_s(\bar g_s) = (b_0 \bar g_s^2)^{-b_1/(2b_0^2)} \mathrm{e}^{-1/(2b_0\bar g_s^2)} \times \exp\left\{ -\int_0^{\bar g_s} \mathrm{d}x \left[\frac{1}{\beta_s(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x} \right] \right\}$$

Results for Λ L_0

fit $\boldsymbol{\sigma}$(u) with a 4-loop coefficient in $\boldsymbol{\beta}$ instead of polynomial in u

fit	u_n	i	$\frac{L}{a}\Big\vert_{\min}$	$n_\rho^{(i)}$	n_c	$L_0\Lambda$ $\times 100$	b_3^{eff} $\times(4\pi)^4$	χ^2	d.o.f.
A	1.193(4)	0	6	2	1	3.04(8)		14.7	16
B	1.194(4)	1	6	2	1	3.07(8)		14.2	16
C	1.193(5)	2	6	2	1	3.03(8)		14.5	16
D	1.192(7)	2	6	2	2	3.03(13)		14.5	15
E		2	6	2	1	3.00(11)	4(3)	14.6	16
F		2	8	1	1	3.01(11)	4(3)	12.7	9
G	1.191(11)	2	8	0	2	3.02(20)		13.0	9
H		1	6	2	1	3.04(10)	3(3)	14.1	16
fit	ν	i	$\frac{L}{a}\Big\vert_{\min}$	$n_\rho^{(i)}$	n_c	$L_0\Lambda$ $\times 100$	$b_{3,\nu}^{\mathrm{eff}}$ $\times(4\pi)^4$	χ^2	d.o.f
H	−0.5	1	6	2	1	3.03(15)	11(5)	10.4	16
H	0.3	1	6	2	1	3.04(10)	0(3)	20.0	16

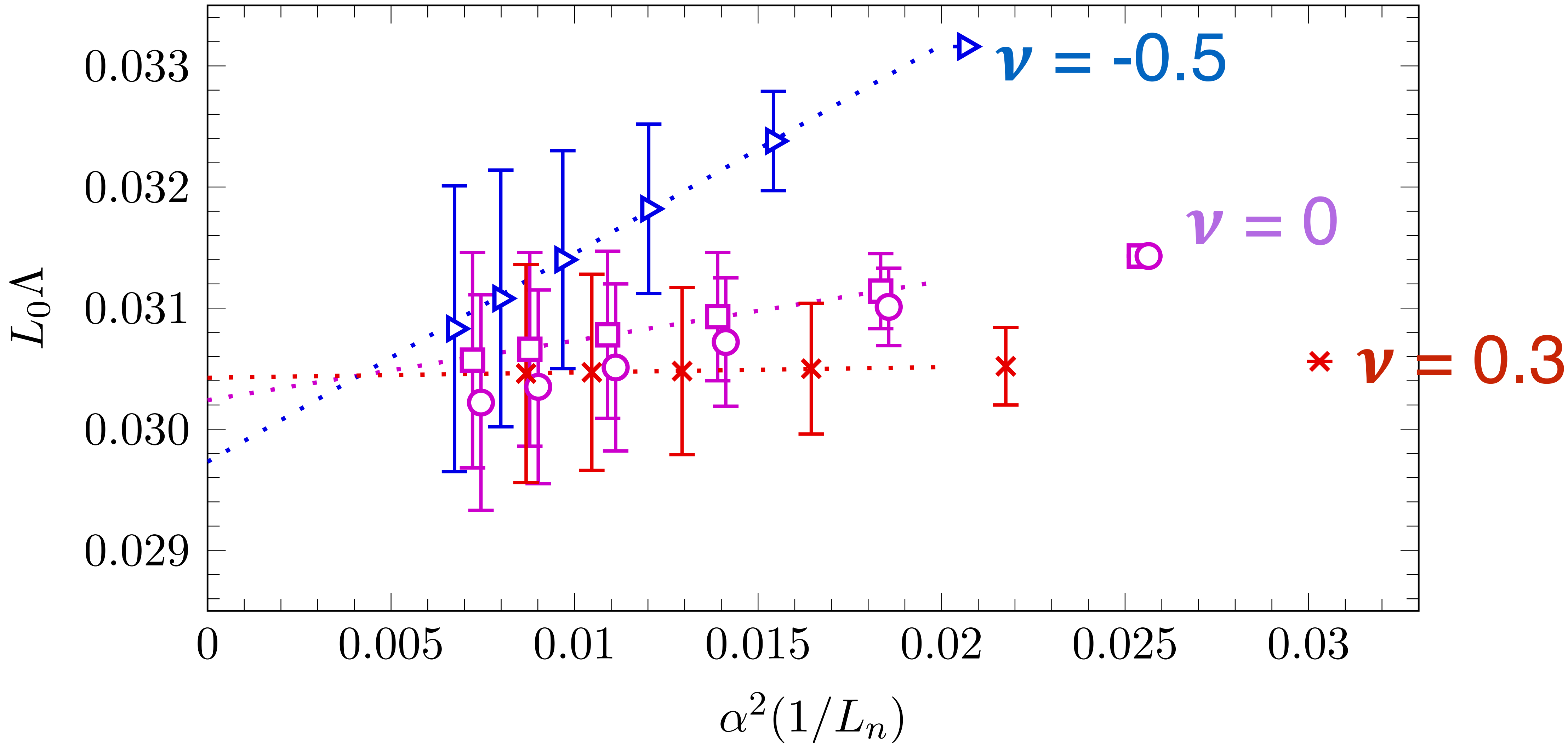
TABLE I. Results for $\nu = 0$ in the upper part.

- all results agree when PT is used at $\boldsymbol{\alpha}$ = 0.1

$$\varphi_s(\bar{g}_s) = (b_0\bar{g}_s^2)^{-b_1/(2b_0^2)} \mathrm{e}^{-1/(2b_0\bar{g}_s^2)} \times \exp\left\{-\int_0^{\bar{g}_s} \mathrm{d}x \left[\frac{1}{\beta_s(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x}\right]\right\}$$

- what happens at larger $\boldsymbol{\alpha}$?

Results for Λ L_0

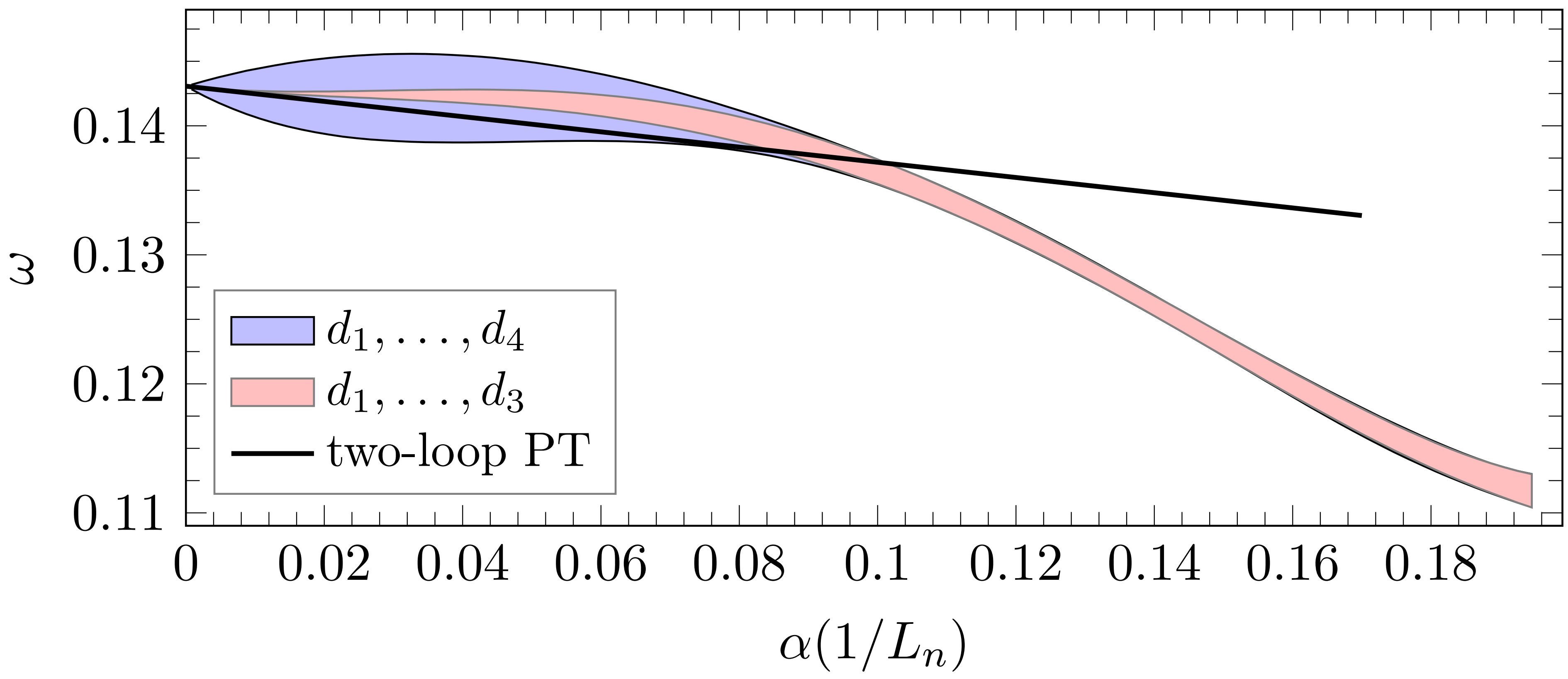


- 3 % accuracy?
- yes, at ***α***=0.1 !
- take a more precise look: ω(u)

$$\sigma(u) = \bar{g}^2(2L) \text{ when } \bar{g}^2(L) = u$$

Results for ω

$$\frac{1}{\bar{g}_\nu^2} = \frac{1}{\bar{g}^2} - \nu \times \omega(\bar{g}^2)$$



- deviation from PT at u_0 ($\boldsymbol{\alpha}$=0.19):

$$(\omega(\bar{g}^2) - v_1 - v_2\bar{g}^2)/v_1 = -3.7(2)\,\alpha^2$$

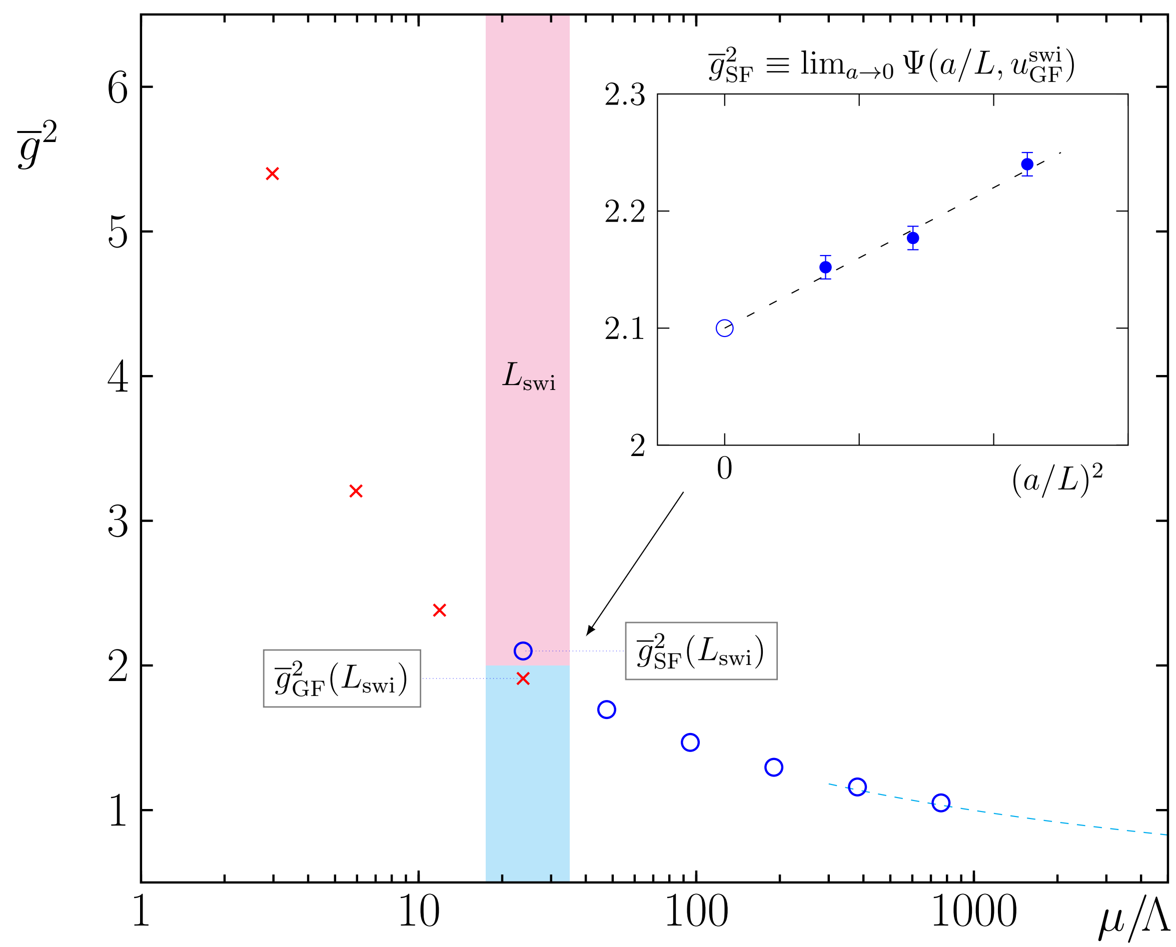
- not small, not perturbative
- statistically very significant

Now: Gradient flow scheme

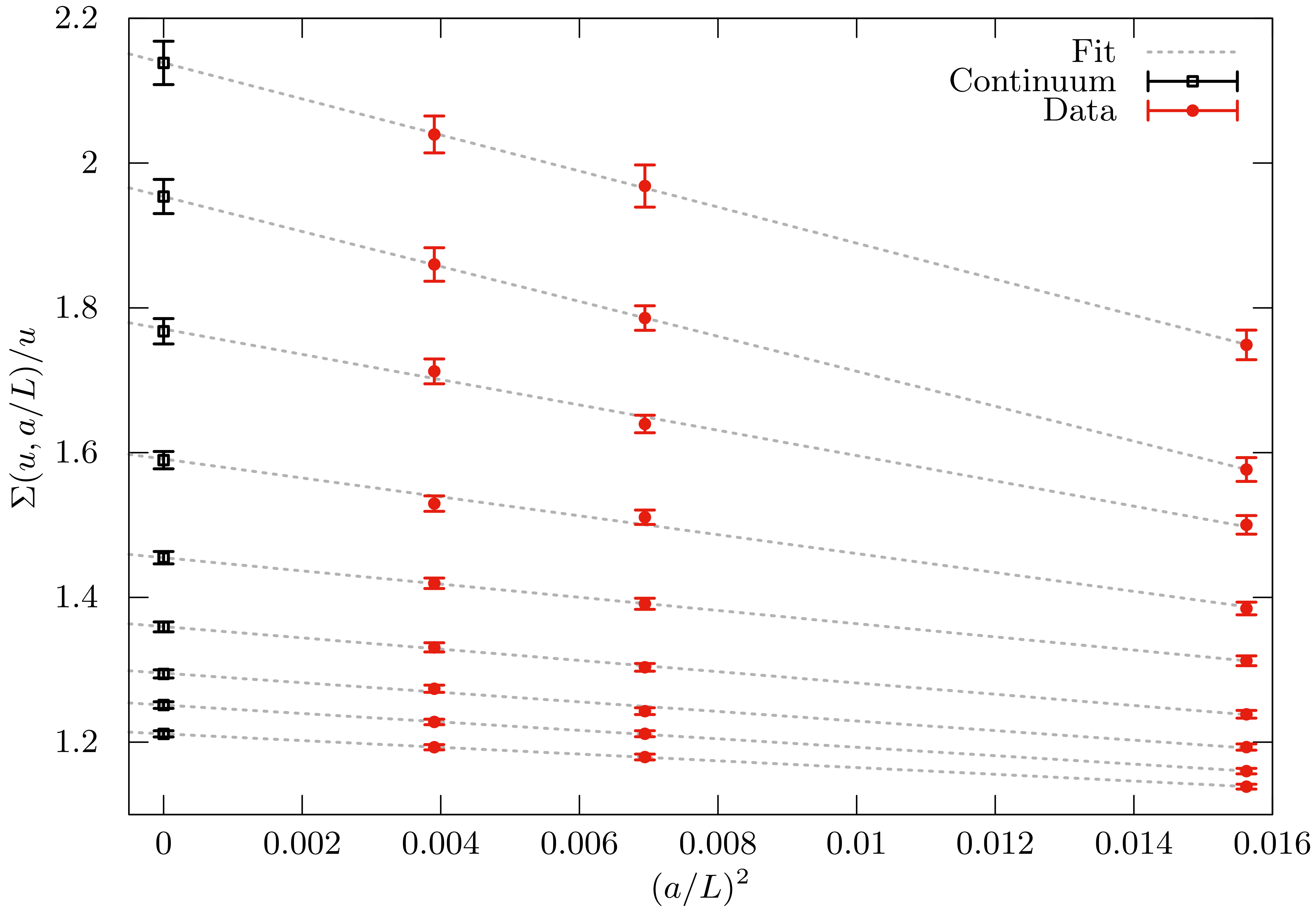
two different schemes

Gradient flow
200 MeV ← 8 GeV

Schrödinger functional
4 GeV ← 200 GeV

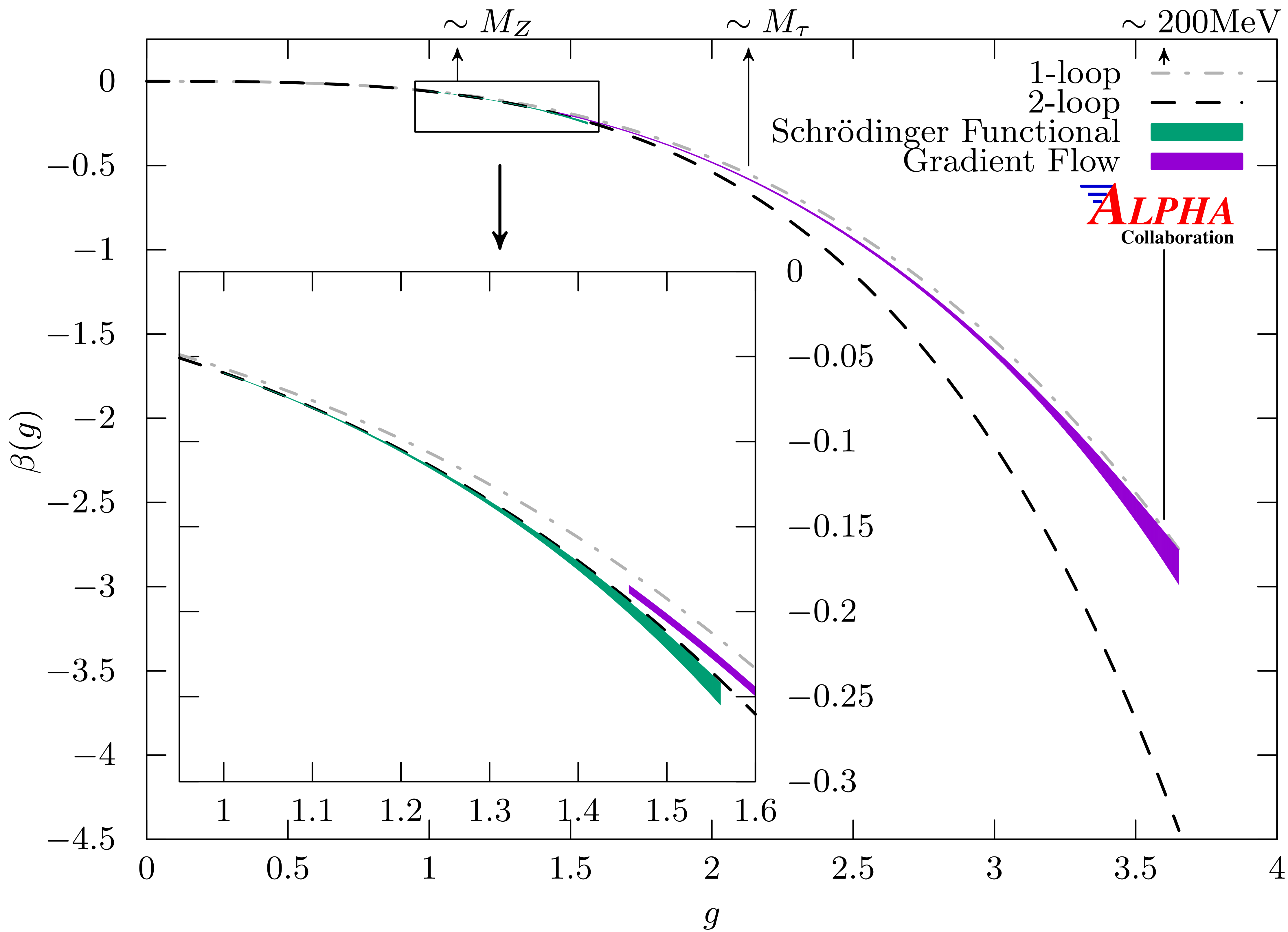


Continuum limit $\sigma(g^2) = \Sigma(g^2, 0)$ in large g^2 region

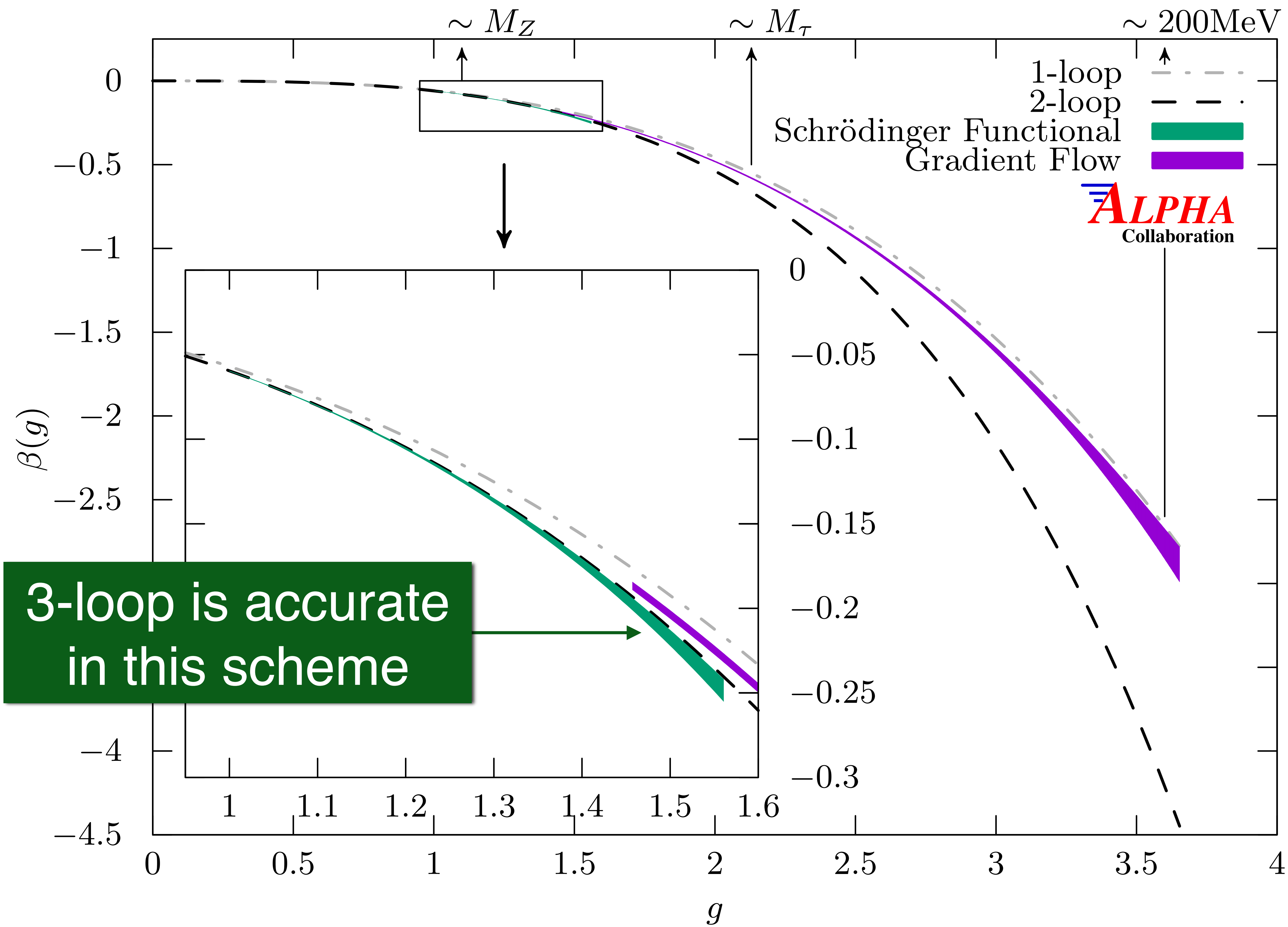


- χ^2 of global fits is good - continuum limit is precise

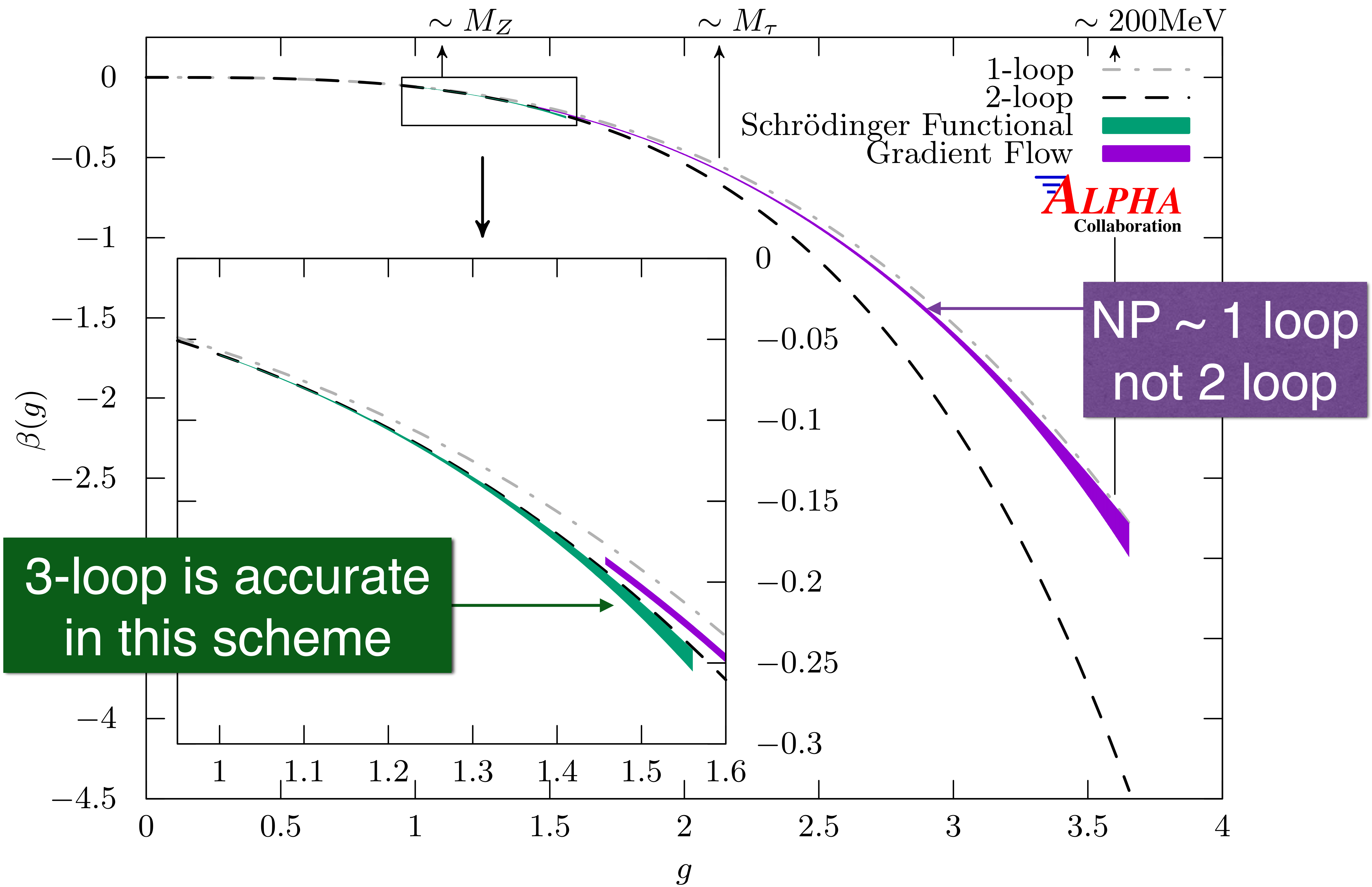
Results (1): the non-perturbative β-functions



Results (1): the non-perturbative β-functions



Results (1): the non-perturbative β-functions



Preliminary result for α

- $$\Lambda^{(3)}_{\overline{\mathrm{MS}}} = 351(14)\mathrm{MeV}$$
- $$\alpha_{\overline{\mathrm{MS}}}(m_Z) = 0.1191(10)$$

quantity	value	error	relative error	comment
$\Lambda^{(3)}_{\overline{\mathrm{MS}}} L_0$	0.0791	0.0021	0.026	arXiv:1604.06193
$L_{2.6712}/(2L_0)$	1	0.0080	0.0080	
$s(11.31, 2.6712)$	10.895	0.170	0.0156	
$t^{1/2}_{0,\mathrm{symm}}/L_{11.31}$	0.1507	0.0015	0.0099	
$t^{-1/2}_{0,\mathrm{symm}}$ [GeV]	1.3524	0.0126	0.0093	symmetric
$\Lambda^{(3)}_{\overline{\mathrm{MS}}}$ [GeV]	0.351	0.012	0.034	11. 6. 2016
$\alpha(m_Z)$	0.1191	0.0008	0.007	no conversion error
$\Lambda^{(3)}_{\overline{\mathrm{MS}}}$ [GeV]	0.336	0.019		FLAG3

- using 3-flavor theory (decoupling)
  - at present no error from conversion from 3 flavors to 4 to 5: negligible uncertainty, assuming estimate from perturbation theory

Conclusions

- errors of (asymptotic) series expansions are difficult to assess

- at α=0.2: we have examples where α=0.2 does not lead to an accurate perturbative result
  — more generally, this may be a reason for differences in determinations in $\alpha(m_Z)$
  — also a warning for some uses of PT in flavor physics

- at α=0.1: PT is accurate

  - SSF technology allows to get there

  - very accurate predictions for LHC (if matching is accurate)

Conclusions

- errors of (asymptotic) series expansions are difficult to assess
- at α=0.2: we have examples where α=0.2 does not lead to an accurate perturbative result
  — more generally, this may be a reason for differences in determinations in $\alpha(m_Z)$
  — also a warning for some uses of PT in flavor physics
- at α=0.1: PT is accurate
  - SSF technology allows to get there
  - very accurate predictions for LHC (if matching is accurate)

EFT (N_f=3 ⇐ N_f=6)

Thank you

Backup

Change of N_f

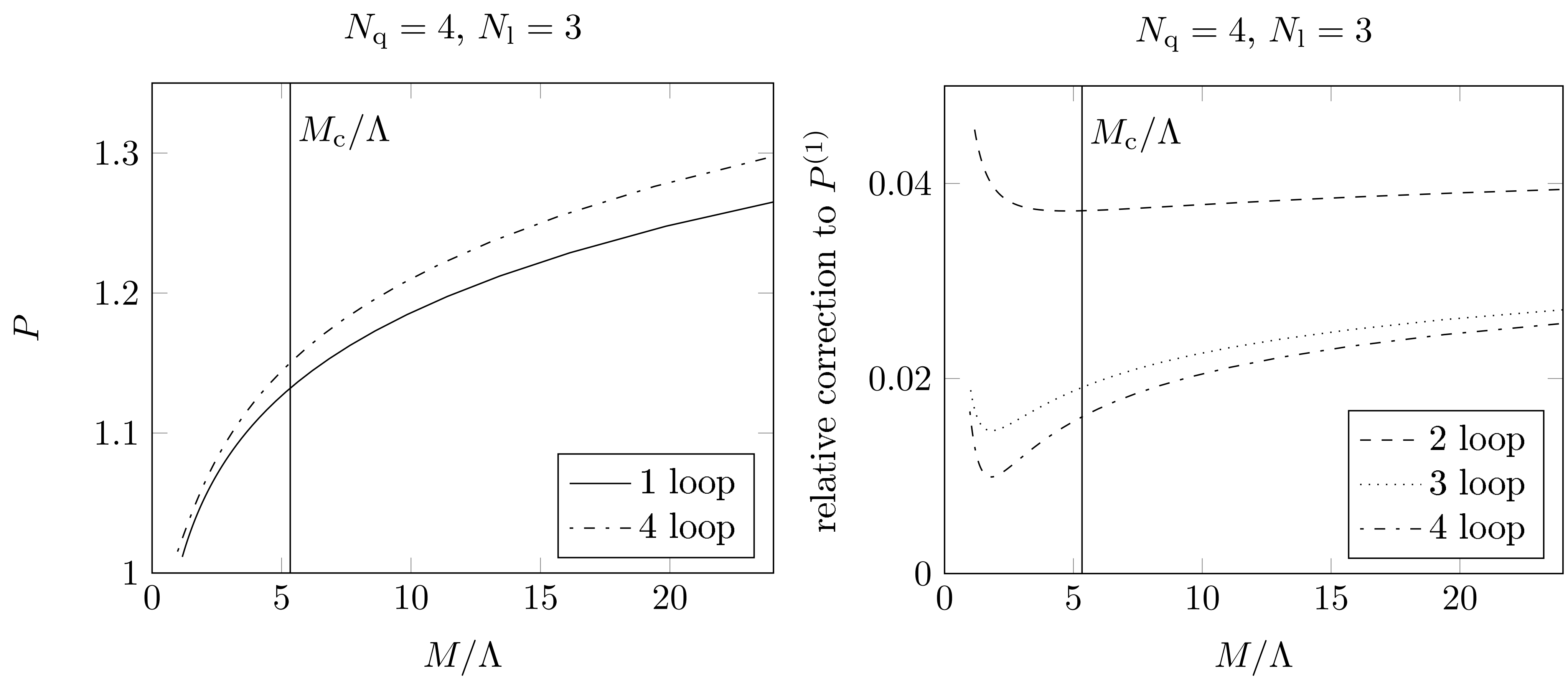


Figure 6: The mass-dependence P at 1-loop formula and at 4-loop (left) as well as 2,3,4-loop correction normalised to the 1-loop approximation (right) for the case $N_q = 4, N_l = 3$.

$$P = \frac{\Lambda^{(N_f - 1)}}{\Lambda^{(N_f)}}$$

it is harmless in perturbation theory

The SF scheme - basic definition

M. Lüscher, R. Narayanan, P. Weisz, and U. Wolff, Nucl. Phys. **B384**, 168 (1992), arXiv:hep-lat/9207009 [hep-lat].

M. Lüscher, R. Sommer, P. Weisz, and U. Wolff, hep-lat/9309005

- Dirichlet bc's

$$A_k(x)|_{x_0=0} = C_k(\eta,\nu)\,, \quad A_k(x)|_{x_0=L} = C_k'(\eta,\nu)$$

$$C_k = \frac{i}{L}[\mathrm{diag}(-\pi/3, 0, \pi/3) + \eta(\lambda_8 + \nu\lambda_3)$$

$$C_k' = \frac{i}{L}[\mathrm{diag}(-\pi, \pi/3, 2\pi/3) - \eta(\lambda_8 - \nu\lambda_3)].$$

$$\langle \partial_\eta S|_{\eta=0}\rangle = \frac{12\pi}{\bar{g}_\nu^2} = 12\pi[\frac{1}{\bar{g}^2} - \nu\,\bar{v}]$$

- similar to Casimir effect

- non-perturbative definition of background field (BF)
  = classical solution with these Dirichlet bc's
  spatially constant, abelian

- each value of ν : a different scheme

The GF scheme - basic definition

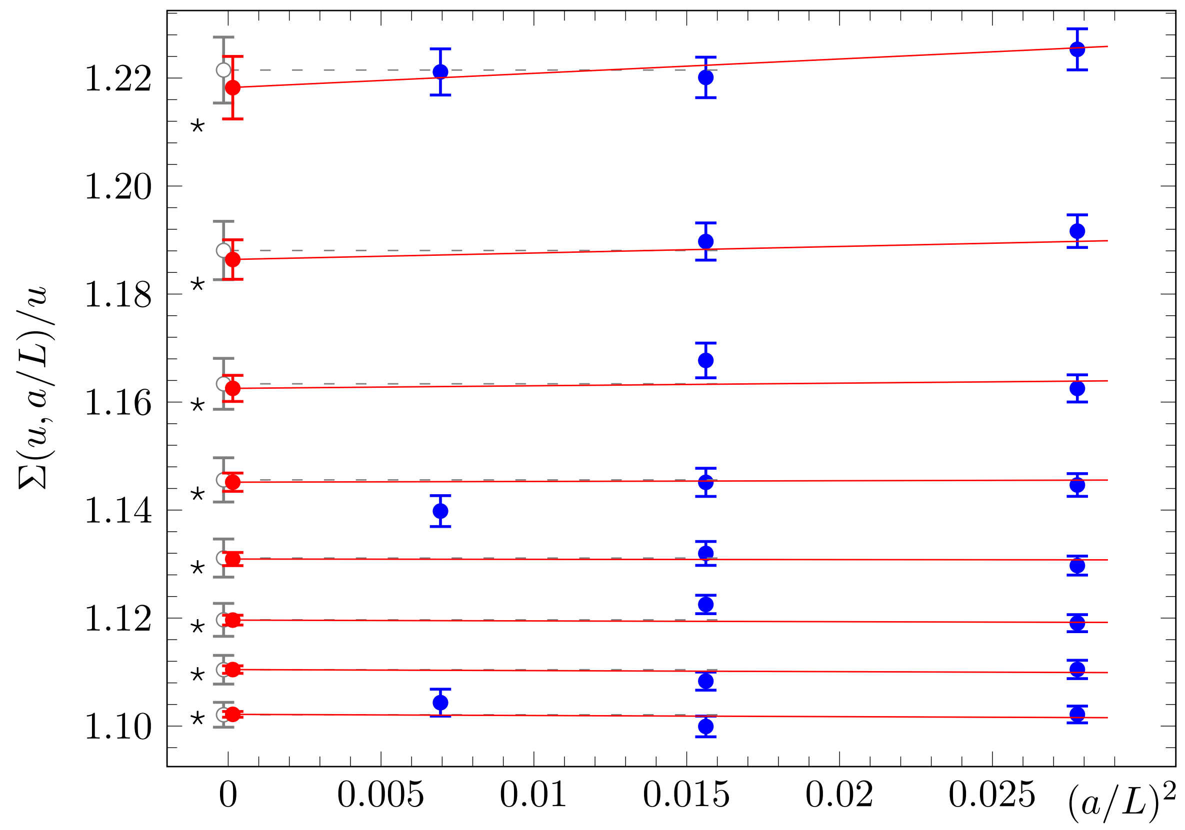
$$\frac{dB_\mu(t,x)}{dt} = D_\nu G_{\nu\mu}(t,x)\,, \qquad B_\mu(0,x) = A_\mu(x)$$

$$G_{\mu\nu}(t,x) = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$$

$$\bar{g}^2_{\rm GF}(1/L) = t^2 \mathcal{N}^{-1}(c) \langle {\rm tr}\,[G_{ij}(x_0,t) G_{ij}(x_0,t)] \Big|_{\sqrt{8t}=cL;x_0=T/2}$$

Continuum limit of Σ

N_f=3 from now on



- linear in a/L discretisation errors suppressed by Symanzik improvement (boundary terms)
  - 2-loop coefficients
  - in weak coupling region
  - taking $1 + c_1 g^2 + (c_2 \pm c_2)\, g^4 \quad (g=g_0)$
- extrapolate with $O(\,(a/L)^2\,)$

Properties of the scheme

- $\Delta_{\mathrm{stat}}\bar{g}_\nu^2 = s(a/L)\bar{g}_\nu^4 + \mathrm{O}(\bar{g}_\nu^6)$: good accuracy for small g

- no μ^{-1}, μ^{-2} renormalons (infrared cutoff)

  instead: secondary minimum of the action

$$\exp(-2.62/\alpha) \sim (\Lambda/\mu)^{3.8}$$

- 3-loop **β**

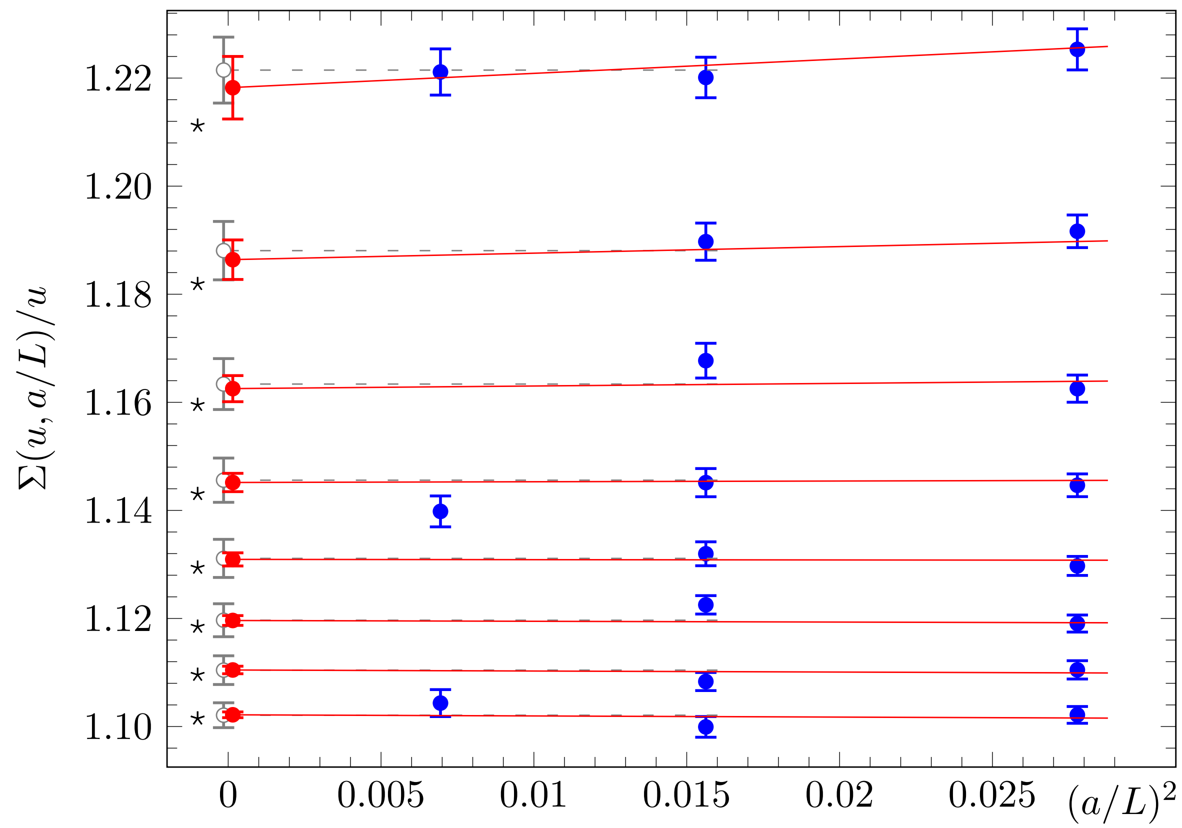
$$(4\pi)^3 \times b_{2,\nu} = -0.06(3) - \nu \times 1.26\,, \ (N_{\mathrm{f}} = 3)$$

- small discretisation effects (a^4 at LO PT)
  we also subtract them including 2-loop terms

  [hep-lat/9911018 Bode, Weisz, Wolff]

- but O(a) discretisation effects due to boundary terms

Continuum limit of Σ



- use perturbative improvement (i=1,2)

$$\Sigma^{(i)}(u, a/L) = \frac{\Sigma(u, a/L)}{1 + \sum_{k=1}^{i} \delta_k(a/L)\, u^k} \,,$$

- and global fit

$$\Sigma_\nu^{(i)}(u, a/L) = \sigma_\nu(u) + \rho_\nu^{(i)}(u)\,(a/L)^2$$

- with

$$\rho_\nu^{(i)}(u) = \sum_{k=1}^{n_\rho^{(i)}} \rho_{\nu,k}^{(i)} u^{i+1+k} \,, \quad \sigma_\nu(u) = u + u^2 \sum_{k=0}^{3} s_k u^k$$

Continuum limit of Σ

- was also tested carefully in pure gauge theory

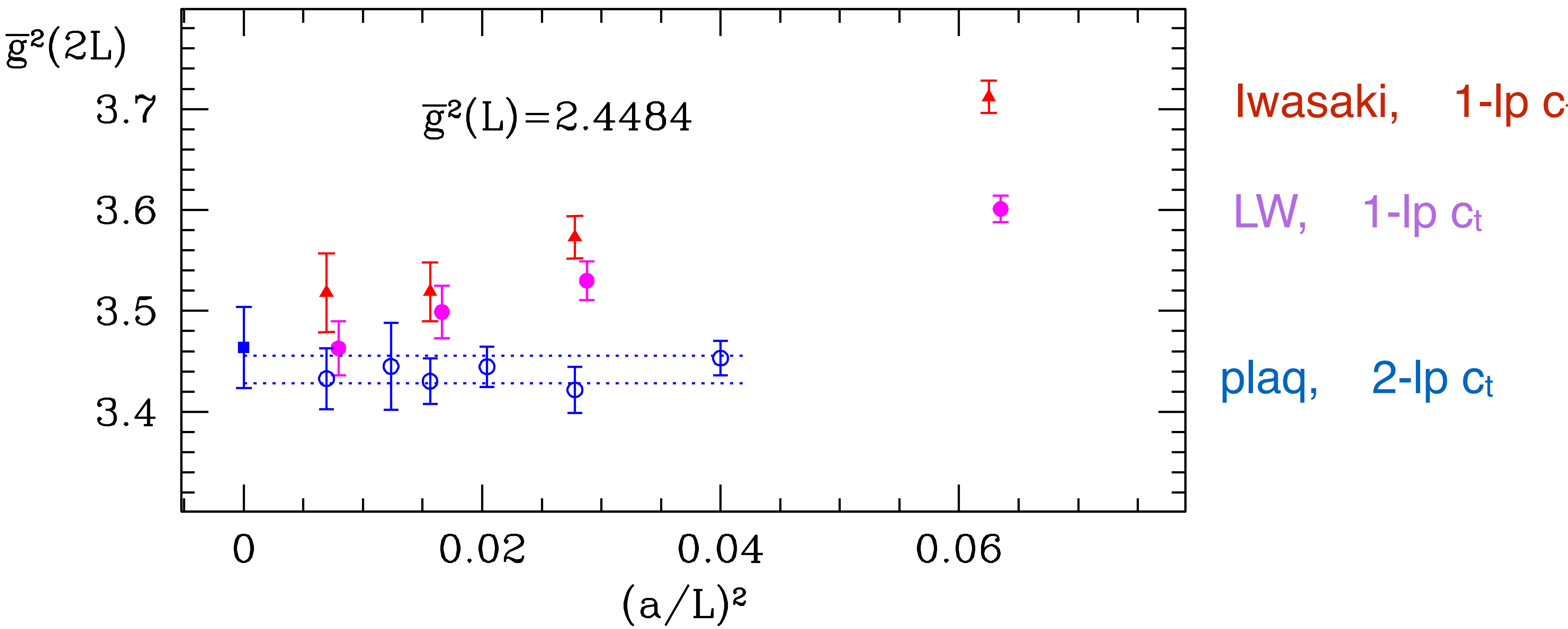
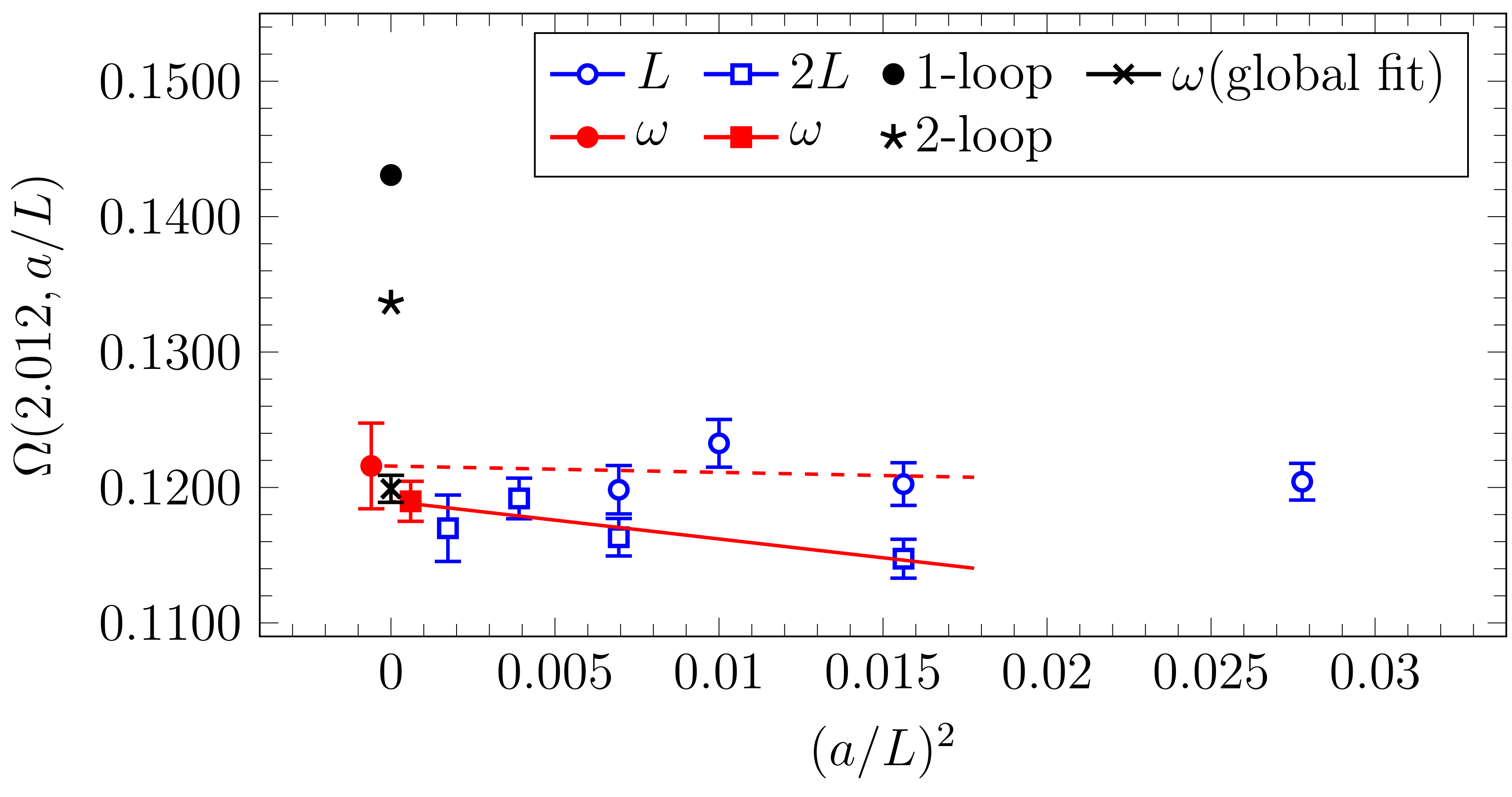


Figure 8: A test of the continuum extrapolations with different actions for $N_f = 0$. The data from top (triangles) to bottom (open circles) are for the Iwasaki, the tree level Lüscher Weisz and the Wilson gauge action. Both the boundary improvement of the action and the improvement of the observables have been included. At present this is possible at the 2-loop level for the Wilson gauge action only, and at the 1-loop level in the two other cases. Figure from [29] based on data from [63, 64].

Continuum limit of Ω

$$\Omega(u, a/L) = \bar{v}|_{\bar{g}^2(L)=u} \quad \omega(u) = \Omega(u, 0)$$

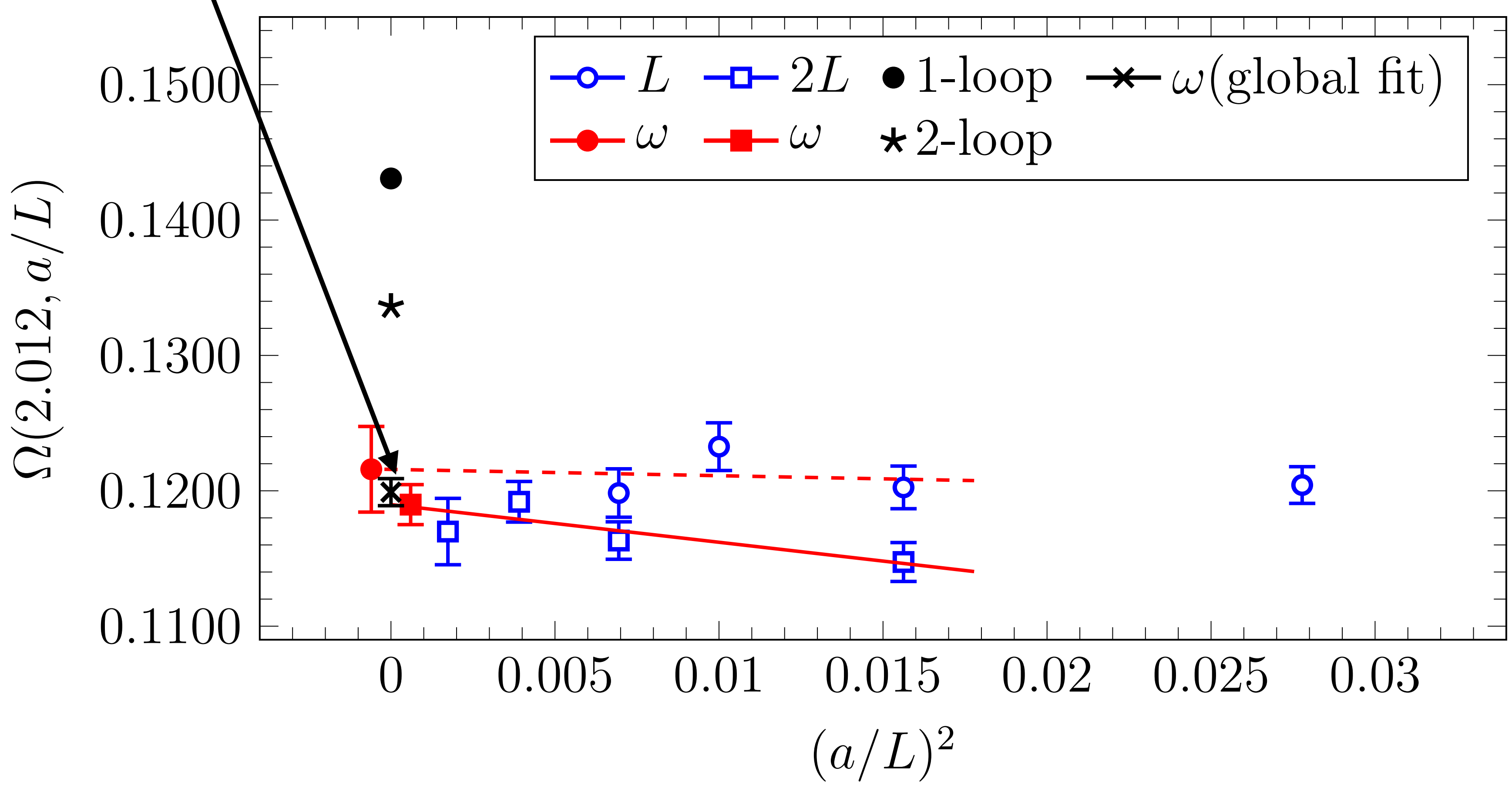
- Global fits, similar to Σ
- but with L/a=6,8,10,12 ("L") and L/a=12,16,24 ("2L")
- a-effects different for "L" vs. "2L" (different def. of m=0)



Continuum limit of Ω

$$\Omega(u, a/L) = \bar{v}|_{\bar{g}^2(L)=u} \quad \omega(u) = \Omega(u, 0)$$

- Global fits, similar to Σ
- but with L/a=6,8,10,12 (“L”) and L/a=12,16,24 (“2L”)
- a-effects different for “L” vs. “2L” (different def. of m=0)



Continuum limit of Ω

$$\Omega(u, a/L) = \bar{v}|_{\bar{g}^2(L)=u} \quad \omega(u) = \Omega(u, 0)$$

- Global fits, similar to Σ
- but with L/a=6,8,10,12 (“L”) and L/a=12,16,24 (“2L”)
- a-effects different for “L” vs. “2L” (different def. of m=0)