Description of neutrinoless double beta decay using IBM-2

Jenni Kotila

Challenges in the investigation of double charge-exchange nuclear reactions: towards neutrino-less double beta decay

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Motivation

- Although proposed more than 70 years ago to establish the nature of neutrinos, neutrinoless double beta decay remains the most sensitive probe to following open questions:
  - What is the absolute neutrino mass scale?
  - Are neutrinos Dirac or Majorana particles?
  - How many neutrino species are there?
- At the moment experiments are scanning half-lives of the order of $10^{25}$ yr:
  $$\left[ \frac{\tau_{1/2}^{0\nu}}{G_{0\nu} g_A^4 |M^{(0\nu)}|^2 |f(m_i, U_{ei})|^2} \right]^{-1}$$
- $f(m_i, U_{ei})$ contains the physics beyond standard model and is different for different scenarios and mechanisms: exchange of light or heavy neutrino, emission of Majoron, exchange of sterile neutrino(s)...
- The fact that $0\nu \beta\beta$-decay is a unique process, and there is no direct probe which connects the initial and final states other than the process itself makes the prediction challenging for theoretical models.
- The reliability of the used wave functions, and eventually $M^{(0\nu)}$, has to be then tested using other available relevant data.
Different models, different assumptions

$M^{(0\nu)}$ are calculated in nuclear models, such as:

- The Quasiparticle random phase approximation, QRPA, constructs ground state correlations by iterating two-quasiparticle excitations on top of a BCS or HFB vacuum. A quasiboson approximation is then imposed on the excitations. The calculations are performed in a large valence space including several major shells. The Hamiltonian is typically based on a realistic G matrix, but modified in the like-particle pairing and particle-hole channels to reproduce experimental pairing gaps and Gamow-Teller resonance energies. Results depend on fine-tuning of the interaction, especially near the spherical-deformed transition, for example $^{150}$Nd.

- In the interacting shell model, ISM, the single-particle Hilbert space is small, typically a few valence orbits. However, the shell model includes all possible correlations within that space through direct diagonalization of the Hamiltonian. The valence-shell interaction usually comes from G-matrix perturbation theory or a renormalization-group treatment, but must be adjusted to reproduce spectra. ISM cannot address nuclei with many particles in the valence shells, for example $^{150}$Nd, due to the exploding size of the Hamiltonian matrices ($>10^9$).
The idea that inspires the microscopic interacting boson model, IBM-2, is a truncation of the very large shell model space to states built from pairs of nucleons with $J = 0$ and 2. These pairs are then assumed to be collective and are taken as bosons. The Hamiltonian is constructed phenomenologically and two- and four valence-nucleon states are generated by a schematic interaction. IBM-2 is known to be very successful in reproducing trends for spectra and E2 transitions involving collective states across isotopic and isotonic chains.

Can be used in any nucleus and thus all nuclei of interest can be calculated within the same model.

Realistic and well checked wave functions (excitation energies, B(E2) values and quadrupole moments, B(M1) values and magnetic moments, occupation probabilities, etc.).
Different models, different assumptions: IBM-2

- In the microscopic IBM the shell model $S, D$ pair states are mapped onto $s, d$ bosons as

$$
S^\dagger_\rho = \sum_j \alpha_j \sqrt{\frac{\Omega_j}{2}} (a^{\dagger}_j \times a^{\dagger}_j)^{(0)} \rightarrow s^\dagger_\rho
$$

$$
D^\dagger_\rho = \sum_{j \leq j'} \beta_{jj'} \frac{1}{\sqrt{1+\delta_{jj'}}} (a^{\dagger}_j \times a^{\dagger}_{j'})^{(2)} \rightarrow d^\dagger_\rho,
$$

with $\Omega_j = j + 1/2$ and pair structure coefficients $\alpha_j$ and $\beta_{jj'}$, that are obtained by diagonalizing the surface delta interaction (SDI) in a chosen valence space.

- Following the method developed by Pittel, Duval and Barrett, $S^\dagger_\rho$ and $D^\dagger_\rho$ create the energetically-lowest $0^+$ and $2^+$ two-fermion states appropriate to the nucleus of interest. By using this method some possible renormalization (polarization) effects induced by the neutron-proton interaction are included approximately.

- The used single particle energies are taken from experiments.

- Isovector strength parameter $A_1$ value is fitted to reproduce the energy difference between the first $2^+$ and the $0^+$ ground state in the corresponding two-valence-particle or two-valence-hole nucleus.
Different models, different assumptions: IBM-2

- The bosonization method, when carried to all orders, produces results that are identical to the fermionic results. In $\beta\beta$-decay the fermion transition operator creates a pair of protons (neutrons) and annihilates a pair of neutrons (protons), so we need the mapping of the coupled pair operator:

\[
\left(\pi^+_{j_\pi} \times \pi^+_{j'_\pi}\right)^{(0)} \rightarrow A^{(01)}_{j_\pi} s^+_{j_\pi} + A^{(11)}_{j_\pi} s^+_{j_\pi} \left(d^+_{j_\pi} \tilde{d}^+_{j_\pi}\right)^{(0)} + ...
\]

\[
\left(\pi^+_{j_\pi} \times \pi^+_{j'_\pi}\right)^{(2)} \rightarrow B^{(01)}_{j_\pi j'_\pi} d^+_{j_\pi} \\
+ B^{(11)}_{j_\pi j'_\pi} s^+_{j_\pi} \left(s^+_{j_\pi} \tilde{d}^+_{j_\pi}\right)^{(2)} + B^{(12)}_{j_\pi j'_\pi} s^+_{j_\pi} \left(d^+_{j_\pi} \tilde{d}^+_{j_\pi}\right)^{(2)} + ...
\]

\[
\left(\tilde{v}_{j_\nu} \times \tilde{v}_{j'_\nu}\right)^{(0)} \rightarrow \tilde{A}^{(01)}_{j_\nu} \tilde{s}_{j_\nu} + \tilde{A}^{(11)}_{j_\nu} \tilde{s}_{j_\nu} \left(d^+_{j_\nu} \tilde{d}^+_{j_\nu}\right)^{(0)} + ...
\]

\[
\left(\tilde{v}_{j_\nu} \times \tilde{v}_{j'_\nu}\right)^{(2)} \rightarrow \tilde{B}^{(01)}_{j_\nu j'_\nu} \tilde{d}_{j_\nu} \\
+ \tilde{B}^{(11)}_{j_\nu j'_\nu} \left(d^+_{j_\nu} \tilde{s}_{j_\nu}\right)^{(2)} \tilde{s}_{j_\nu} + \tilde{B}^{(12)}_{j_\nu j'_\nu} \left(d^+_{j_\nu} \tilde{d}_{j_\nu}\right)^{(2)} \tilde{s}_{j_\nu} + ...
\]

- The nuclear matrix elements of proper operators are then obtained between realistic wave functions obtained from IBM-2, which in addition to spherical nuclei is also capable of describing medium and heavy deformed nuclei as $^{150}$Nd and $^{150}$Sm.
**Some tests of wave functions**

**Case $^{154}$Gd: Granddaughter of $^{154}$Sm**

- Shape transitional region $\Rightarrow$ rapid changes of nuclear deformation
- Old calculation: No experimental information about $1^+$ scissors mode
- New experimental data $\Rightarrow$ parameters of Majorana operator can be fitted
  - Little effect on low-lying full-symmetric states
  - BUT significant effect on the mixed symmetry state wave function, like $0^+_2 \Rightarrow$ new $M^{0\nu}(0^+_2) = 0.37$ (old $M^{0\nu}(0^+_2) = 0.02$)

<table>
<thead>
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<th></th>
<th>$1^+$</th>
<th>$2^+$</th>
<th>$3^+$</th>
<th>$4^+$</th>
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</table>
Some tests of wave functions

Occupation probabilities: $A=100$ system, neutrons

- **IBM-2**: $1d$ overfilled for both $^{100}$Mo and $^{100}$Ru
- The change appears to be dominated by the $1d$ and $0h_{11/2}$ orbitals with a small contributions from $2s_{1/2}$ and $0g_{7/2}$
- **IBM-2**: Agreement good with experiments
- **BCS**: more complex rearrangement of nucleons, differs both from experiments and IBM-2 results

EXP: D. K Sharp at MEDEX'15; BCS: J. Suhonen and O. Civitarese, NPA 924 (2014) 1
Some tests of wave functions

Occupation probabilities: A=100 system, protons

- Individual $^{100}$Mo and $^{100}$Ru proton occupancies, as well as the difference in proton occupancy are in proper agreement with the experiments.
- Change is dominated by $0g_{9/2}$ orbital, where $1p$ orbitals play a lesser role, and $0f_{5/2}$ orbital gives only a small contribution.
- Comparison with BCS calculation reveals complex differences.

∴ Overall agreement good between IBM-2 and experiments for $A = 100$ system.
Nuclear Matrix Elements

- Transition operator for $\beta\beta$ decay: $T(p) = H(p)f(m_i, U_{ei})$, where

$$H(p) = \sum_{n, n'} n^+ n'^+ \left[ -h_F(p) + h^{GT}(p)\vec{\sigma}_n \cdot \vec{\sigma}_{n'} + h^T(p)S_{nn'}^p \right]$$

(in momentum space, including higher order corrections)

- Truncated transition operator in IBM-2

$$h_{IBM}^{F, GT, T} = h_{s\bar{s}}^{F, GT, T} s^{\dagger}_\pi \cdot \tilde{S}_\nu + h_{d\bar{d}}^{F, GT, T} d^{\dagger}_\pi \cdot \tilde{d}_\nu$$

where coefficients

$$h_{s\bar{s}}^{F, GT, T} = -\sum_{j_\pi} \sum_{j_\nu} G^{F, GT, T} \left( j_\pi j_\pi j_\nu j_\nu; J = 0 \right) A_{j_\pi}^{(01)} \tilde{A}_{j_\nu}^{(01)}$$

$$h_{d\bar{d}}^{F, GT, T} = -\frac{1}{2} \sum_{j_\pi j'_\pi} \sum_{j_\nu j'_\nu} \sqrt{1 + \delta_{j_\pi j'_\pi}} \sqrt{1 + \delta_{j_\nu j'_\nu}}$$

$$\times G^{F, GT, T} \left( j_\pi j'_\pi j_\nu j'_\nu; J = 2 \right) B_{j_\pi j'_\pi}^{(01)} \tilde{B}_{j_\nu j'_\nu}^{(01)}$$
Nuclear Matrix Elements

Using the above defined operators we obtain some general trends:

- Shell effects: The matrix elements are smaller at the closed shells than in the middle of the shell

- Deformation effects always decrease the matrix elements

- Isospin restoration reduces matrix elements
  - The offending isospin violating NME is the Fermi NME in $2\nu\beta\beta$ which should be zero, since the Fermi part of the transition operator can not change isospin
  - Isospin restoration makes the Fermi NME vanish for $2\nu\beta\beta$ and for $0\nu\beta\beta$ is reduced by subtraction of the monopole term in the expansion of the matrix element multipoles
Nuclear Matrix Elements: $^0\nu\beta^−\beta^−$

**ISOSPIN RESTORATION** reduces matrix elements

\[
\chi_F = \left(\frac{g_V}{g_A}\right)^2 \frac{M_F^{(0\nu)}}{M_{GT}^{(0\nu)}}
\]

<table>
<thead>
<tr>
<th>Decay</th>
<th>IBM-2</th>
<th>QRPA</th>
<th>ISM</th>
</tr>
</thead>
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<tr>
<td>$^{48}\text{Ca}$</td>
<td>-0.10(-0.39)</td>
<td>-0.32(-0.93)</td>
<td></td>
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<tr>
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<td>-0.09(-0.37)</td>
<td>-0.21(-0.34)</td>
<td>-0.12</td>
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<tr>
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<td>-0.23(-0.35)</td>
<td>-0.11</td>
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<tr>
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<td>-0.23(-0.38)</td>
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<td>$^{100}\text{Mo}$</td>
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<td>-0.30(-0.30)</td>
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<tr>
<td>$^{110}\text{Pd}$</td>
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<td>-0.27(-0.33)</td>
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<td>$^{116}\text{Cd}$</td>
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<td>$^{124}\text{Sn}$</td>
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<td>$^{128}\text{Te}$</td>
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<tr>
<td>$^{130}\text{Te}$</td>
<td>-0.12(-0.33)</td>
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<td>$^{238}\text{U}$</td>
<td>-0.08(-0.08)</td>
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</tbody>
</table>

$^0\nu\beta^−\beta^−$:

\[
\chi_F = \left(\frac{g_V}{g_A}\right)^2 \frac{M_F^{(0\nu)}}{M_{GT}^{(0\nu)}}
\]

(old values in parentheses):

- Considerable reduction obtained!
- Isospin restored $\chi_F$ values very close to the ones obtained from ISM, where isospin is a good quantum number by construction
- Similar prescription has been used for QRPA (Simkovic et al., PRC 87 045501 (2013) and Suhonen et al., PRC 91 024613 (2015))
Nuclear Matrix Elements: $0\nu\beta^-\beta^-$

- Light neutrino exchange: $\nu(p) = \frac{2}{\pi} \frac{1}{p(p+A)}$, $f = \frac{\langle m_\nu \rangle}{m_e}$

\[
M^{(0\nu)} = M^{(0\nu)}_{GT} - \left( \frac{g_V}{g_A} \right)^2 M^{(0\nu)}_F + M^{(0\nu)}_T
\]

**Figure: Neutron number vs. $M^{(0\nu)}$ for various nuclides.**

Nuclear Matrix Elements: $0\nu\beta^-\beta^-$

\[ M^{(0\nu)} = M_{\text{GT}}^{(0\nu)} - \left( \frac{g_V}{g_A} \right)^2 M_F^{(0\nu)} + M_T^{(0\nu)} \]

- Comparison of IBM-2, QRPA, ISM NMEs for light neutrinos
- IBM-2/QRPA/ISM similar trend
- Larger values at the middle of the shell than at closed shells
- The ISM is a factor of $\sim 2$ smaller than both the IBM-2 and QRPA in the lighter nuclei and the difference is smaller for heavier
  - Effective value of $g_A$?

Nuclear Matrix Elements: $0\nu\beta^-\beta^-$

Estimate of error

- Sensitivity to input parameter changes
  - Single particle energies: 10%
  - Strengths of interactions: 5%
  - Oscillator parameter (SP wave functions): 5%
  - Closure energy in the neutrino potential: 5%
  - Nuclear radius (If NMEs in dimensionless units): 5%

- Sensitivity to model assumptions
  - Truncation to S-D space: 1% (spherical) - 10% (deformed)
  - Isospin purity: 2%

- Sensitivity to operator assumptions
  - Form of the transition operator: 5%
  - Finite nuclear size: 1%
  - Short range correlations (SRC): 5%

- The total error estimate is 16%
Nuclear Matrix Elements: \(0\nu_h\beta^-\beta^-\)

- In heavy neutrino exchange scenario the transition operator has same form as for light neutrinos, but with:
  \[
  f \propto m_p \langle m_{\nu_h}^{-1} \rangle
  \]
  \[
  \langle m_{\nu_h}^{-1} \rangle = \sum_{k=\text{heavy}} (U_{ekh})^2 \frac{1}{m_{kh}}
  \]
  - involves the mass eigenstates \(m_{kh}\) of heavy neutrinos
  - and \(m_{\nu_h} \gg 1\text{GeV}\)
  - The Fourier transform of the neutrino "potential" is:
    \[
    v(p) = \frac{2}{\pi} \frac{1}{m_pm_e}
    \]
    - Contact interaction in configuration space ⇒ strongly influenced by short range correlations
Nuclear Matrix Elements: $0\nu \beta^-\beta^-$

- Comparison of IBM-2, QRPA, and ISM matrix elements for heavy neutrinos

- IBM-2/QRPA/ISM similar trend, factor of $\sim 2$ difference between IBM-2 and QRPA-Tü

- Comment: Ratio for QRPA-Jy/QRPA-Tü results varies from 1 up to 2.5, reason for this discrepancy is not clear

- Note: IBM-2 error estimate in this case is 56% mostly coming from SRC

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**Half-Life Predictions: $0\nu\beta^{-}\beta^{-}$**

- Predictions calculated with $g_A=1.269$ (and $|\langle m_\nu \rangle| = 1\text{eV}$)

- Judging by the half-life, best candidates $^{150}\text{Nd}$, $^{100}\text{Mo}$, and $^{130}\text{Te}$, where half-lives $\sim 10^{23}\text{yr}$
Half-Life Predictions: $0\nu\beta^{-}\beta^{-}$

**DECAYS TO FIRST EXCITED $0^+$ STATES**

- In some cases, the matrix elements to the first excited $0^+$ state are large.
- Although the PSFs are smaller to the excited state, large matrix elements offer the possibility of a direct detection, by looking at the $\gamma$-ray de-exciting the excited $0^+$ level.

**Best candidates** $^{100}$Mo and $^{150}$Nd, $\tau_{1/2}^{(0\nu)} \sim 10^{24}$ yr.

- $2\nu\beta\beta$-decay observed to excited $0^+$ state in these nuclei!
Comment about $0\nu\beta^+\beta^+$, $0\nu\text{EC}\beta^+$, and $R0\nu\text{ECEC}$

- $\beta^+\beta^+$ and $0\nu\text{EC}\beta^+$: available kinetic energy much smaller since
  \[ T_{\beta^+\beta^+} = M(A, Z) - M(A, Z - 2) - 4m_e c^2 \]
  \[ T_{\text{EC}\beta^+} = M(A, Z) - M(A, Z - 2) - 2m_e c^2 - \epsilon_b \]
  \[ \Rightarrow \text{much smaller phase space} \Rightarrow \text{much longer } \tau_{1/2}^{(0\nu)} > 10^{26}\text{yr} \]

- For $0\nu\text{ECEC}$ available energy larger, but since all the energies are fixed, additional requirement that Q-value matches the final state energy

- Resonance enhancement:
  \[ \left[ \tau_{1/2}^{0\nu} \right]^{-1} = g_A^4 G_{0\nu} \left| M^{0\nu} \right|^2 |f|^2 \frac{(m_e c^2) \Gamma}{\Delta^2 + \Gamma^2/4}, \]
  where $\Delta = |Q - B_{2h} - E|$ is the degeneracy parameter, and $\Gamma$ is the two-hole width

- So in principle, if $\Delta \sim 0$ and $\Gamma \sim 1\text{eV}$ we could obtain up to $10^6$ enhancement

  - Unfortunately this is not the case and $\tau_{1/2}^{(0\nu)} > 10^{27}\text{yr}$
This mechanism requires the emission of one or two additional bosons, Majorons, so it has similarities with $2\nu\beta\beta$.

There are many different models, where $m$, the number of emitted Majorons and $n$, the spectral index of the decay take different values:

$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = g_A^4 G_{m\chi_0 n}^{(0)} \left|\langle g_{\chi_{ee}}^{M}\rangle\right|^{2m} \left|M_{0\nu M}^{(m,n)}\right|^2$$

Comparison with experimental limits on $\tau_{1/2,\text{exp}}^{0\nu M}$ gives information about $\langle g_{ee}^{M}\rangle$, the majoron-neutrino coupling constant.

Ordinary Majoron decay $m = 1, n = 1$: If the Majoron couples only to light neutrino, the NME needed to calculate the half-life are the same as for light neutrino exchange.

There are cosmologic constraints on $\langle g_{ee}^{M}\rangle$, such as values $3 \times 10^{-7} \lesssim g_{ee}^{M} \lesssim 2 \times 10^{-5}$ or $g_{ee}^{M} \gtrsim 3 \times 10^{-4}$ are excluded by the observation of SN 1987A.

The most stringent of the current limits are at these regions.
Sterile neutrinos

- Another scenario, currently being extensively discussed, is the mixing of additional “sterile” neutrinos

- The NME for sterile neutrinos of arbitrary mass can be calculated using a transition operator as in $\nu_{\text{light}}$ and $\nu_{\text{heavy}}$ exchange but with

\[
f = \frac{m_N}{m_e}, \quad v(p) = \frac{2}{\pi} \frac{1}{\sqrt{p^2 + m_N^2} \left(\sqrt{p^2 + m_N^2 + \tilde{A}}\right)},
\]

where $m_N$ is the mass of the sterile neutrino

- The product

\[
f v(p) = \frac{m_N}{m_e} \frac{2}{\pi} \frac{1}{\sqrt{p^2 + m_N^2} \left(\sqrt{p^2 + m_N^2 + \tilde{A}}\right)}
\]

has the limits:

\[
m_N \to 0 \quad f v(p) = \frac{m_N}{m_e} \frac{2}{\pi} \frac{1}{p(p+\tilde{A})}
\]

\[
m_N \to \infty \quad f v(p) = \frac{m_N}{m_e} \frac{2}{\pi} \frac{1}{m_N^2} = \frac{2}{\pi} \frac{1}{m_e m_N}
\]
Sterile neutrinos

- Several types of sterile neutrinos have been suggested.
  - Light sterile neutrinos
    - Neutrino masses are $m_N \sim 1\text{eV}$
    - These neutrinos account for the reactor anomaly in oscillation experiments and for the gallium anomaly *
  - Heavy sterile neutrinos
    - Neutrino masses are $m_N \gg 1\text{eV}$
    - keV mass range, MeV-GeV mass range, TeV mass range

- When the mass $m_N$ is intermediate the factorization is not possible, and physics beyond the standard model is entangled with nuclear physics. In this case, the half-life can be written as

\[
[\tau_{1/2}^{0\nu}]^{-1} = G_{0\nu} \sum_N (U_{eN})^2 M_{0\nu}(m_N) \frac{m_N}{m_e}^2
\]
Sterile neutrinos

- The corresponding nuclear matrix elements can be written as

\[ M_{0\nu}(m_N) = g_A^2 M^{(0\nu)}(m_N), \]

\[ M^{(0\nu)}(m_N) = M_{GT}^{(0\nu)}(m_N) - \left( \frac{g_V}{g_A} \right)^2 M_F^{(0\nu)}(m_N) + M_T^{(0\nu)}(m_N) \]

- The NMEs can be calculated exactly, but a simple formula

\[ \left[ \tau_{1/2}^{0\nu} \right]^{-1} = G_{0\nu} g_A^4 \left| M^{(0\nu_h)} \right|^2 m_p \sum_N (U_{eN})^2 \frac{m_N}{\left\langle p^2 \right\rangle + m_N^2} \],

with

\[ \left\langle p^2 \right\rangle = \frac{M^{(0\nu_h)}}{M^{(0\nu)}} m_p m_e, \]

gives a very good approximation
Sterile neutrinos

- IBM-2 NMEs for neutrinos of arbitrary mass plotted as a function of $m_N$ in a) $^{76}$Ge, b) $^{130}$Te, and c) $^{136}$Xe. Blue squares represent the exact calculation for $m_N = 0.001\text{GeV}, 0.01\text{GeV}, 0.1\text{GeV}, 1\text{GeV}, 10\text{GeV}$, joined together by a Mathematica interpolating formula. The curve is obtained using the simple formula.

- The interesting aspect is that the curves peak at $m_N \sim 100\text{MeV}$, the scale set by the nucleon Fermi momentum in the nucleus, $p_F$. If sterile neutrinos of this mass exist, their contribution to the half-life is enhanced.
• Expected half-life for a single neutrino of mass $m_N$ with coupling $U_{\nu e N}^2 = 10^{-2} - 10^{-8}$ and $g_A = 1.269$ for a) $^{76}\text{Ge}$, b) $^{130}\text{Te}$, and c) $^{136}\text{Xe}$. Blue squares represent the exact calculation for $m_N = 0.001\text{GeV}, 0.01\text{GeV}, 0.1\text{GeV}, 1\text{GeV}, 10\text{GeV}$. The smooth curve is obtained using the simple formula. The experimental limits from GERDA, CUORE-0, KamLAND-Zen, and EXO are also shown. The excluded zone is that below these limits.
Excluded values of $|U_{eN}|^2$ and $m_N$ in the $m_N-|U_{eN}|^2$ plane, for $g_A = 1.269$. For each experiment, GERDA, CUORE-0, KamLAND-Zen, and EXO, a band of values is given, corresponding to our error estimate.
Limits on Average Light Neutrino Mass

Reminder:

$$\left[ \tau_{1/2}^{0\nu} \right]^{-1} = G_{0\nu} g_A^4 |M^{0\nu}|^2 |f(m_i, U_{ei})|^2$$

- Light neutrinos:

$$f(m_i, U_{ei}) = \frac{\langle m_{\nu} \rangle}{m_e} = \frac{1}{m_e} \sum_{k=\text{light}} (U_{ek})^2 m_k$$

- The average light neutrino mass is now well constrained by atmospheric, solar, reactor and accelerator neutrino oscillation experiments

- Obtained information on mass differences and their mixing leaves two possibilities: Normal and inverted hierarchy
Limits on Average Light Neutrino Mass

- The average light neutrino mass is then written as:

\[
\langle m_\nu \rangle = \left| c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\varphi_2} + s_{13}^2 e^{i\varphi_3} \right|,
\]

\[
c_{ij} = \cos \theta_{ij}, \quad s_{ij} = \sin \theta_{ij}, \quad \varphi_{2,3} = [0, 2\pi],
\]

\[
(m_1^2, m_2^2, m_3^2) = \frac{m_1^2 + m_2^2}{2} + \left( -\frac{\delta m_2^2}{2}, +\frac{\delta m_2^2}{2}, \pm \Delta m^2 \right)
\]

- \(\theta_{12}, \theta_{13}, \theta_{23}\) and \(\delta m, \Delta m\) fitted to oscillation experiments*

- Phases \(\varphi_2\) and \(\varphi_3\) may vary from 0 to \(2\pi\)

---

* \(\sin^2 \theta_{12} = 0.308, \delta m^2 = 7.54 \times 10^{-5} \text{ eV}^2\)

NH: \(\Delta m^2 = 2.43 \times 10^{-3} \text{ eV}^2, \sin^2 \theta_{13} = 0.0234, \sin^2 \theta_{23} = 0.437\)

IH: \(\Delta m^2 = 2.38 \times 10^{-3} \text{ eV}^2, \sin^2 \theta_{13} = 0.0240, \sin^2 \theta_{23} = 0.455\)
Limits on Average Light Neutrino Mass

Current lower half-life limits coming from different experiments

<table>
<thead>
<tr>
<th>Experiment</th>
<th>nucleus</th>
<th>$\tau_{1/2}$</th>
<th>$\langle m_\nu \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IGEX</td>
<td>$^{76}$Ge</td>
<td>$&gt; 1.57 \times 10^{25}$ yr</td>
<td>$&lt; 0.35$ eV</td>
</tr>
<tr>
<td>GERDA</td>
<td>$^{76}$Ge</td>
<td>$&gt; 2.1 \times 10^{25}$ yr</td>
<td>$&lt; 0.30$ eV</td>
</tr>
<tr>
<td>NEMO-3</td>
<td>$^{100}$Mo</td>
<td>$&gt; 1.1 \times 10^{24}$ yr</td>
<td>$&lt; 0.56$ eV</td>
</tr>
<tr>
<td>CUORE</td>
<td>$^{130}$Te</td>
<td>$&gt; 4.0 \times 10^{24}$ yr</td>
<td>$&lt; 0.35$ eV</td>
</tr>
<tr>
<td>EXO</td>
<td>$^{136}$Xe</td>
<td>$&gt; 1.1 \times 10^{25}$ yr</td>
<td>$&lt; 0.25$ eV</td>
</tr>
<tr>
<td>Kamland-Zen</td>
<td>$^{136}$Xe</td>
<td>$&gt; 1.9 \times 10^{25}$ yr</td>
<td>$&lt; 0.20$ eV</td>
</tr>
</tbody>
</table>

$\tau_{1/2} \Rightarrow \langle m_\nu \rangle < \frac{m_e}{\sqrt{\tau_{1/2}^{\text{exp}} G_0 g_\alpha^2 |M(0\nu)|}}$

Limits on Average Light Neutrino Mass

- Current limits to $\langle m_\nu \rangle$ from CUORE, IGEX, NEMO-3, KamLAND-Zen, EXO, and GERDA $0\nu\beta\beta$ experiments for light neutrino exchange

Limits on Average Light Neutrino Mass

- If, however, there are sterile neutrinos, this picture is different.
- Considering, for example, a suggested of a 4th neutrino with mass $m_4 = 1\text{eV}$ and $|U_{e4}|^2 = 0.03$, we have

$$\langle m_{N,\text{light}} \rangle = \sum_{k=1}^{3} U_{ek}^2 m_k + U_{e4}^2 e^{i\alpha_4} m_4, \text{ with } 0 \leq \alpha_4 \leq 2\pi$$
Limits on Average Light Neutrino Mass: Remarks

- We do not know what is the mechanisms of $0
\nu \beta \beta$ -decay and several mechanisms may contribute with different relative phases.

- The question of effective value of $g_A$ is still open. Three suggested scenarios are:
  
  - Free value: 1.269
  - Quark value: 1
  - Even stronger quenching: $g_{A,\text{eff}} < 1$
Quenching of $g_A$

- It is well-known from single $\beta$ decay/EC and $2\nu\beta\beta$ that $g_A$ is renormalized in nuclei. Reasons:
  - Limited model space
  - Omission of non-nucleonic degrees of freedom ($\Delta, N^*, ...$)
- The effective value of $g_A$ can be
  - defined as

$$M_{2\nu}^{\text{eff}} = \left( \frac{g_{A,\text{eff}}}{g_A} \right)^2 M_{2\nu}$$

$$M_{\beta/EC}^{\text{eff}} = \left( \frac{g_{A,\text{eff}}}{g_A} \right) M_{\beta/EC}$$

- and obtained by comparing the calculated and measured half-lives for $\beta/EC$ and/or for $2\nu\beta\beta$

Quenching of $g_A$

Maximally quenched value from $2\nu\beta^-\beta^-$ experiments:

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$\tau_{1/2}^{2\nu}(10^{18}\text{ y})\ exp^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{48}\text{Ca}$</td>
<td>$44^{+6}_{-5}$</td>
</tr>
<tr>
<td>$^{76}\text{Ge}$</td>
<td>$1650^{+140}_{-120}$</td>
</tr>
<tr>
<td>$^{82}\text{Se}$</td>
<td>$92 \pm 7$</td>
</tr>
<tr>
<td>$^{96}\text{Zr}$</td>
<td>$23 \pm 2$</td>
</tr>
<tr>
<td>$^{100}\text{Mo}$</td>
<td>$7.1 \pm 0.4$</td>
</tr>
<tr>
<td>$^{100}\text{Mo}-^{100}\text{Ru}(0^+_2)$</td>
<td>$670^{+50}_{-40}$</td>
</tr>
<tr>
<td>$^{116}\text{Cd}$</td>
<td>$28.7 \pm 1.3$</td>
</tr>
<tr>
<td>$^{128}\text{Te}$</td>
<td>$2000000 \pm 300000$</td>
</tr>
<tr>
<td>$^{130}\text{Te}$</td>
<td>$690 \pm 130$</td>
</tr>
<tr>
<td>$^{136}\text{Xe}$</td>
<td>$2110 \pm 250$</td>
</tr>
<tr>
<td>$^{150}\text{Nd}$</td>
<td>$8.2 \pm 0.9$</td>
</tr>
<tr>
<td>$^{150}\text{Nd}-^{150}\text{Sm}(0^+_2)$</td>
<td>$120^{+30}_{-20}$</td>
</tr>
<tr>
<td>$^{238}\text{U}$</td>
<td>$2000 \pm 600$</td>
</tr>
</tbody>
</table>

$|\mathbf{M}_{2\nu}^{\text{eff}}|^2$ is obtained from the measured half-life by

$$|\mathbf{M}_{2\nu}^{\text{eff}}|^2 = \left[\frac{\tau_{1/2}^{2\nu}}{G_{2\nu}}\right]^{-1}$$

Smallest $\mathbf{M}_{2\nu}^{\text{eff}}$ for $^{136}\text{Xe}$, the newest one measured!

Quenching of $g_A$

$$g_{A,\text{eff}} = g_A \sqrt{\frac{M_{2\nu}}{M_{2\nu}}}$$

- Extracted $g_{A,\text{eff}}$:
  - IBM-2 $\sim 0.6 - 0.5$
  - QRPA $\sim 0.7 - 0.6$
  - ISM $\sim 0.8 - 0.7$

- Similar values found by analyzing $\beta^-/EC$ for IBFM-2$^a$ and for QRPA$^b$

- Assumption: $g_{A,\text{eff}}$ is a smooth function of $A$

- Parametrization:
  $$g_{A,\text{eff}} = 1.269A^{-\gamma}$$
  - IBM-2: $\gamma = 0.18$
  - QRPA: $\gamma = 0.16$
  - ISM: $\gamma = 0.12$

  a Yoshida and Iachello, PTEP 2013, 043D01 (2013).
Let's return to $0^{\nu}\beta\beta$ NMEs:

\[ M_{0\nu} = g_{A,\text{eff}}^2 M^{(0\nu)} \] for IBM-2, QRPA, and ISM

Taking into account the 16% error estimate for IBM-2:
Agreement quite good

Looks promising...
Quenching of $g_A$

Effective value of $g_A$ is a work in progress, since:

- Is the renormalization of $g_A$ the same in $2\nu\beta\beta$ as in $0\nu\beta\beta$?
  - In $2\nu\beta\beta$ only the $1^+$ (GT) multipole contributes. In $0\nu\beta\beta$ all multipoles $1^+, 2^-, ...; 0^+, 1^-, ...$ contribute. Some of which could be even unquenched.
  - This is a critical issue, since half-life predictions with maximally quenched $g_A$ are $\sim 6 - 34$ times longer due to the fact that $g_A$ enters the equations to the power of 4!

- Additional ways to study quenching of $g_A$:
  - Theoretical studies by using effective field theory (EFT) to estimate the effect of non-nucleonic degrees of freedom (two-body currents)
  - Experimental and theoretical studies of single beta decay and single charge exchange reactions involving the intermediate odd-odd nuclei
  - Double charge exchange reactions
Double charge exchange reactions

A lot of similarities with $0\nu\beta\beta$:

- Same initial and final states: Parent/daughter states of the $0\nu\beta\beta = \text{target/residual nuclei}$ in the DCE

- Structure of the transition operator: Fermi, Gamow-Teller and rank-2 tensor present in both cases

- Large momentum transfer: A linear momentum transfer as high as 100 MeV/c or so is characteristic of both processes

- In-medium processes: both processes happen in the same nuclear medium, thus we can learn about quenching phenomena

...
Double charge exchange reactions

However, a simple relation between DCE cross sections and $\beta\beta$-decay half-lives is by no means trivial:

- DCE and $0\nu\beta\beta$ processes are mediated by different interactions, so the comparison is not straightforward.
- The theory of DCE is much more complicated than the theory of $0\nu\beta\beta$-decay.
- DCE reaction, to its leading order, is a two-step process involving projectile and target internal structure as well as the full nucleus-nucleus interaction and the details of the theory have not yet been fully worked out.
- Both theoretical and experimental work is needed.
- ...

In any case the involved nuclear matrix elements are connected, and valuable information about the reliability of NMEs and quenching of $g_A$ may be learned from the study of DCE reactions.
Conclusions

• We have studied several scenarios and mechanisms suggested to describe double beta decay
  ▶ This includes two neutrino and neutrinoless decays, exchange of light and heavy neutrinos, majoron emitting $\beta\beta$, decays to ground states as well as to first excited $0^+$ states, and possible contributions of sterile neutrinos

• The next generation of experiments should be able to reach at least the inverted mass hierarchy. In case there are sterile neutrinos, the situation might be more complicated
  ▶ With or without sterile neutrinos, the reliability of nuclear matrix elements as well as the quenching of $g_A$ are becoming more and more important and the NUMEN project is expected to bring valuable information on these issues

Motivation for the work is clear: No matter what the mechanism of neutrinoless DBD is, its observation will answer the fundamental questions

• What is the absolute neutrino mass scale?
• What is the nature of neutrinos?
• How many neutrino species are there?
THANK YOU!

Image: The Royal Swedish Academy of Sciences

Collaborators: Francesco Iachello (Yale)
Jose Barea (Universidad de Concepción)

More information: nucleartheory.yale.edu