Coherent Multi–Nucleon Transfer Reactions

Grégory Potel Aguilar (MSU/LLNL)

Catania, December 1st, 2015

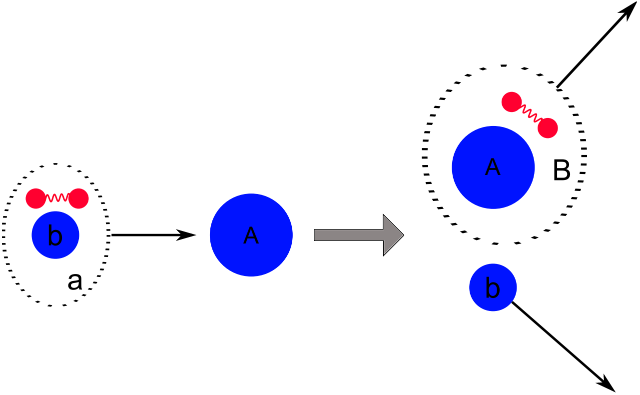
Outline

1. Two–Nucleon Transfer: Reaction Mechanism
   - Mechanism of Cooper pair transfer

2. Two–Nucleon Transfer: Structure
   - Two–nucleon transfer in stable nuclei
   - Two–nucleon transfer in exotic nuclei

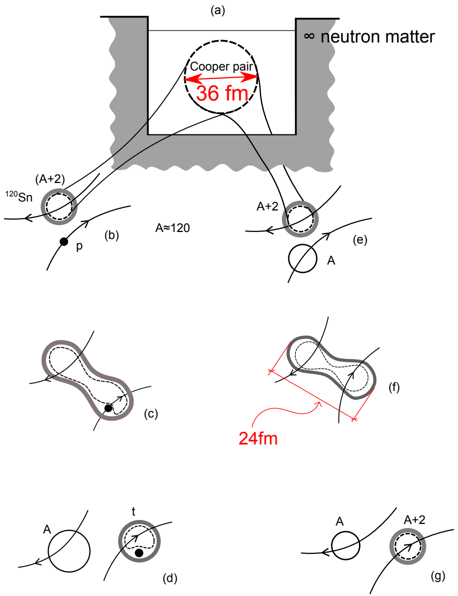
3. Results across the Nuclear Chart

4. Conclusions



- Reaction $A + a(\equiv b + 2) \longrightarrow a + B(\equiv A + 2)$.
- Measure of the pairing correlations between the transferred nucleons.
- Need to correctly account for the correlated wavefunction.

Delocalization of the pair transfer process



Two–nucleon transfer in a nutshell

$$|\phi_{el}\rangle = \Psi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2)\Psi_A(\xi_A);$$
$$|\phi_{1NT}\rangle = \Psi_f(\xi_b, \mathbf{r}_1)\Psi_F(\xi_A, \mathbf{r}_2);$$
$$|\phi_{2NT}\rangle = \Psi_b(\xi_b)\Psi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2); \quad V_1 \equiv V_{NA}(r_{1A}), V_2 \equiv V_{NA}(r_{2A})$$

$$\begin{aligned}
(H_1 - E_1)\chi_{el} &= 0, \\
(H_2 - E_2^i)\chi_{1NT}^i &= -\left\langle \phi_{1NT}^i | V_1 | \phi_{el} \right\rangle \chi_{el}, \\
&\Rightarrow \chi_{1NT}^i = (E_2^i - H_2)^{-1} \left\langle \phi_{1NT}^i | V_1 | \phi_{el} \right\rangle \chi_{el} \\
(H_3 - E_3)\chi_{2NT} &= -\left\langle \phi_{2NT} | V_1 | \phi_{el} \right\rangle \chi_{el} - \sum_i \left\langle \phi_{2NT} | V_2 | \phi_{1NT}^i \right\rangle \chi_{1NT}^i \\
&\quad - \sum_i \left\langle \phi_{2NT} | \phi_{1NT}^i \right\rangle (H_2 - E_2^i)\chi_{1NT}^i,
\end{aligned}$$

Two–nucleon transfer in a nutshell

$$|\phi_{el}\rangle = \Psi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2)\Psi_A(\xi_A);$$
$$|\phi_{1NT}\rangle = \Psi_f(\xi_b, \mathbf{r}_1)\Psi_F(\xi_A, \mathbf{r}_2);$$
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$$\begin{aligned}
(H_1 - E_1)\chi_{el} &= 0, \\
(H_2 - E_2^i)\chi_{1NT}^i &= -\langle \phi_{1NT}^i | V_1 | \phi_{el} \rangle \chi_{el}, \\
&\Rightarrow \chi_{1NT}^i = (E_2^i - H_2)^{-1} \langle \phi_{1NT}^i | V_1 | \phi_{el} \rangle \chi_{el} \\
(H_3 - E_3)\chi_{2NT} &= -\langle \phi_{2NT} | V_1 | \phi_{el} \rangle \chi_{el} \\
&- \sum_i \langle \phi_{2NT} | V_2 | \phi_{1NT}^i \rangle (E_2^i - H_2)^{-1} \langle \phi_{1NT}^i | V_1 | \phi_{el} \rangle \chi_{el} \\
&+ \sum_i \langle \phi_{2NT} | \phi_{1NT}^i \rangle \langle \phi_{1NT}^i | V_1 | \phi_{el} \rangle \chi_{el},
\end{aligned}$$

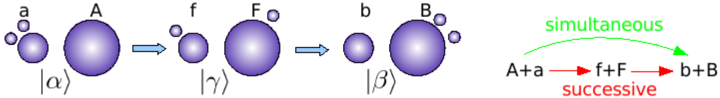
Two–nucleon transfer in a nutshell

$$|\phi_{el}\rangle = \Psi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2)\Psi_A(\xi_A);$$
$$|\phi_{1NT}\rangle = \Psi_f(\xi_b, \mathbf{r}_1)\Psi_F(\xi_A, \mathbf{r}_2);$$
$$|\phi_{2NT}\rangle = \Psi_b(\xi_b)\Psi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2); \quad V_1 \equiv V_{NA}(r_{1A}), V_2 \equiv V_{NA}(r_{2A})$$

$$(H_1 - E_1)\chi_{el} = 0,$$
$$(H_2 - E_2^i)\chi_{1NT}^i = -\langle \phi_{1NT}^i | V_1 | \phi_{el} \rangle \chi_{el},$$
$$\Rightarrow \chi_{1NT}^i = (E_2^i - H_2)^{-1} \langle \phi_{1NT}^i | V_1 | \phi_{el} \rangle \chi_{el}$$
$$(H_3 - E_3)\chi_{2NT} = -\sum_i \langle \phi_{2NT} | V_2 | \phi_{1NT}^i \rangle (E_2^i - H_2)^{-1} \langle \phi_{1NT}^i | V_1 | \phi_{el} \rangle \chi_{el}$$

V is a single particle operator (mean field potential)$\Rightarrow$ 2NT is a sequential process

The final cross section is the result of a coherent sum of many amplitudes



$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T^{(2)}_{succ}(j_i, j_f) - T^{(2)}_{NO}(j_i, j_f) \right)$$

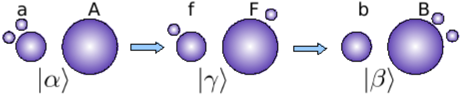
$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(4\pi\hbar^2)^2} \frac{k_f}{k_i} |T_{2NT}|^2$$

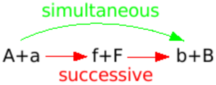
Simultaneous transfer

$$T^{(1)}(j_i, j_f) = 2 \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]^{0*}_{0} \chi^{(-)*}_{bB}(\mathbf{r}_{bB})$$

$$\times\ v(\mathbf{r}_{b1}) [\Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1) \Psi^{j_i}(\mathbf{r}_{b2}, \sigma_2)]^{\Lambda}_{\mu} \chi^{(+)}_{aA}(\mathbf{r}_{aA})$$

The final cross section is the result of a coherent sum of many amplitudes



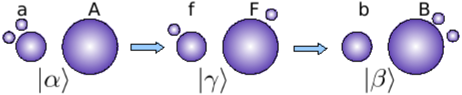


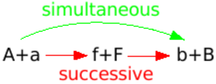
$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T^{(2)}_{succ}(j_i, j_f) - T^{(2)}_{NO}(j_i, j_f) \right)$$

Successive transfer

$$\begin{aligned} T^{(2)}_{succ}(j_i, j_f) = 2 \sum_{K,M} \sum_{\substack{\sigma_1\sigma_2 \\ \sigma'_1\sigma'_2}} \int & d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1)\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]^{0*}_{0} \\ & \times \chi^{(-)*}_{bB}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)\Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]^{K}_{M} \\ & \times \int d\mathbf{r}'_{fF} d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} G(\mathbf{r}_{fF}, \mathbf{r}'_{fF}) [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2)\Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^{K}_{M} \\ & \times \frac{2\mu_{fF}}{\hbar^2} v(\mathbf{r}'_{f2}) [\Psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2)\Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^{\Lambda}_{\mu} \chi^{(+)}_{aA}(\mathbf{r}'_{aA}) \end{aligned}$$

The final cross section is the result of a coherent sum of many amplitudes



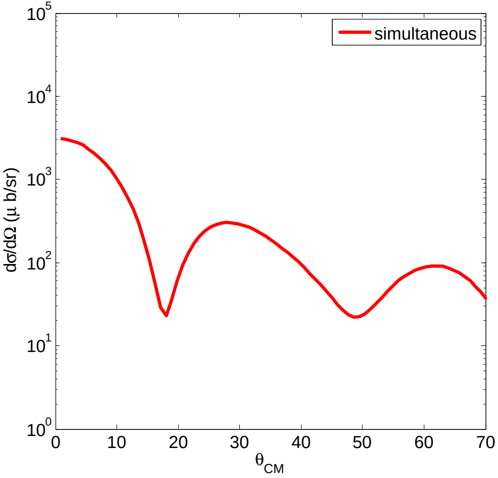


$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T^{(2)}_{succ}(j_i, j_f) - T^{(2)}_{NO}(j_i, j_f) \right)$$

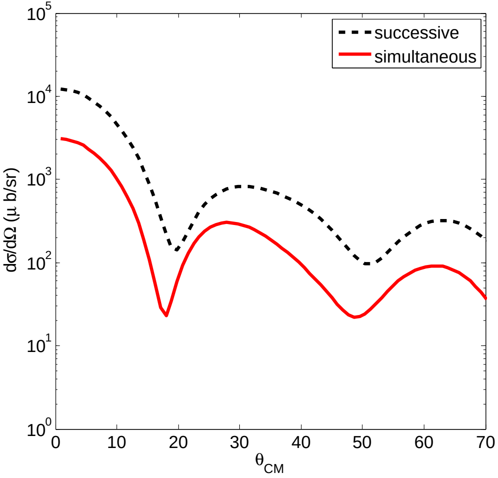
Non–orthogonality term

$$T^{(2)}_{NO}(j_i, j_f) = 2 \sum_{K,M} \sum_{\substack{\sigma_1\sigma_2 \\ \sigma'_1\sigma'_2}} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1)\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]^{0*}_0$$
$$\times \chi^{(-)*}_{bB}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)\Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]^K_M$$
$$\times \int d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2)\Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^K_M$$
$$\times [\Psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2)\Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^\Lambda_\mu \chi^{(+)}_{aA}(\mathbf{r}'_{aA})$$

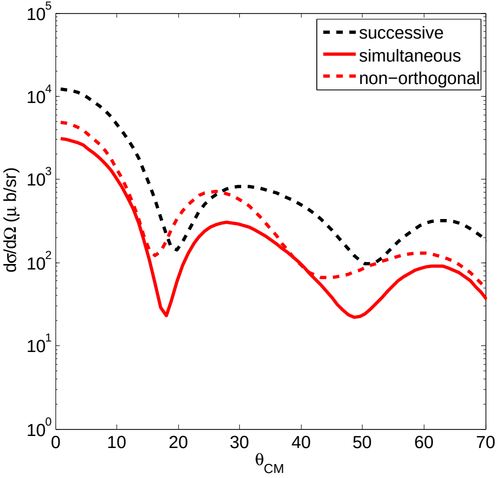
Contributions to the $^{112}Sn(p,t)^{110}Sn$ total cross section



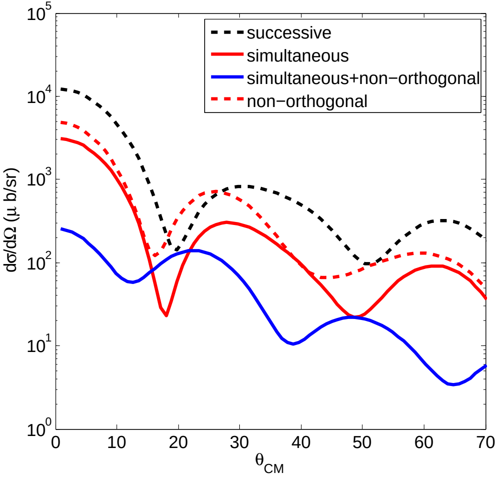
Contributions to the ^{112}Sn(p,t)^{110}Sn total cross section



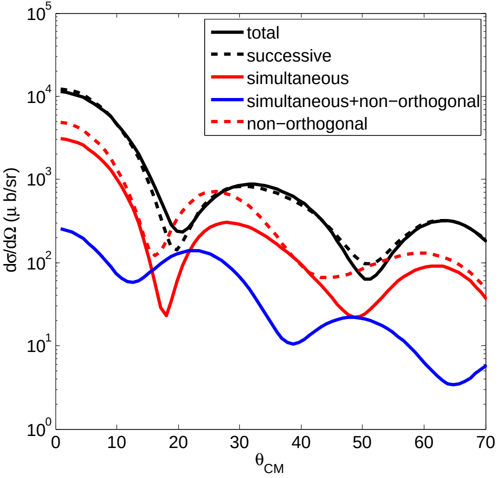
Contributions to the ^{112}Sn(p,t)^{110}Sn total cross section



Contributions to the $^{112}Sn(p,t)^{110}Sn$ total cross section



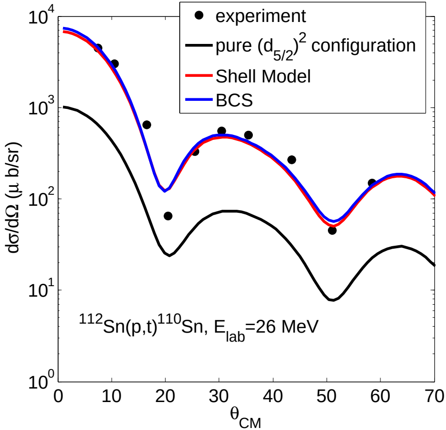
Contributions to the $^{112}Sn(p,t)^{110}Sn$ total cross section



Essentially a sequential process!

Outline

Probing pairing with 2–transfer: ^{112}Sn(p,t)^{110}Sn @ 26 MeV



enhancement factor with respect to the transfer of uncorrelated neutrons: $\varepsilon = 20.6$

G.P., Barranco, Marini, Idini, Vigezzi, Broglia, PRL **107** (2011) 092501

G.P., Idini, Barranco, Vigezzi, Broglia, PRC **87** (2013) 054321

experiment very well reproduced with mean field (BCS) wavefunctions

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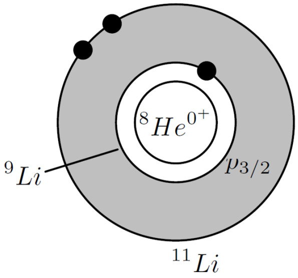
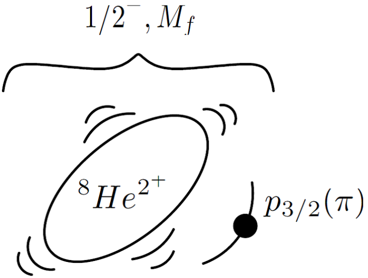
We will try to draw information about the halo structure of ^{11}Li from the reactions $^1H(^{11}Li,^9Li)^3H$ and $^1H(^{11}Li,^9Li^*(2.69\ MeV))^3H$ (I. Tanihata *et al.*, Phys. Rev. Lett. **100**, 192502 (2008))

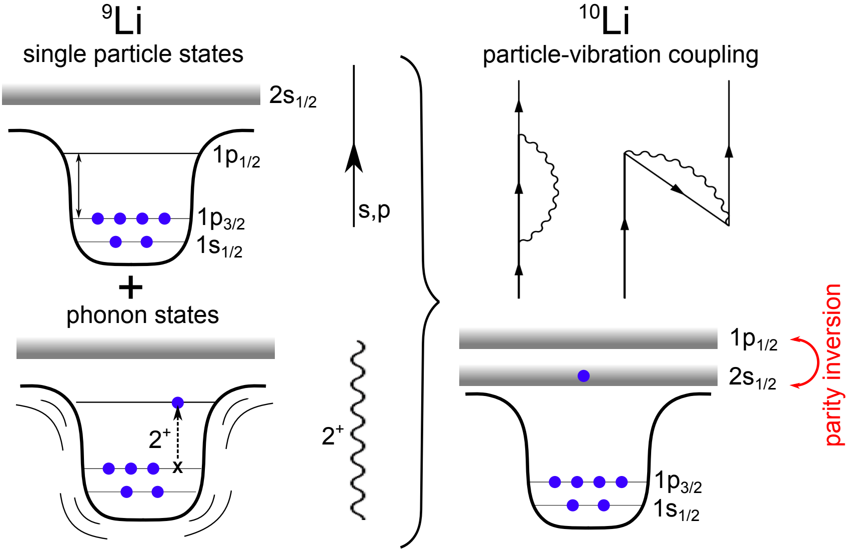


Schematic depiction of ^{11}Li

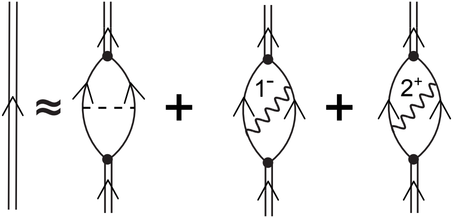


First excited state of ^9Li

Beyond mean field: particle–vibration coupling



^{11}Li=^{9}Li core+2–neutron halo (single Cooper pair). According to Barranco *et al.* (2001), the two neutrons correlate by means of the bare interaction (accounting for $\approx$ 20% of the ^{11}Li binding energy) and by exchanging 1^- and 2^+ phonons ($\approx$ 80% of the binding energy)

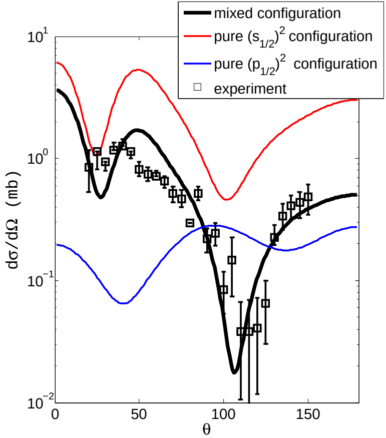


Within this model, the ^{11}Li wavefunction can be written as

$$|\tilde{0}\rangle = 0.45|s^2_{1/2}(0)\rangle + 0.55|p^2_{1/2}(0)\rangle + 0.04|d^2_{5/2}(0)\rangle + 0.70|(ps)_{1^-} \otimes 1^-; 0\rangle + 0.10|(sd)_{2^+} \otimes 2^+; 0\rangle.$$

highly renormalized single particle states coupled to excited states of the core

Transition to the ground state of ^{9}Li



differential cross section calculated with three ^{11}Li ground state model wavefunctions:

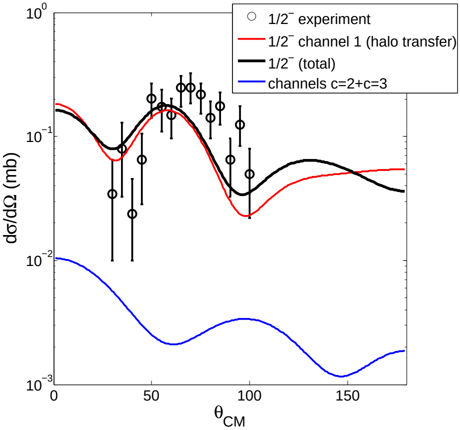
- pure $(s_{1/2})^2$ configuration
- pure $(p_{1/2})^2$ configuration
- $20\%(s_{1/2})^2 + 30\%(p_{1/2})^2$ configuration (Barranco *et al.* (2001)).

compared with experimental data.

^{1}H(^{11}Li,^{9}Li)^{3}H at 33 MeV. Data from Tanihata *et.al.* (2008).

G.P., Barranco, Vigezzi, Broglia, PRL **105** (2010) 172502

Transition to the first $1/2^-$(2.69 MeV) excited state of ^{9}Li



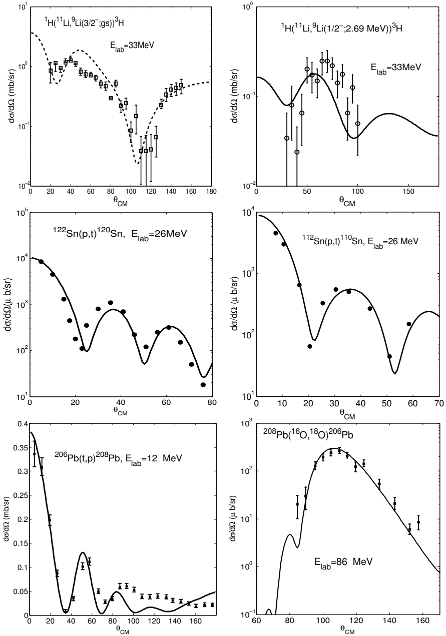
differential cross section calculated with the Barranco *et. al.* (2001) ^{11}Li ground state wavefunction, compared with experimental data. According to this model, the ^{9}Li excited state is found after the transfer reaction because it is already present in the ^{11}Li ground state.

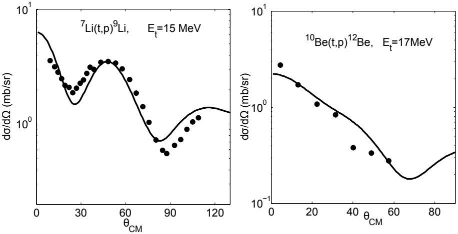
^{1}H(^{11}Li,^{9}Li*(2.69 MeV))^{3}H at 33 MeV. Data from Tanihata *et.al.* (2008).

G.P., Barranco, Vigezzi, Broglia, PRL **105** (2010) 172502

Outline

Cooper pair transfer across the nuclear chart





good results obtained for halo nuclei,
population of excited states,
superfluid nuclei,
normal nuclei (pairing vibrations),
heavy ion reactions...

G.P., Idini, Barranco, Vigezzi, Broglia, Rep. Prog. Phys. **76** (2013) 106301

Outline

Conclusions

- A quantitative account of two–neutron transfer cross sections can be achieved for a wide variety of nuclear species.
- A correct simultaneous description of both reaction and structure aspects is essential in order to obtain the absolute value of the transfer cross sections.
- The reaction mechanism of Cooper pair transfer is described in terms of the sequential transfer of two nucleons.
- Pairing correlations are preserved during the transfer process despite the localization of Cooper pair partners close in different nuclear species.
- An independent Cooper pair model gives a very good account for Cooper pair transfer in superfluid nuclei.

Structure:

$$\Phi_i(\mathbf{r}_1,\sigma_1,\mathbf{r}_2,\sigma_2) = \sum_{j_i} B_{j_i}\left[\psi^{j_i}(\mathbf{r}_1,\sigma_1)\psi^{j_i}(\mathbf{r}_2,\sigma_2)\right]^{\Lambda}_{\mu}$$

$$\Phi_f(\mathbf{r}_1,\sigma_1,\mathbf{r}_2,\sigma_2) = \sum_{j_f} B_{j_f}\left[\psi^{j_f}(\mathbf{r}_1,\sigma_1)\psi^{j_f}(\mathbf{r}_2,\sigma_2)\right]^{0}_{0}$$

Reaction:

$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i}\left(T^{(1)}(j_i,j_f) + T^{(2)}_{succ}(j_i,j_f) - T^{(2)}_{NO}(j_i,j_f)\right)$$

$$\frac{d\sigma}{d\Omega} = \frac{\mu_i\mu_f}{(4\pi\hbar^2)^2}\frac{k_f}{k_i}|T_{2NT}|^2$$

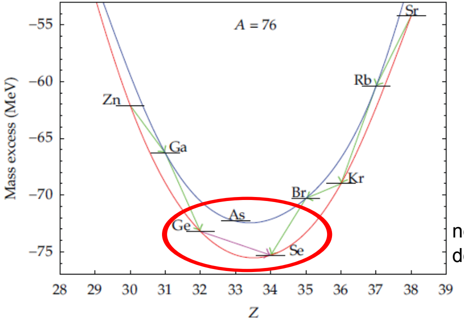
with:

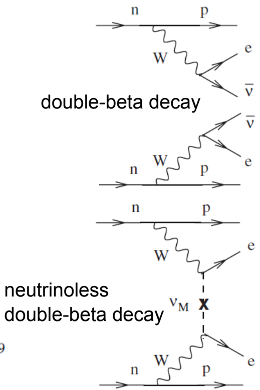
$$T^{(1)}(j_i,j_f) = 2\sum_{\sigma_1\sigma_2}\int d\mathbf{r}_{fF}\, d\mathbf{r}_{b1}\, d\mathbf{r}_{A2}[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)]^{0*}_{0}\chi^{(-)*}_{bB}(\mathbf{r}_{bB})$$
$$\times\, v(\mathbf{r}_{b1})[\psi^{j_i}(\mathbf{r}_{b1},\sigma_1)\psi^{j_i}(\mathbf{r}_{b2},\sigma_2)]^{\Lambda}_{\mu}\chi^{(+)}_{aA}(\mathbf{r}_{aA})$$

etc...

Neutrinoless double β–decay ($\beta\beta 0\nu$)

- neutrino mass?
- is the neutrino a Majorana particle?





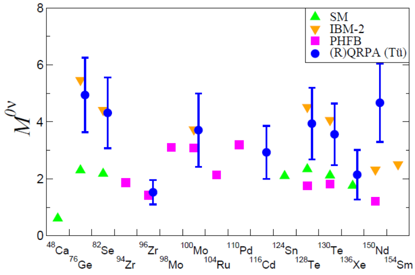
$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}(Q_{\beta\beta}, Z)|M_{0\nu}|^2 \langle m_{\beta\beta}\rangle^2$$

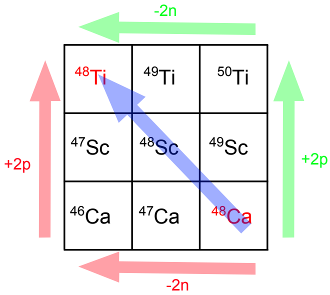
Nuclear matrix elements $M_{0\nu}$ needed from low–energy nuclear physics.

2015 LRP

We recommend the timely development and deployment of a US–led ton–scale neutrinoless double beta decay experiment.

Experimental and theoretical challenges (Nuclear matrix elements)





Experimental efforts:

- 2–proton and 2–neutron transfer reactions.
- Single and double charge exchange reactions.

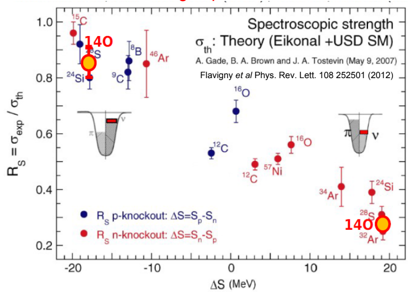
Theory efforts:

- Relationship between $M_{0\nu}$ and Fermi and Gamow–Teller matrix elements.
- Theory for charge exchange reactions.

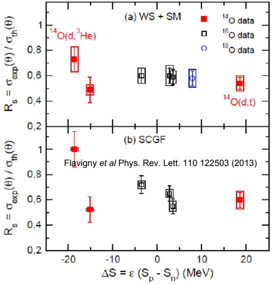
Outline

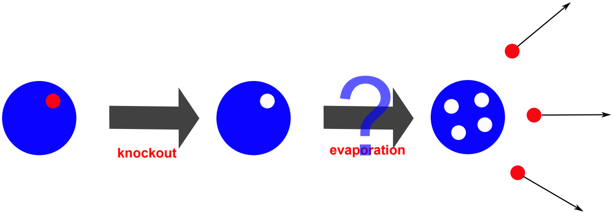
Assymetry dependence of spectroscopic factors

knock-out experiments:
strong dependence



transfer experiments:
weak dependence





Theory for eikonal inclusive breakup needed

Outline

nature COMMUNICATIONS

ARTICLE

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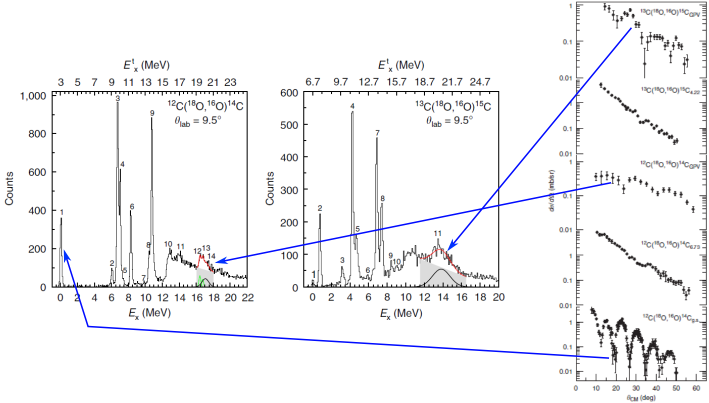
OPEN

Signatures of the Giant Pairing Vibration in the ^{14}C and ^{15}C atomic nuclei

F. Cappuzzello[1,2], D. Carbone[2], M. Cavallaro[2], M. Bondì[1,2], C. Agodi[2], F. Azaiez[3], A. Bonaccorso[4], A. Cunsolo[2], L. Fortunato[5,6], A. Foti[1,7], S. Franchoo[3], E. Khan[3], R. Linares[8], J. Lubian[8], J.A. Scarpaci[9] & A. Vitturi[5,6]

Experimental and theoretical challenges (GPV)



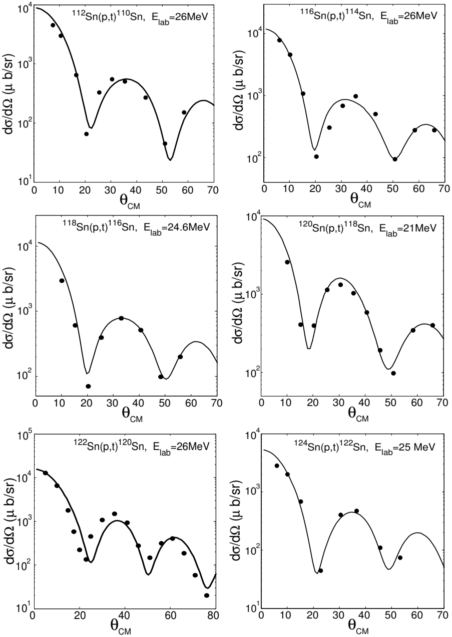
Experimental efforts:

- 2–nucleon transfer reactions

Theory efforts:

- Nuclear structure (pairing correlations) in the continuum
- 2–nucleon transfer in the continuum.

2–transfer in well bound nuclei ^{A}Sn(p,t)$^{A-2}$Sn



Comparison with the experimental data available so far for superfluid tin isotopes

Potel *et al.*, PRL **107**, 092501 (2011)

Cancellation of simultaneous and non-orthogonal contributions

very schematically, the first order (*simultaneous*) contribution is

$$T^{(1)} = \langle \beta | V | \alpha \rangle,$$

while the second order contribution can be separated in a *successive* and a *non-orthogonality* term

$$T^{(2)} = T^{(2)}_{succ} + T^{(2)}_{NO}$$
$$= \sum_{\gamma} \langle \beta | V | \gamma \rangle G \langle \gamma | V | \alpha \rangle - \sum_{\gamma} \langle \beta | \gamma \rangle \langle \gamma | V | \alpha \rangle.$$

If we sum over a complete basis of intermediate states γ, we can apply the closure condition and $T^{(2)}_{NO}$ exactly cancels $T^{(1)}$

the transition potential being single particle, two-nucleon transfer is a second order process.

Structure:

$$\Phi_i(\mathbf{r}_1,\sigma_1,\mathbf{r}_2,\sigma_2) = \sum_{j_i} B_{j_i}\left[\psi^{j_i}(\mathbf{r}_1,\sigma_1)\psi^{j_i}(\mathbf{r}_2,\sigma_2)\right]^{\Lambda}_{\mu}$$

$$\Phi_f(\mathbf{r}_1,\sigma_1,\mathbf{r}_2,\sigma_2) = \sum_{j_f} B_{j_f}\left[\psi^{j_f}(\mathbf{r}_1,\sigma_1)\psi^{j_f}(\mathbf{r}_2,\sigma_2)\right]^{0}_{0}$$

Reaction:

$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i}\left(T^{(1)}(j_i,j_f) + T^{(2)}_{succ}(j_i,j_f) - T^{(2)}_{NO}(j_i,j_f)\right)$$

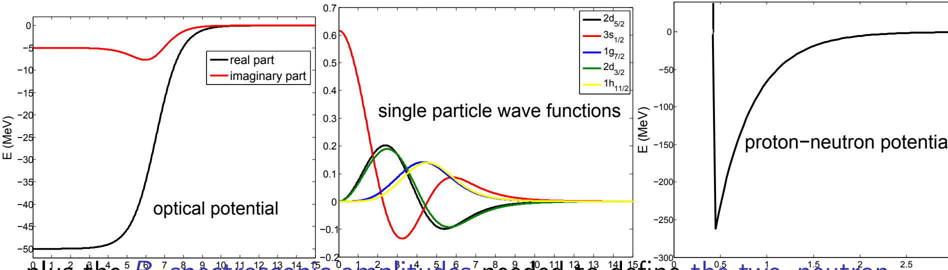
$$\frac{d\sigma}{d\Omega} = \frac{\mu_i\mu_f}{(4\pi\hbar^2)^2}\frac{k_f}{k_i}|T_{2NT}|^2$$

with:

$$T^{(1)}(j_i,j_f) = 2\sum_{\sigma_1\sigma_2}\int d\mathbf{r}_{fF}\, d\mathbf{r}_{b1}\, d\mathbf{r}_{A2}[\psi^{j_f}(\mathbf{r}_{A1},\sigma_1)\psi^{j_f}(\mathbf{r}_{A2},\sigma_2)]^{0*}_{0}\chi^{(-)*}_{bB}(\mathbf{r}_{bB})$$
$$\times\, v(\mathbf{r}_{b1})[\psi^{j_i}(\mathbf{r}_{b1},\sigma_1)\psi^{j_i}(\mathbf{r}_{b2},\sigma_2)]^{\Lambda}_{\mu}\chi^{(+)}_{aA}(\mathbf{r}_{aA})$$

etc...

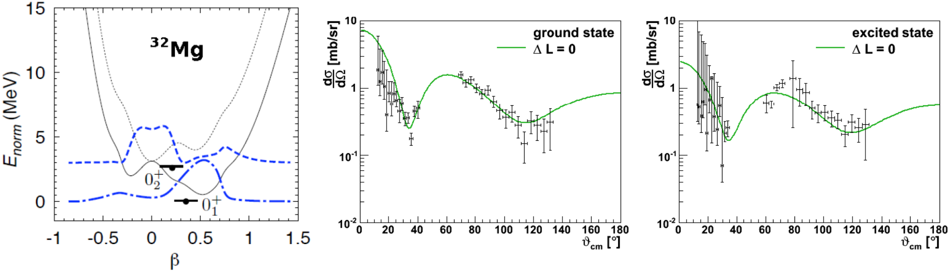
Structure input for, e.g., the ^{112}Sn(p,t)^{110}Sn reaction:



plus the B_j spectroscopic amplitudes needed to define the two−neutron wavefunction:

$$\Phi(\mathbf{r}_1, \sigma_1, \mathbf{r}_2, \sigma_2) = \sum_j B_j \left[\psi^j(\mathbf{r}_1, \sigma_1)\psi^j(\mathbf{r}_2, \sigma_2)\right]^0_0$$

Shape coexistence and 2–neutron transfer

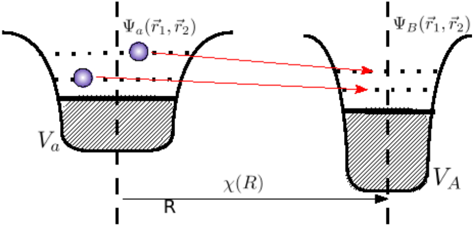


- Recent $t(^{32}\mathrm{Mg},p)^{30}\mathrm{Mg}$ @ 1.8 MeV.A at ISOLDE (Wimmer *et.al.*) reaction.
- Shape coexistence (low–lying 0^+ excited state).
- Ground state and first excited 0^+ populated with 2–neutron transfer

Elements of the calculation

$\Psi_a(\vec{r}_1, \vec{r}_2)$, $\Psi_B(\vec{r}_1, \vec{r}_2)$: internal wave functions of the transferred nucleons in each nucleus

$\chi(R)$: distorted wave describing the relative motion in the optical potential $U(R) = V(R) + iW(R)$ $\left(\frac{P_R^2}{2\mu} + U(R)\right)\chi(R) = E_{CM}\chi(R)$



V_A, V_a: mean field potentials of the two nuclei

$\Psi_a(\vec{r}_1, \vec{r}_2)$, $\Psi_B(\vec{r}_1, \vec{r}_2)$: internal wave functions of the transferred nucleons in each nucleus

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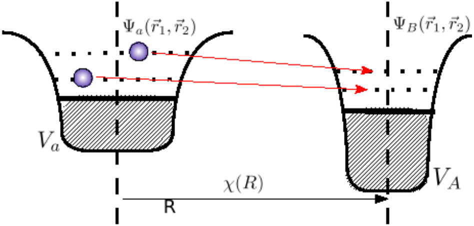

V_A, V_a: mean field potentials of the two nuclei

V_A (V_a) is the interaction potential that transfers the nucleons from one nucleus to the other in the *prior* (*post*) representation

Elements of the calculation

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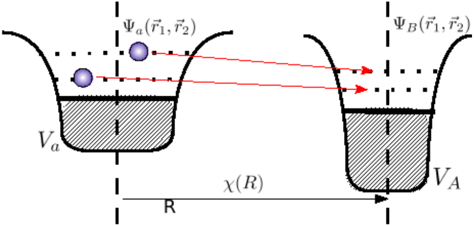


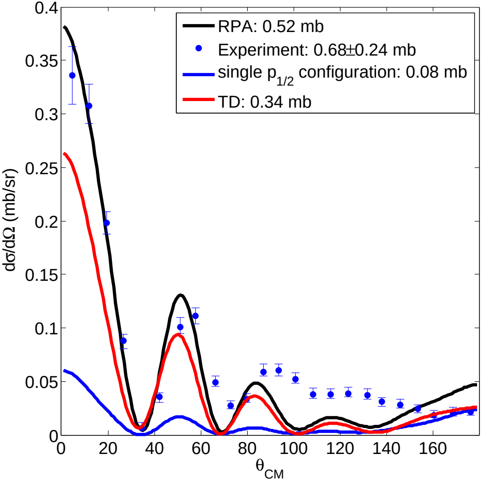
V_A, V_a: mean field potentials of the two nuclei

V_A (V_a) is the interaction potential that transfers the nucleons from one nucleus to the other in the *prior* (*post*) representation

it is a single particle potential!!

^{206}Pb$(t,p)^{208}$Pb (gs): pairing in normal nuclei

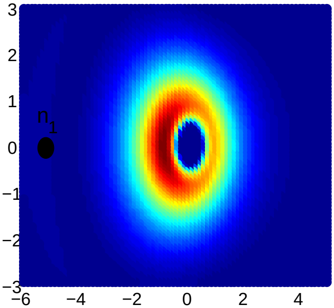
^{206}Pb$(t,p)^{208}$Pb at 12 MeV. Data from Bjerregaard *et.al.* (1966)

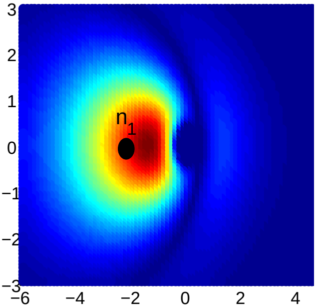


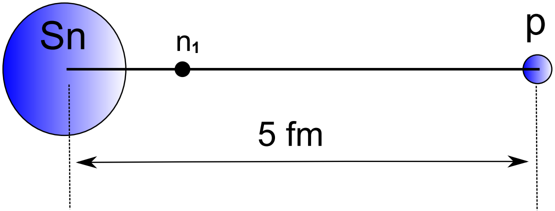
	B_{nlj}	
state *nlj*	*pp*RPA	(TDA)
$1h_{9/2}$	0.15	(0.14)
$2f_{7/2}$	0.21	(0.26)
$1i_{13/2}$	0.29	(0.28)
$3p_{3/2}$	0.23	(0.22)
$2f_{5/2}$	0.32	(0.31)
$3p_{1/2}$	0.89	(0.85)
$2g_{9/2}$	0.18	
$1i_{11/2}$	0.15	
$1j_{15/2}$	0.13	
$3d_{5/2}$	0.06	(–)
$4s_{1/2}$	0.06	
$2g_{7/2}$	0.10	
$3d_{3/2}$	0.05	

Non–local, correlated form factor

$$F(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{Ap}) = \phi_f(\mathbf{r}_{p1}, \mathbf{r}_{p2}) V_{pn}(\mathbf{r}_{p1}) V_{pn}(\mathbf{r}_{p2}) \phi_i(\mathbf{r}_{A1}, \mathbf{r}_{A2})$$

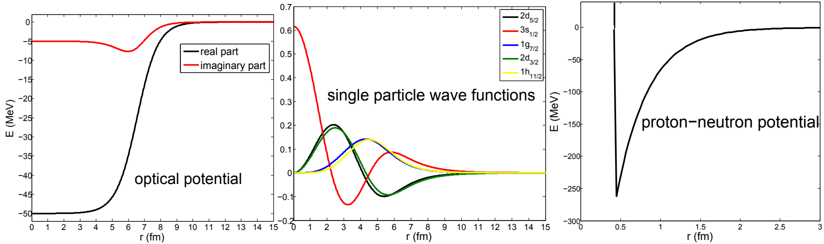






Ingredients of the calculation

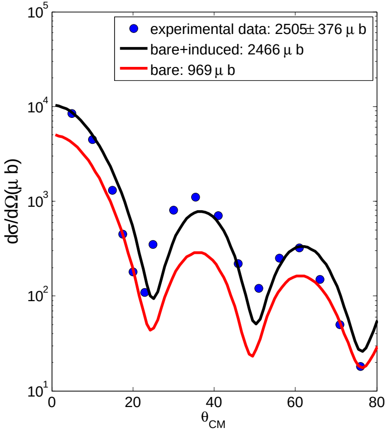
Structure input for, e.g., the ^{112}Sn(p,t)^{110}Sn reaction:



plus the B_j spectroscopic amplitudes needed to define the two–neutron wavefunction:

$$\Phi(\mathbf{r}_1, \sigma_1, \mathbf{r}_2, \sigma_2) = \sum_j B_j \left[\psi^j(\mathbf{r}_1, \sigma_1)\psi^j(\mathbf{r}_2, \sigma_2)\right]^0_0$$

^{122}Sn$(p,t)^{120}$Sn (gs): role of induced interaction



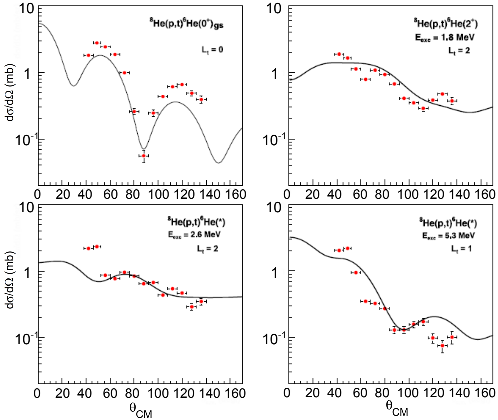
Differential cross section worked out making use of two different structure calculations:

- Skyrme in $p-h$ channel (mean field)+collective vibrations+bare v_{14} Argonne interaction and particle–vibration coupling (induced interaction) in $p-p$ channel (black line),
- Skyrme in $p-h$ channel (mean field)+bare v_{14} Argonne in $p-p$ channel (red line),

compared with experimental data.

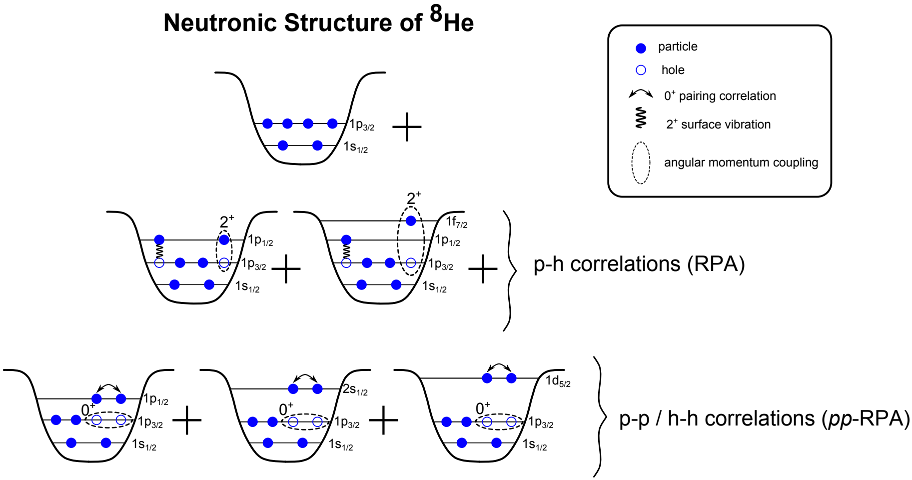
^{122}Sn$(p,t)^{120}$Sn at 26 MeV. Data from Guazzoni *et.al.* (1999).

Two–neutron transfer with ^{8}He

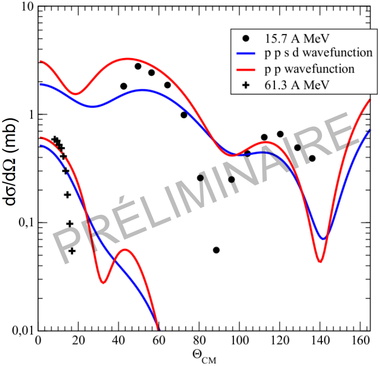


- X. Mougeot *et al.* PLB **718**, 441 (2012) ^{8}He(p,t)^{6}He(gs),^{8}He(2$^+$) with SPIRAL and MUST2;
- Coupled Reaction Channels (CRC) analysis by N .Keeley.

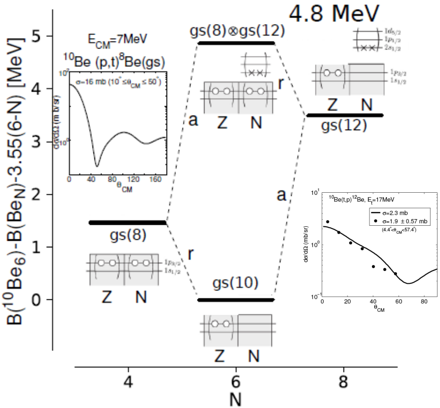
schematic structure of ^{8}He in NFT



^{8}He(p, t) reaction in 2–step DWBA



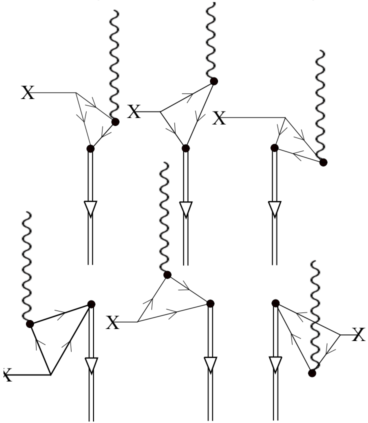
- Sensitive to ^{8}He structure.
- Nuclear Fied Theory calculations for ^{8}He(g.s.),^{6}He(g.s,2^{+}) (^{6}He as a pair removal mode of ^{8}He?).
- Consistent description of elastic and one–neutron transfer channels and the overlap ^{8}He(g.s.)/^{6}He(2^{+}) is essential.

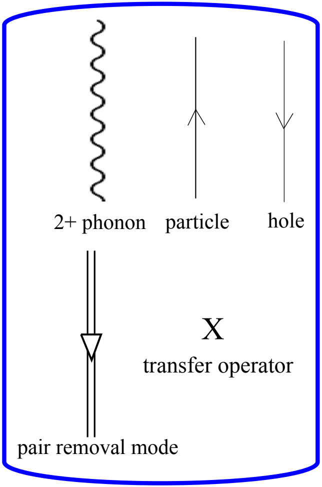


^{8}Be(p,t)^{10}Be and ^{10}Be(t,p)^{12}Be reactions can probe the pairing vibrations around ^{10}Be ($N = 6$ shell closure).
^{10}Be(t,p)^{12}Be data by Fortune *et al*, PRC **50** (1994) 1355.

Population of excited 2^+ state with (t,p) reaction

Diagrams contributing





Prediction of GPV

Volume 69B, number 2 PHYSICS LETTERS 1 August 1977

HIGH-LYING PAIRING RESONANCES★

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State University of New York, Department of Physics, Stony Brook, New York 11794, USA

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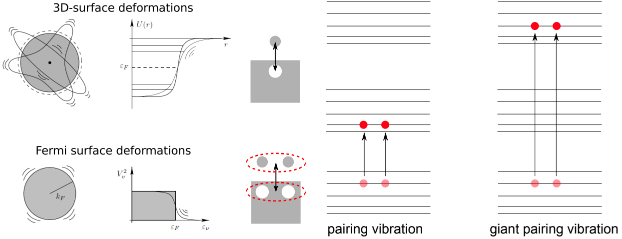
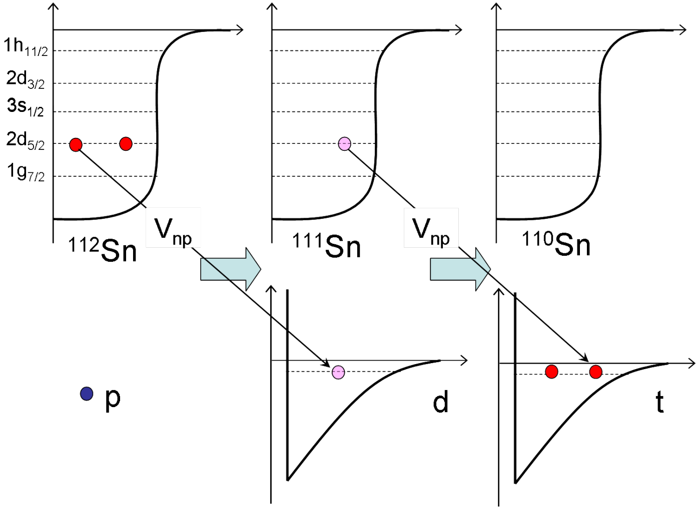
D.R. BES[2]
NORDITA, DK-2100 Copenhagen Ø, Denmark

Received 1 April 1977

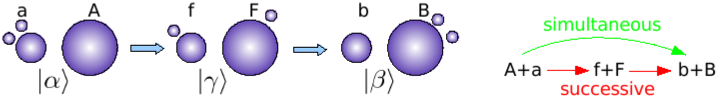
Pairing vibrations based on the excitation of pairs of particles and holes across major shells are predicted at an excitation energy of about $70/A^{1/3}$ MeV and carrying a cross section which is 20%–100% the ground state cross section.



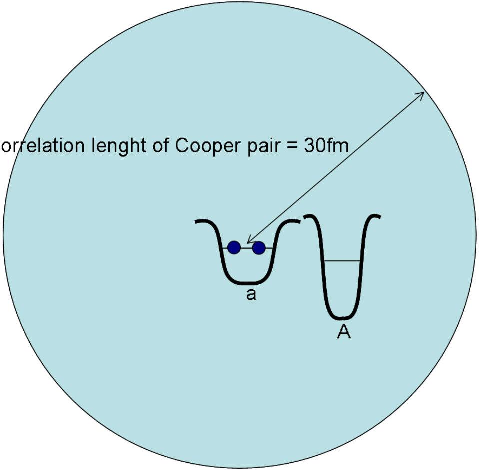
Example: $^{112}Sn(p,t)^{110}Sn$ in 2–step DWBA



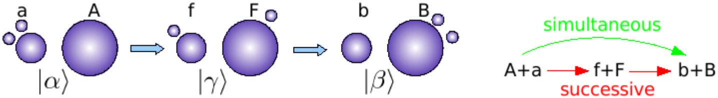
simultaneous and successive contributions

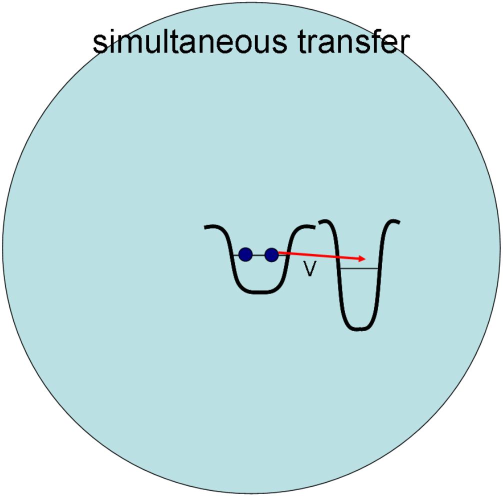


$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2)\times$$
$$\phi_A(\xi_A)\chi_{aA}(\mathbf{r}_{aA})$$
$$|\beta\rangle = \phi_b(\xi_b)\phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2)\times$$
$$\chi_{bB}(\mathbf{r}_{bB})$$



simultaneous and successive contributions





$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2)\times$$
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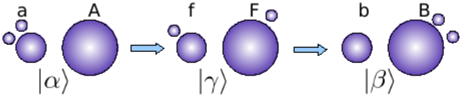

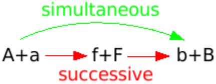

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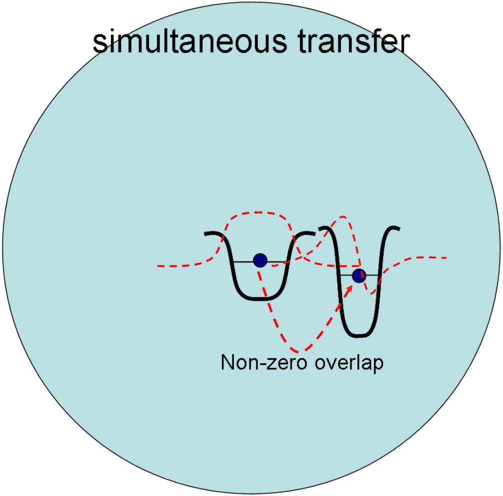
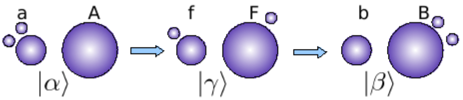
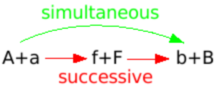
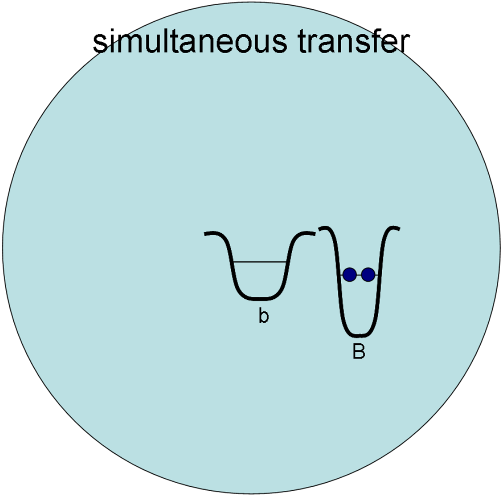


simultaneous and successive contributions



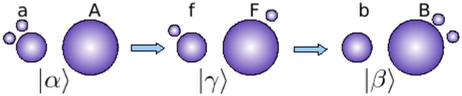


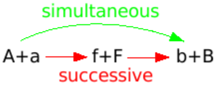


$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2)\times \phi_A(\xi_A)\chi_{aA}(\mathbf{r}_{aA})$$

$$|\beta\rangle = \phi_b(\xi_b)\phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2)\times \chi_{bB}(\mathbf{r}_{bB})$$

simultaneous and successive contributions





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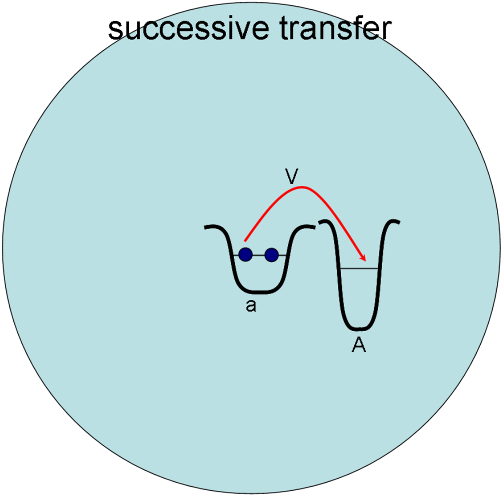
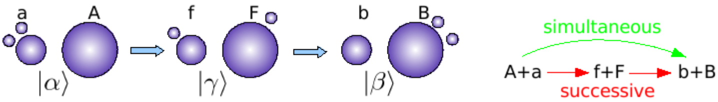


simultaneous and successive contributions



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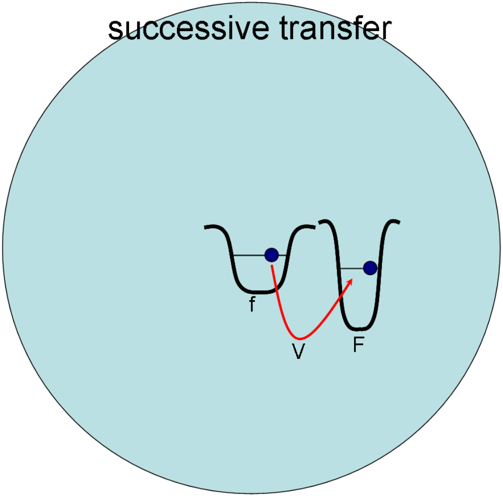

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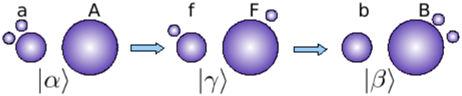

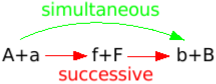

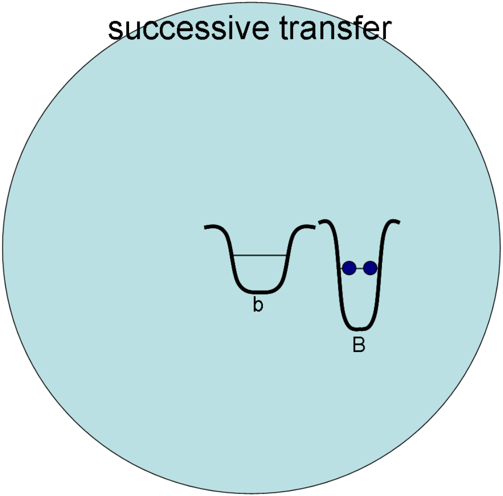

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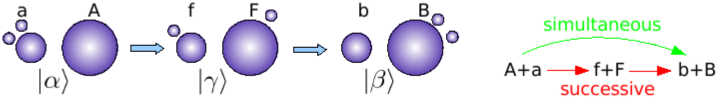
simultaneous and successive contributions



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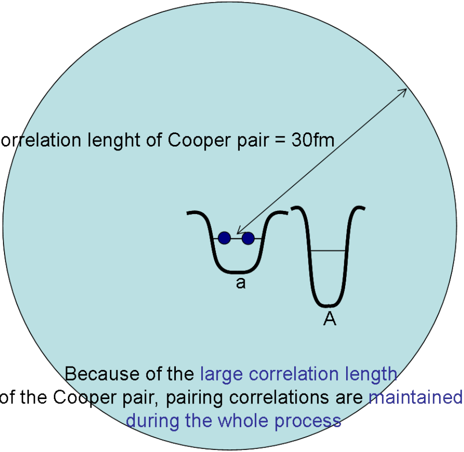
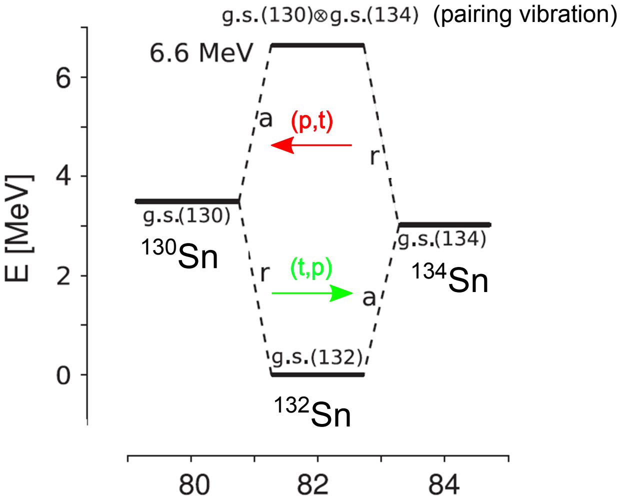


Because of the large correlation length of the Cooper pair, pairing correlations are maintained during the whole process

Pairing vibrations in exotic nuclei: ^{132}Sn



$^{130}Sn(t,p)^{132}Sn(2^+)$

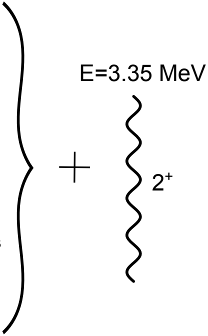
^{132}Sn

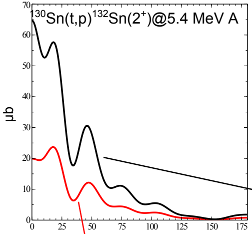
	Ω_j	ε_j	X_{rem}	Y_{add}	$X_{rem}Y_{add}$	$\sqrt{\Omega_j}\Lambda_{rem}$	$\sqrt{\Omega_j}\Lambda_{add}$
$g_{7/2}$	4	−9.78	0.229	0.080	0.018	3.20	2.16
$d_{5/2}$	3	−9.01	0.255	0.078	0.020	2.78	1.88
$s_{1/2}$	1	−7.68	0.286	0.058	0.017	1.60	1.08
$h_{11/2}$	6	−7.52	0.791	0.147	0.116	3.92	2.64
$d_{3/2}$	2	−7.35	0.529	0.088	0.047	2.26	1.52
			Y_{rem}	X_{add}	$X_{add}Y_{rem}$	$\sqrt{\Omega_j}\Lambda_{add}$	$\sqrt{\Omega_j}\Lambda_{rem}$
$f_{7/2}$	4	−2.44	0.209	0.922	0.192	3.20	2.16
$p_{3/2}$	2	−1.59	0.121	0.265	0.032	2.26	1.52
$h_{9/2}$	5	−0.88	0.166	0.281	0.046	3.58	2.42
$p_{1/2}$	1	−0.78	0.073	0.120	0.009	1.60	1.08
$f_{5/2}$	3	−0.44	0.119	0.180	0.021	2.78	1.88

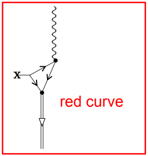
hole states

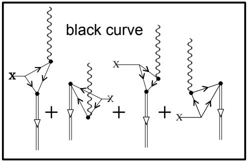
E_F

particle states

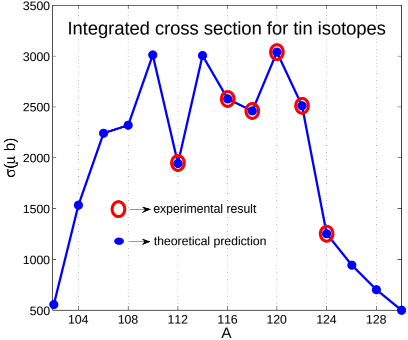


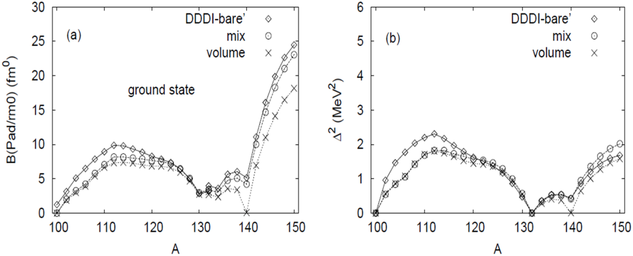






^{A}Sn(p,t)$^{A-2}$Sn, superfluid isotopic chain





Shimoyama and Matsuo, nucl-th/1106.1715