IUCAA
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Stochastic Gravitational Wave Background: Sources and Searches

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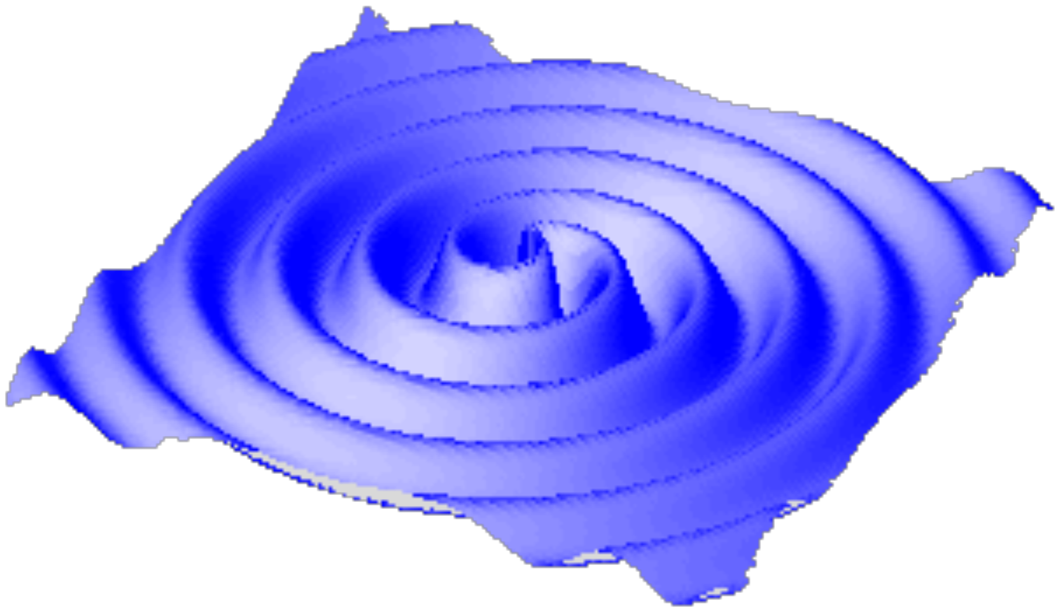
Gravitational Waves

- The existence of gravitational waves is one of the most intriguing predictions of the General Theory of Relativity

- GWs are distortions in the spacetime geometry that propagate with the speed of light, analogous to ripples on the surface of a pond.



Sources of Gravitational Waves

- Inspiral
- Continuous
- Burst
- Stochastic

Sources of Gravitational Waves

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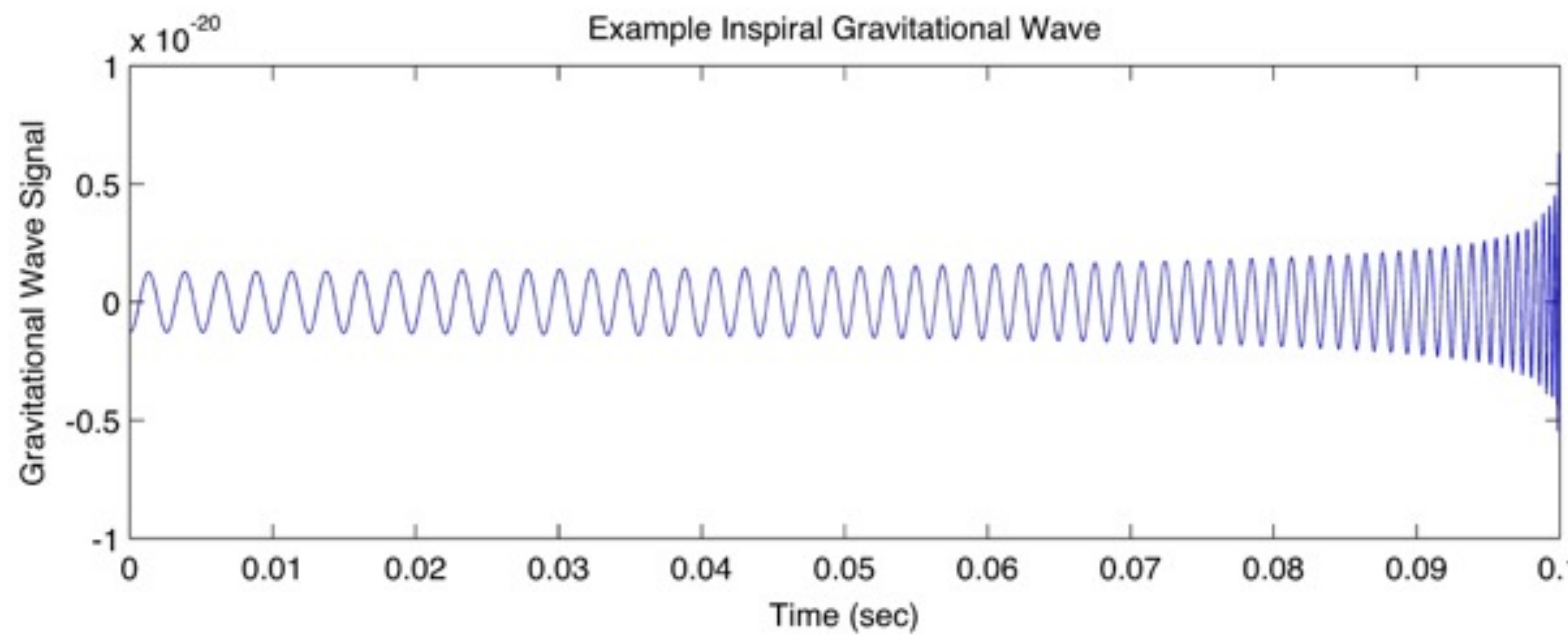

Inspiral GWs are generated during the end-of-life stage of binary systems where the two objects merge into one.

Sources of Gravitational Waves

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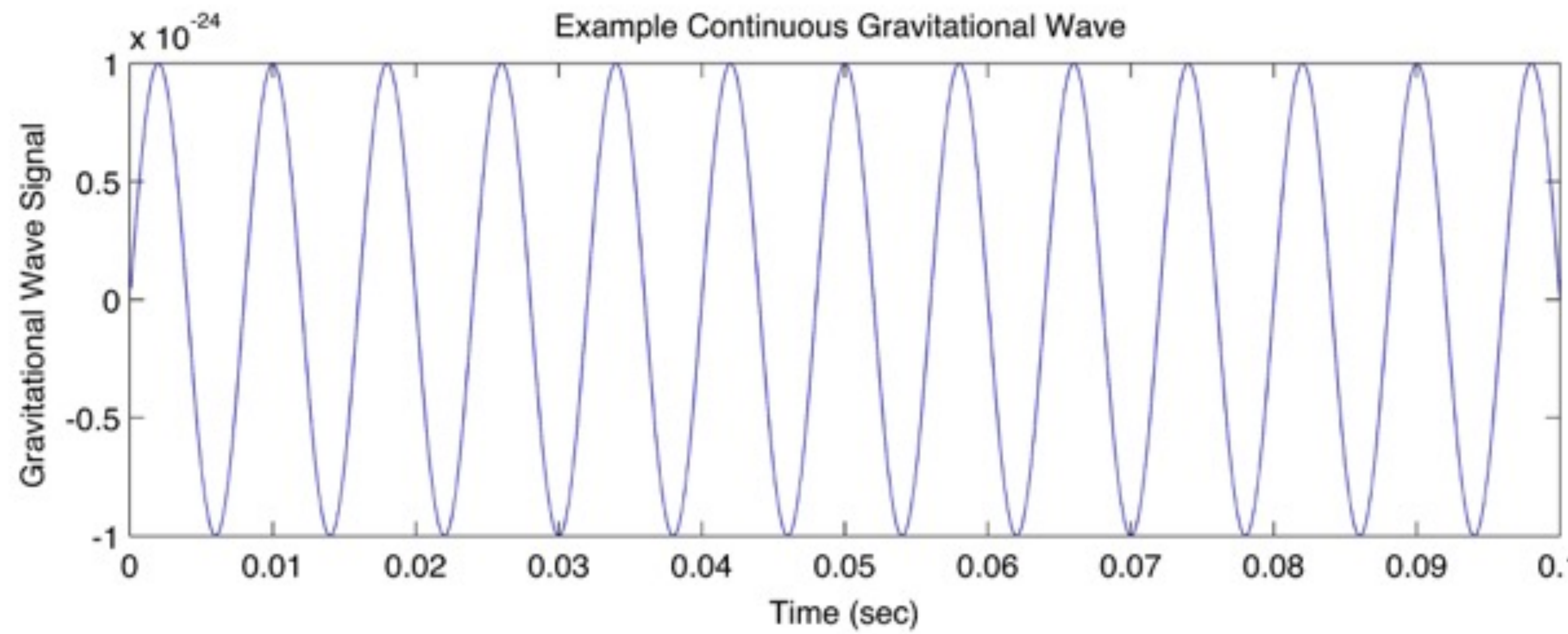

Continuous GWs are produced by systems that have a fairly constant and well-defined frequency.

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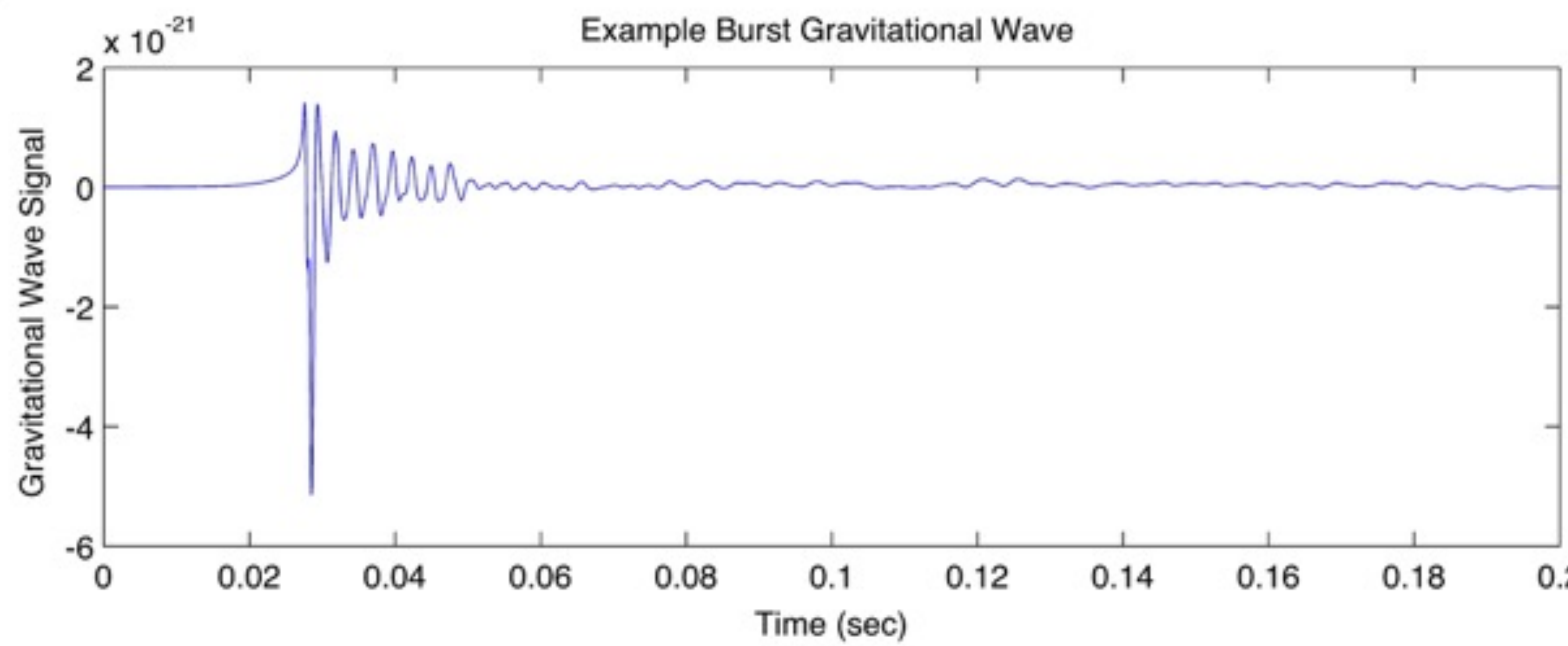

Burst GWs come from short-duration unknown or unanticipated sources. (These are expected unexpected)

Sources of Gravitational Waves

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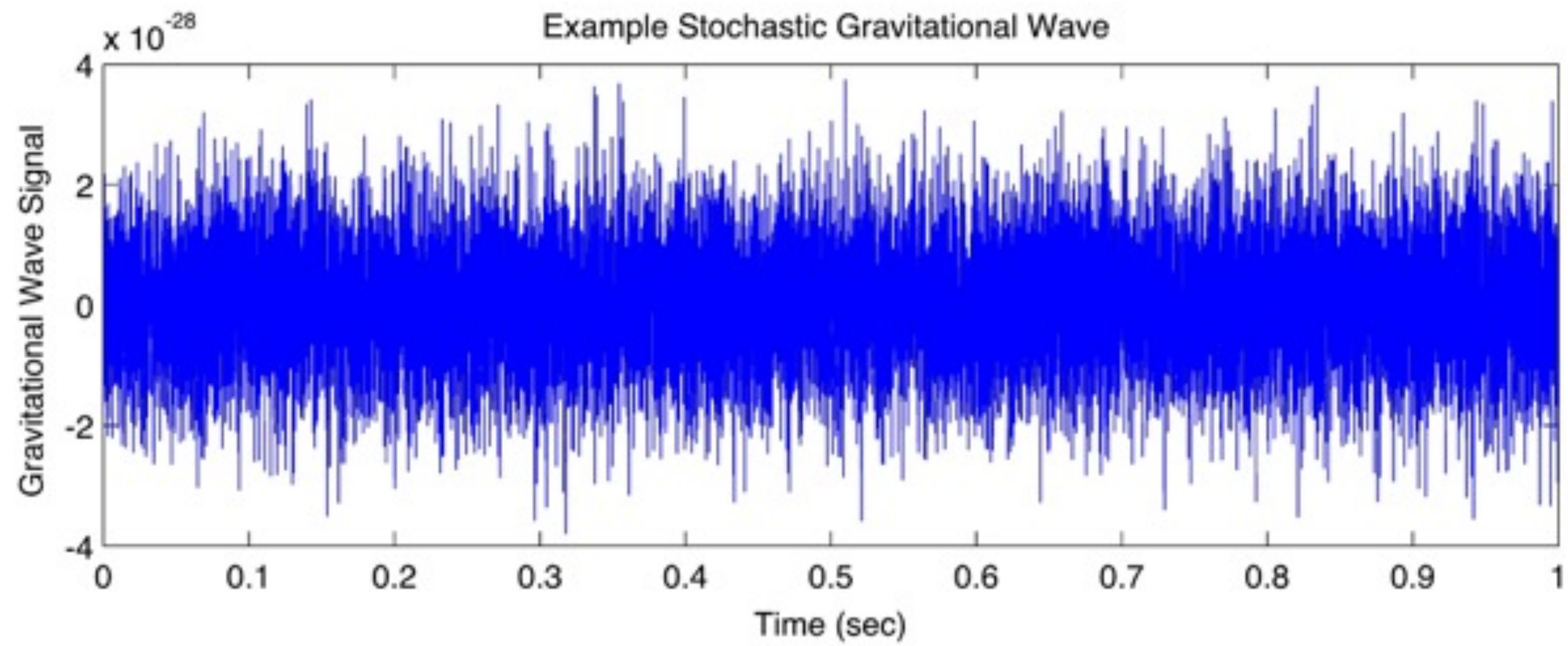
- Continuous
- Burst
- **Stochastic**



SGWB is a combination of GWs which are relic of early universe phenomenas and unresolved astrophysical sources.

Stochastic Gravitational Wave Background

- A background to all gravitational wave signals.
- Source is cosmological or unresolved independent and uncorrected events.

$$\Omega_{gw} = \frac{1}{\rho_c} \frac{d\rho_{gw}}{d\ln(f)}$$

$$h_c(f) = 3 \times 10^{-20} h_{100} \sqrt{\Omega(f)} \frac{100 \text{ Hz}}{f}$$

Gravitational Energy from Gravitationally Bound Objects

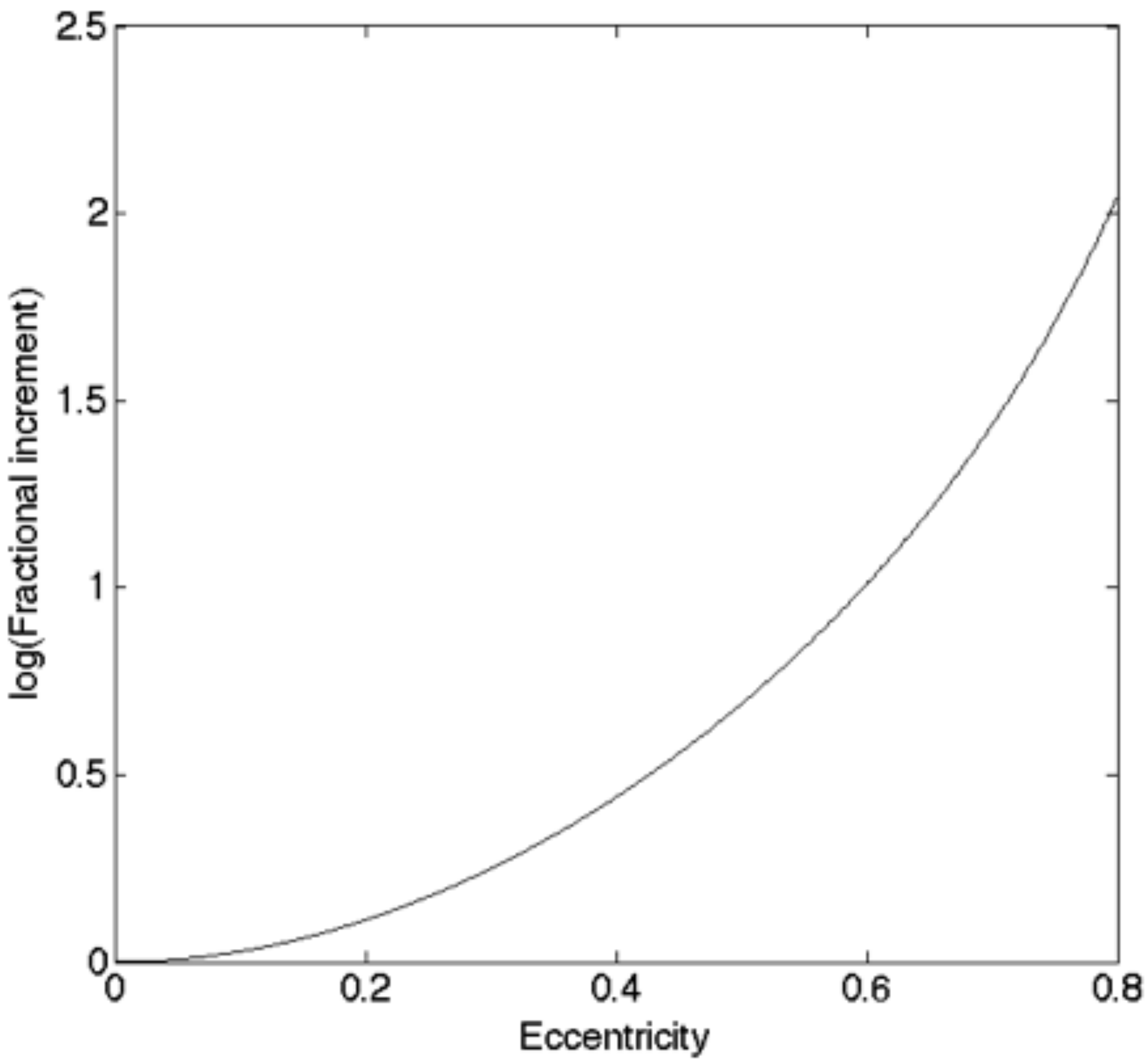
- Two objects in a circular Keplerian orbit radiate gravitational waves with luminosity

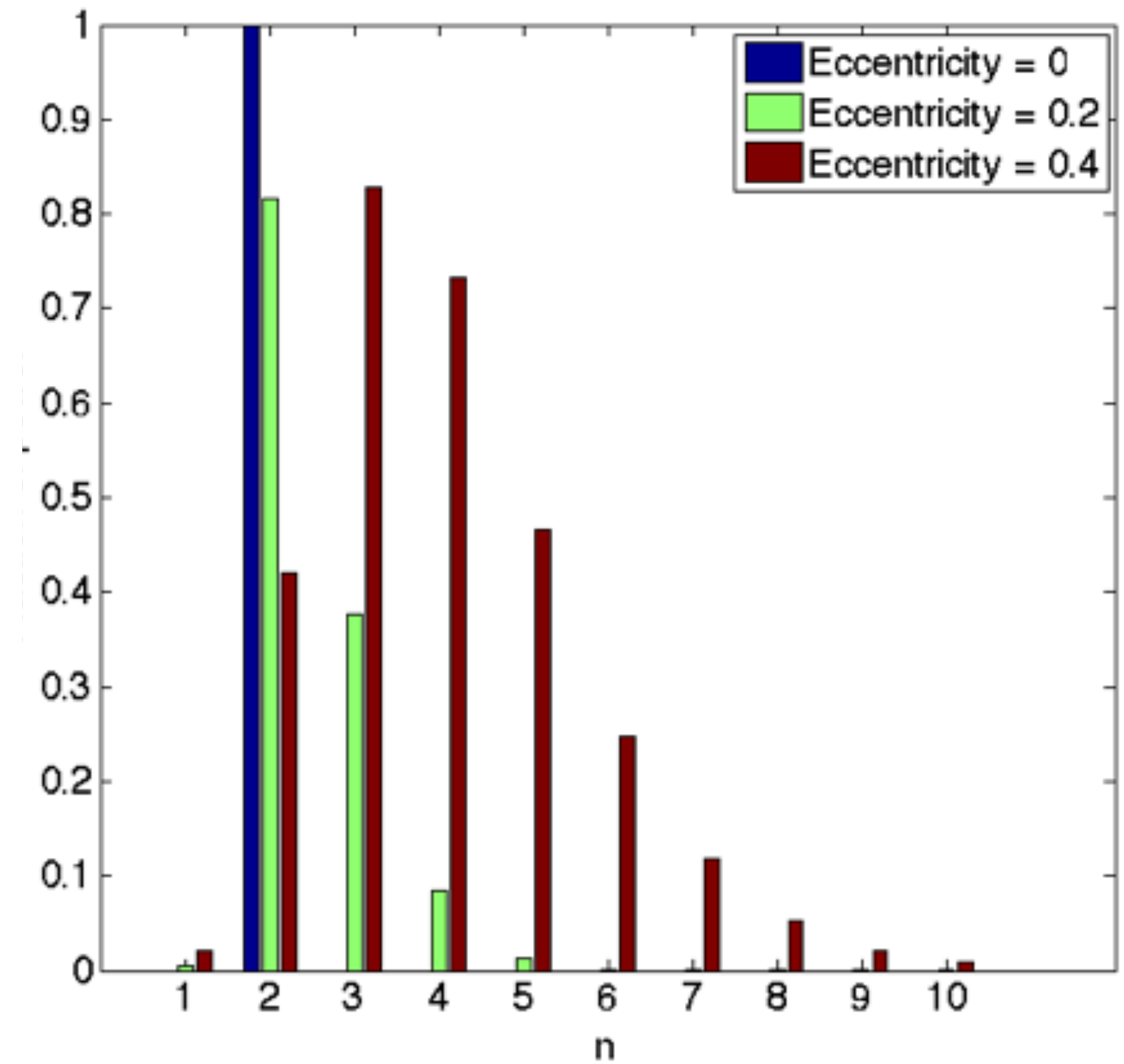
$$L_0 = \frac{32}{5}\frac{G^4}{c^5}\frac{M^2 m^2 (M+m)}{a^5}$$

- The gravitational waves has a frequency twice of the orbital frequency

$$f_{GW} = 2\frac{1}{2\pi}\sqrt{\frac{G(M+m)}{a^3}}$$

Gravitational Wave From Eccentric Orbit





Total Power

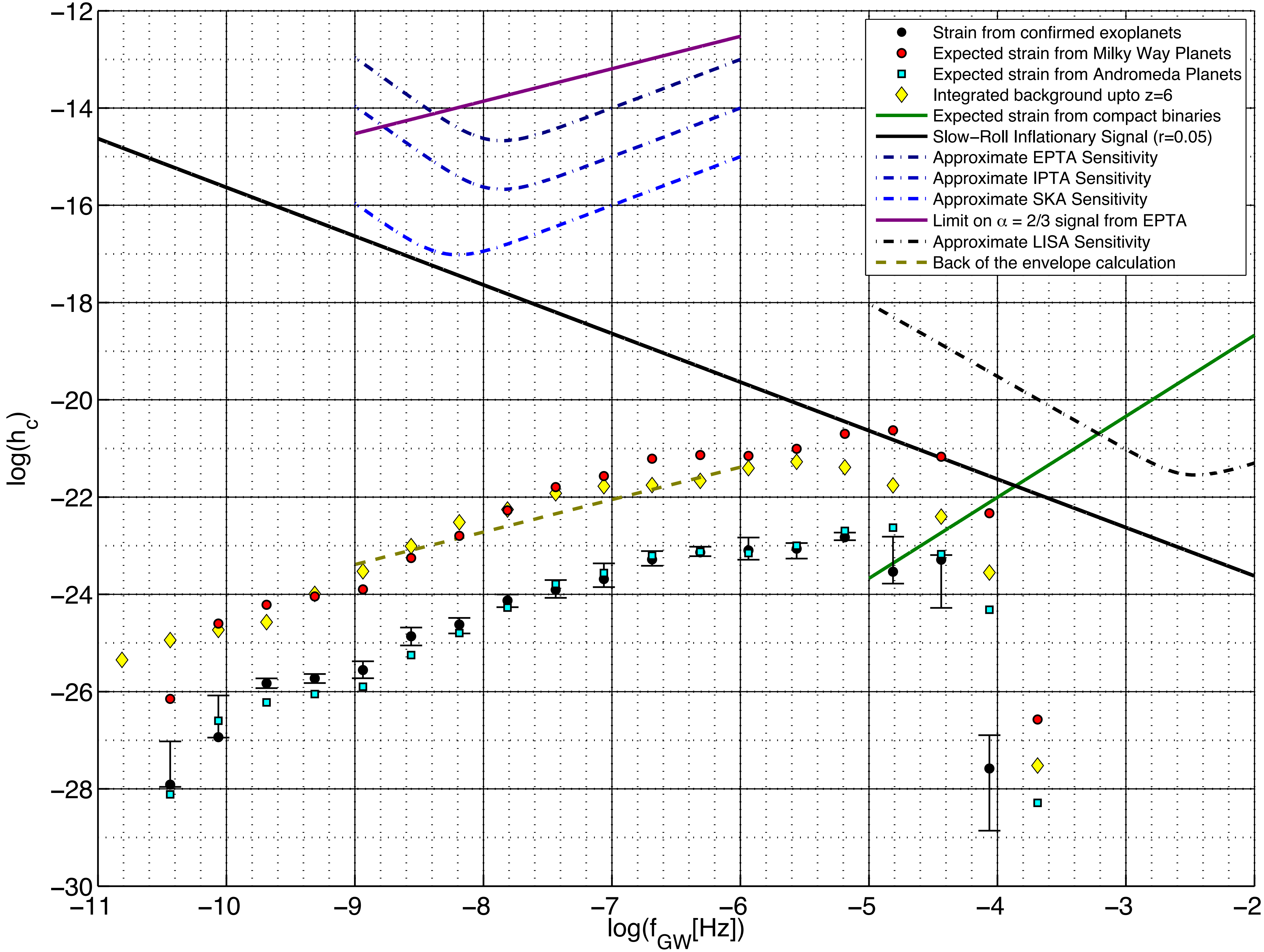
g(n,e)

Exoplanets

- An exoplanet or extrasolar planet is a planet that does not orbit the Sun.
- 1523 confirmed exoplanets, total 4826 candidates.
- “One or more (1.4) bound planets per Milky Way star.”

Exoplanet Databases

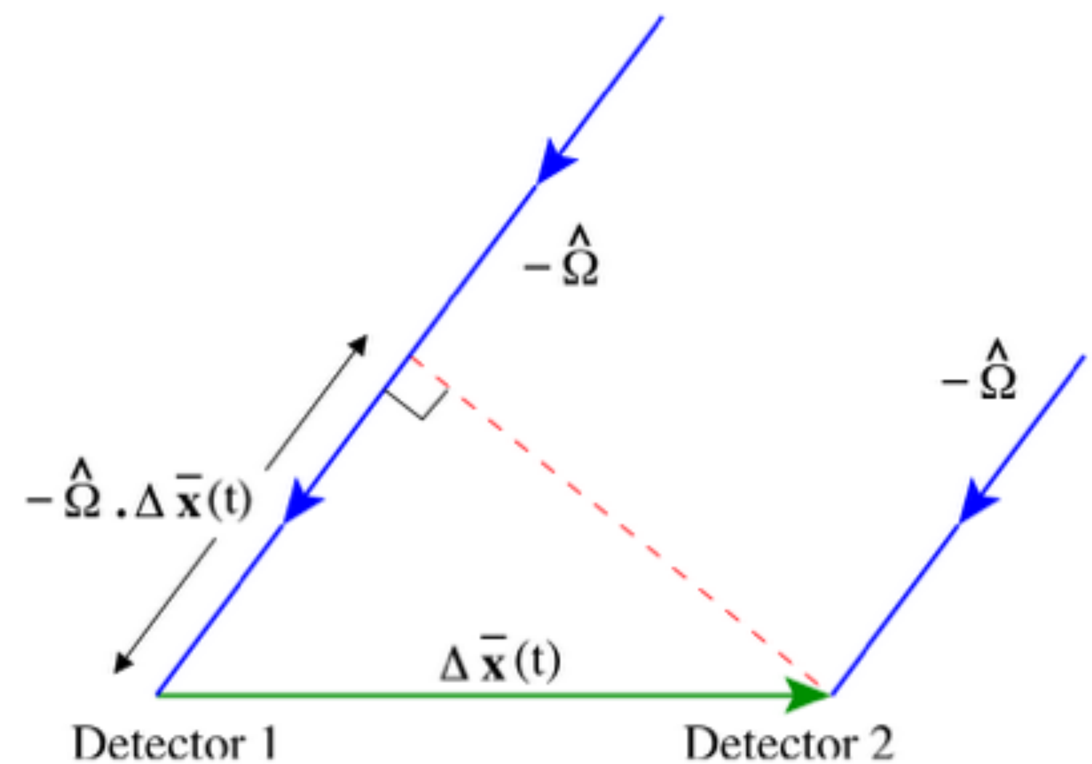
- Many publicly available databases
  - Exoplanet Orbit Database (exoplanets.org)
  - The Extrasolar Planets Encyclopaedia (exoplanet.eu)
  - NASA Exoplanet Archive (exoplanetarchive.ipac.caltech.edu)


Strain from confirmed exoplanets
Expected strain from Milky Way Planets
Expected strain from Andromeda Planets
Integrated background upto z=6
Expected strain from compact binaries
Slow–Roll Inflationary Signal (r=0.05)
Approximate EPTA Sensitivity
Approximate IPTA Sensitivity
Approximate SKA Sensitivity
Limit on α = 2/3 signal from EPTA
Approximate LISA Sensitivity
Back of the envelope calculation
log(h_c)
log(f_GW[Hz])
−12
−14
−16
−18
−20
−22
−24
−26
−28
−30
−11
−10
−9
−8
−7
−6
−5
−4
−3
−2

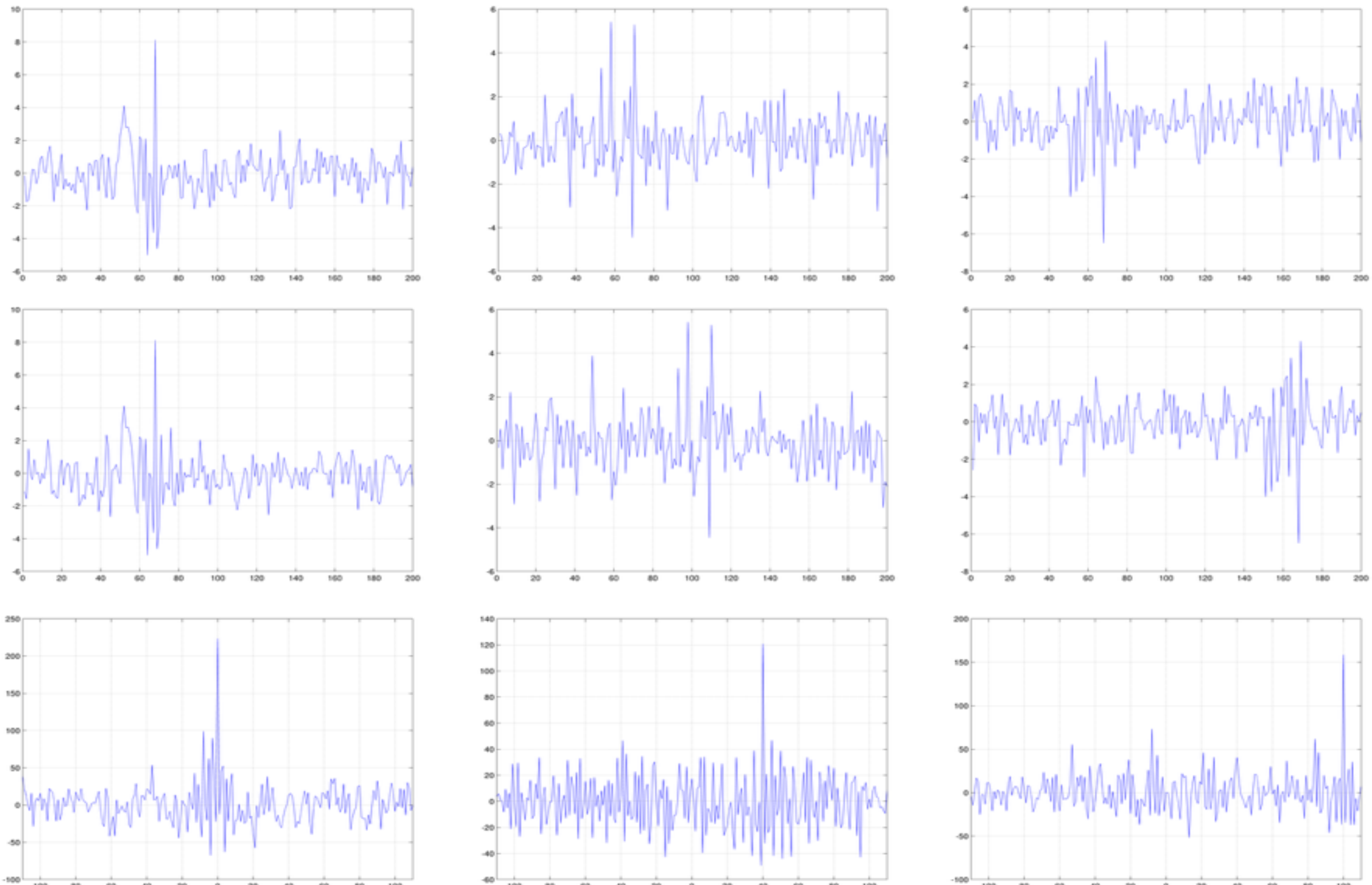
Radiometric Mapping of SGWB

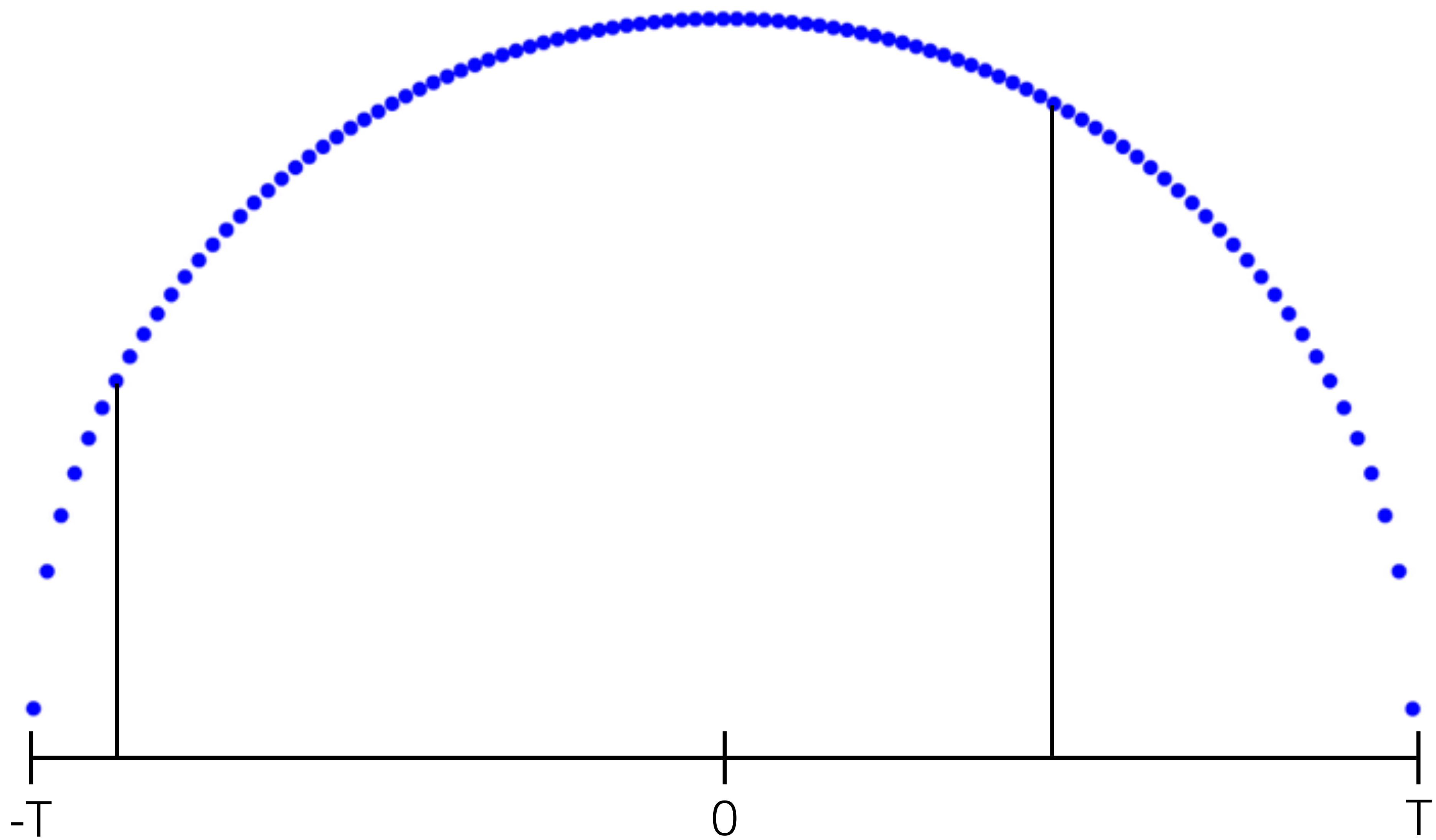
With data from 2 detectors using interferometry (similar to radio astronomy) it is possible to map the entire sky using Aperture synthesis techniques.

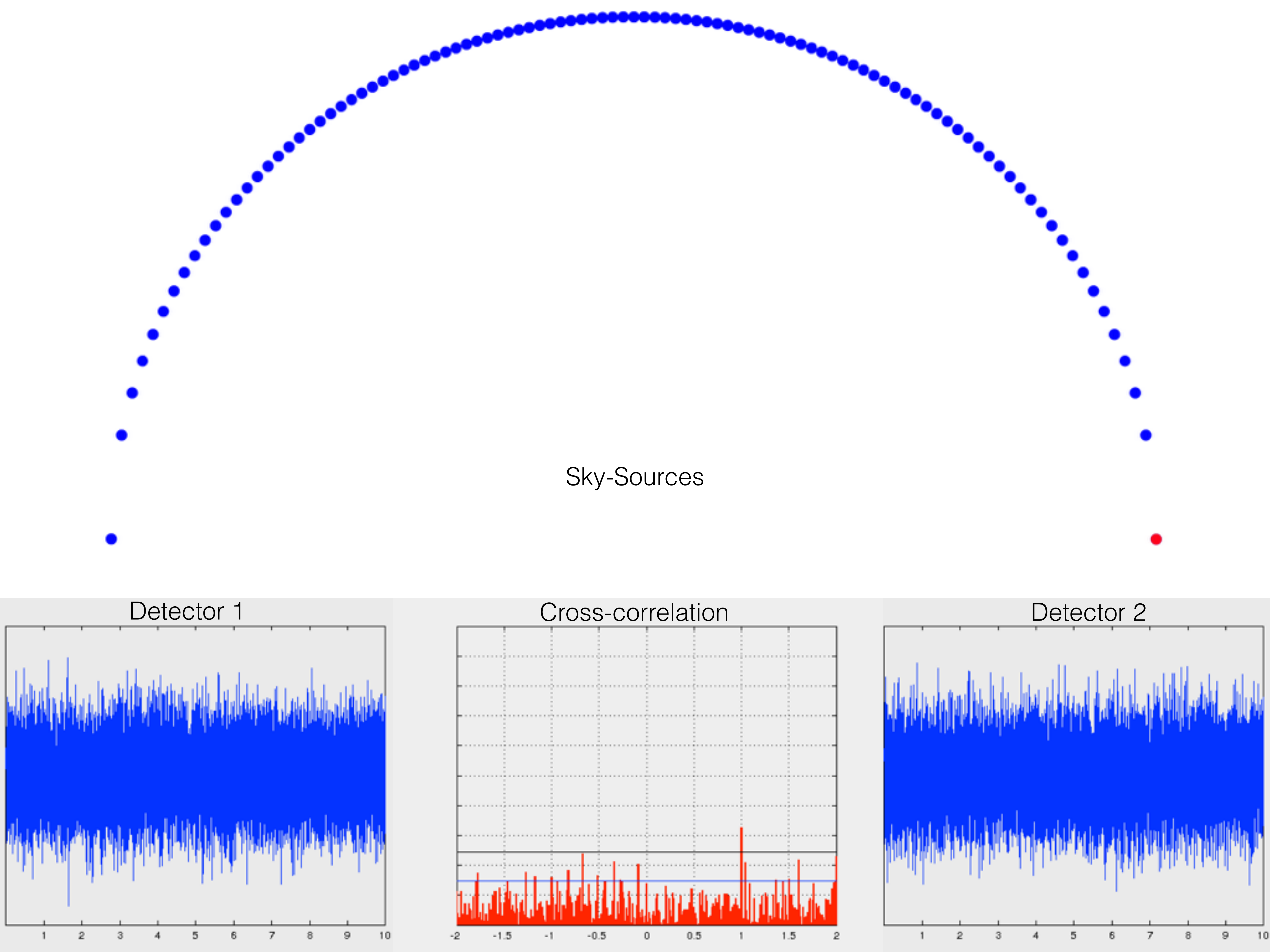
- Cross-correlate detector outputs
- Make maps using time-dependent phase delay

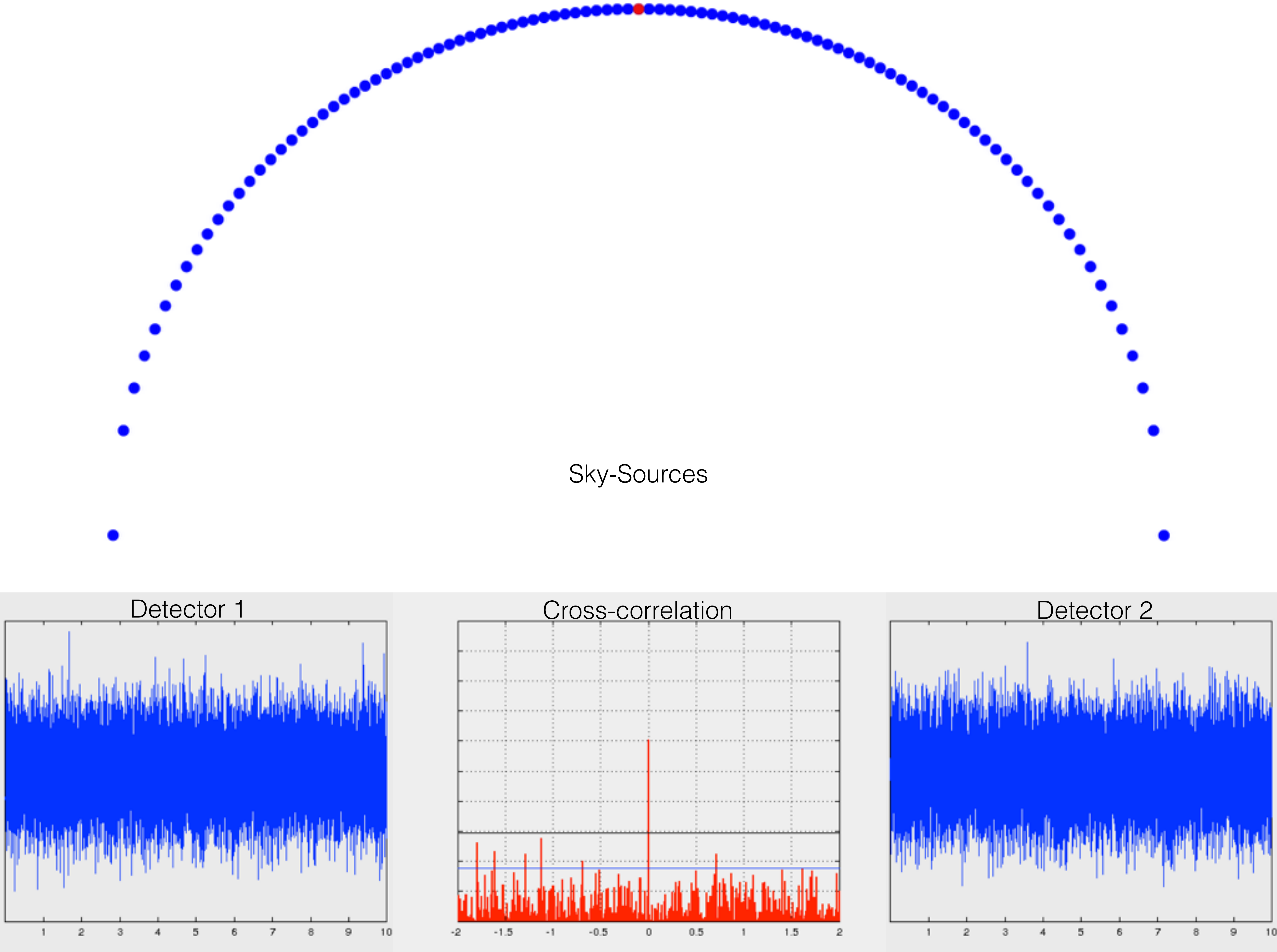


Cross-Correlation to detect pattern




-T
0
T


Sky-Sources
Detector 1
Cross-correlation
Detector 2
1 2 3 4 5 6 7 8 9 10
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2
1 2 3 4 5 6 7 8 9 10



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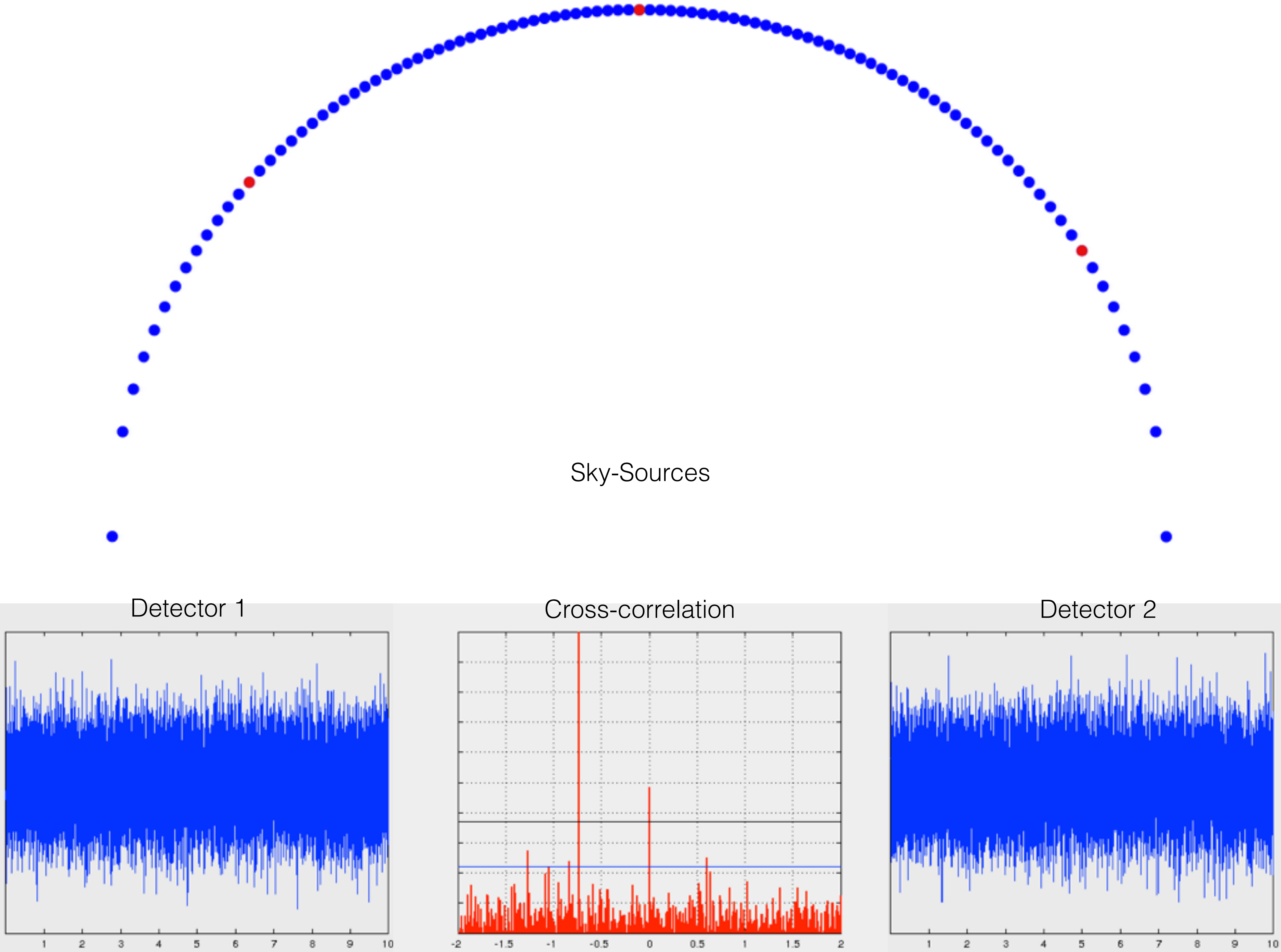
Sky-Sources
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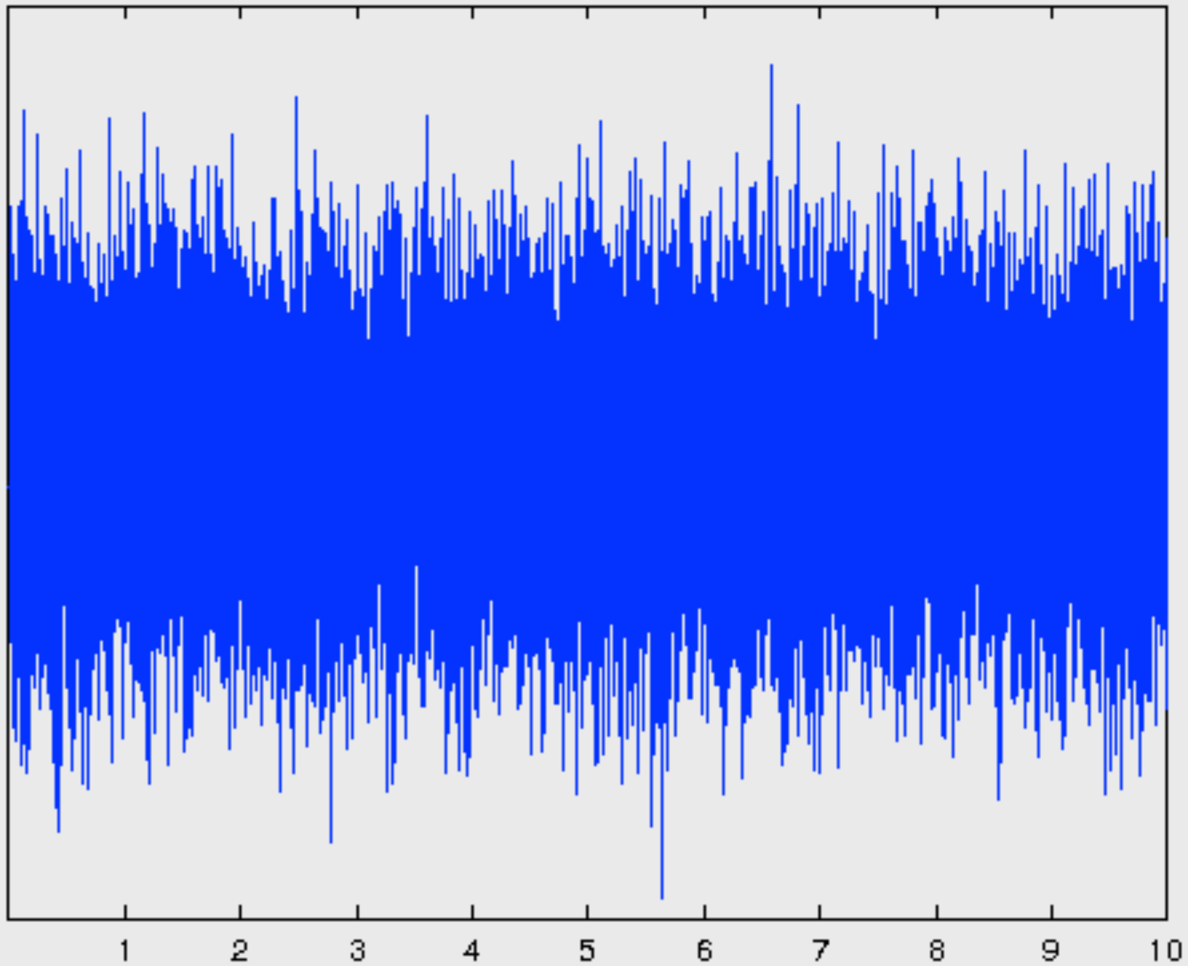
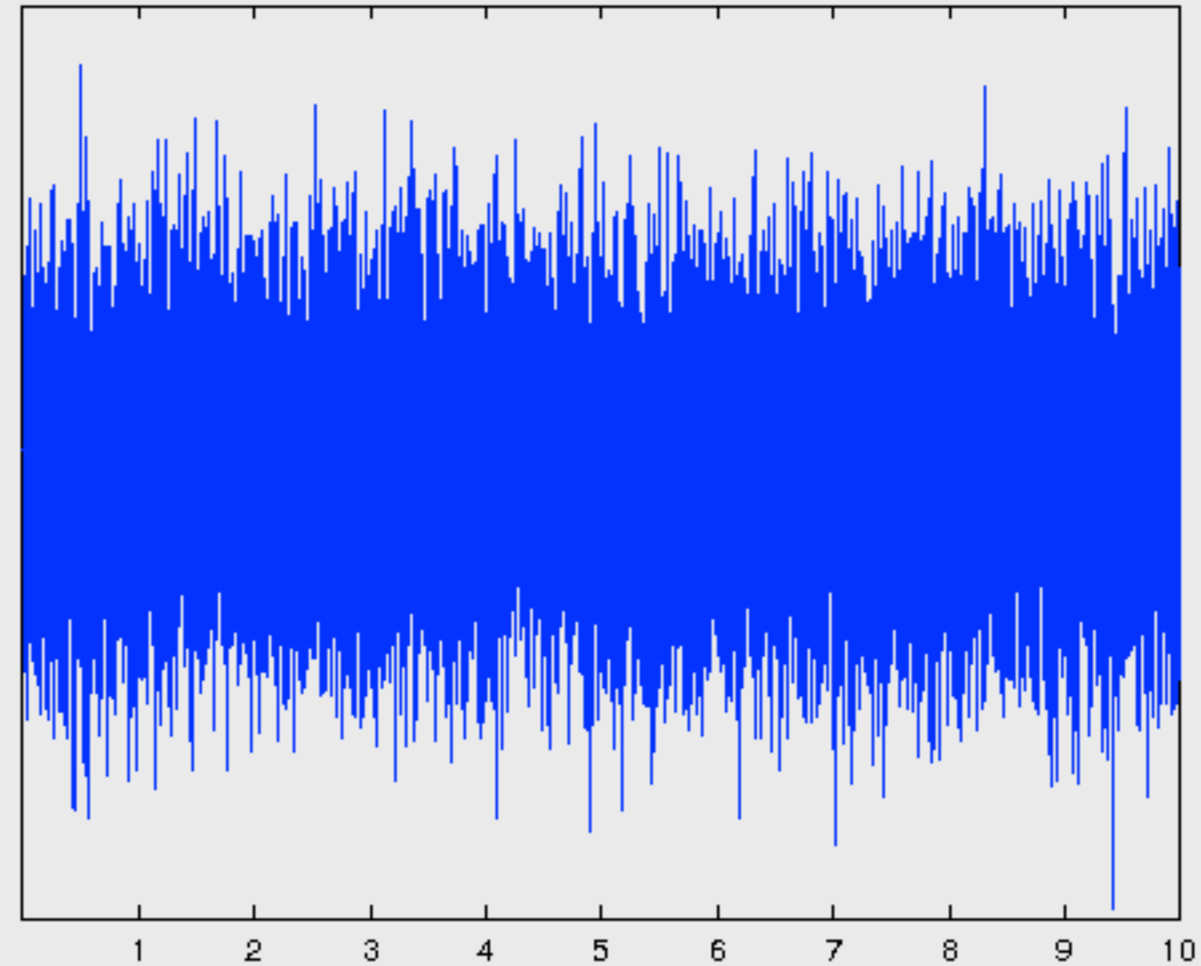
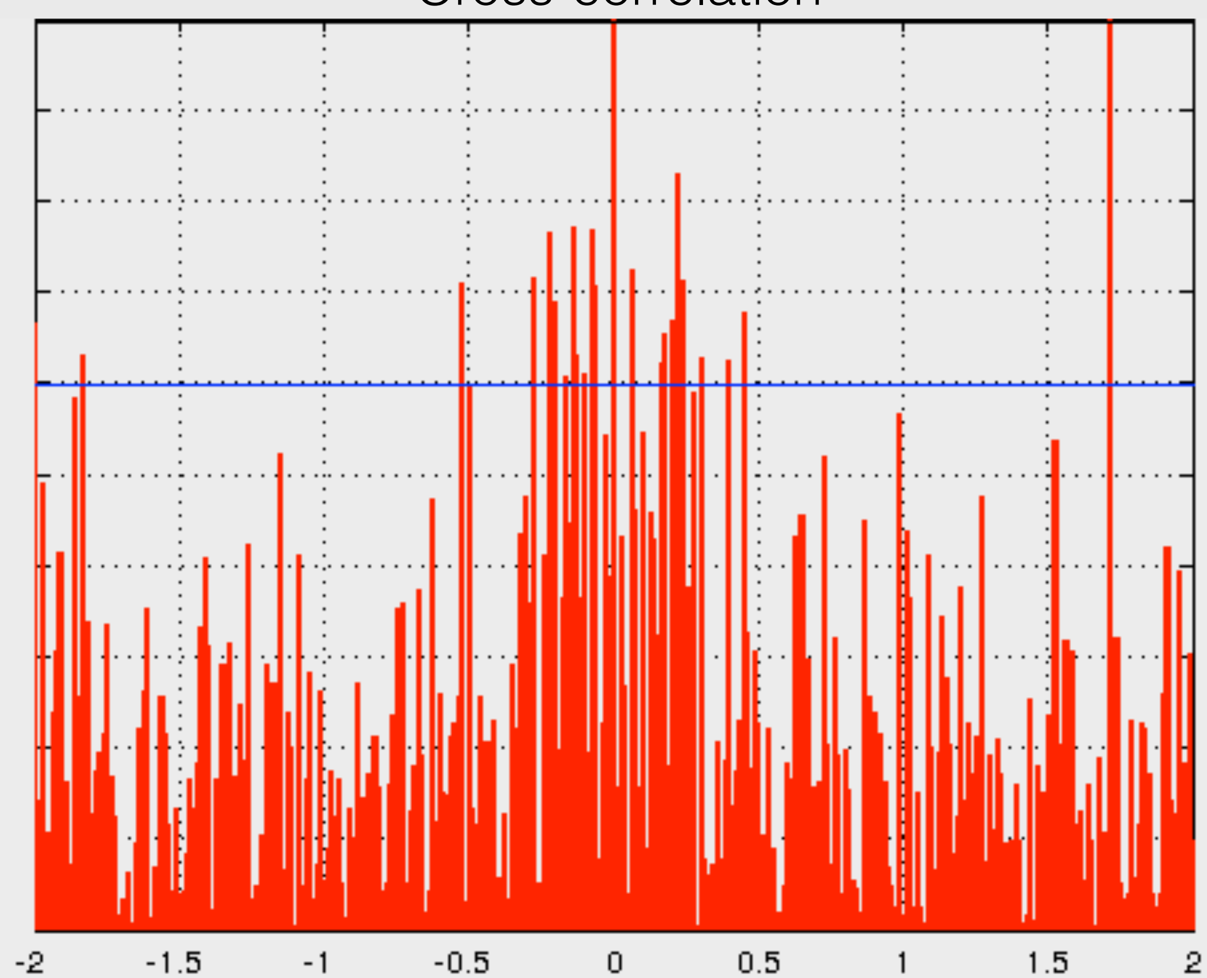


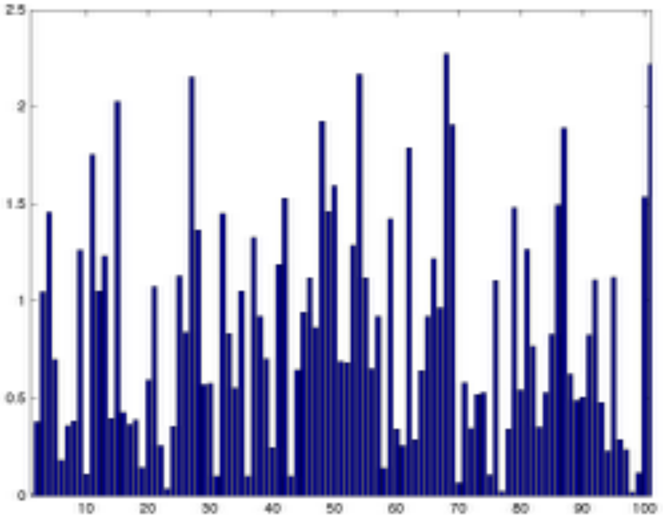
Detector 1



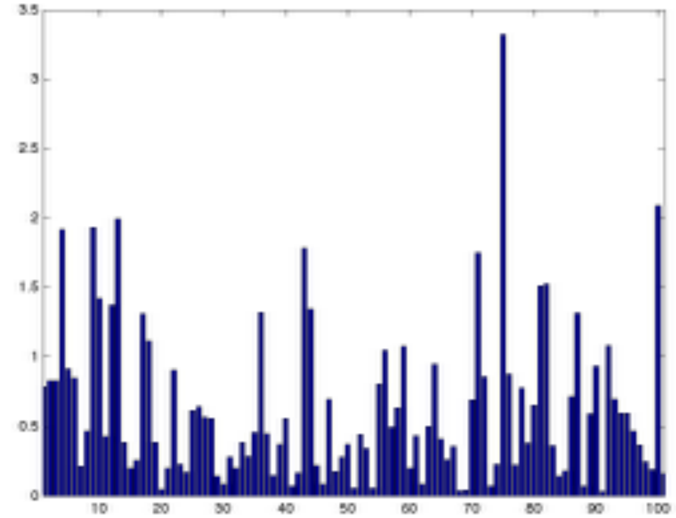
Detector 2

Cross-correlation

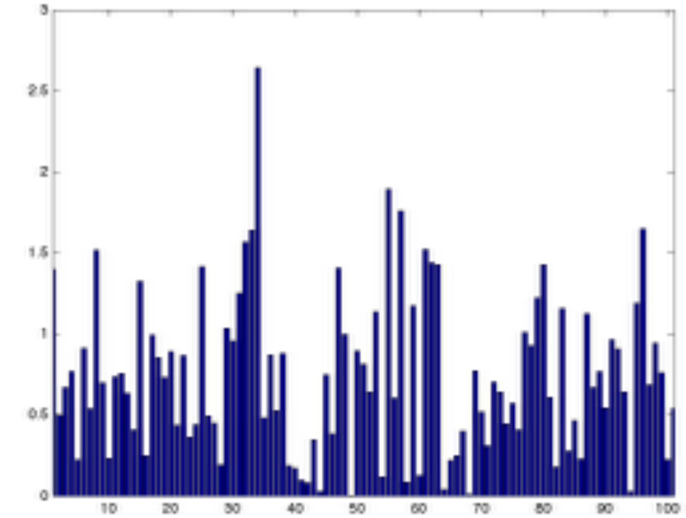




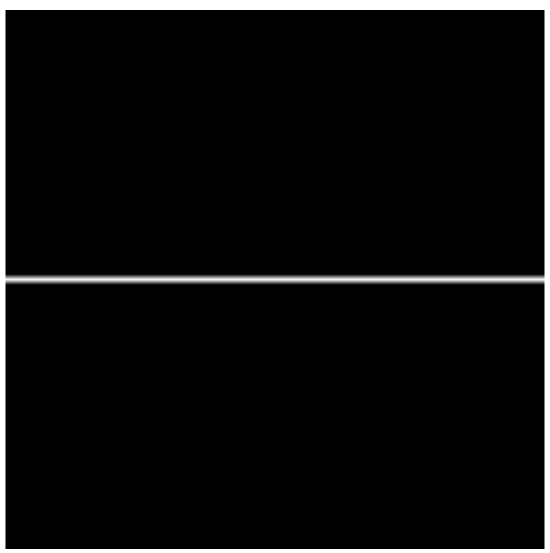
Cross-correlation 1



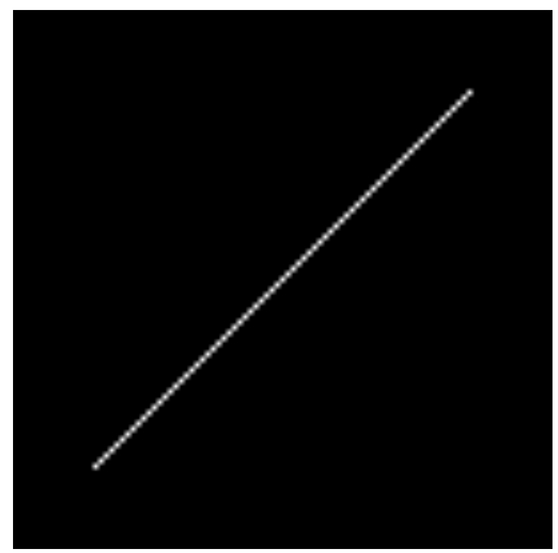
Cross-correlation 2



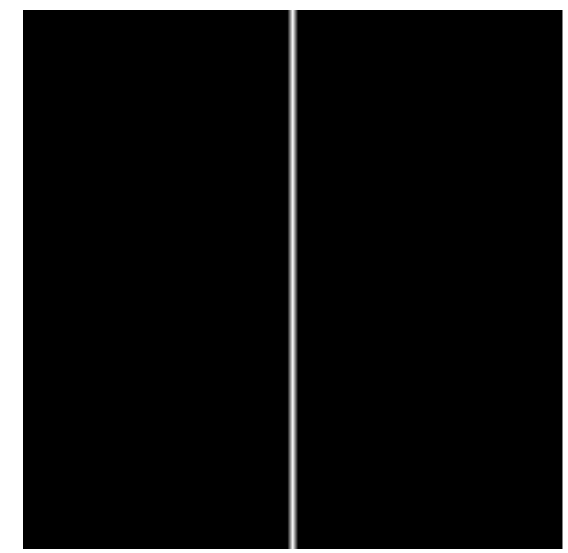
Cross-correlation 3



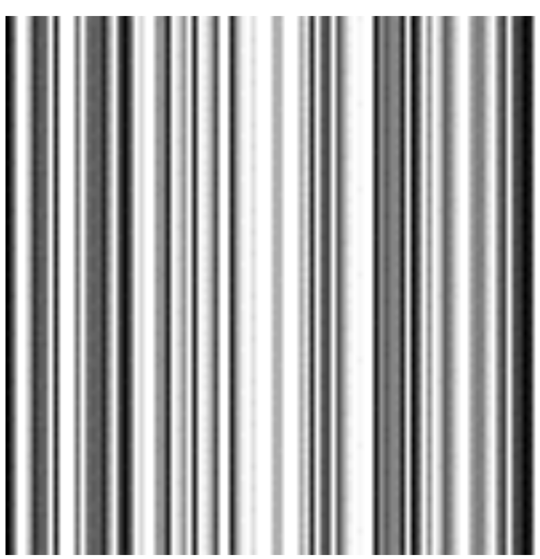
Baseline 1



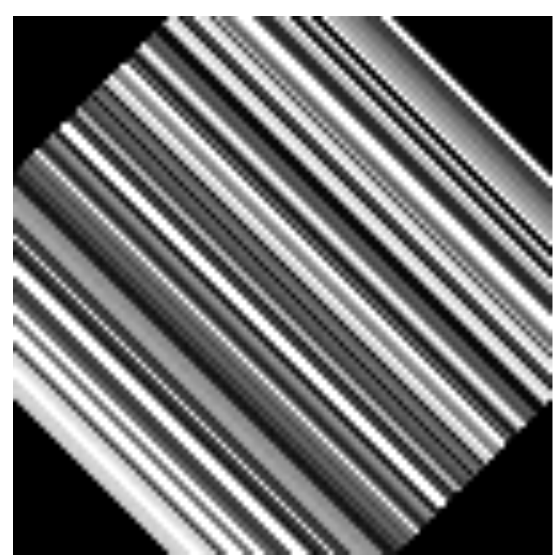
Baseline 2



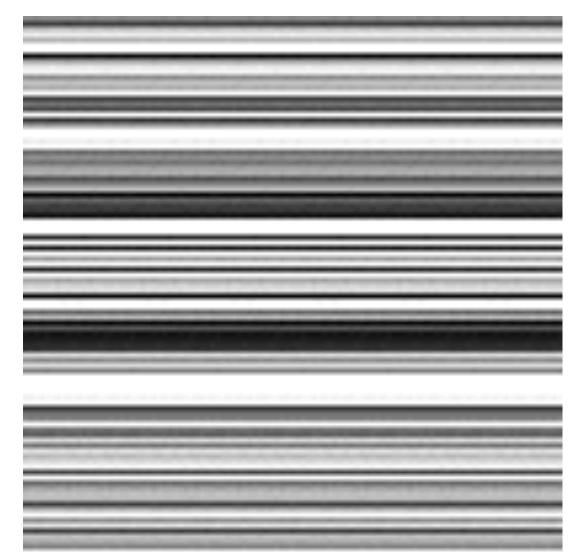
Baseline 3



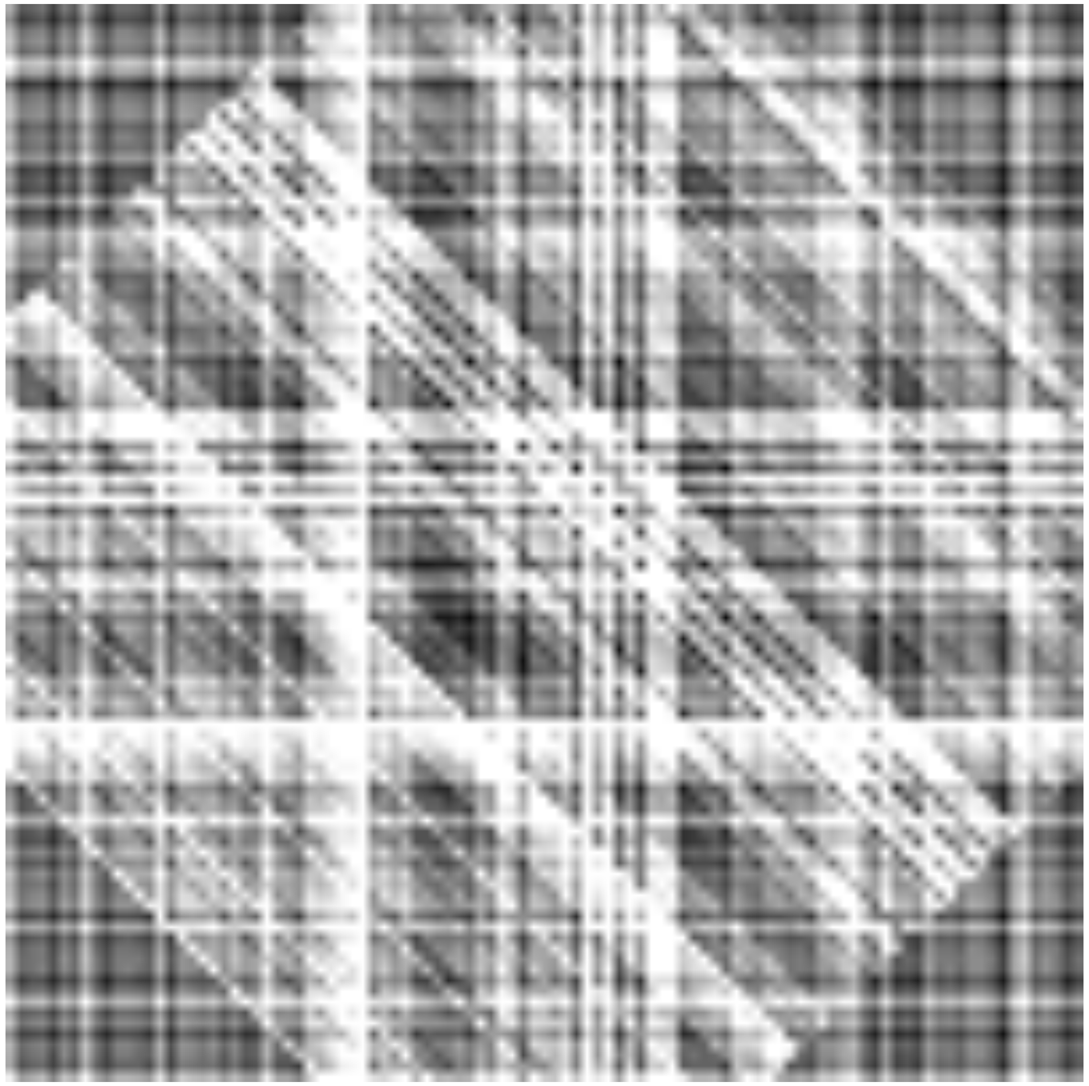
sky-map 1



sky-map 2



sky-map 3



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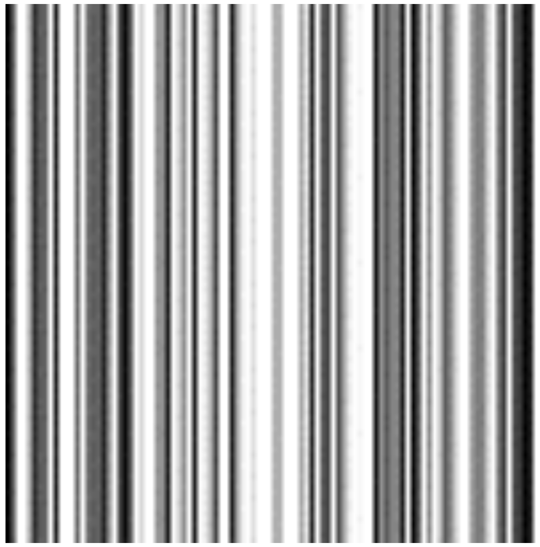

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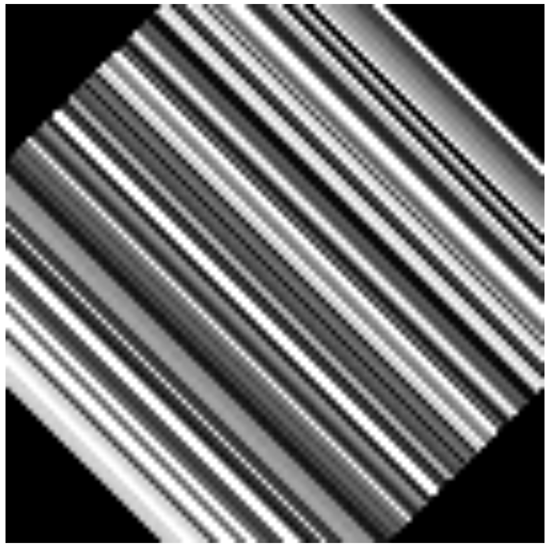
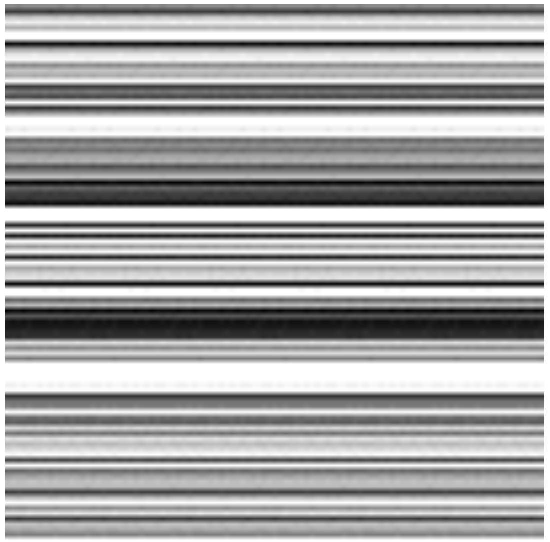


sky-map 2



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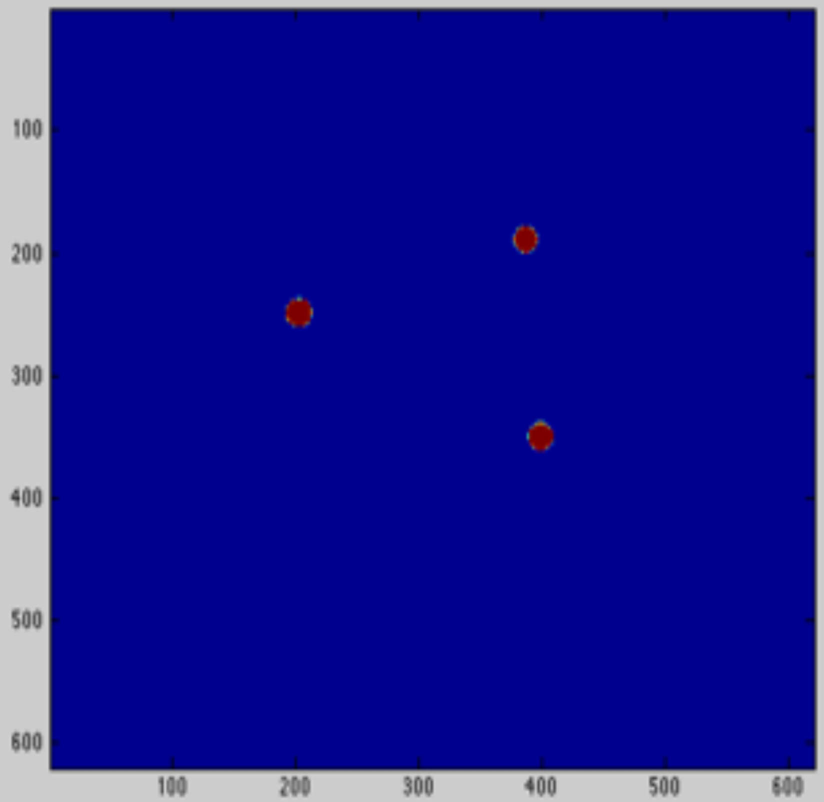
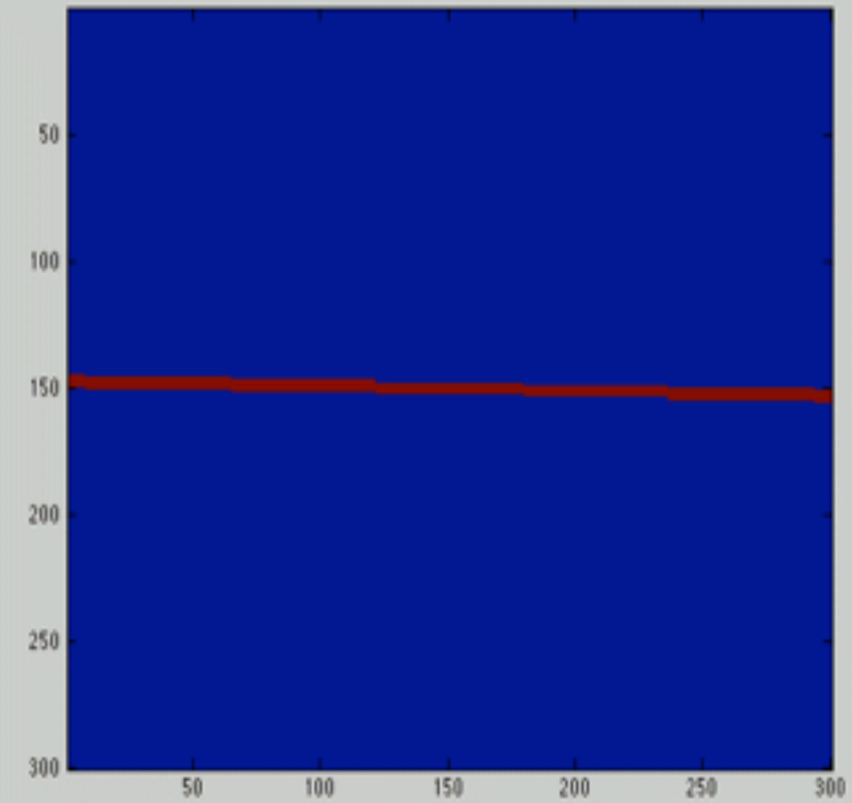
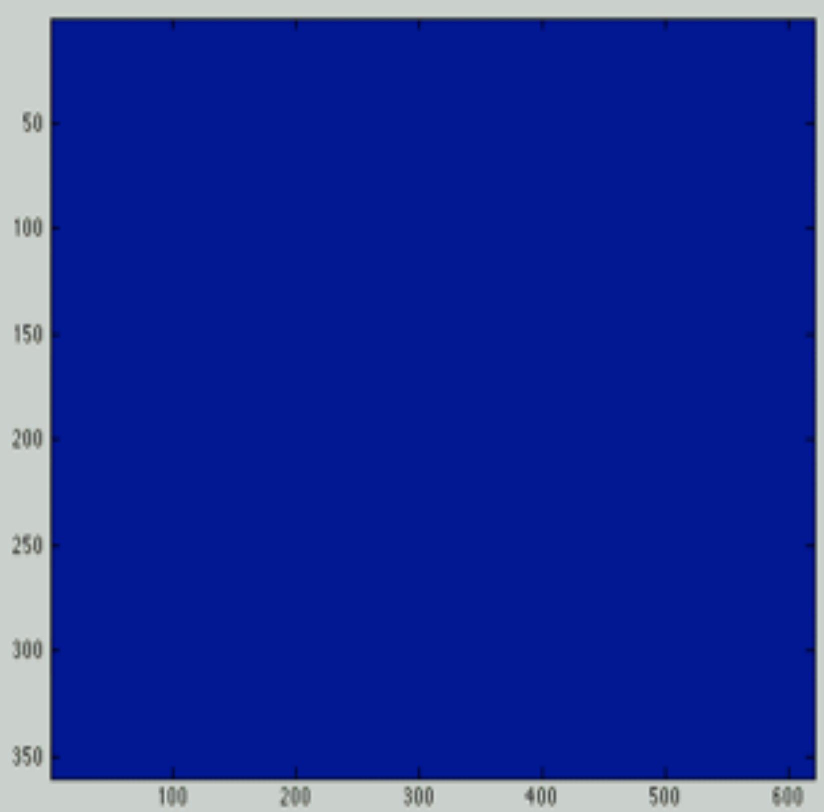


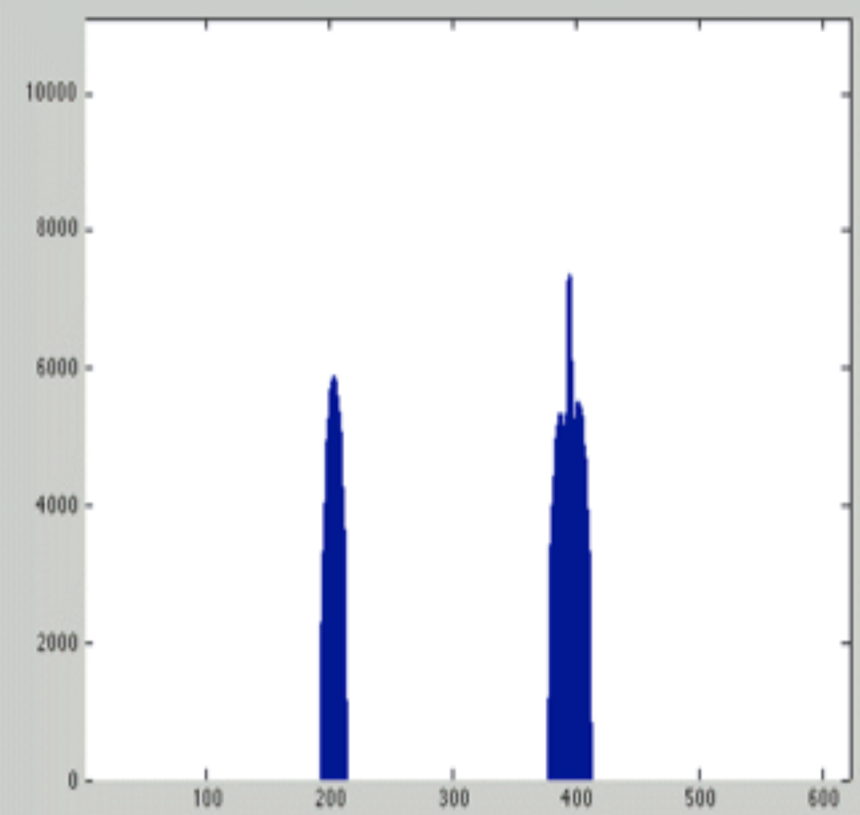
Actual Sky



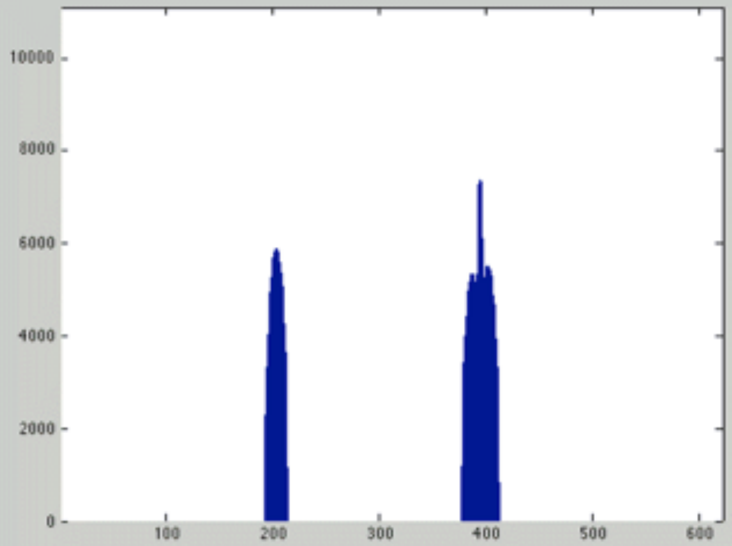
Baseline



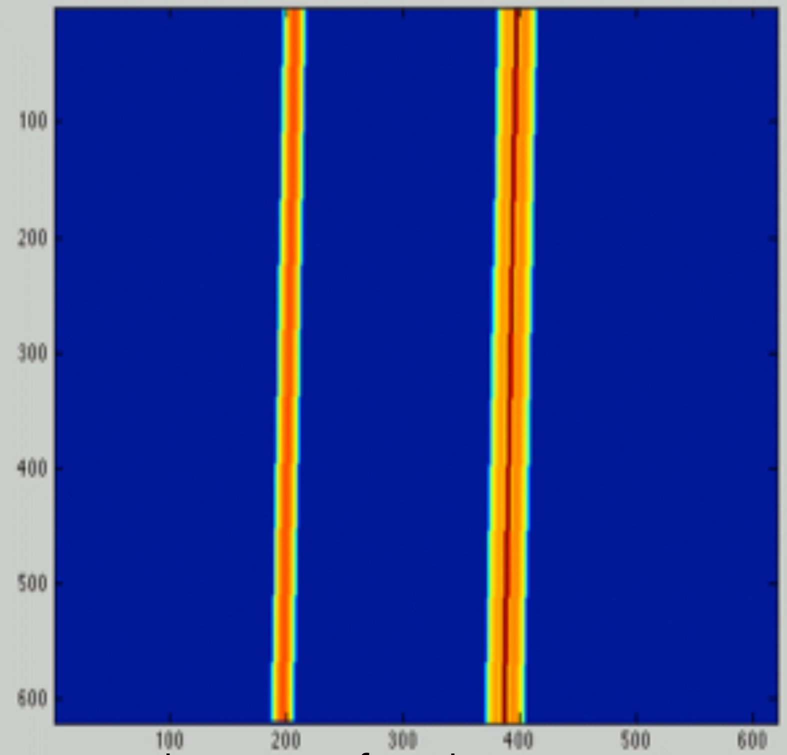
CC Throughout the day



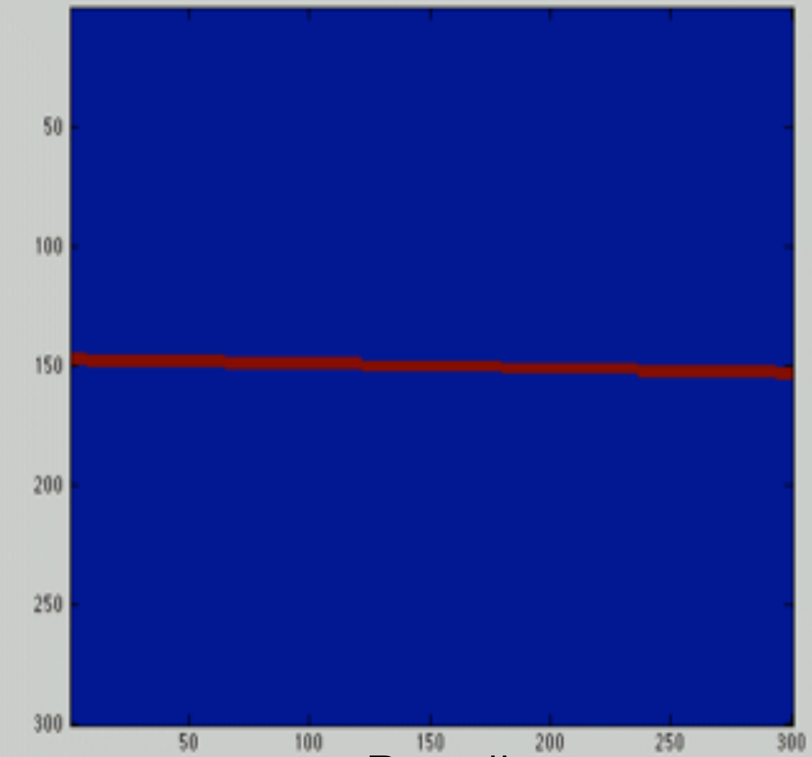
CC of signals
(projection on baseline)



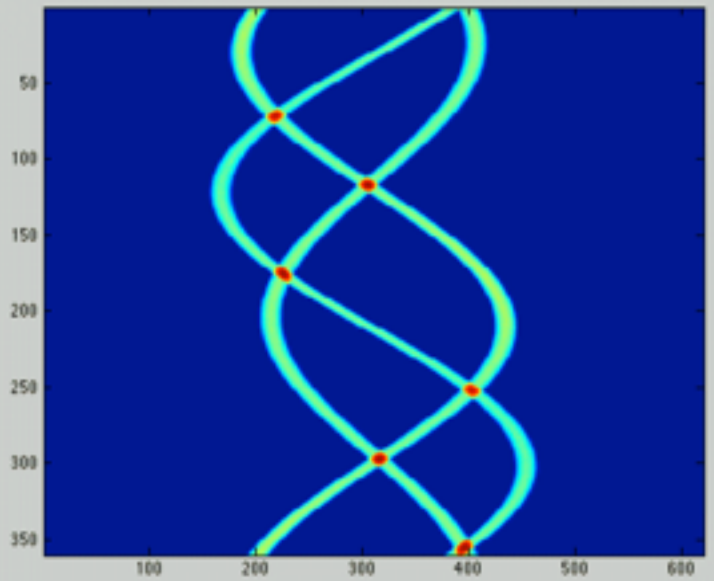
CC of one segment



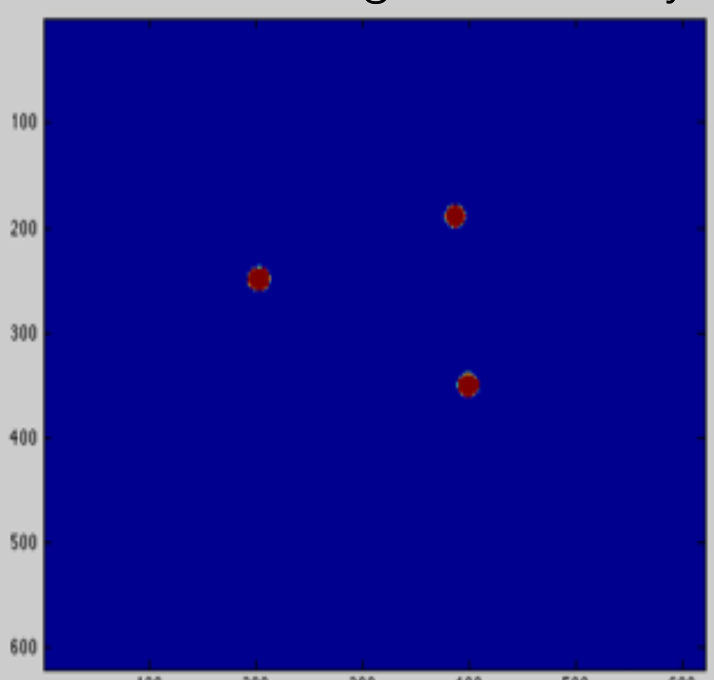
sky-map for that segment



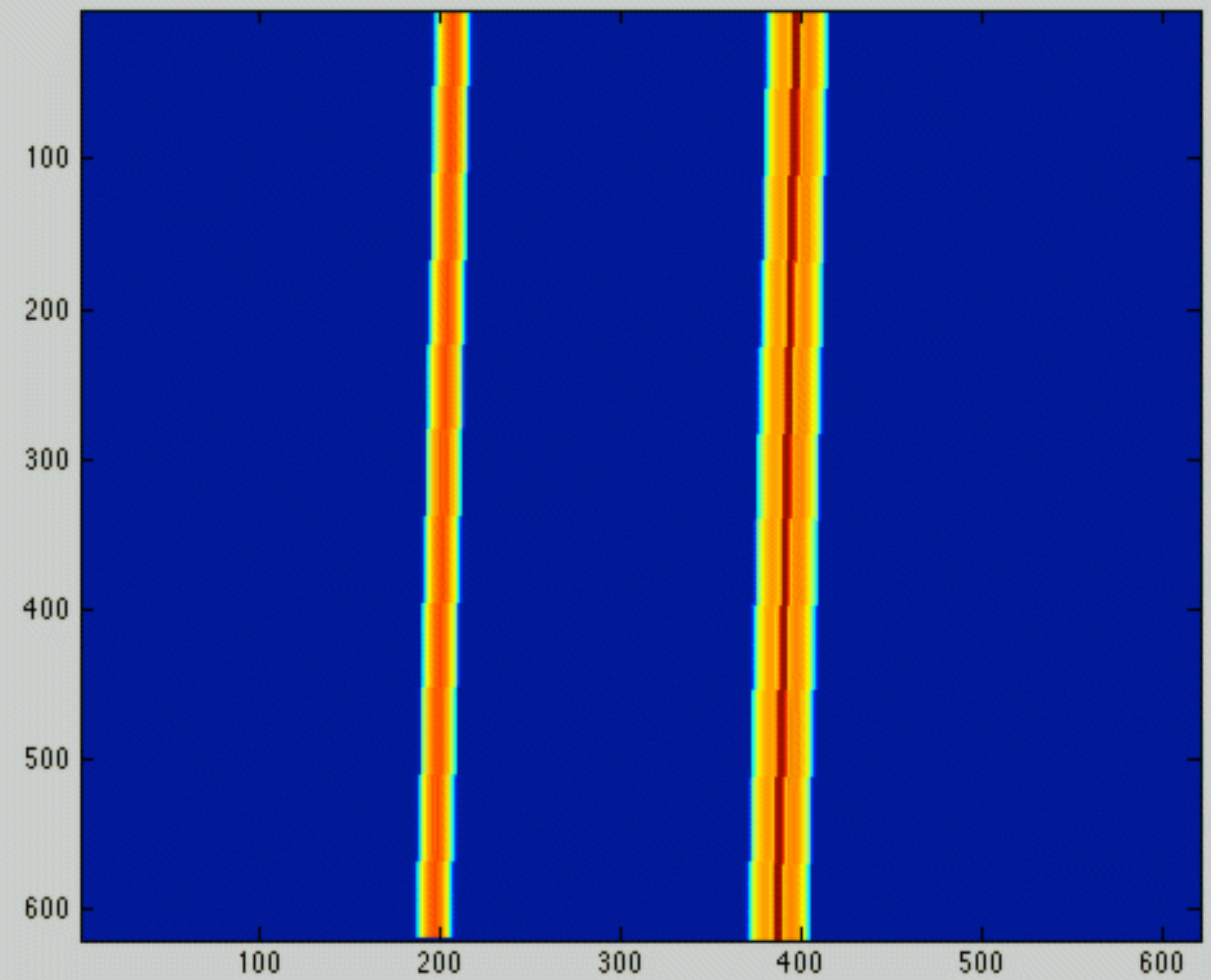
Baseline



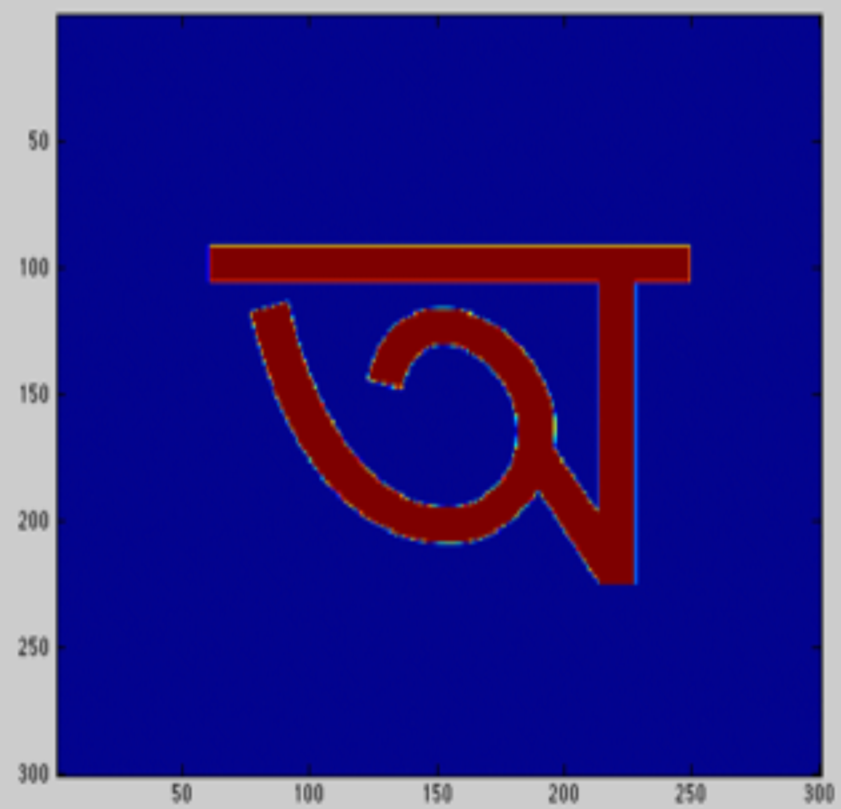
CC Throughout the day



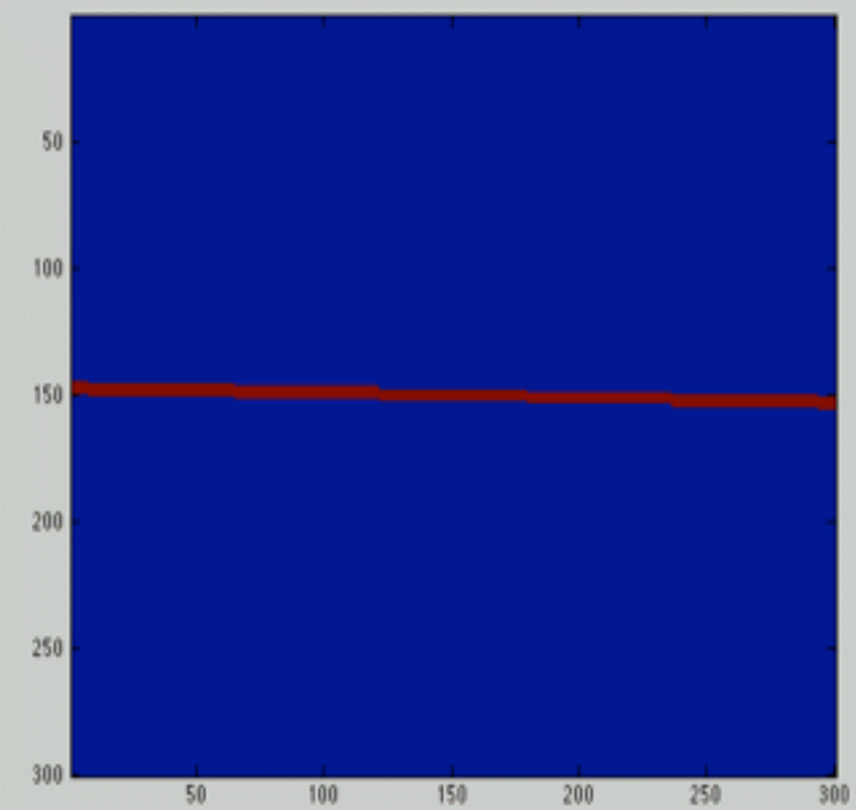
Actual Sky



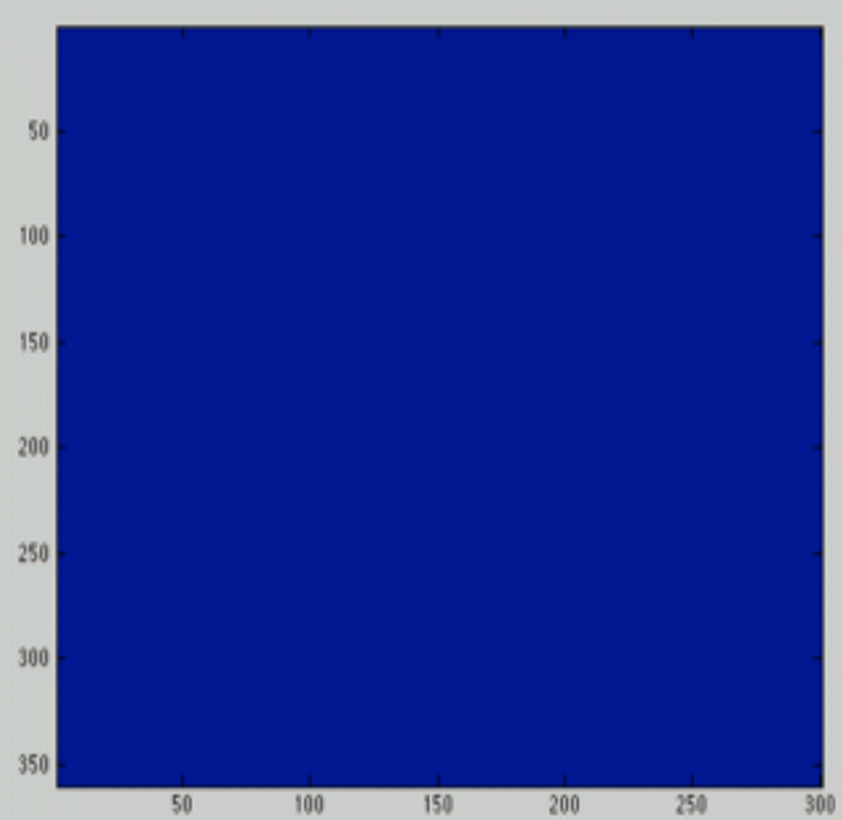
sum of the sky-maps (dirty map)



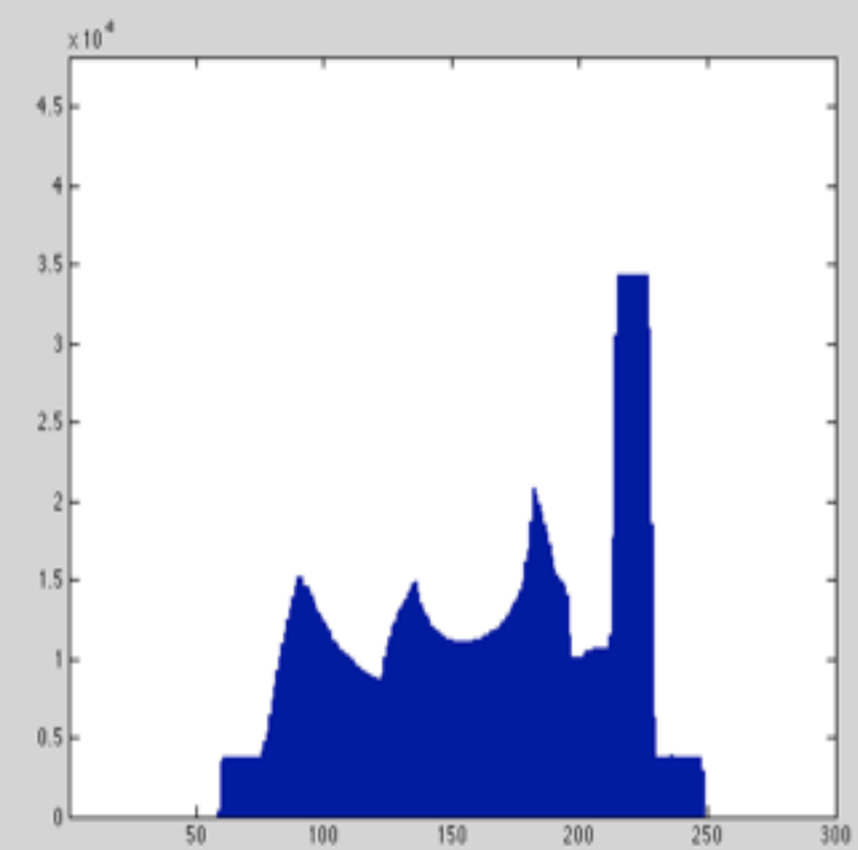
Actual Sky



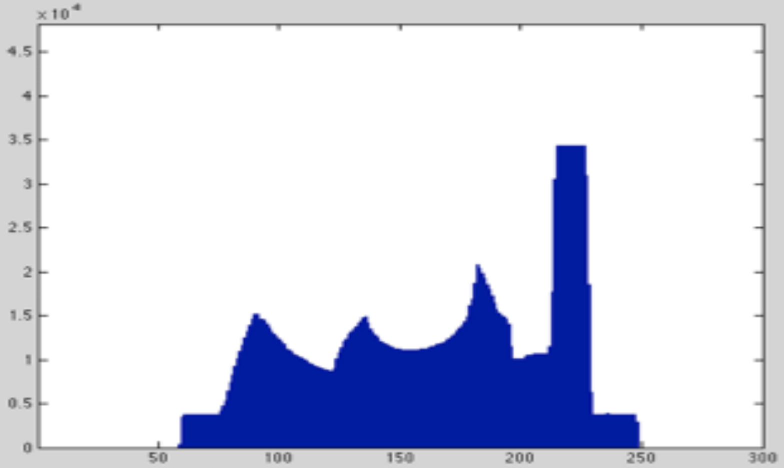
Baseline



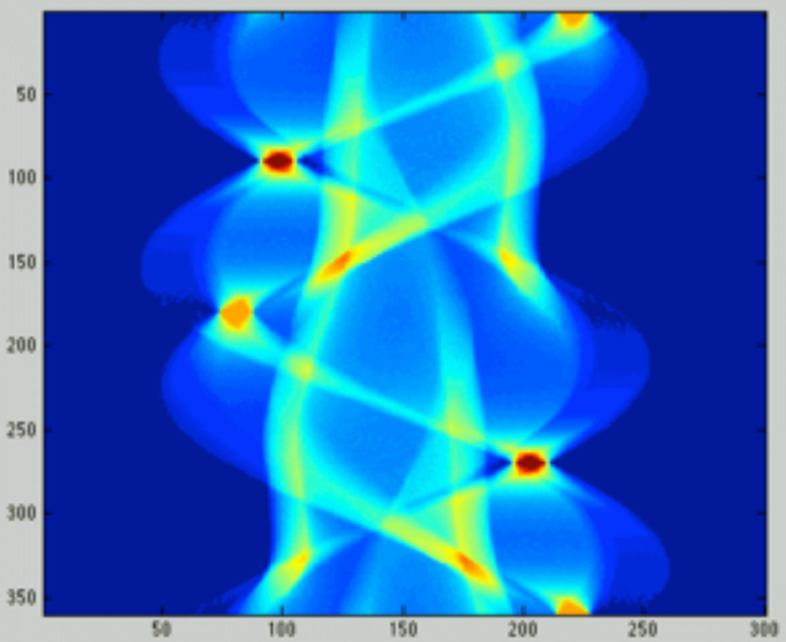
CC Throughout the day



CC of signals
(projection on baseline)

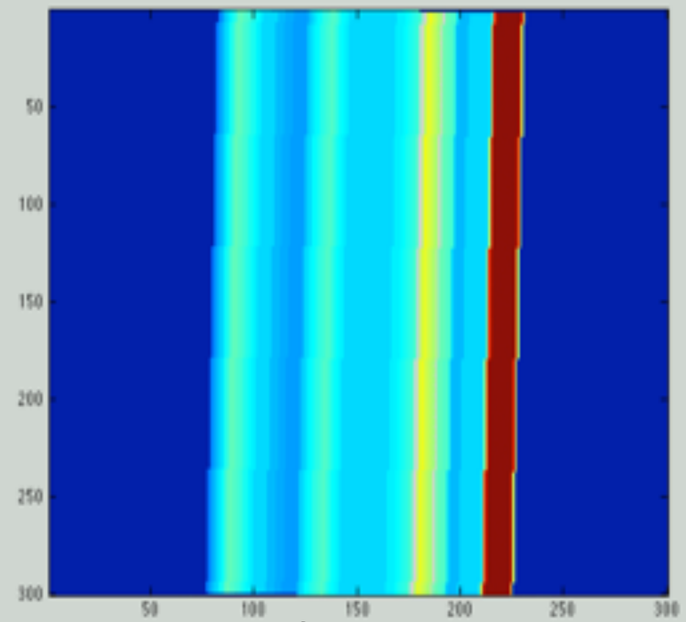


CC of one segment

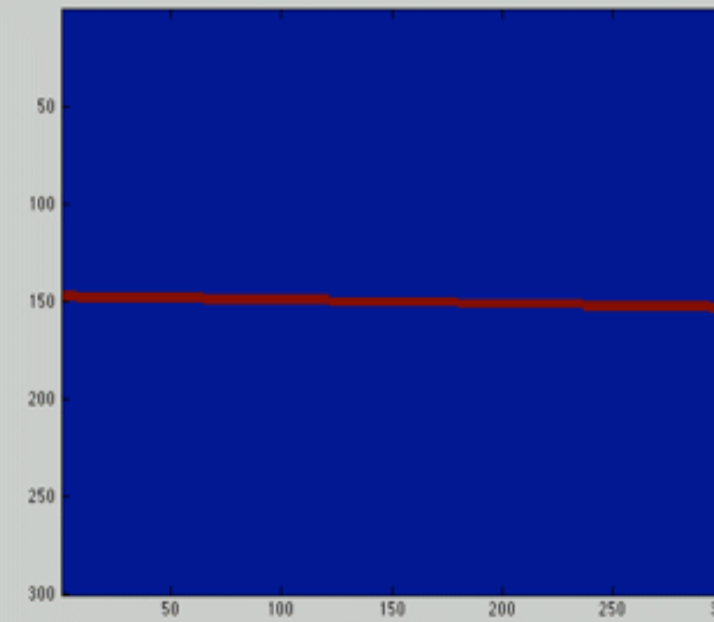


CC Throughout the day

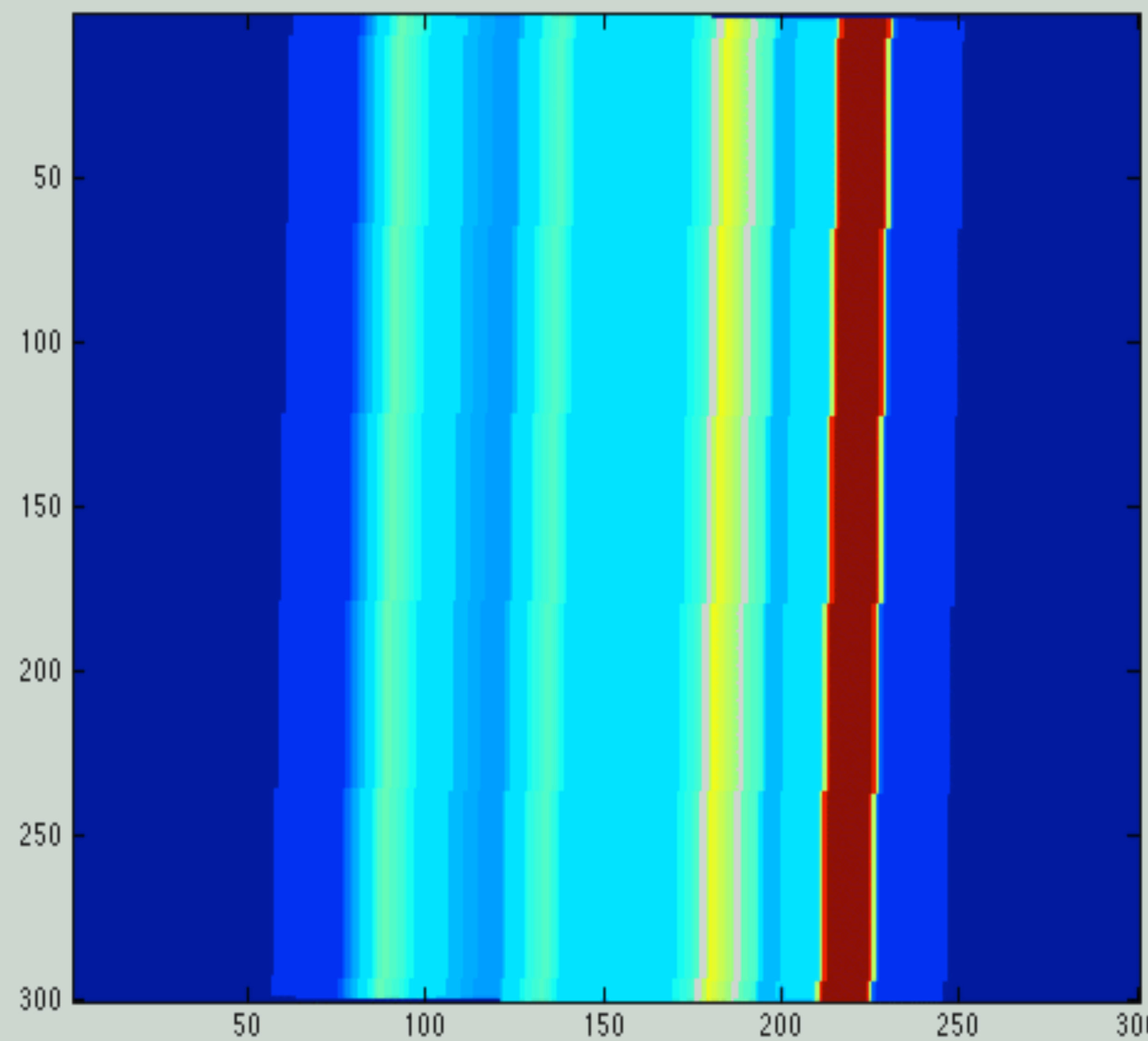
Actual Sky



sky-map for that segment

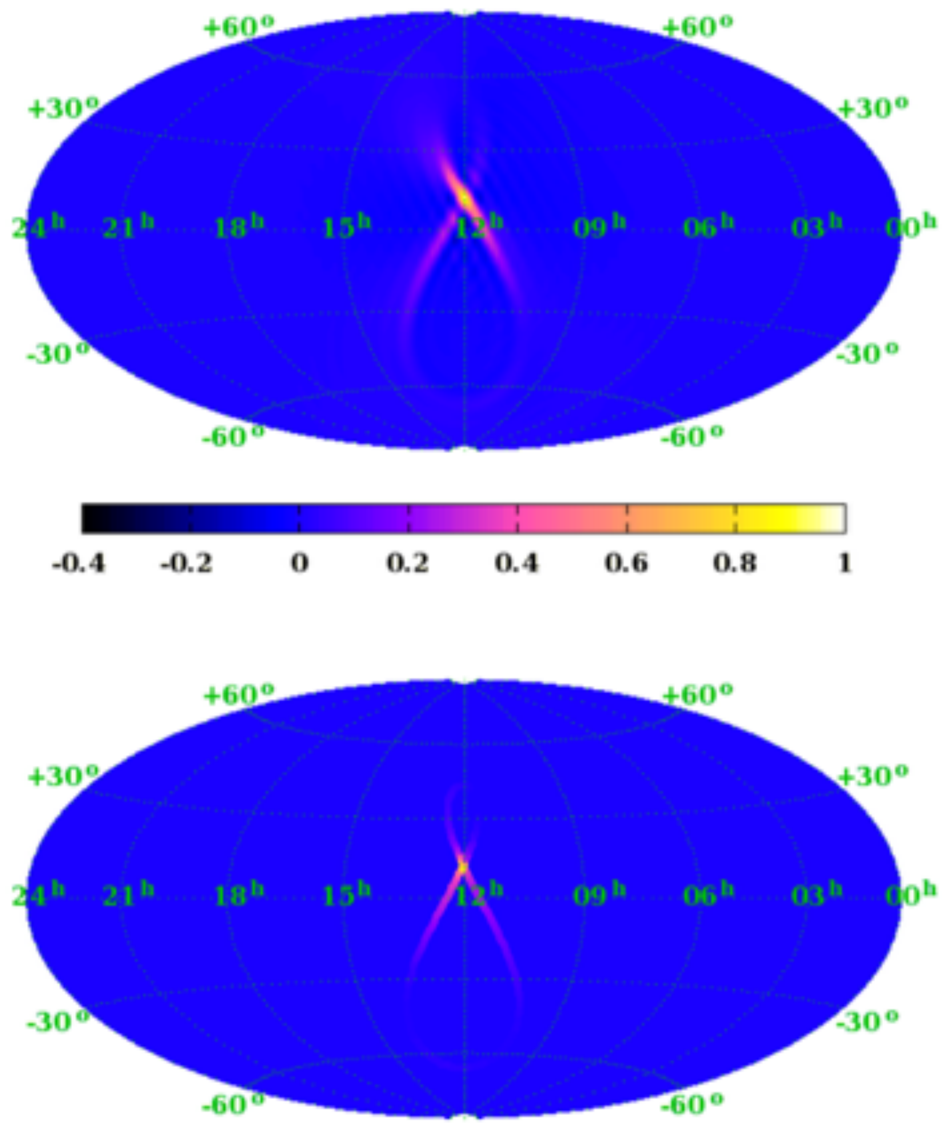


Baseline



sum of the sky-maps (dirty map)

Cleaning the dirty-map



Gravitational wave radiometry: Mapping a stochastic gravitational wave background

Sanjit Mitra et.al.

What the Interferometers Observe

$$s_{\mathcal{I}_1}(t) \; = \; h_{\mathcal{I}_1}(t) \; + \; n_{\mathcal{I}_1}(t)$$

$$s_{\mathcal{I}_2}(t) \; = \; h_{\mathcal{I}_2}(t) \; + \; n_{\mathcal{I}_2}(t)$$

$$\tilde{s}(t;f) \; := \; \int_{t-\tau/2}^{t+\tau/2} \mathrm{d}t' \, s(t') \, e^{-i2\pi f t'}$$

The Cross-Spectral Density

$$\mathbf{C}^I \equiv C^I_{ft} := \widetilde{s}^*_{\mathcal{I}_1}(t;f)\,\widetilde{s}_{\mathcal{I}_2}(t;f)$$

$$\mathbf{n}^I \equiv n^I_{ft} := \widetilde{n}^*_{\mathcal{I}_1}(t;f)\,\widetilde{n}_{\mathcal{I}_2}(t;f)$$

$$\sigma^2_{Ift} := \langle n^{I*}_{ft}\, n^I_{ft}\rangle = \frac{\tau^2}{4} P_{\mathcal{I}_1}(t;f)\, P_{\mathcal{I}_2}(t;f)$$

Making a map of the SGWB

Anisotropy Map $\mathcal{P}(\widehat{\mathbf{\Omega}}) := \sum_{\alpha} \mathcal{P}_\alpha \, e_\alpha(\widehat{\mathbf{\Omega}})$

Spectrum of Sky $H(f)$

What CSD is expected

$$\langle C^I_{ft} \rangle \; := \; \tau \, H(f) \sum_\alpha \mathcal{P}_\alpha \, \gamma^I_{ft,\alpha}$$

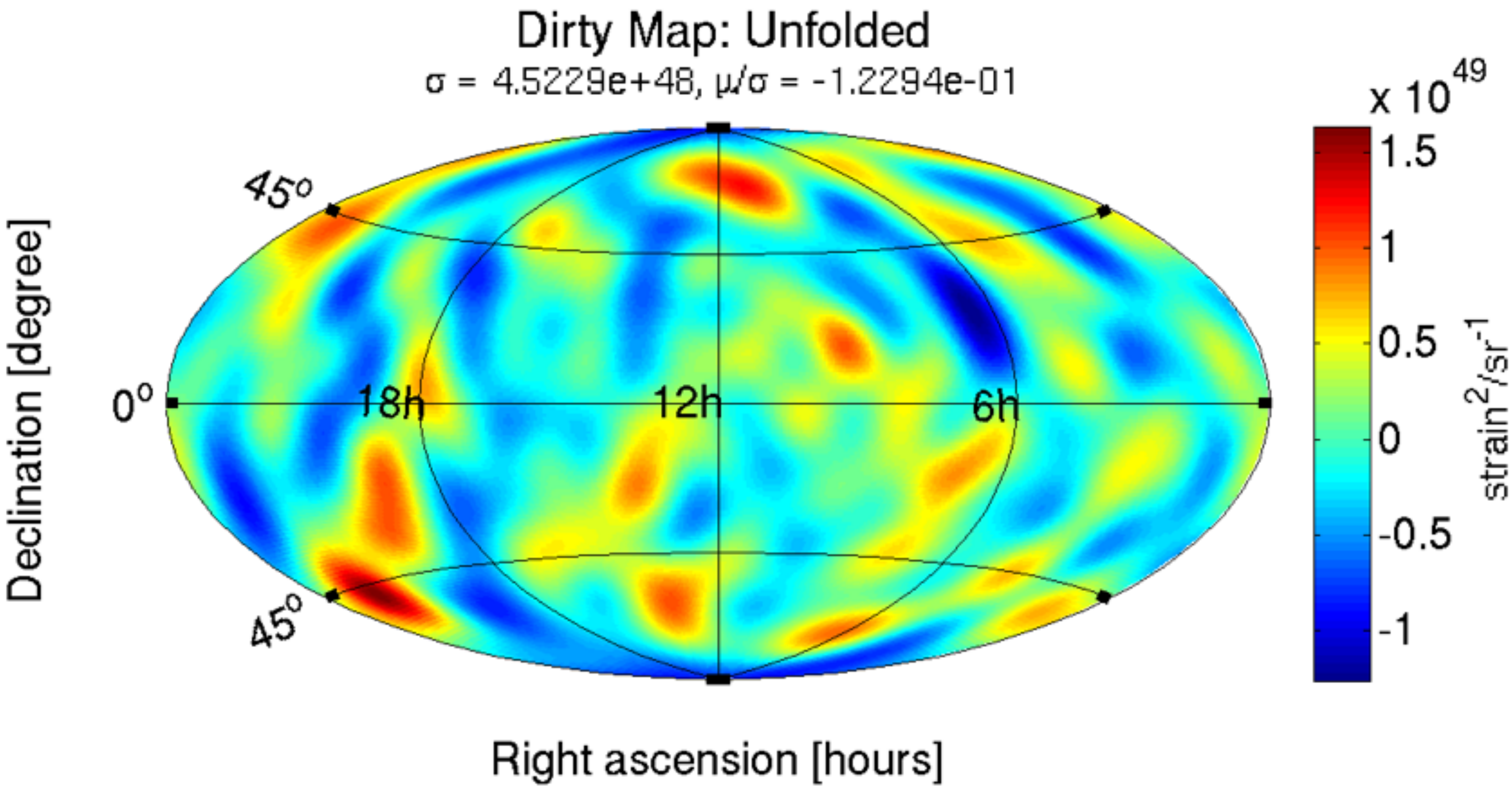
$$\gamma^I_{ft,\alpha} := \sum_A \int_{S^2} d\hat{\mathbf{\Omega}} F^A_{\mathcal{I}_1}(\hat{\mathbf{\Omega}}, t) F^A_{\mathcal{I}_2}(\hat{\mathbf{\Omega}}, t) e^{2\pi i f \frac{\hat{\mathbf{\Omega}} \cdot \mathbf{\Delta x}_I(t)}{c}} e_\alpha(\hat{\mathbf{\Omega}})$$

Maximum Likelihood Estimation of The SGWB

$$\hat{\mathcal{P}}_\alpha \equiv \hat{\mathcal{P}} = \mathbf{\Gamma}^{-1} \cdot \mathbf{X}$$

$$\mathbf{X} = \frac{4}{\tau} \sum_{Ift} \frac{H(f)\gamma^{I*}_{ft,\alpha}}{P_{\mathcal{I}_1}(t;f)P_{\mathcal{I}_2}(t;f)} \widetilde{s}^*_{\mathcal{I}_1}(t;f)\widetilde{s}_{\mathcal{I}_2}(t;f)$$

$$\mathbf{\Gamma} = 4 \sum_{Ift} \frac{H^2(f)}{P_{\mathcal{I}_1}(t;f)\, P_{\mathcal{I}_2}(t;f)} \gamma^{I*}_{ft,\alpha}\, \gamma^{I}_{ft,\alpha'}$$


Dirty Map: Unfolded
σ = 4.5229e+48, μ/σ = -1.2294e-01
x 10^49
1.5
1
0.5
0
-0.5
-1
strain^2/sr^-1
Declination [degree]
45°
0°
45°
18h
12h
6h
Right ascension [hours]

Folding

The main idea behind folding the data is that the radiometric data can be folded into data from 1 sidereal day (i.e. 23 hr 56 min 4 sec). This folding reduces computation for a search on the data of a run by few hundred fold.

indigo

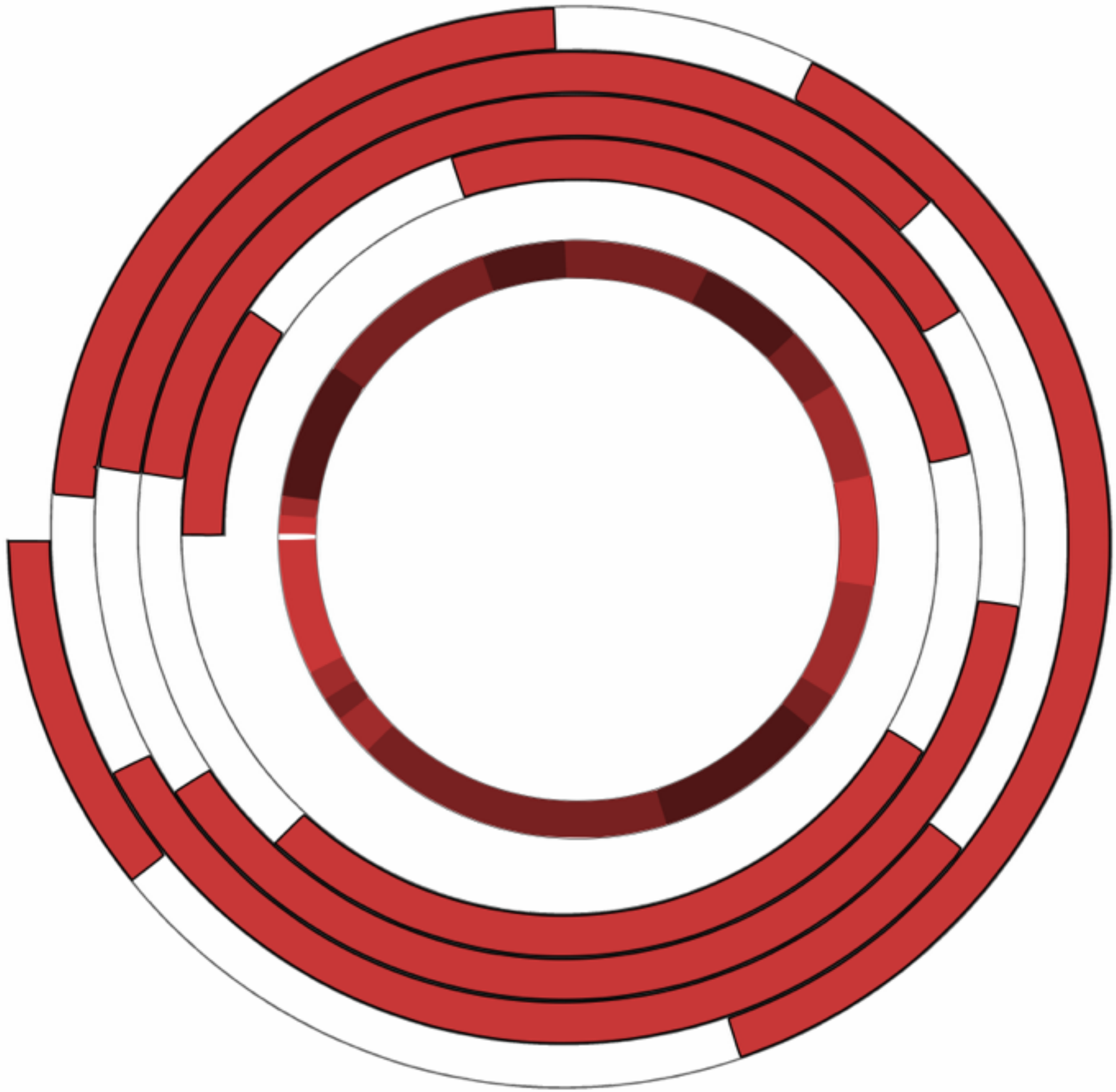
Folding Formula

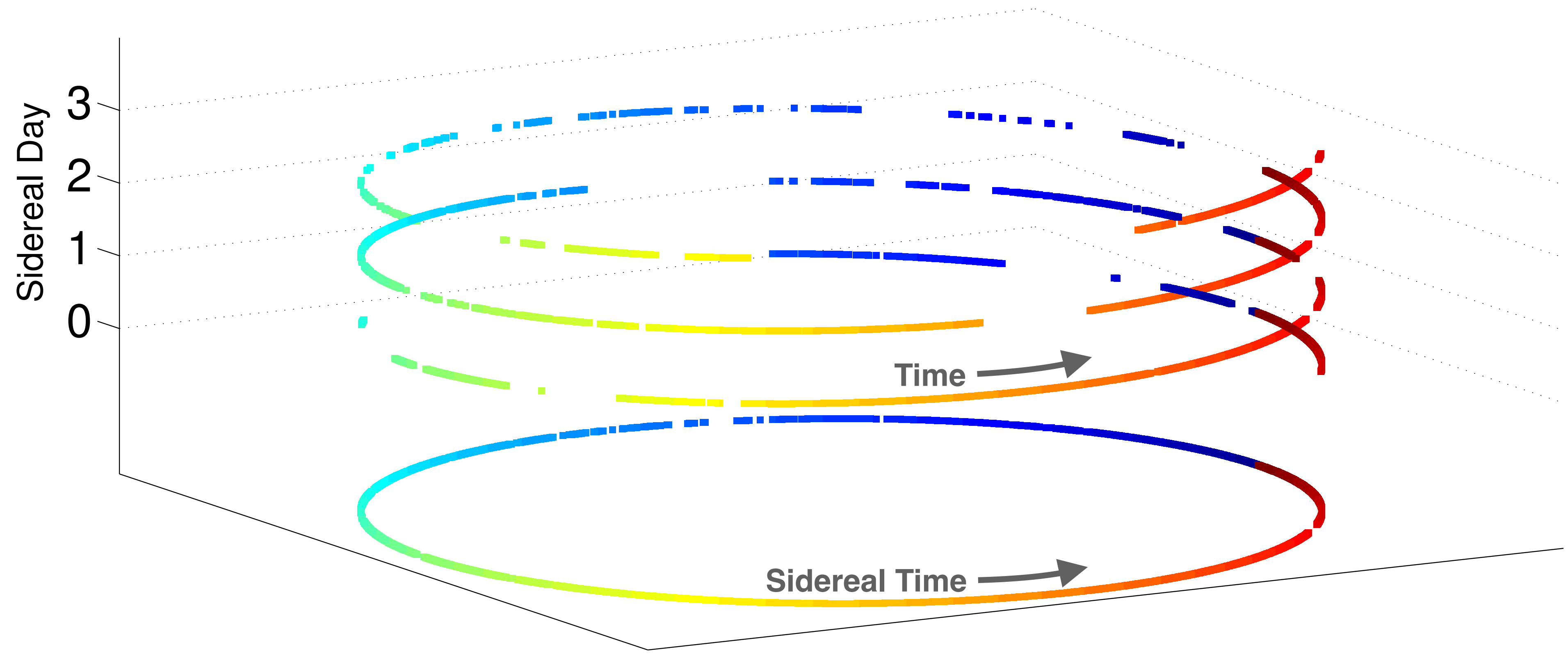
$$X_\alpha = \sum_{Ift} K^{I*}_{\alpha,ft}\, \sigma^{-2}_{Ift}\, C^I_{ft}$$

$$= \sum_{Ift_{sid}} K^{I*}_{\alpha,ft_{sid}} \sum_{i_{day}} \sigma^{-2}_{If(i_{day}+t_{sid})}\, C^I_{f(i_{day}+t_{sid})}$$

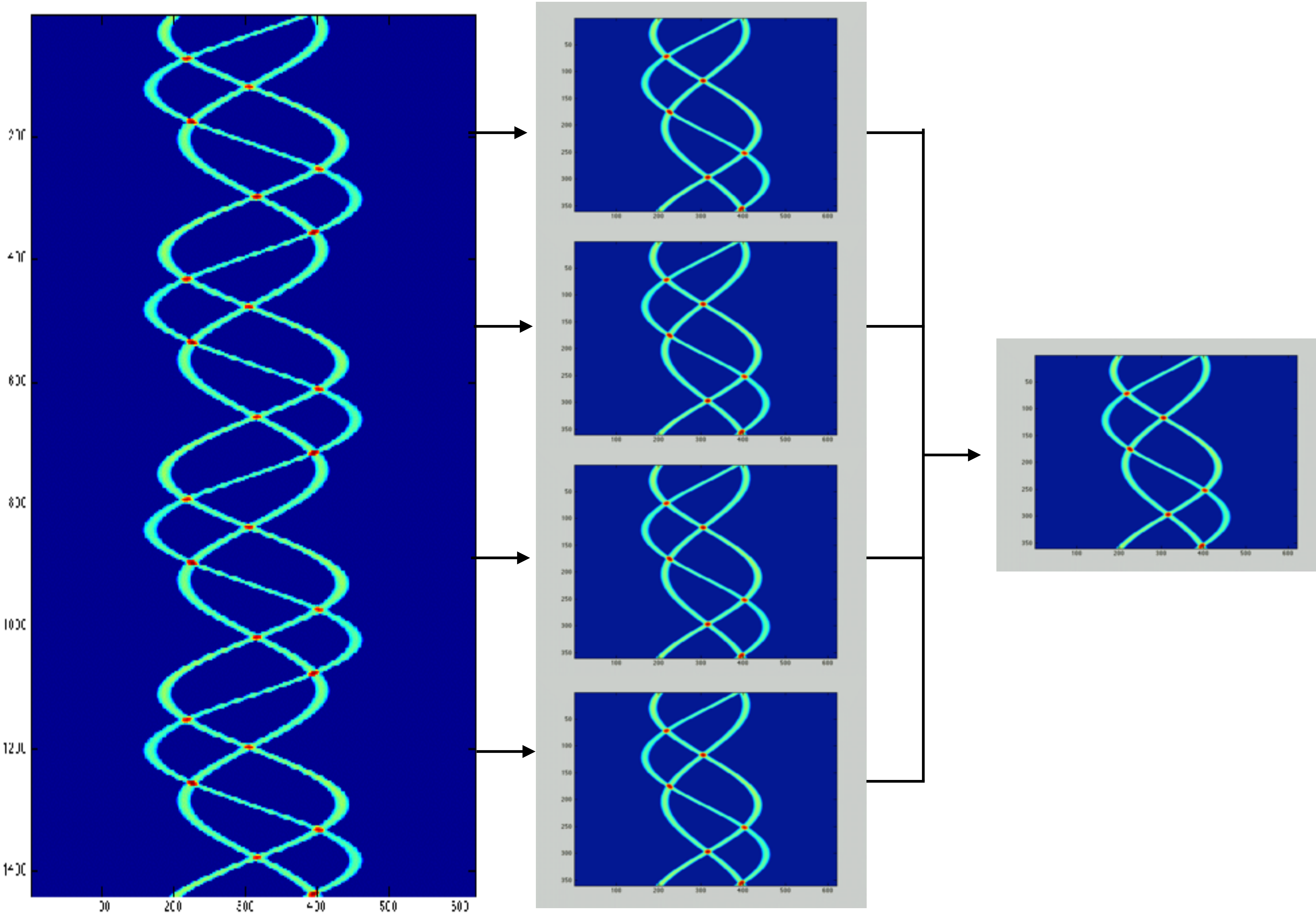
$$\Gamma_{\alpha\alpha'} = \sum_{Ift} K^{I*}_{\alpha,ft}\, \sigma^{-2}_{Ift}\, K^I_{ft,\alpha'}$$

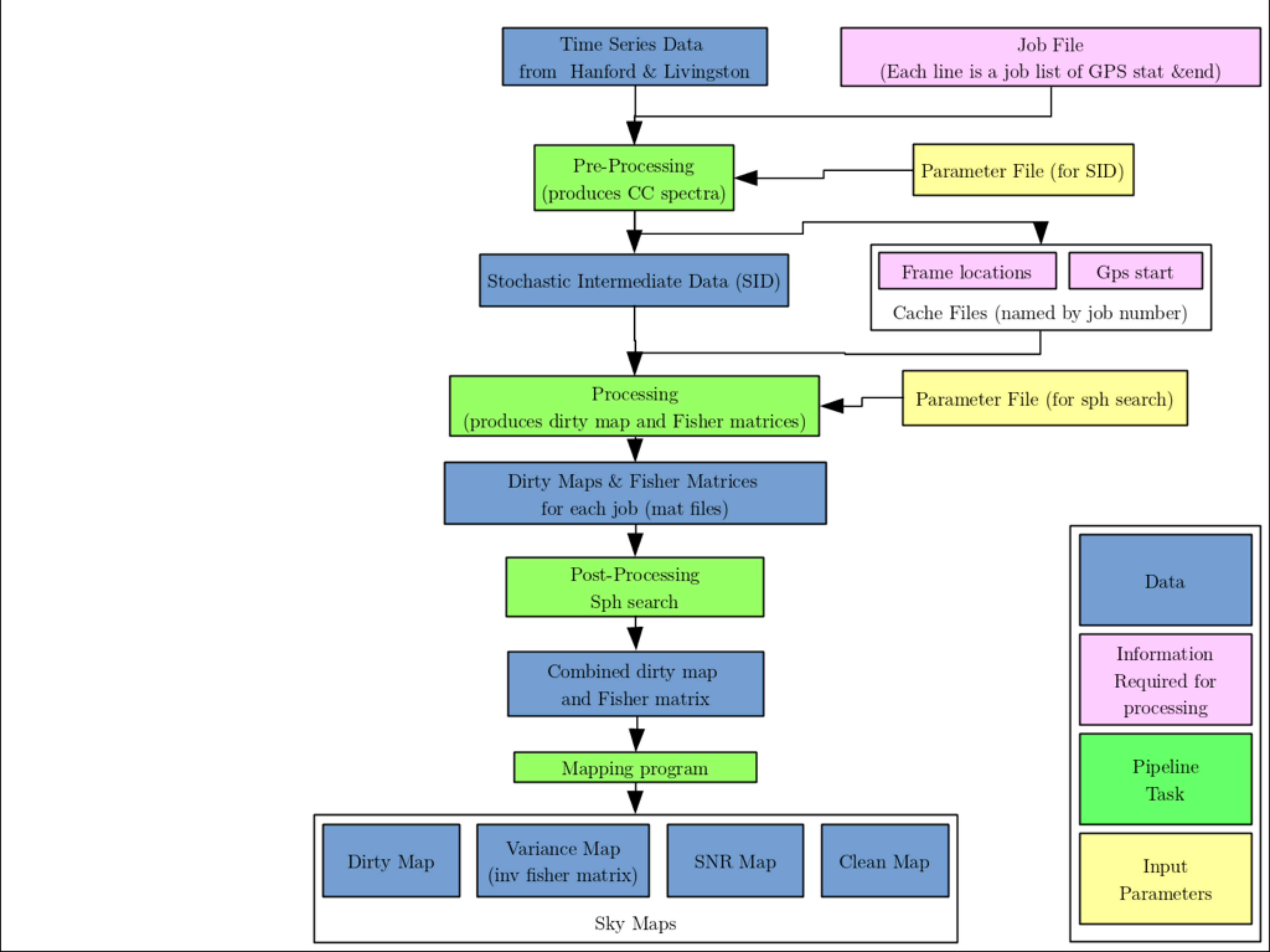
$$= \sum_{I,ft_{sid}} K^{I*}_{\alpha,ft_{sid}}\, K^I_{ft_{sid},\alpha'} \sum_{i_{day}} \sigma^{-2}_{If(i_{day}+t_{sid})}$$

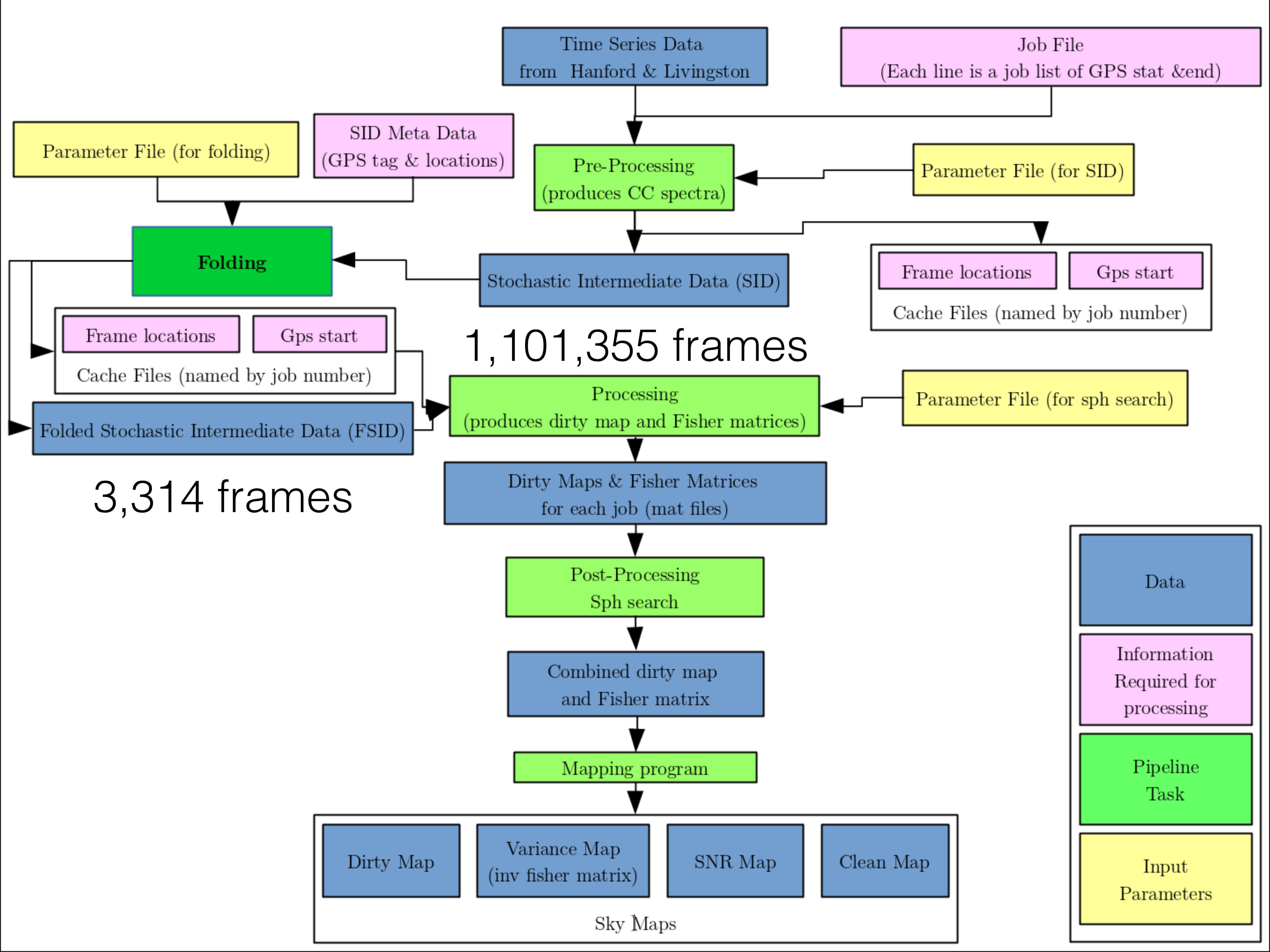




Three days of LIGO S5 data from GPS time 860832366 sec to 861090858 sec. Each point on the spiral represents one segment marked by a GPS time and its colour represents the corresponding sidereal time. The projected ring at the bottom represents folded data.




Time Series Data
from Hanford & Livingston
Job File
(Each line is a job list of GPS stat &end)
Pre-Processing
(produces CC spectra)
Parameter File (for SID)
Stochastic Intermediate Data (SID)
Frame locations
Gps start
Cache Files (named by job number)
Processing
(produces dirty map and Fisher matrices)
Parameter File (for sph search)
Dirty Maps & Fisher Matrices
for each job (mat files)
Post-Processing
Sph search
Combined dirty map
and Fisher matrix
Mapping program
Dirty Map
Variance Map
(inv fisher matrix)
SNR Map
Clean Map
Sky Maps
Data
Information
Required for
processing
Pipeline
Task
Input
Parameters


Time Series Data
from Hanford & Livingston
Job File
(Each line is a job list of GPS stat &end)
Parameter File (for folding)
SID Meta Data
(GPS tag & locations)
Pre-Processing
(produces CC spectra)
Parameter File (for SID)
Folding
Stochastic Intermediate Data (SID)
Frame locations
Gps start
Cache Files (named by job number)
Frame locations
Gps start
Cache Files (named by job number)
1,101,355 frames
Processing
(produces dirty map and Fisher matrices)
Parameter File (for sph search)
Folded Stochastic Intermediate Data (FSID)
Dirty Maps & Fisher Matrices
for each job (mat files)
3,314 frames
Post-Processing
Sph search
Combined dirty map
and Fisher matrix
Mapping program
Dirty Map
Variance Map
(inv fisher matrix)
SNR Map
Clean Map
Sky Maps
Data
Information
Required for
processing
Pipeline
Task
Input
Parameters

Data Folding at a Glance

CSD (C): The **C**ross **S**pectral **D**ensity is the cross-correlation of the data from the two interferometers.

$$C = \widetilde{s}_1^* \widetilde{s}_2$$

PSD (P): It is the one-sided **P**ower **S**pectral **D**ensity of the noise from the two individual interferometers.

$$\sigma^2 = \frac{T^2}{4} P_1 P_2$$

Dirty Map
for stochastic search

$$\mathbf{X} = \sum_{time} K\sigma^{-2}C = \sum_{time\ of\ day} K \sum_{\substack{same\ time\ of\\ different\ days}} \sigma^{-2}C$$

Fisher Matrix
for stochastic search

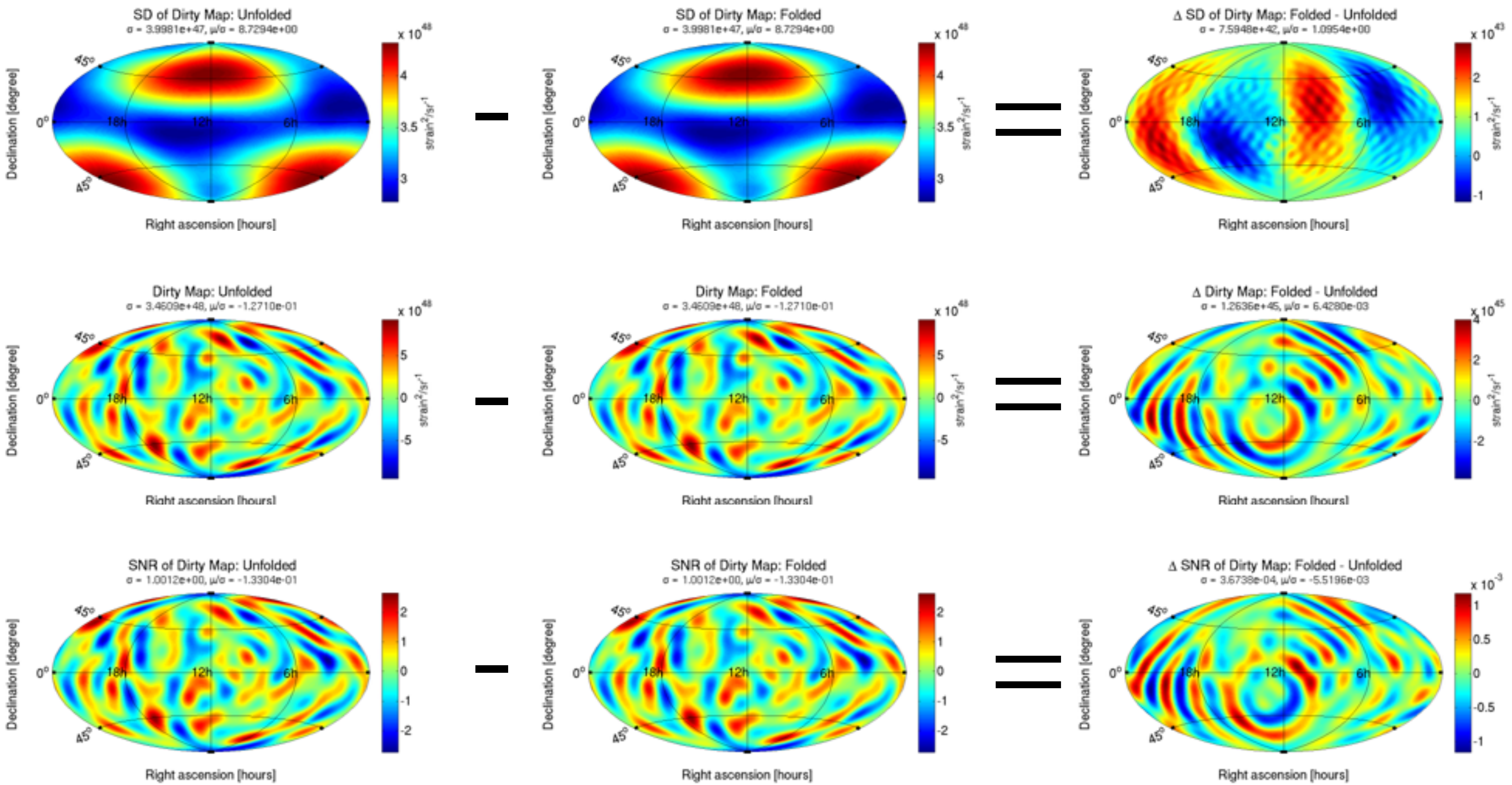
$$\mathbf{\Gamma} = \sum_{time} K^*\sigma^{-2}K = \sum_{time\ of\ day} K^*K \sum_{\substack{same\ time\ of\\ different\ days}} \sigma^{-2}$$

Folding is applicable to any analysis which applies a periodic linear operator on a function of CSD and PSD with a sum over several days.

$$\sum_{time} QF(C,P) = \sum_{time\ of\ day} Q \sum_{\substack{same\ time\ of\\ different\ days}} F(C,P)$$

indigo

Comparison of maps



Comparison of Numbers for Folded and Unfolded Maps

Errors in [RMS(FSID-SID)/RMS(SID)]

Real parts of FSID Fisher matrix	= 2.50e-05
Imaginary parts of FSID Fisher matrix	= 3.65e-05
Dirty SpH	= 4.24e-04
Clean SpH	= 4.49e-04
Dirty map	= 3.62e-04
Clean map	= 4.25e-04
Variance map	= 3.21e-06
SNR map	= 3.64e-04

The Advantages of Folded Data

Advantages of Folding

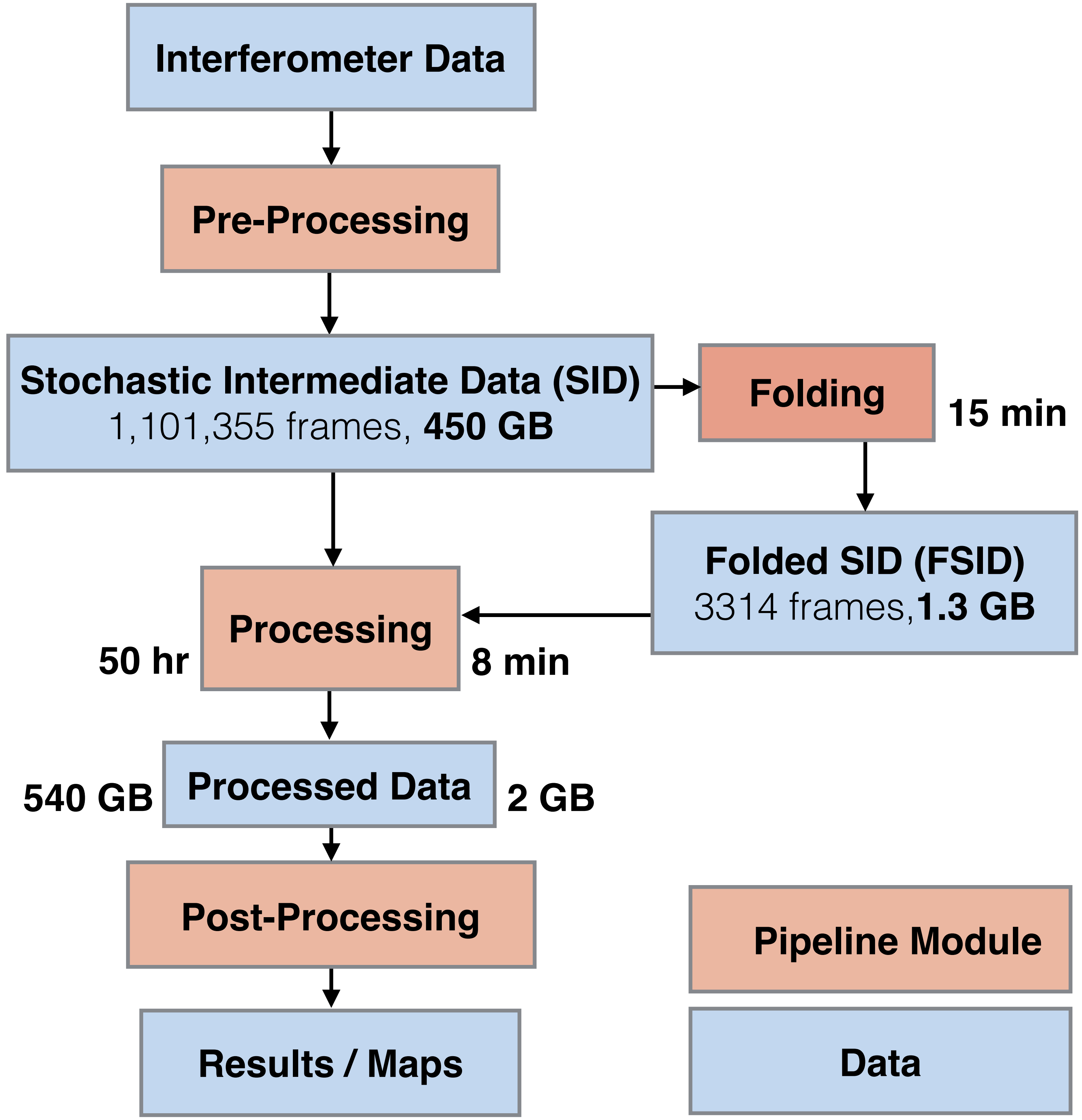
- Efficiency: Processing time reduced by ~300 fold for full S5 data.
- Portability: Total S5 data becomes 1.3 GB, can fit in a USB key.
- Convenience: Dirty details are already included (overlapping window, data quality cuts). Parallel processing is not needed for standard searches.
- Modularity: Folding is independent of analysis pipeline. FSID can be implemented in `MATLAB/Python` codes in `mat/gwf/HDF` format.
- Management: Results are for 1day, easy to predict computation time and data size of outputs.

A full radiometer search can be done on a personal computer.

The folding code is ready.

The differences in the results from SID and FSID are less than ~0.01% for full S5.

The Advantages of Folded Data



Dirty steps are all included

- Overlapping window correction
  - was already incorporated in the algebra
- Stationarity cuts (recently implemented in the code)
  - provides the current integrated cross-PSD option
  - plus single detector and narrow band options
- **This makes analysis of folded data very clean**

Search for Narrowband Sources

- Using the present search methods, detection of monochromatic sources is surely impossible since it integrates over all frequencies.

- To detect monochromatic sources, instead of calculating,

$$\mathbf{X} = \frac{4}{\tau} \sum_{Ift} \frac{H(f)\gamma^{I*}_{ft,\alpha}}{P_{\mathcal{I}_1}(t;f)P_{\mathcal{I}_2}(t;f)} \widetilde{s}^*_{\mathcal{I}_1}(t;f)\widetilde{s}_{\mathcal{I}_2}(t;f)$$

- We calculate

$$\mathbf{X}(f) = \frac{4}{\tau} \sum_{It} \frac{\gamma^{I*}_{ft,\alpha}}{P_{\mathcal{I}_1}(t;f)P_{\mathcal{I}_2}(t;f)} \widetilde{s}^*_{\mathcal{I}_1}(t;f)\widetilde{s}_{\mathcal{I}_2}(t;f)$$

for each frequency bin

Challenges of Blind All-Sky Search for Narrowband Sources

- The computational time increases by factor of 2000 approximately.
- Full S5 analysis takes 50 hours and produces 540 GB of intermediate data approximately.
- So the narrowband search will take 10 cpu years and require 800 TB of disk space.

Blind All-Sky Search for Narrowband Sources using Folded Data

- We have already folded the entire S5 data into one sidereal day 1.3 GB.
- All-sky search using Matlab on IUCAA cluster takes only 8 minutes for one frequency bin. It produces 2 GB of intermediate data.
- The entire analysis will take 10 CPU days and produce 2.5 TB of processed data.

Current Status of the search

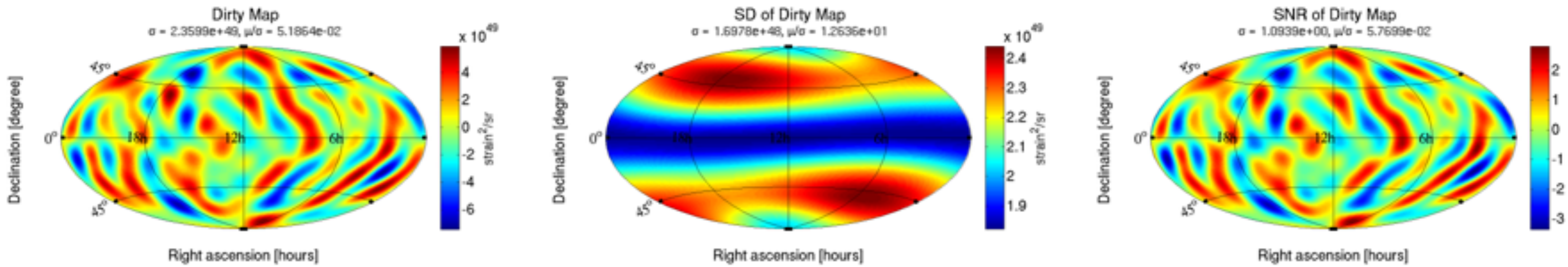
- We have incorporated data quality cuts (existing scheme and **few more**) in the folding code
- We have demonstrated using the stochastic pipeline that a narrowband search can be done in reasonable time.
  - using a very narrow frequency filter. (example in next slides)
  - but we will not do it this way, **we are writing a new python code.**

Folding pipeline has been extensively validated
- code review will start soon

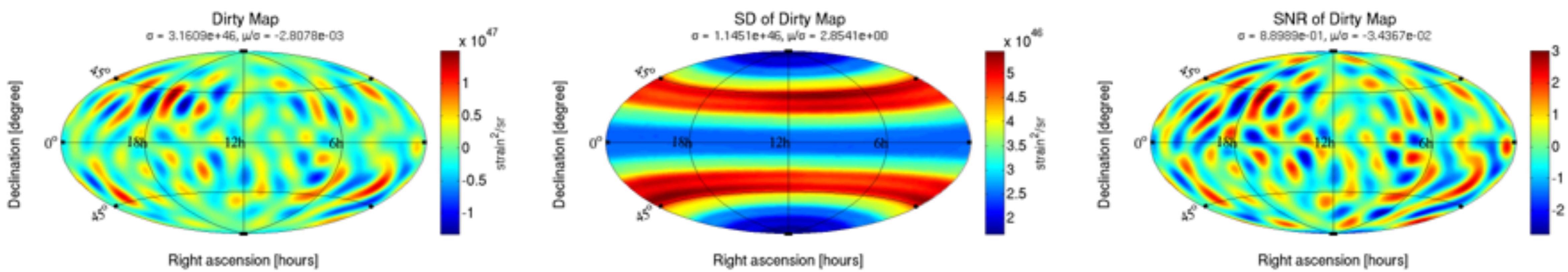
Narrowband search will be ready before the end of O1

Maps from full S5 for different frequency

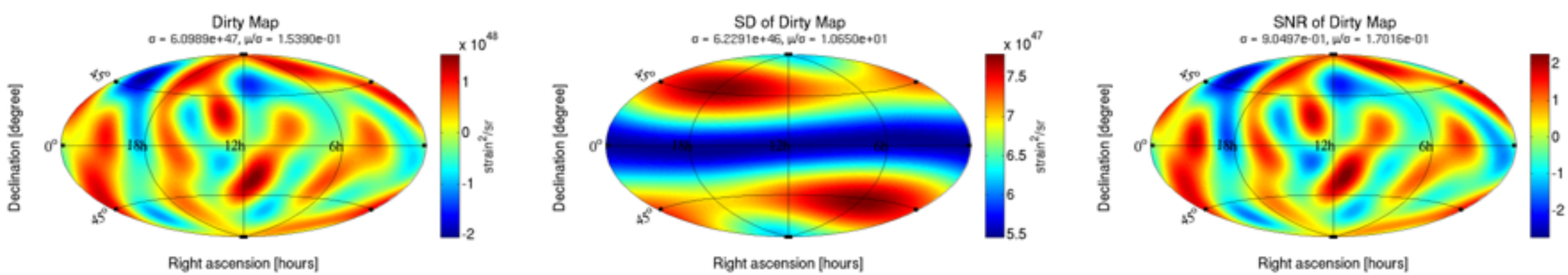
f = 30 ~ 500 Hz



f = 500 Hz



f = 100 Hz



A new pipeline for folded data

- These calculations only requires a set of the general overlap reduction function.
- Once we have that set, the blind all sky search is just few matrix multiplications.
- We are planning to switch to python for better use of HEALPix and parallelisation of the code.
- This may also be applicable to the standard stochastic searches

Tentative outcome of the search

- Processed data will be **thousands of skymaps**
  - ✳ one for each frequency
- Planning an **easy interface** to browse through this data
- We are also exploring ways of **quoting key characteristics** of this data for easy presentation, e.g.,
  - ✳ max upper limits and the corresponding frequency at each frequency (i.e., 2 maps)
  - ✳ histograms at each pixel (make a map of std dev?)
  - ✳ find expected statistics of the histograms and outliers
  - ✳ place joint constraints on physical parameters of narrow-band sources

summary

SGWB from exoplanets are not detectable with present detectors, but hope remains for the future.

Now you can run stochastic search on your laptop.