Nuclear Structure Aspects of Double Charge Exchange

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Isotensor giant resonances

- A "model" giant resonance (state) built on a state $|n\rangle$ is :
- $|Q_\alpha; n\rangle = Q_\alpha|n\rangle / |\langle n|Q_\alpha^+ Q_\alpha|n\rangle|^{-1/2}$
- Q_α is a one-body operator, $Q_\alpha = \sum_i q_\alpha(i)$
- For $|0\rangle$, the g.s. we have the "usual" giant resonances.
- When $|n\rangle$ is itself a giant resonance state, then the giant state built on it will be a ***double giant state***.

Double Giant Resonances

- The operator Q_α might depend on spin and isospin. For example the electric dipole is an isovector and the corresponding operator is:
- $Q_\alpha \equiv D = \sum_i r_i Y_1(\theta_i)\tau_\mu(i), with\ \mu = 0, \pm 1$
- In addition to the "common" $\mu = 0$ dipole one has also the *charge-exchange analogs* $\mu = \pm 1$
- The action of the operator with $\mu = -1$ (or $\mu = 1$) twice will lead to states with $\Delta T_z = \pm 2$
- These states can be reached in *double* charge-exchange (DCX) processes.
- One of the best examples, are the pion DCX reactions $[(\pi^+, \pi^-), (\pi^-, \pi^+)]$, studied extensively in the past.
- Because we deal with isotensor transitions, the selectivity of the DCX is large and enhances the possibilities to observe double giant resonances (states).

PARTICLE–HOLE CALCULATION OF THE ISOBARIC ANALOG AND ISOVECTOR MONOPOLE RESONANCES

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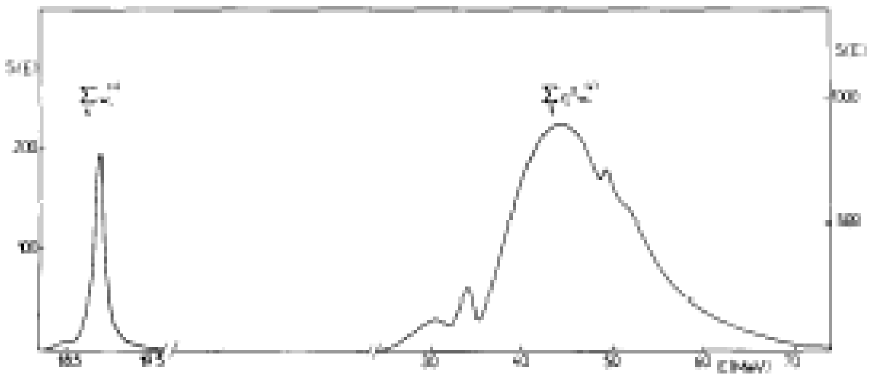

- THE GIANT DIPOLE RESONANCE AND ITS CHARGE-EXCHANGE ANALOGS
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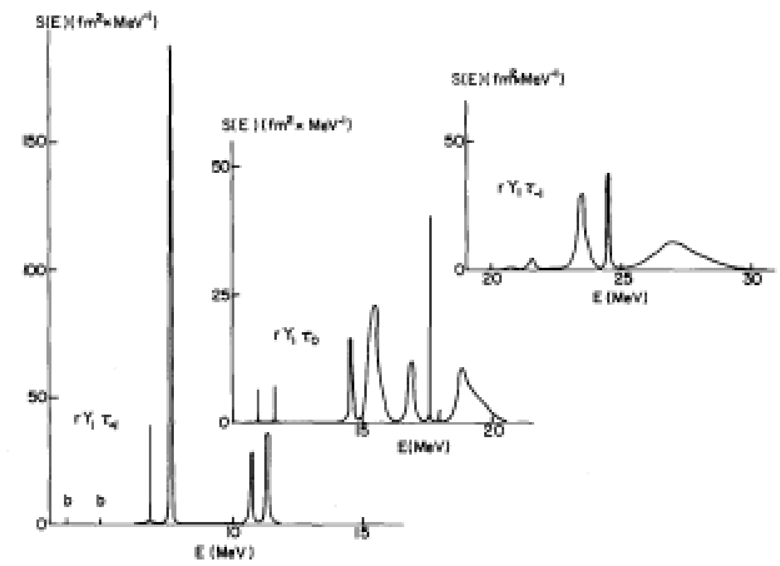


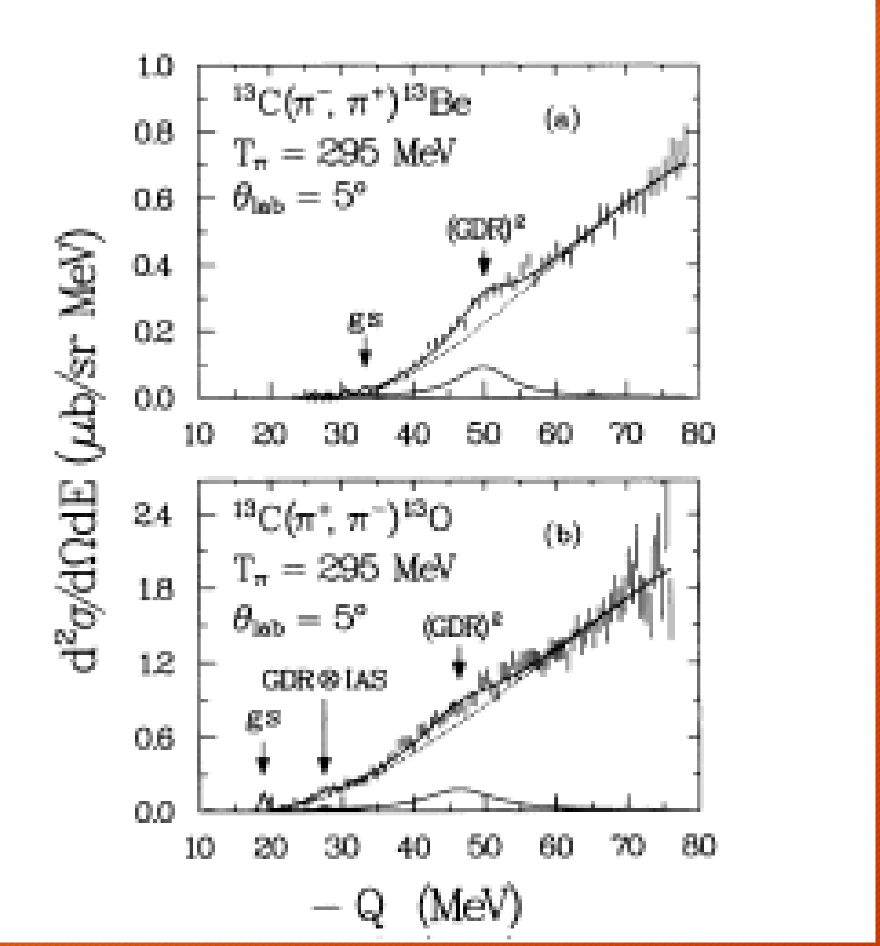
Fig. 1. Distribution of dipole strength in ^{90}Zr and its charge-exchange analogs calculated in the RPA.

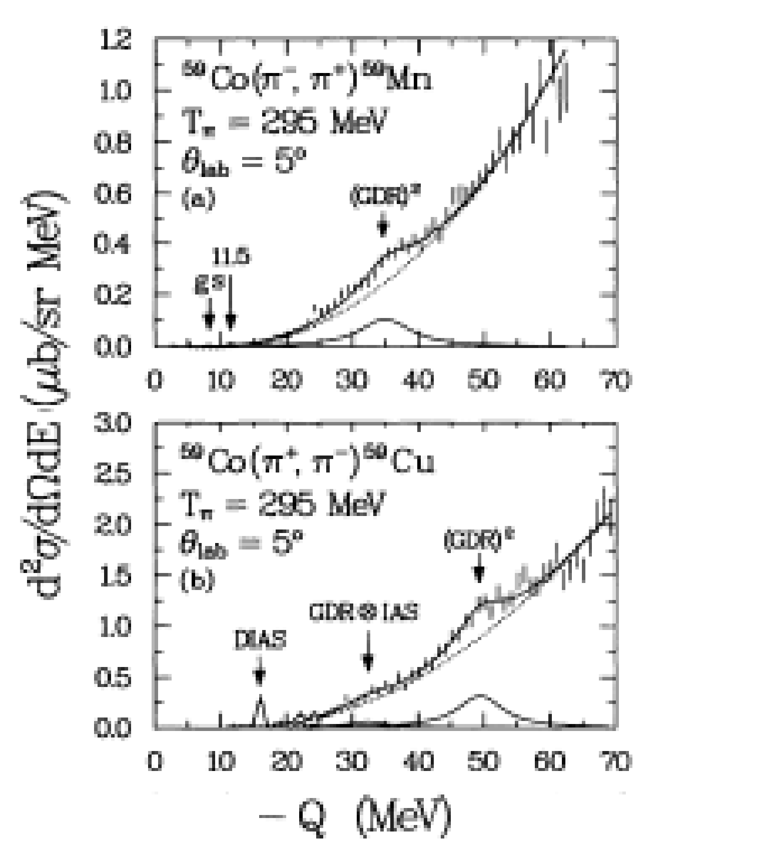
- The isovector dipole strength for all three components $\tau_z = \pm 1,0$ is calculated in the RPA.

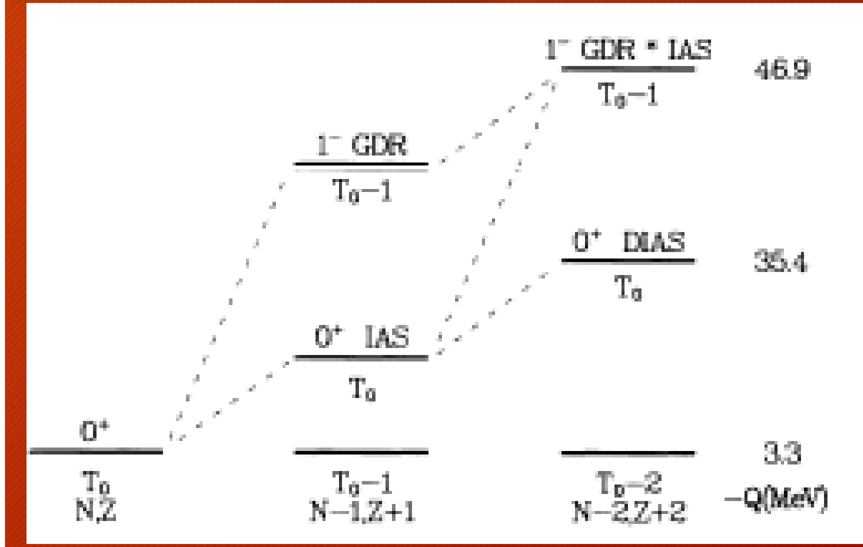
Examples of simple isotensor states

- Double isobaric analog state (DIAS).
- The IAS is defined as: $|A_1\rangle = T_{-}|0\rangle/(N-Z)^{1/2}$
- The DIAS is: $|A_2\rangle = T_{-}^2|0\rangle/[(N-Z)(N-Z-1)]^{1/2}$
- Dipole built on an analog: $|D_{-1}; A_1\rangle = \sum_i r_i Y_1(\theta_i)\,\tau_{-1}(i)|A_1\rangle/\tilde{N}$
- Yet another example, the double dipole (for the $\Delta T_z = -2$)
- $|D_{-1}; D_{-1}\rangle$=$\sum_i r_i\, Y_1(\theta_i)\tau_{-1}(i)|D_{-1}\rangle/\tilde{N}$

Pion DCX experiments







Spin degrees of freedom

- $\boldsymbol{G_{\pm} = \sum_i \sigma(i)\, \tau_{\pm}(i)}$
- The giant Gamow-Teller (GT) state: $|\boldsymbol{GT}\rangle = \boldsymbol{G_{-}|0\rangle/N}$
- The double Gamow-Teller (DGT) state: $|\boldsymbol{DGT; J}\rangle = \boldsymbol{G_{-}^{2}(\mathrm{J})|0\rangle/N(J)}$
- J is the total spin of the DGT
- Simple selection rules for the total spins of isotensor transitions.
- For two identical phonons the wave function is symmetric in space-spin-isospin, therefore for $\Delta T = 0,2 \quad J = 0^{+}, 2^{+}$ …..
- For $\Delta T = 1, \quad J = 1^{+}, 3^{+}$. For the DGT $J = 0^{+}, 2^{+}$

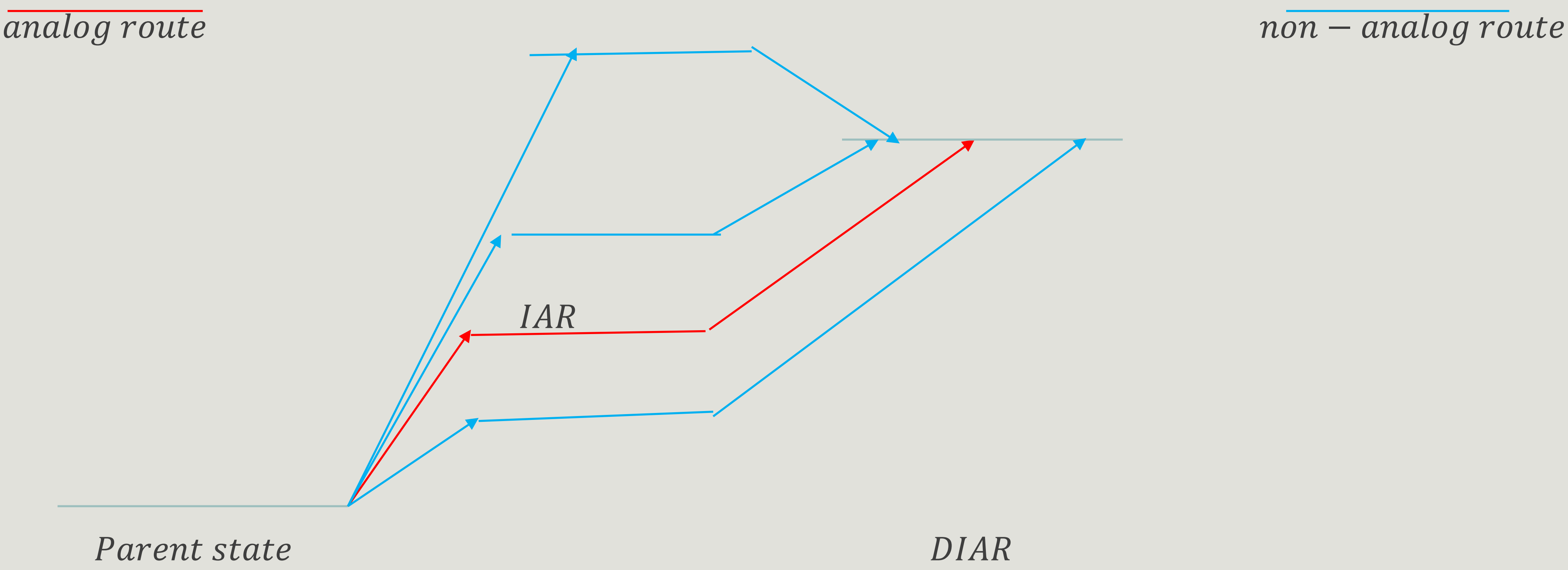
Some properties

- Excitation energies: $E_{Q_1Q_2} \approx E_{Q_1} + E_{Q_2}$
- Widths *(spreading widths)* $\Gamma_{n;\lambda} = n\Gamma_{1;\lambda}$. In some theories n is replaced by $\sqrt{n}$
- Isospin and intensities: for an isotensor excitation $\Delta T_z = -2$
- and $(N - Z) \geq 2$ there are five isospin values $T' = T + 2,$
- $T + 1,\ T,\ T - 1,\ T - 2.$
- For an isotensor od rank k $F_\mu^{(k)}$ the intensities for the various isospin components T' are given by the corresponding Clebsh-Gordan (CG) cofficients and reduced matrix elements.
- $S_{T'} = |\langle T, T, k, \mu | T', T + \mu\rangle|^2 \langle T' \| F^{(k)} \| T\rangle^2$
- The CG coefficient for $T \gg k$ (in our case $T \gg 2$) are dominated by the
- CG of the aligned isospin $T' = T + \mu$. For $\mu = -k,\ \ (CG)^2 = \frac{2(T-k)+1}{2T+1}$.

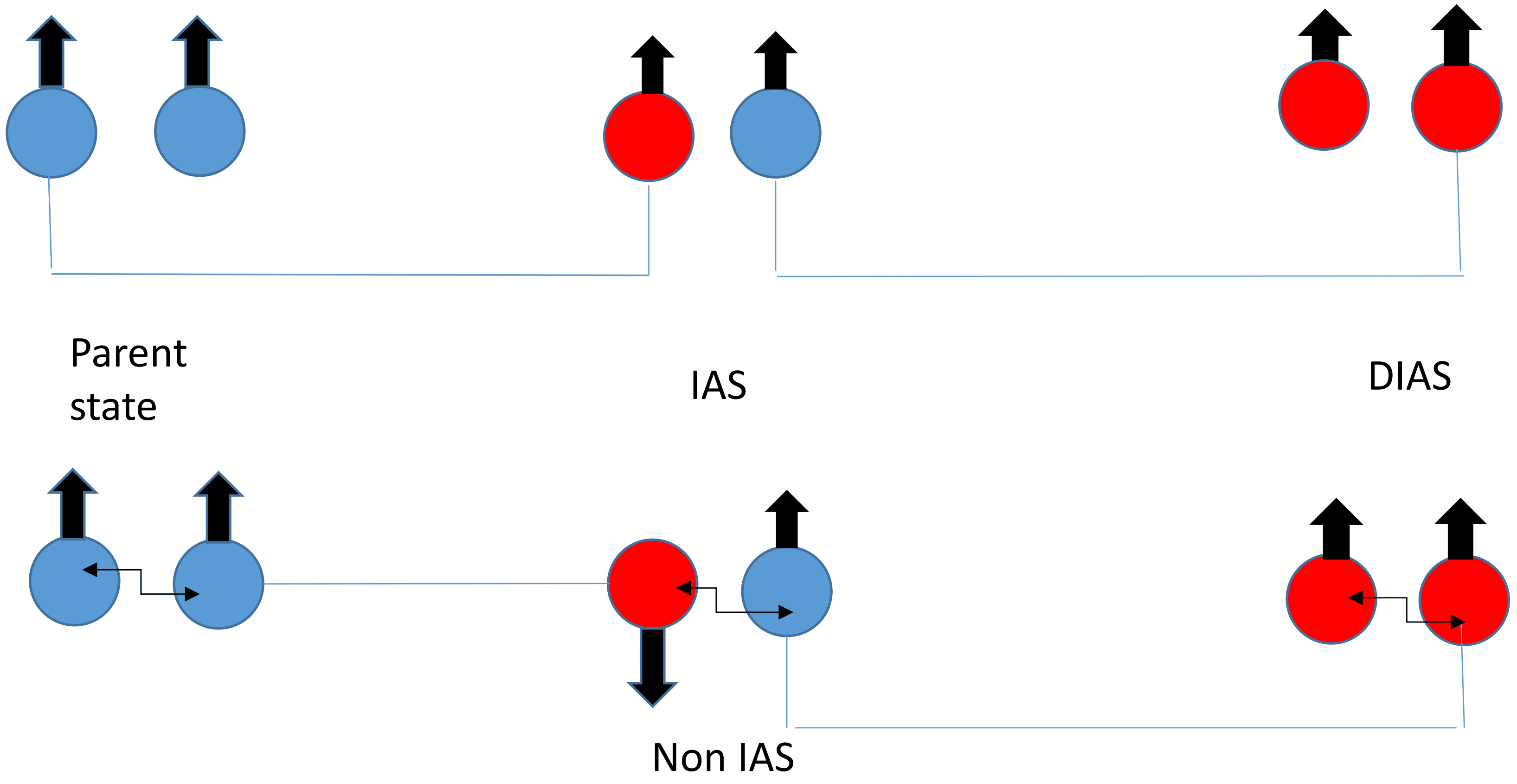
The DCX process.

- The DCX process has held out the hope that it would be a means of probing two-body correlations in nuclei. In the initial studies it was found that that uncorrelated nuclear wave functions produced qualitative agreement with experiment at higher energies (pion energies around 300 MeV.) This was the situation when the double giant resonances were studied. However at low pion energies (around 50 MeV) the disagreement with experiment was very large, (sometimes a factor of 50) when uncorrelated wave functions were used. It was necessary to include wave functions with correlated nucleons.
- The DCX process, as determined in pion charge-exchange reactions involves two basic routes.
- 1. For uncorrelated n paricles in the transition to a double analog state the route from the initial state to the final state goes via the excitation of the single analog in the intermediate stage , and from there in a charge-exchange to the double analog. This is termed as the analog route or more generally as the *"sequential"* process. The cross section is proportional to the number of pairs one can form from the n nucleons, $n(n-1)/2$.
- 2.The second process involves correlated nucleons and the process proceedes through intermediate states that are not the analog. This we term the "non-analog route".

Analog vs non-analog routes



Correlations and non-analog transitions



The DCX process

In order to obtain the desired relation, we consider the reaction to be caused by a two-body operator between two identical (in the same isospin multiplet) 0^+ states. Thus we need to calculate the matrix element for the transition:

$$M = \langle 0^+ | \textstyle\sum_{i,j} \theta_{ij}(\mathbf{r}_i, \mathbf{r}_j, \mathbf{k}, \mathbf{k}') | 0^+ \rangle. \tag{1}$$

- The operator depends also on the spins of the two particles

The DCX for the even Ca isotopes (the AGP formula) (Single- j, n-even). (NA, W.R. Gibbs, E. Piasetzky, PRL, 59, 1076 (1987)

Nucleus	σ_{DCX}
^{42}Ca	$\lvert A+B \rvert^2$
^{44}Ca	$6\lvert A+\frac{1}{9}B \rvert^2$
^{46}Ca	$15\lvert A-\frac{1}{15}B \rvert^2$
^{48}Ca	$28\lvert A-\frac{1}{7}B \rvert^2$

$$\sigma_{DCX}(\theta)=\frac{n(n-1)}{2}\left|A+\frac{(2j+3-2n)}{(n-1)(2j-1)}B\right|^2,$$

$$A=\alpha+[2/(2j+1)]\beta,\quad B=[(2j-1)/(2j+1)]\beta,$$

$$\sigma_{DCX}(\theta)=\tfrac{1}{2}n(n-1)\,|\alpha+\beta/(n-1)|^2.$$

TABLE 1

Nucleus	$n(n-1)/2$ Scaling	Exp.
^{42}Ca	1	1
^{44}Ca	6	0.5
^{46}Ca	15	—
^{48}Ca	28	1

We see that the counting rule given by the $n(n-1)/2$ independent pairs is completely destroyed, indicating that the correlations play a very important role in the DCX process at these low pion energies. Fitting the data, one finds that the ratio $|B|/|A| \simeq 3$. For higher energies[4] $E_\pi \geq 150$ MeV this ratio is found to be $\simeq 1$.

The DCX process (cont'd)

- In general, the DCX cross section will contain the sequential term and the correlation term. Both amplitudes could be complex.
- The above formula is extended to the generalized seniority scheme when several orbits are involved. Then:
- $\sigma_{DCX} = \frac{n(n-1)}{2}\left|A + \frac{(\Omega+1-n)}{(\Omega-1)(n-1)}B\right|^2$; with $\Omega = \sum_i (j_i + {}^1/_2)$
- This formalism was applied to the Te 128 and 130 nuclei, relevant to the double beta-decay, (H.C. Wu, et.al., Phys. Rev. C**54**, 1208 (1996)).
- See also N. Auerbach, et. al., Phys. Rev. C**38**, 1277(1988),
- J. Barea and F. Iachello, Phys. Rev. C**79**, 044301 (2009).

Spin

- The two-body DCX operator is also a function of the spin of the two nucleons σ_1 and σ_2. In fact one of the important non-analog routes leading to the DIAS is the route with intermediate state being the Gamow-Teller (GT).

- [In the case of good SU_4 symmetry the Double GT (DGT) strength obeys the relation:
- $\frac{S_{DGT}(DIAS)}{\sum_f S_{DGT}\left(all\ 0_f^+\right)} = \frac{3}{(N-Z)^2-1}$. For $N - Z = 2$ all the $J = 0^+$ DGT strength
- is in the DIAS. As N-Z increases most of the DGT strength is contained in states that are not the DIAS. For example in Ca48 the DIAS contains only 1/21 of DGT strength. The giant DGT resonance.]

“Pairing” property and short range operators.

In terms of the matrix elements of a two-body DCX transition operator $\hat{O}_{12}$ one can write [4]

$$A = \frac{1}{2j+1}(2j\bar{O} + O_0) ,$$
$$B = \frac{2j}{2j+1}(O_0 - \bar{O}) , \quad (4)$$

where the following definitions were used:

$$O_0 = \langle j^2 J=0 | \hat{O}_{12} | j^2 J=0 \rangle ,$$
$$\bar{O} = \frac{\sum_{J\,even}(2J+1)\langle j^2 J | \hat{O}_{12} | j^2 J \rangle}{\sum_{J\,even}(2J+1)} . \quad (5)$$

One sees from these expressions that the relative size of A and B depends on the range of $\hat{O}_{12}$. Consider a real DCX transition operator which possesses the “pairing” property [4], i.e., an operator that for even n satisfies

$$\langle j^n J=0 | \hat{O}_{12} | j^n J=0 \rangle = \frac{n}{2} O_0 , \quad (6)$$

where $n/2$ is the number of neutron pairs. Operators such as $\delta(\mathbf{r}_1 - \mathbf{r}_2)$ and $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$ fall into this category. For such an operator, $\bar{O} = O_0/(2j)$, $\alpha = 0$, $\beta = O_0$, and

$$B/A = j - \tfrac{1}{2} . \quad (7)$$

Double β –decay

- $(A, Z) \rightarrow (A, Z+2) + 2e^{-} + 2\bar{\boldsymbol{\nu}}$
- $(A, Z) \rightarrow (A, Z+2) + 2e^{-}$

2-beta decay amplitudes

The amplitudes A in these expressions contain the nuclear structure ingredients.

The inverse half–lives of the $2\nu\beta\beta$ decay and the $0\nu\beta\beta$ decay (due to the massive neutrino) can be written [5, 18, 19]:

$$\frac{1}{T_{2\nu}} = G^{2\nu}|A^{2\nu}|^2, \tag{1}$$

$$\frac{1}{T_{0\nu}} = G^{0\nu}|R_0 A^{0\nu}|^2 \left(\frac{m_\nu}{m_e}\right)^2, \tag{2}$$

Motivation

The (ββ-decay) matrix element, however, still remains very small and accounts for only a 10−4 to 10−3 of the total DGT sum rule. A precise calculation of such hindered transition is, of course, very difficult and is inherently a subject of large percent uncertainties. At present there is no direct way to "calibrate" such complicated nuclear structure calculations involving miniature fractions of the two-body DGT transitions. By studying the stronger DGT transitions and, in particular, the giant DGT states experimentally and as we do here, theoretically, one may be able to "calibrate" the calculations of ββ-decay nuclear elements.

N. Auerbach, L. Zamick, and D. Zheng, Annals of Physics 192, 77 (1989).

DGT strength and the double β-decay

- For example the 2-neutrino amplitude can be written as:
- $A_{GT}^{2\nu} = \sum_n \frac{\langle f|G_-|n\rangle\langle n|G_-|i\rangle}{E_i - E_n - \varepsilon}$
- (The 0-neutrino amplitude includes also a Fermi amplitude.)
- The total coherent DGT strength is:
- $S_{DGT} = \sum_f \underline{|\sum_n \langle f|G_-|n\rangle\langle n|G_-|i\rangle|^2}$
- (Coherent sum)
- $S_{2\beta} = |\sum_n \langle f_0|G_-|n\rangle\langle n|G_-|i\rangle|^2$ is the double beta strength (in the closure approximation) and is a ***coherent*** sum. Thus: $S_{2\beta} \sim |\langle f|G_-(1)G_-(2)|i\rangle|^2$
- The incoherent DGT strength for a given final state f_0:
- $S_{DGT}^{inc} = \sum_n |\langle f_0|G_-|n\rangle|^2 |\langle n|G_-|i\rangle|^2$

D.C. Zheng et.al., Annals of Physics, 197, 343 (1990)

- We approached this problem of the distribution of DGT strength in the most
- straightforward way. We calculate the shell-model states in an extended model
- space in the parent nucleus, (N, Z), in the intermediate nucleus (N - 1, Z + 1), and
- in the final nucleus (N- 2, Z + 2). Having determined the nuclear wave functions
- we then evaluate directly the *incoherent* sum and *coherent* sum.

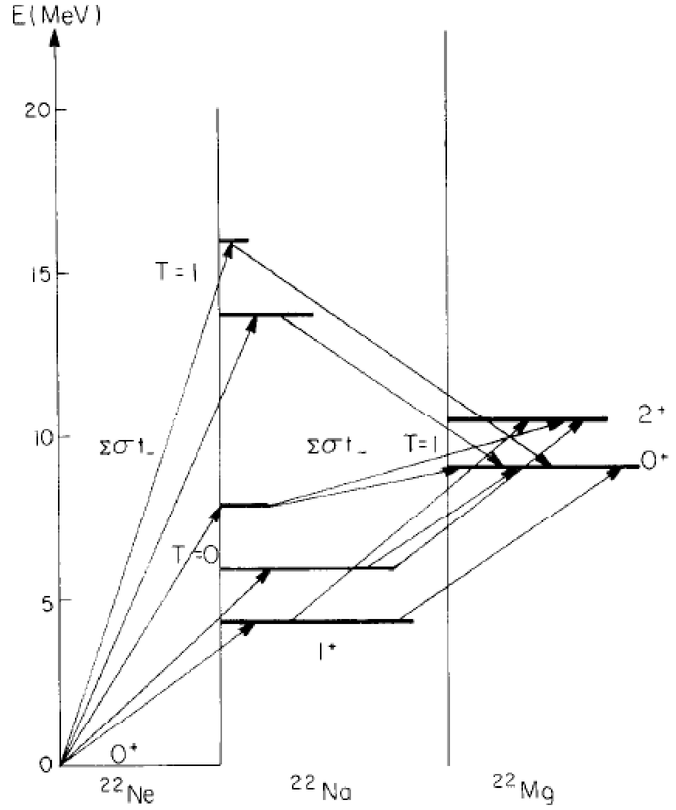


FIG. 2. The calculated important G_- and G^2_- transitions in the $A = 22$ system. The length of each bar represents the relative G_- and G^2_- strength.

^{44}Ca $\qquad$ ^{48}Ca

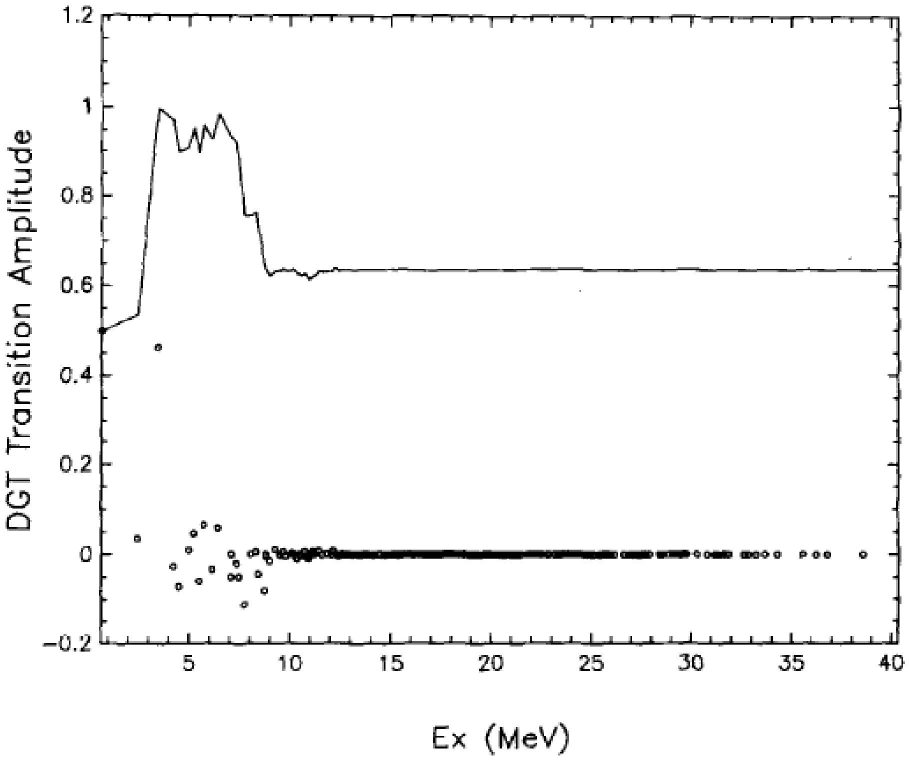
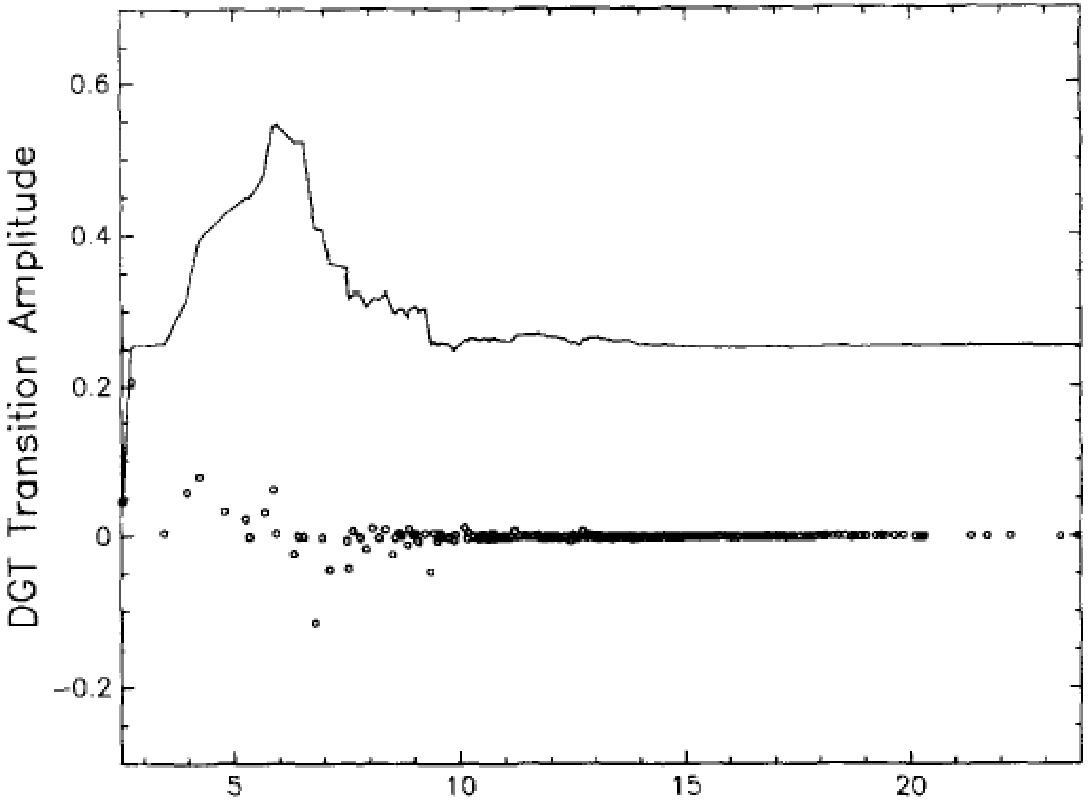


FIG. 4. The DGT ^{44}Ca g.s. to ^{44}Ti g.s. transition amplitudes $M^{C}_{+\beta\beta}(i)$ (the individual contribution from the ith intermediate state, open circles) and $\sum M^{C}_{+\beta\beta}(n)$ (sum of $M^{C}_{+\beta\beta}(i)$ for $i = 1$ to n, solid line) vs excitation energy (Ex) of the intermediate 1^+ state in ^{44}Sc. The modified renormalized Kuo–Brown interaction is used in the 0f–1p shell.



DCX and 2β decay

- Some (n, p) studies were initiated in which one chooses as the target states the final states of the relevant double beta decay process. In these (n, p) experiments one measures the GT strength to the various intermediate single-charge exchange states. One can then take and multiply the two GT strengths obtained in (p,n) and (n,p) to the same intermediate states obtained in each reaction and try to sum over the intermediate states observed. For example, let us take the A = 48 nuclei. The (p, n) and (n, p) reactions are performed correspondingly on the 48Ca and 48Ti targets. The intermediate states measured are in both reactions located in the 48Sc nucleus. The actual experiments were performed at the initial nucleon energies of 200 MeV. At these energies the spin states and in the forward direction GT states in particular are excited predominantly.
- Using the DWIA analysis, one is able to extract the GT strength for the various intermediate states observed in these two reactions. Note that only the squares of the matrix elements of the type $\langle J_i|\sigma\tau|n\rangle$ $and\langle n|\sigma\tau|J_f\rangle$ can be deduced from these two one-body experiments. The relative signs are not determined in such processes. By measuring transitions to a great number of intermediate states, one in principle can determine the incoherent sum for the g.s. to g.s. transition. There are several difficulties in this procedure even when one tries to determine the incoherent decay sum. First, because of final experimental resolution on the one hand and large density of intermediate states on the other, one is not always sure that it is the same matching intermediate state that is excited when the (p,n) and (n, p) experiments are compared. Second, and related to it is the fact that when a strong GT transition in (p, n) is observed to a given intermediate state, the (n, p) transition to the same intermediate state might be (and often is) weak, and vice versa. This of course makes the experimental exploration of the double-beta decay amplitude difficult. The ground states in the two target nuclei are not related by any simple transformation and therefore the action of the $\sigma\tau_{\pm}$ operators leads to mismatched distributions of strength in the intermediate nucleus.

DCX and double beta decay

- It is clear that the combined studies of (p, n) and (n, p) may
- provide some information concerning the nuclear structure aspects of the 2β decay
- matrix elements. It is also clear that because of the coherence of the 2β decay
- matrix elements, the use of (n, p) and (p, n) strength cannot determine the value of
- such matrix elements. Also, quite obviously in such experiments, one cannot excite
- all DGT strengths. In order to be able experimentally to study the DGT strength
- (including the 2β decay g.s. to g.s. transition), one must employ processes in which
- the leading terms are genuine two-body transitions. Double charge-exchange
- (DCX) reactions are the natural candidates for such a study.

Comment

- One of the difficult problems in the theoretical studies of 2-beta decay is the "universal" quenching of the single Gamow-Teller strength.
- Experiments show that 30-40% of GT strength is missing in the main GT peak. This affects the 2-beta decay transition matrix element. The source of the above quenching is not certain. There are several ideas. One is that the missing strength is due to the $\Delta - h$ configurations interacting with the GT states and the missing strength is 300 MeV above the GT peak. Another theory is that the GT strength is fragmented and the missing strength is several tens of MeV above the main GT peak. These two different theories will affect the 2-beta decay matrix element differently. The uncertainty, (because of this quenching), could be as large as a factor of 2 for double beta decay.
- (L.Zamick and N. Auerbach , Phys.Rev. C26, 2185 (1982).)

Interesting problems (circa 1990)

Interesting problems.

1. Can one observe other types of isotensor DGRs in DCX reactions.

2. In particular the DGTR.

3. The reaction theory. Is the 3-channel approach sufficient? Role of correlations.

4. More specific questions, (why only the $J=2^+$ (?) of the DGDR is observed, etc).

5. DCX reactions with Ions.

The stable $T=1$ nuclei that can be used as probes such as ^{14}C, ^{18}O, etc. have an excess of two neutrons ($T_z=+1$) and the DCX will involve $\Delta T_z = 2$ transitions in the target. Such transitions do not involve the DIAR and usually also not have the DGTR. Nevertheless the use of targets such as ^{12}C, ^{54}Fe, ^{60}Ni may be of interest. (In these nuclei the DGT transitions are allowed). An intriguing possibility is the use of radioactive beams such as ^{14}O.

Nuclear structure properties of the double-charge-exchange transition amplitudes

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For single j, n-even, in the seniority scheme –the AGP formula holds.

$$M=\left[\frac{n(n-1)}{2}\right]^{1/2}\left[A+\frac{2j+3-2n}{(n-1)(2j-1)}B\right], \quad (1)$$

$$M=\left[\frac{n(n-1)}{2}\right]^{1/2}\left[\alpha+\frac{\beta}{n-1}\right],$$

$$A=\alpha+\frac{2}{2j+1}\beta,$$
$$B=\frac{2j-1}{2j+1}\beta.$$

$$A=\frac{1}{2j+1}(2j\bar{O}+O_0),$$
$$B=\frac{2j}{2j+1}(O_0-\bar{O}),$$

$$O_0=\langle j^2J=0|\hat{O}_{12}|j^2J=0\rangle,$$
$$\bar{O}=\frac{\sum_{J\,\mathrm{even}}(2J+1)\langle j^2J|\hat{O}_{12}|j^2J\rangle}{\sum_{J\,\mathrm{even}}(2J+1)}.$$

DCX transition operator which possesses the "pairing" property [4], i.e., an operator that for even n satisfies

$$\langle j^nJ=0|\hat{O}_{12}|j^nJ=0\rangle=\frac{n}{2}O_0, \quad (6)$$

where $n/2$ is the number of neutron pairs. Operators such as $\delta(\mathbf{r}_1-\mathbf{r}_2)$ and $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ fall into this category. For such an operator, $\bar{O}=O_0/(2j)$, $\alpha=0$, $\beta=O_0$, and

$$B/A=j-\tfrac{1}{2}. \quad (7)$$

The DCX amplitude M then takes the following simple form:

$$M=\left[\frac{n}{2(n-1)}\right]^{1/2}\beta. \quad (8)$$

Configuration mixing in Ca isotopes

- The configuration mixing is weak. In this case one can still use the AGP equation with $j=7/2$, however the coefficients A and B are changed, which means that the transition operator has changed. Calculations indicate that the range of the effective operator has a shorter range.

TABLE I. $^{40+n}$Ca($0^+_{g.s.}$)$\rightarrow$$^{40+n}$Ti(DIAS) transition amplitudes M for various types of operators in the single-$f_{7/2}$ shell-model (SJ) and in the extended fp space shell-model (CM) calculations using the "FPD6" interaction. In the column labeled "AGP" is shown the fit obtained using Eq. (1) with A,B parameters (denoted by $\bar{A},\bar{B}$) determined by the $n=2$ and 8 CM values. The ratios B/A and $\bar{B}/\bar{A}$ are also shown for each operator.

n	$\hat{O}_{12}$	SJ	CM	AGP	$\hat{O}_{12}$	SJ	CM	AGP
2	δtt	2.000	3.090	3.090	$\delta\sigma\sigma tt$	−2.571	−3.973	−3.973
4		1.633	2.340	2.33		−2.099	−3.008	−3.00
6		1.549	1.974	2.03		−1.992	−2.538	−2.61
8		1.512	1.800	1.800		−1.944	−2.314	−2.314
$B/A(\bar{B}/\bar{A})$		3.00		3.52		3.00		3.52
2	$Y_\rho tt$	2.000	3.008	3.008	$Y_\rho\sigma\sigma tt$	−2.571	−3.992	−3.992
4		1.707	2.382	2.35		−2.099	−3.023	−3.01
6		1.690	2.099	2.14		−1.992	−2.549	−2.62
8		1.718	1.990	1.990		−1.944	−2.324	−2.324
$B/A(\bar{B}/\bar{A})$		2.75		3.27		3.00		3.52
2	$Y_\pi tt$	2.000	2.581	2.581	$Y_\pi\sigma\sigma tt$	−2.571	−4.183	−4.183
4		2.539	3.026	2.92		−2.099	−3.166	−3.15
6		3.268	3.577	3.54		−1.992	−2.648	−2.73
8		4.028	4.206	4.206		−1.944	−2.413	−2.413
$B/A(\bar{B}/\bar{A})$		1.18		1.54		3.00		3.54
2	Ctt	2.000	2.185	2.185	$C\sigma\sigma tt$	−2.571	−4.147	−4.147
4		4.101	4.276	4.22		−2.099	−3.172	−3.15
6		6.232	6.345	6.32		−1.992	−2.680	−2.76
8		8.367	8.428	8.428		−1.944	−2.471	−2.471
$B/A(\bar{B}/\bar{A})$		0.22		0.31		3.00		3.47
2	tt	2.000	2.000	2.000	$\sigma\sigma tt$	−2.571	−4.021	−4.021
4		4.899	4.899	4.90		−2.099	−3.109	−3.08
6		4.746	7.746	7.75		−1.992	−2.662	−2.73
8		10.583	10.583	10.583		−1.944	−2.481	−2.481
$B/A(\bar{B}/\bar{A})$		0.00		0.00		3.00		3.41

Ni isotopes

- Here the configuration mixing is strong and the AGP equation is approximately valid when one uses the generalized seniority scheme. Instead of j one must use
- $\tilde{j} = \Omega - \frac{1}{2}$ with $\Omega = \sum_i \left(j_i + \frac{1}{2}\right)$.
- In this case of the Ni isotopes $\tilde{j} = {}^{11}/_{2}$

TABLE II. $^{56+n}\mathrm{Ni}(0^+_{g.s.}) \to {}^{56+n}\mathrm{Zn}(\mathrm{DIAS})$ transition amplitudes M for various types of operators in the full $(1p_{3/2}, 0f_{5/2}, 1p_{1/2})$ space shell-model (SM) calculation using the "ASDI" interaction. In the column labeled "AGP" is shown the fit obtained using Eq. (27) or (28) with $\bar{A}, \bar{B}$ and $\bar{\alpha}, \bar{\beta}$ amplitudes determined by the $n=2$ and 8 SM values. The ratios $\bar{B}/\bar{A}$ and $\bar{\beta}/\bar{\alpha}$ are also shown for each operator.

n	$\hat{O}_{12}$	SM	AGP	$\hat{O}_{12}$	SM	AGP
2	δtt	3.611	3.611	$\delta\sigma\sigma tt$	−6.018	−6.018
4		2.969	2.97		−4.948	−4.94
6		2.835	2.83		−4.724	−4.71
8		2.776	2.776		−4.626	−4.626
10		2.734	2.75		−4.557	−4.59
12		2.698	2.74		−4.497	−4.57
$\bar{B}/\bar{A}$			4.92			4.92
$\bar{\beta}/\bar{\alpha}$			352			354
2	$Y_\rho tt$	3.597	3.597	$Y_\rho\sigma\sigma tt$	−6.050	−6.050
4		3.099	3.08		−4.973	−4.97
6		3.075	3.06		−4.747	−4.74
8		3.118	3.118		−4.648	−4.648
10		3.173	3.21		−4.578	−4.61
12		3.226	3.31		−4.517	−4.59
$\bar{B}/\bar{A}$			4.35			4.92
$\bar{\beta}/\bar{\alpha}$			39.9			367
2	$Y_\nu tt$	3.077	3.077	$Y_\nu\sigma\sigma tt$	−5.536	−5.536
4		3.847	3.80		−4.488	−4.50
6		4.858	4.82		−4.251	−4.26
8		5.894	5.894		−4.140	−4.140
10		6.923	6.98		−4.054	−4.07
12		7.932	8.08		−3.968	−4.02
$\bar{B}/\bar{A}$			1.63			5.05
$\bar{\beta}/\bar{\alpha}$			2.91			−561
2	Ctt	2.290	2.290	$C\sigma\sigma tt$	−4.650	−4.650
4		4.589	4.57		−3.688	−3.73
6		6.912	6.90		−3.455	−3.48
8		9.234	9.234		−3.338	−3.338
10		11.550	11.57		−3.236	−3.23
12		13.858	13.91		−3.122	−3.15
$\bar{B}/\bar{A}$			0.30			5.26
$\bar{\beta}/\bar{\alpha}$			0.38			−120
2	tt	2.000	2.000	$\sigma\sigma tt$	−4.239	−4.239
4		4.899	4.90		−3.322	−3.38
6		7.746	7.75		−3.094	−3.13
8		10.583	10.583		−2.978	−2.978
10		13.416	13.42		−2.869	−2.86
12		16.248	16.25		−2.743	−2.76
$\bar{B}/\bar{A}$			0.00			5.38
$\bar{\beta}/\bar{\alpha}$			0.00			−85.9

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Table II. Same as Table I but for the extended fp space calculations. For n=2 and 4 the full fp space is used while for n=6 and 8 only up to two valence nucleons are allowed to occupy the $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ orbits. Only the transition amplitudes to five lowest 0^+ states and to the DIAS are shown.

n	Final State	0_1^+	0_2^+	0_3^+	0_4^+	0_5^+	DIAS
2	E_x	0.000	5.961	10.931	14.906	—	0.000
	$R_0 A_F^{0\nu}$	1.965	0.223	0.109	0.199		1.965
	$R_0 A_F^{0\nu}(0)$	2.539	0.238	0.114	0.207		2.539
	$A_F^{2\nu}$	2.000	0.000	0.000	0.000		2.000
	$R_0 A_{GT}^{0\nu}$	-4.398	-1.128	-1.089	-3.053		-4.398
	$R_0 A_{GT}^{0\nu}(0)$	-5.573	-1.353	-1.360	-3.980		-5.573
	$A_{GT}^{2\nu}$	-4.021	-0.664	-0.884	-3.270		-4.021
4	E_x	0.000	5.273	8.460	9.749	10.596	9.243
	$R_0 A_F^{0\nu}$	0.433	0.254	0.216	-0.211	0.218	3.629
	$R_0 A_F^{0\nu}(0)$	0.457	0.268	0.231	-0.225	0.230	4.968
	$A_F^{2\nu}$	0.000	0.000	0.000	0.000	0.000	4.899
	$R_0 A_{GT}^{0\nu}$	-1.696	-1.321	-0.927	0.129	-2.404	-3.357
	$R_0 A_{GT}^{0\nu}(0)$	-1.974	-1.629	-1.205	-0.034	-3.161	-4.263
	$A_{GT}^{2\nu}$	-0.808	-1.011	-0.975	-0.755	-2.661	-3.109
6	E_x	0.000	4.891	5.712	6.967	7.340	12.522
	$R_0 A_F^{0\nu}$	0.437	0.372	0.167	0.005	-0.153	5.283
	$R_0 A_F^{0\nu}(0)$	0.460	0.395	0.179	0.005	-0.164	7.372
	$A_F^{2\nu}$	0.000	0.000	0.000	0.000	0.000	7.746
	$R_0 A_{GT}^{0\nu}$	-1.598	-1.596	-0.759	-0.272	0.214	-2.830
	$R_0 A_{GT}^{0\nu}(0)$	-1.832	-1.986	-0.994	-0.363	0.156	-3.601
	$A_{GT}^{2\nu}$	-0.642	-1.309	-0.828	-0.340	-0.323	-2.662
8	E_x	0.000	4.575	6.818	7.464	7.620	15.933
	$R_0 A_F^{0\nu}$	0.368	0.364	-0.006	-0.009	0.322	6.955
	$R_0 A_F^{0\nu}(0)$	0.387	0.389	-0.007	-0.012	0.342	9.791
	$A_F^{2\nu}$	0.000	0.000	0.000	0.000	0.000	10.583
	$R_0 A_{GT}^{0\nu}$	-1.287	-1.143	-0.025	-0.212	-1.480	-2.603
	$R_0 A_{GT}^{0\nu}(0)$	-1.454	-1.378	-0.012	-0.245	-1.890	-3.320
	$A_{GT}^{2\nu}$	-0.430	-0.721	0.057	-0.104	-1.427	-2.481