Nuclear Structure Aspects of Double Charge Exchange

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Isotensor giant resonances

- A “model” giant resonance (state) built on a state $|n\rangle$ is:
  \[ |Q_\alpha; n\rangle = Q_\alpha |n\rangle / \sqrt{\langle n | Q_\alpha^+ Q_\alpha | n\rangle} \]
- $Q_\alpha$ is a one-body operator, $Q_\alpha = \sum_i q_\alpha(i)$
- For $|0\rangle$, the g.s. we have the “usual” giant resonances.
- When $|n\rangle$ is itself a giant resonance state, then the giant state built on it will be a **double giant state**.
Double Giant Resonances

- The operator $Q_\alpha$ might depend on spin and isospin. For example the electric dipole is an isovector and the corresponding operator is:
  
  $Q_\alpha \equiv D = \sum_i r_i Y_1(\theta_i) \tau_\mu(i), \text{with } \mu = 0, \pm 1$

- In addition to the “common” $\mu = 0$ dipole one has also the charge-exchange analogs $\mu = \pm 1$

- The action of the operator with $\mu = -1$ (or $\mu = 1$) twice will lead to states with $\Delta T_z = \pm 2$

- These states can be reached in double charge-exchange (DCX) processes.

- One of the best examples, are the pion DCX reactions $[(\pi^+, \pi^-), (\pi^-, \pi^+)]$, studied extensively in the past.

- Because we deal with isotensor transitions, the selectivity of the DCX is large and enhances the possibilities to observe double giant resonances (states).
The isovector dipole strength for all three components $\gamma_1 = \pm 1.0$ is calculated in the RPA.
Examples of simple isotensor states

- Double isobaric analog state (DIAS).
- The IAS is defined as: $| A_1 \rangle = T_- |0\rangle / (N - Z)^{1/2}$
- The DIAS is: $| A_2 \rangle = T_-^2 |0\rangle / [(N - Z)(N - Z - 1)]^{1/2}$
- Dipole built on an analog: $| D_{-1}; A_1 \rangle = \sum_i r_i Y_1 (\theta_i) \tau_{-1} (i) |A_1\rangle / \tilde{N}$
- Yet another example, the double dipole (for the $\Delta T_z = -2$)
  - $| D_{-1}; D_{-1} \rangle = \sum_i r_i Y_1 (\theta_i) \tau_{-1} (i) |D_{-1}\rangle / \tilde{N}$
Pion DCX experiments
Spin degrees of freedom

• $G_\pm = \sum_i \sigma(i) \tau_\pm(i)$

• The giant Gamow-Teller (GT) state: $|GT\rangle = G_+ |0\rangle/N$

• The double Gamow-Teller (DGT) state: $|DGT; J\rangle = G_\pm^2(J) |0\rangle/N(J)$

• $J$ is the total spin of the DGT

• Simple selection rules for the total spins of isotensor transitions.

• For two identical phonons the wave function is symmetric in space-spin-isospin, therefore for $\Delta T = 0, 2$ \( J = 0^+, 2^+ \ldots \)

• For $\Delta T = 1$, \( J = 1^+, 3^+ \). For the DGT \( J = 0^+, 2^+ \)
Some properties

• Excitation energies: $E_{Q_1Q_2} \approx E_{Q_1} + E_{Q_2}$

• Widths (spreading widths) $\Gamma_{n;\lambda} = n\Gamma_{1;\lambda}$. In some theories $n$ is replaced by $\sqrt{n}$

• Isospin and intensities: for an isotensor excitation $\Delta T_Z = -2$

  and $(N - Z) \geq 2$ there are five isospin values $T' = T + 2$,

  $T + 1$, $T$, $T - 1$, $T - 2$.

• For an isotensor od rank $k$ $F^{(k)}_{\mu}$ the intensities for the various isospin components $T'$ are given by the corresponding Clebsh-Gordan (CG) coefficients and reduced matrix elements.

  $S_{T'} = |\langle T, T, k, \mu | T', T + \mu \rangle|^2 \langle T' \| F^{(k)} \| T \rangle^2$

• The CG coefficient for $T \gg k$ (in our case $T \gg 2$) are dominated by the

• CG of the aligned isospin $T' = T + \mu$. For $\mu = -k$, $(CG)^2 = \frac{2(T-k)+1}{2T+1}$. 
The DCX process.

• The DCX process has held out the hope that it would be a means of probing two-body correlations in nuclei. In the initial studies it was found that uncorrelated nuclear wave functions produced qualitative agreement with experiment at higher energies (pion energies around 300 MeV.) This was the situation when the double giant resonances were studied. However at low pion energies (around 50 MeV) the disagreement with experiment was very large, (sometimes a factor of 50) when uncorrelated wave functions were used. It was necessary to include wave functions with correlated nucleons.

• The DCX process, as determined in pion charge-exchange reactions involves two basic routes.

• 1. For uncorrelated $n$ paricles in the transition to a double analog state the route from the initial state to the final state goes via the excitation of the single analog in the intermediate stage, and from there in a charge-exchange to the double analog. This is termed as the analog route or more generally as the “sequential” process. The cross section is proportional to the number of pairs one can form from the $n$ nucleons, $n(n-1)/2$.

• 2. The second process involves correlated nucleons and the process proceeds through intermediate states that are not the analog. This we term the “non-analog route”.
Analog vs non-analog routes

analog route

non-analog route

Parent state

DIAR

IAR
Correlations and non-analog transitions
The DCX process

In order to obtain the desired relation, we consider the reaction to be caused by a two-body operator between two identical (in the same isospin multiplet) $0^+$ states. Thus we need to calculate the matrix element for the transition:

$$M = (0^+ | \sum_{i,j} \theta_{ij} (r_i, r_j, k, k') | 0^+) \quad (1)$$

• The operator depends also on the spins of the two particles
The DCX for the even Ca isotopes (the AGP formula) (Single- $j$, n-even). (NA, W.R. Gibbs, E. Piasetzky, PRL, 59, 1076 (1987)

\[
\sigma_{\text{DCX}}(\theta) = \frac{n(n-1)}{2} \left| A + \frac{(2j+3-2n)}{(n-1)(2j-1)} B \right|^2,
\]

\[ A = a + \frac{2}{(2j+1)} \beta, \quad B = \frac{1}{(2j+1)(2j-1)} \beta, \]

\[
\sigma_{\text{DCX}}(\theta) = \frac{1}{2} n(n-1) \left| a + \beta/(n-1) \right|^2.
\]

We see that the counting rule given by the $n(n-1)/2$ independent pairs is completely destroyed, indicating that the correlations play a very important role in the DCX process at these low pion energies. Fitting the data, one finds that the ratio $|B|/|A| \simeq 3$. For higher energies $E_{\pi} \geq 150$ MeV this ratio is found to be $\simeq 1$.
The DCX process (cont’d)

• In general, the DCX cross section will contain the sequential term and the correlation term. Both amplitudes could be complex.

• The above formula is extended to the generalized seniority scheme when several orbits are involved. Then:

$$\sigma_{DCX} = \frac{n(n-1)}{2} A + \frac{(\Omega+1-n)}{(\Omega-1)(n-1)} B \right|^2$$; with $$\Omega = \sum_i (j_i + \frac{1}{2})$$

• This formalism was applied to the Te 128 and 130 nuclei, relevant to the double beta-decay, (H.C. Wu, et.al., Phys. Rev. C54, 1208 (1996)).

• See also N. Auerbach, et. al., Phys. Rev. C38, 1277(1988),
Spin

• The two-body DCX operator is also a function of the spin of the two nucleons $\sigma_1$ and $\sigma_2$. In fact one of the important non-analog routes leading to the DIAS is the route with intermediate state being the Gamow-Teller (GT).

• [In the case of good $SU_4$ symmetry the Double GT (DGT) strength obeys the relation:

\[
\frac{S_{DGT(DIAS)}}{\sum_f S_{DGT(all 0^+_f)}} = \frac{3}{(N-Z)^2-1}.
\]

For $N - Z = 2$ all the $J = 0^+$ DGT strength is in the DIAS. As $N-Z$ increases most of the DGT strength is contained in states that are not the DIAS. For example in Ca48 the DIAS contains only $1/21$ of DGT strength. The giant DGT resonance.]
“Pairing” property and short range operators.

In terms of the matrix elements of a two-body DCX transition operator \( \hat{\sigma}_{12} \) one can write [4]

\[
A = \frac{1}{2j+1} (2j\bar{\sigma} + O_0),
\]

\[
B = \frac{2j}{2j+1} (O_0 - \bar{\sigma}),
\]

where the following definitions were used:

\[
O_0 = \langle j^2J=0|\hat{\sigma}_{12}|j^2J=0 \rangle,
\]

\[
\bar{\sigma} = \sum_{J_{\text{even}}}(2J+1)\langle j^2J|\hat{\sigma}_{12}|j^2J \rangle \sum_{J_{\text{even}}}(2J+1).
\]

One sees from these expressions that the relative size of \( A \) and \( B \) depends on the range of \( \hat{\sigma}_{12} \). Consider a real DCX transition operator which possesses the “pairing” property [4], i.e., an operator that for even \( n \) satisfies

\[
\langle j^nJ=0|\hat{\sigma}_{12}|j^nJ=0 \rangle = \frac{n}{2} O_0,
\]

where \( n/2 \) is the number of neutron pairs. Operators such as \( \delta(r_1-r_2) \) and \( \sigma_1 \sigma_2 \) fall into this category. For such an operator, \( \bar{\sigma} = O_0/(2j) \), \( \alpha = 0 \), \( \beta = O_0 \), and

\[
B/A = j - \frac{1}{2}.
\]
Double $\beta$–decay

- $(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}$

- $(A, Z) \rightarrow (A, Z + 2) + 2e^-$
2-beta decay amplitudes

The amplitudes $A$ in these expressions contain the nuclear structure ingredients.

The inverse half-lives of the $2\nu\beta\beta$ decay and the $0\nu\beta\beta$ decay (due to the massive neutrino) can be written [5, 18, 19]:

\[
\frac{1}{T_{2\nu}} = G^{2\nu} |A^{2\nu}|^2, \tag{1}
\]

\[
\frac{1}{T_{0\nu}} = G^{0\nu} |R_0 A^{0\nu}|^2 \left( \frac{m_\nu}{m_e} \right)^2, \tag{2}
\]
Motivation

The (ββ-decay) matrix element, however, still remains very small and accounts for only a $10^{-4}$ to $10^{-3}$ of the total DGT sum rule. A precise calculation of such hindered transition is, of course, very difficult and is inherently a subject of large percent uncertainties. At present there is no direct way to “calibrate” such complicated nuclear structure calculations involving miniature fractions of the two-body DGT transitions. By studying the stronger DGT transitions and, in particular, the giant DGT states experimentally and as we do here, theoretically, one may be able to “calibrate” the calculations of 88-decay nuclear elements.

DGT strength and the double $\beta$-decay

- For example the 2-neutrino amplitude can be written as:
  $$A_{GT}^{2\nu} = \sum_n \frac{\langle f | G_- | n \rangle \langle n | G_- | i \rangle}{E_i - E_n - \epsilon}$$
- (The 0-neutrino amplitude includes also a Fermi amplitude.)
- The total coherent DGT strength is:
  $$S_{DGT} = \sum_f |\sum_n \langle f | G_- | n \rangle \langle n | G_- | i \rangle|^2$$
  (Coherent sum)
- $S_{2\beta} = |\sum_n \langle f_0 | G_- | n \rangle \langle n | G_- | i \rangle|^2$ is the double beta strength (in the closure approximation) and is a coherent sum. Thus: $S_{2\beta} \sim |\langle f | G_- (1) G_- (2) | i \rangle|^2$
- The incoherent DGT strength for a given final state $f_0$:
  $$S_{DGT}^{inc} = \sum_n |\langle f_0 | G_- | n \rangle|^2 |\langle n | G_- | i \rangle|^2$$
We approached this problem of the distribution of DGT strength in the most straightforward way. We calculate the shell-model states in an extended model space in the parent nucleus, (N, Z), in the intermediate nucleus (N - 1, Z + 1), and in the final nucleus (N - 2, Z + 2). Having determined the nuclear wave functions, we then evaluate directly the incoherent sum and coherent sum.
Fig. 2. The calculated important $G_0$ and $G_0^2$ transitions in the $A=22$ system. The length of each bar represents the relative $G_0$ and $G_0^2$ strength.


**Fig. 4.** The DGT $^{44}$Ca g.s. to $^{44}$Ti g.s. transition amplitudes $M_{i,j}^I(i)$ (the individual contribution from the $i$th intermediate state, open circles) and $\sum M_{i,j}^I(n)$ (sum of $M_{i,j}^I(i)$ for $i = 1$ to $n$, solid line) vs excitation energy ($Ex$) of the intermediate $1^n$ state in $^{48}$Sc. The modified renormalized Kuo-Brown interaction is used in the $0f-1p$ shell.
DCX and $2\beta$ decay

- Some (n, p) studies were initiated in which one chooses as the target states the final states of the relevant double-beta decay process. In these (n, p) experiments one measures the GT strength to the various intermediate single-charge exchange states. One can then take and multiply the two GT strengths obtained in (p,n) and (n,p) to the same intermediate states obtained in each reaction and try to sum over the intermediate states observed. For example, let us take the $A = 48$ nuclei. The (p, n) and (n, p) reactions are performed correspondingly on the $^{48}$Ca and $^{48}$Ti targets. The intermediate states measured are in both reactions located in the $^{48}$Sc nucleus. The actual experiments were performed at the initial nucleon energies of 200 MeV. At these energies the spin states and in the forward direction GT states in particular are excited predominantly.

- Using the DWIA analysis, one is able to extract the GT strength for the various intermediate states observed in these two reactions. Note that only the squares of the matrix elements of the type $\langle f_i | \sigma \tau | n \rangle$ and $\langle n | \sigma \tau | f_f \rangle$ can be deduced from these two one-body experiments. The relative signs are not determined in such processes. By measuring transitions to a great number of intermediate states, one in principle can determine the incoherent sum for the g.s. to g.s. transition. There are several difficulties in this procedure even when one tries to determine the incoherent decay sum. First, because of final experimental resolution on the one hand and large density of intermediate states on the other, one is not always sure that it is the same matching intermediate state that is excited when the (p,n) and (n, p) experiments are compared. Second, and related to it is the fact that when a strong GT transition in (p, n) is observed to a given intermediate state, the (n, p) transition to the same intermediate state might be (and often is) weak, and vice versa. This of course makes the experimental exploration of the double-beta decay amplitude difficult. The ground states in the two target nuclei are not related by any simple transformation and therefore the action of the $\sigma \tau_\pm$ operators leads to mismatched distributions of strength in the intermediate nucleus.
DCX and double beta decay

- It is clear that the combined studies of $(p, n)$ and $(n, p)$ may
- provide some information concerning the nuclear structure aspects of the $2\beta$ decay
- matrix elements. It is also clear that because of the coherence of the $2\beta$ decay
- matrix elements, the use of $(n, p)$ and $(p, n)$ strength cannot determine the value of
- such matrix elements. Also, quite obviously in such experiments, one cannot excite
- all DGT strengths. In order to be able experimentally to study the DGT strength
- (including the $2\beta$ decay g.s. to g.s. transition), one must employ processes in which
- the leading terms are genuine two-body transitions. Double charge-exchange
- (DCX) reactions are the natural candidates for such a study.
Comment

- One of the difficult problems in the theoretical studies of 2-beta decay is the “universal” quenching of the single Gamow-Teller strength.

- Experiments show that 30-40% of GT strength is missing in the main GT peak. This affects the 2-beta decay transition matrix element. The source of the above quenching is not certain. There are several ideas. One is that the missing strength is due to the $\Delta - h$ configurations interacting with the GT states and the missing strength is 300 MeV above the GT peak. Another theory is that the GT strength is fragmented and the missing strength is several tens of MeV above the main GT peak. These two different theories will affect the 2-beta decay matrix element differently. The uncertainty, (because of this quenching), could be as large as a factor of 2 for double beta decay.

Interesting problems (circa 1990)

1. Can one observe other types of resonator DEGRs in DCX reactions?
2. In particular the DEGR.
3. The reaction theory. Is the 3-channel approach sufficient? Role of correlations.
4. More specific questions (why only the \(302^+\) of the DEGR is observed, etc.).
5. DCX reactions with ions.

The stable T1 nuclei that can be used as probes such as \(^{16}\)O, \(^{16}\)O, etc. have an excess of two neutrons (T_z=1) and the DCX will involve A_T=2 transitions in the target. Such transitions do not involve the D1AR and usually also not the DEGR. Nevertheless the use of targets such as \(^{12}\)C, \(^{34}\)S, \(^{56}\)Fe, \(^{56}\)Ni may be of interest (in these nuclei the D1R transitions are allowed).

An intriguing possibility is the use of radioactive beams such as \(^{10}\)O.
Nuclear structure properties of the double-charge-exchange transition amplitudes

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For single \( j \), n-even, in the seniority scheme –the AGP formula holds.

\[
M = \left[ \frac{n(n-1)}{2} \right]^{1/2} \left[ A + \frac{2j+3-2n}{(n-1)(2j-1)} B \right],
\]

\[
A = \alpha + \frac{2}{2j+1} \beta,
\]

\[
B = \frac{2j-1}{2j+1} \beta.
\]

\[
M = \left[ \frac{n(n-1)}{2} \right]^{1/2} \left[ A + \frac{2j+3-2n}{(n-1)(2j-1)} B \right],
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\]

DCX transition operator which possesses the “pairing” property [4], i.e., an operator that for even \( n \) satisfies

\[
\langle j^n J = 0 | \hat{\mathcal{O}}_{12} | j^n J = 0 \rangle = \frac{n}{2} O_0,
\]

where \( n/2 \) is the number of neutron pairs. Operators such as \( \delta(r_1 - r_2) \) and \( \sigma_1 \cdot \sigma_2 \) fall into this category. For such an operator, \( \overline{O} = O_0 / (2j) \), \( \alpha = 0 \), \( \beta = O_0 \), and

\[
\frac{B}{A} = j - \frac{1}{2}.
\]

The DCX amplitude \( M \) then takes the following simple form:

\[
M = \left[ \frac{n}{2(n-1)} \right]^{1/2} \beta.
\]
Configuration mixing in Ca isotopes

- The configuration mixing is weak. In this case one can still use the AGP equation with $j=7/2$, however the coefficients $A$ and $B$ are changed, which means that the transition operator has changed. Calculations indicate that the range of the effective operator has a shorter range.
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Ni isotopes

• Here the configuration mixing is strong and the AGP equation is approximately valid when one uses the generalized seniority scheme. Instead of \( j \) one must use

\[
\tilde{j} = \Omega - \frac{1}{2} \quad \text{with} \quad \Omega = \sum_i (j_i + \frac{1}{2}).
\]

• In this case of the Ni isotopes \( \tilde{j} = \frac{11}{2} \)
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\( \beta/\alpha \) | 352 | 354 |

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\( B/A \) | 4.35 | 4.92 |
\( \beta/\alpha \) | 367 | 367 |

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\( B/A \) | 0.35 | 120 |
\( \beta/\alpha \) | 0.00 | 5.38 |

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\( B/A \) | 0.00 | 5.38 |
\( \beta/\alpha \) | 0.00 | -5.38 |
N. Auerbach, W. R. Gibbs and E. Piasetzky
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N. Auerbach,
"The Pion Double Charge Exchange Reaction"

N. Auerbach, and D.C. Zheng
"Nuclear Structure Properties of the Double Charge Exchange Transition Amplitudes"
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