

Solid Applications of Massive Gravity

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MENU



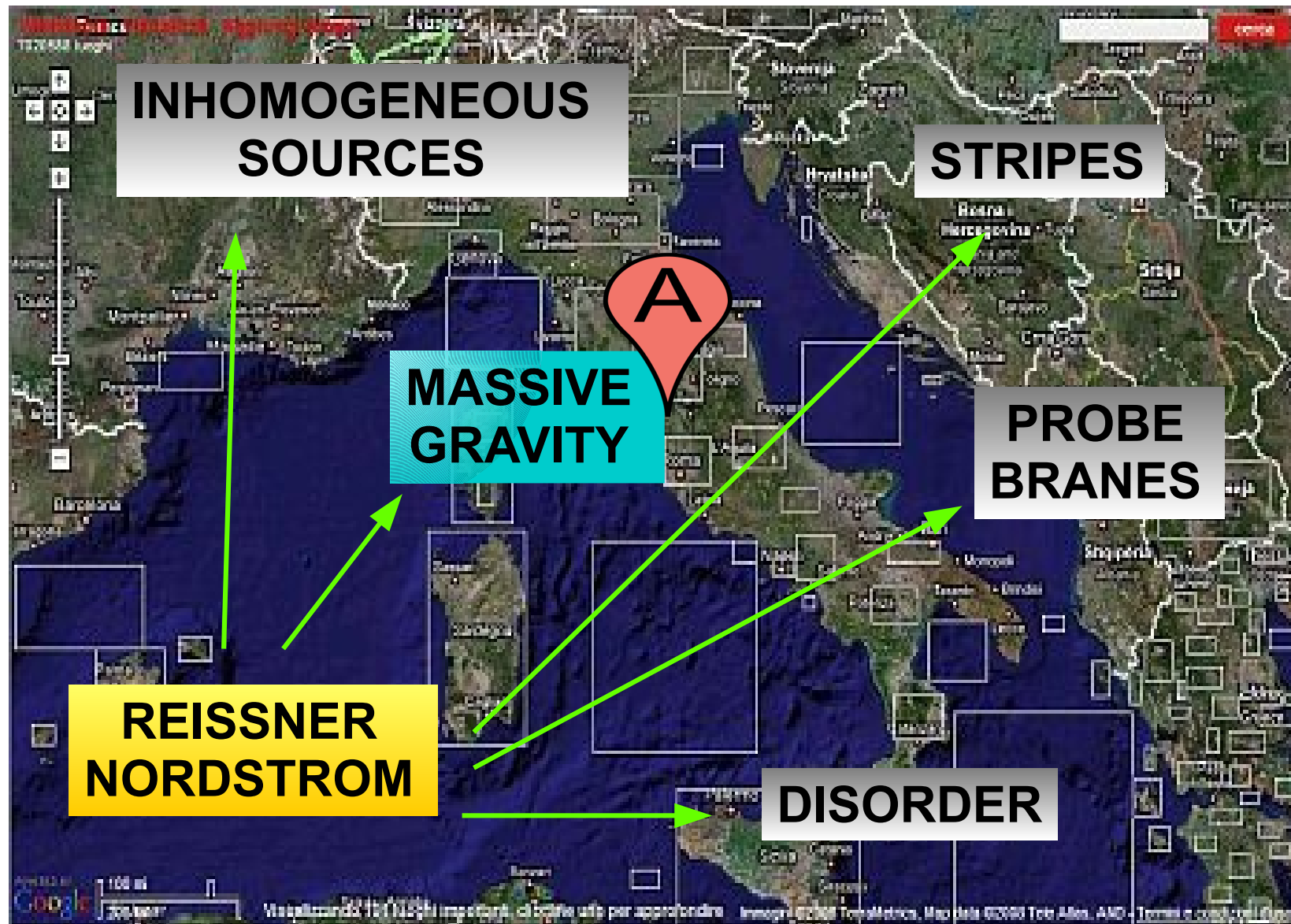
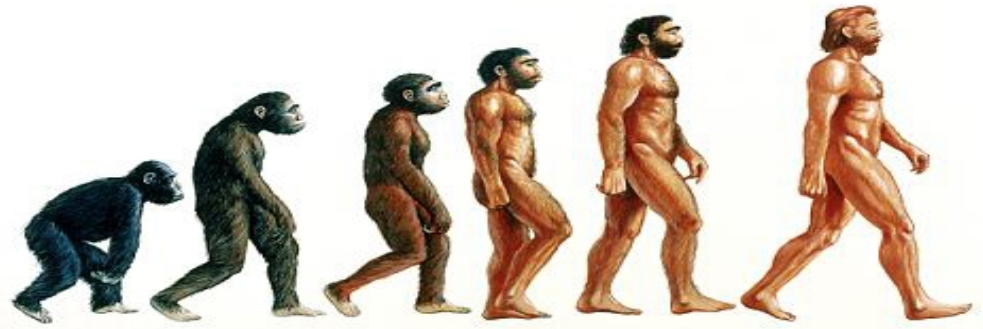
MENU

***Massive Gravity and
effective field theories
for phonons in solids***

***Electron-Phonon
Interactions &
Holographic Response***

***Conclusions
&
future***

Holographic



Massive Gravity is a wonderful effective approach, but ...



Who are we talking about ??

Spoiling the end

LORENTZ VIOLATING

MASSIVE GRAVITY

ENCODES VIA DUALITY

PHONONS BEHAVIOUR

IN SOLIDS

Old School Massive Gravity



Infrared modification of
General Relativity

= “Large distance” modification

$$S = M_{\text{Pl}}^2 \int d^4x \left\{ \underbrace{L_{EH}^{(2)}(h_{\mu\nu})}_{\text{GR}} + \underbrace{\frac{\alpha}{4} h_{\mu\nu} h^{\mu\nu} + \frac{\beta}{4} (h^\mu_\mu)^2}_{\text{FIERZ PAULI}} \right\} \quad \beta = -\alpha = m_G^2.$$

GR

FIERZ PAULI

BREAKING OF DIFFEOMORPHISM INVARIANCE !!

GRAVITON WITH EXTRA DEGREES OF FREEDOM

Attention:

- Ghosts
- dVDZ discontinuity
- Strong Coupling



Goldstones ...

Massive Gravity Nightmare

LINEAR LEVEL: OK !

NON LINEAR LEVEL:



BOULWARE DESER GHOST

Recently (probably) solved by :

dRGT MODEL

But this is for the Lorentz Invariant case

And we are doing Condensed Matter ...

Non Relativistic Massive Gravity

arXiv:0802.4379

$$S^{(2)} = S_{EH}^{(2)} + S_m,$$

arXiv:hep-th/0409124

$$L_m = \frac{1}{4} [m_0^2 h_{00} h_{00} + 2m_1^2 h_{0i} h_{0i} - m_2^2 h_{ij} h_{ij} + m_3^2 h_{ii} h_{jj} - 2m_4^2 h_{00} h_{ii}].$$

+ mass terms in the holographic direction ...

SUPER HEALTHY THEORIES AT NON LINEAR LEVEL
(provided some weak constraints)

WAY MORE GENERAL THAN dRGT MODEL !!

Not all the choices healthy, need to impose constraints on the masses...

For Example : $m_2^2 \geq 0.$ $m_1^2 \geq 0.$

Residual Gauge Symmetries

One elegant way to avoid fine tuning the masses is thinking of various Residual gauge symmetries...

In several cases this approach does lead to healthy theories...

Example:

$$t \rightarrow t + \zeta^0(x^i, t) . \longrightarrow m_0 = m_1 = m_4 = 0.$$

**Our case: we want to preserve SO(3) and homogeneity
And break Just translation in the spatial boundary directions...**

More elegant way: COSET constructions

Solids, fluids, superfluids, supersolid

arXiv:1307.0517

Stuckelberg Trick

**IDEA : “ENLARGE” THE FIELD CONTENT
RESTORE GAUGE INVARIANCE**

introduces a new, Stückelberg field ξ_μ and a new field $\bar{h}_{\mu\nu}$ by writing

$$h_{\mu\nu} = \bar{h}_{\mu\nu} + \partial_\mu \xi_\nu(x) + \partial_\nu \xi_\mu(x).$$

Then the linearized theory is invariant under gauge transformations

$$\begin{aligned}\bar{h}_{\mu\nu}(x) &\rightarrow \bar{h}_{\mu\nu}(x) + \partial_\mu \zeta_\nu(x) + \partial_\nu \zeta_\mu(x), \\ \xi_\mu &\rightarrow \xi_\mu - \zeta_\mu.\end{aligned}$$

**MASSIVE
GRAVITY**



GR + new Sector



Scalars : GOLDSTONES

Lorentz Violating Scalars and Massive Gravity

Adding additional scalar fields : $\phi^\alpha, \alpha = 0, 1, 2, 3,$

GOLDSTONE FIELDS

Breaking Lorentz Invariance
= spacetime coordinates
dependence

Example:

$$S = S_{EH} + S_\phi,$$



$$\bar{\phi}^0 = a\Lambda^2 t,$$

$$\bar{\phi}^i = b\Lambda^2 x^i,$$

To be solution : Lagrangian contains their derivatives only



SHIFT SYM : $\phi^\alpha(x) \rightarrow \phi^\alpha(x) + \lambda^\alpha$

Eventually internal Sym :

$$SO(3) \quad \phi^i \rightarrow \Lambda_j^i \phi^j.$$

Avoiding issues with the scalars

In Minkowski background (cosmology for example) these scalars are dangerous

Very hard to neglect their contribution to stress energy tensor

Without violating null energy condition !!

Here we are happy of having them backreacting!

Note that they dont screw up the UV AdS Asymptotics
needed for the dictionary .

HAPPY END :
LOTS OF SUPER HEALTHY LV MASSIVE GRAVITIES
AVAILABLE !

LV Massive Gravity in stuckelberg form

$$S_\phi = \int d^4x \sqrt{-g} \Lambda^4 F(X, V^i, Y^{ij}, Q)$$

**This is
Way More
than A solid**

$$X = \frac{1}{\Lambda^4} g^{\mu\nu} \partial_\mu \phi^0 \partial_\nu \phi^0,$$

*Even preserving homogeneity
Lot of freedom and opportunities...*

$$V^i = \frac{1}{\Lambda^4} g^{\mu\nu} \partial_\mu \phi^0 \partial_\nu \phi^i,$$

$$Y^{ij} = \frac{1}{\Lambda^4} g^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi^j,$$

**Effective Field Theory
Valid at Low energy**

The UV cutoff in this theory Λ_{UV} is to be somewhat below Λ

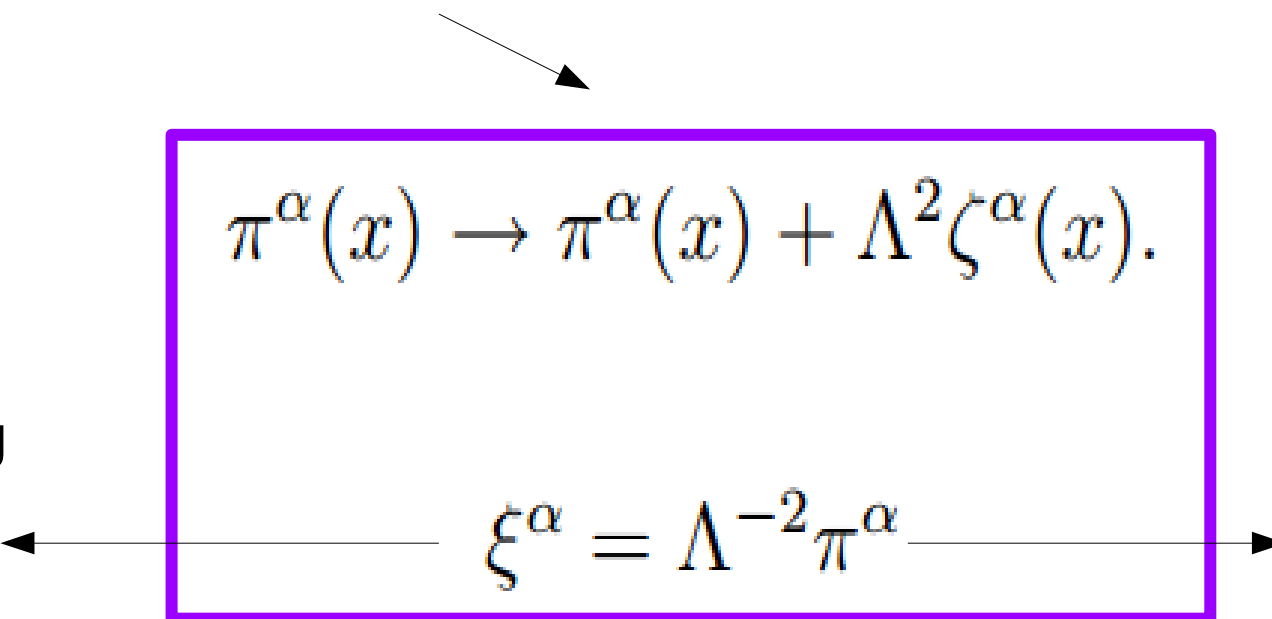
$$\phi^\alpha = \bar{\phi}^\alpha + \underline{\pi}^\alpha, \quad L_\pi = (\partial\pi)^2 + \frac{1}{\Lambda} (\partial\pi)^3 + \dots$$

GOLDSTONES

See: Chiral Perturbation Theory and Pions

Stuckelbergs = Goldstones

Gravity ON : $x^\mu \rightarrow x^\mu + \zeta^\mu(x)$


$$\pi^\alpha(x) \rightarrow \pi^\alpha(x) + \Lambda^2 \zeta^\alpha(x).$$

Stuckelberg
Fields
Of previous
sections

$$\xi^\alpha = \Lambda^{-2} \pi^\alpha$$

Goldstone
Fields

STUCKELBERGS = GOLDSTONES

Unitary Gauge : $\pi^\alpha = 0$

Usual Massive Gravity with : $m = \frac{\Lambda^2}{M_{Pl}}$

**“ WE ARE
HIGGSING
DIFFEOS “**

LV MASSIVE GRAVITY FOR SOLIDS

$$S_\phi = \int d^4x \sqrt{-g} \Lambda^4 F(\cancel{X}, \cancel{V^i}, \boxed{Y^{ij}}, \cancel{Q})$$

arXiv:1307.0517

Note: Symmetries allow for a generic function F

$$SO(3) \longrightarrow \text{JUST SCALARS COMING FROM } Y^{ij} = \frac{1}{\Lambda^4} g^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi^j,$$

$$\boxed{Tr[Y], Tr[Y^2], \dots}$$

arXiv:1311.5157

$$S_0 = \int_M \sqrt{-g} \left[R - 2\Lambda - \frac{1}{2} \sum_I^{d-1} (\partial \psi_I)^2 - \frac{1}{4} F^2 \right] d^{d+1}x$$

A simple holographic model of momentum relaxation

Effective field theory for solids

COMOVING COORDINATES OF THE VOLUME ELEMENTS :

arXiv:hep-th/0512260

TRIPLETS OF SCALARS $\phi^I(\vec{x}, t)$

arXiv:1501.03845

GROUND STATE = $\langle \phi^I(x) \rangle = x^I$. = EQUILIBRIUM

HOMOGENEITY AND ISOTROPY OF SOLIDS AT LARGE SCALES $\longrightarrow \phi^I \rightarrow \phi^I + a^I, \quad \phi^I \rightarrow SO(3) \cdot \phi^I,$

SPATIAL DEFORMATIONS DEGREES OF FREEDOM

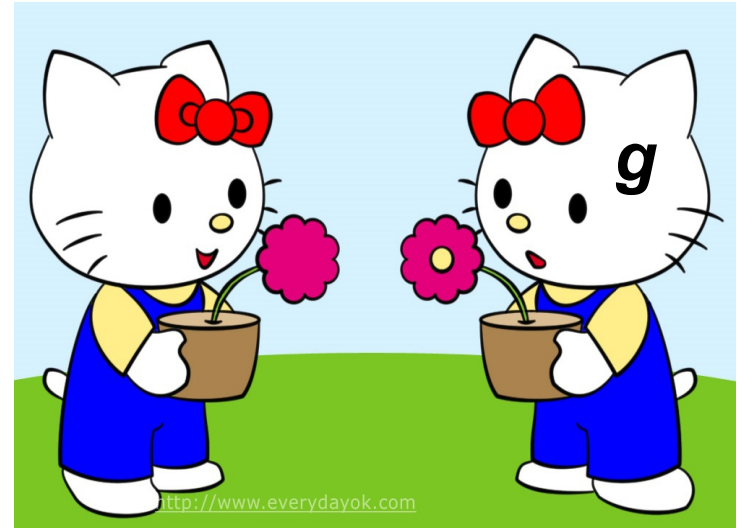
PHONONS

GOLDSTONES BOSONS FOR BROKEN SIMMETRIES

$\phi^a = x^a + \underline{\pi}^a$ describe the three (acoustic) phonons of the solid,

From Simmetry Breaking Pattern $\longrightarrow$ Effective field theory

Find the differences



EFT for Solids : $S_\phi = \int d^4x \quad \Lambda^4 F(\cancel{X}, \cancel{V}^i \boxed{Y^{ij}} \cancel{Q})$

LV MGravity: $S_\phi = \int d^4x \sqrt{-g} \Lambda^4 F(\cancel{X}, \cancel{V}^i \boxed{Y^{ij}} \cancel{Q})$

RECIPE :

WE ARE JUST GAUGING IT

AND PUT IT IN ANTI DE SITTER !

Getting Concrete: the Model

$$S_{HMG} = M_P^2 \int d^4x \sqrt{-g} \left[\frac{R}{2} + \frac{3}{\ell^2} - \underline{m^2 V(X)} \right]$$

$$S = S_{HMG} - \frac{M_P^2}{4} \int d^4x \sqrt{-g} F_{\mu\nu} F^{\mu\nu}$$

Generalizing:

arXiv:1311.5157

arXiv:1406.4870

$$X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi^I \partial_\nu \Phi^I$$

$$\bar{\Phi}^I = \alpha \delta_i^I x^i$$

arXiv:1411.1003

With Oriol Pujolàs

**MOMENTUM DISSIPATION
BUT
ENERGY CONSERVATION**

Consistency and stability

NO GHOSTS :

$$V'(\bar{X}) > 0 .$$

NO GRADIENT INSTABILITY :

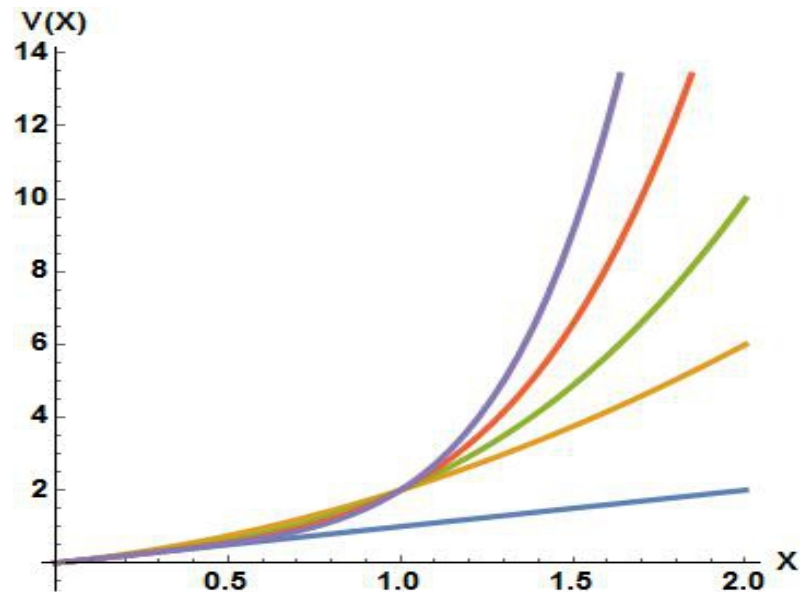
$$c_s^2 = 1 + \frac{\bar{X} V''(\bar{X})}{V'(\bar{X})} > 0 .$$

AdS UV ASYMPTOTICS :

$$V(0) = 0.$$

**WEAKLY COUPLED
GOLDSTONES SECTOR:**

$V'(0)$ is not too small.



The Physics of the model

GRAVITON + NEW GOLDSTONE DEGREES OF FREEDOM

↓
Operators of the CFT with shift symmetries
“Somehow” related to Phonons and Impurities

$$\Phi^I = \bar{\Phi}^I + \phi^I$$

ELECTROMAGNETIC SECTOR + NEW “PHONONS” SECTOR



**GENERIC POTENTIAL FOR THE PHONONS
NON TRIVIAL DYNAMICS
NON ZERO SELF INTERACTIONS**

**Phonon-Phonon interactions
Electron-Phonon interactions**

Where are the phonons?

PHONONS = FLUCTUATIONS OF THE SCALAR FIELDS

Transversal acoustic phonons

Transversal spatial components of stress tensor T_{xy}

FLUCTUATIONS OF THE PHI FIELDS

ARE THE GOLDSTONES OF THE RELATIVE BROKEN SIMMETRY

$$f(u) = u^3 \int_u^{u_H} dv \left[\frac{3}{v^4} - \frac{\rho^2}{2\ell^2} - \frac{(m\ell)^2}{v^4} V \left(\frac{\alpha^2 v^2}{\ell^2} \right) \right]$$

**We can keep track of the phonon pole in T_{xy} operator...
And measure the elastic response...**



With O.Pujolàs



Analytical DC conductivity

Using usual trick:

arXiv:1308.4970

$$\sigma_{\text{DC}} = 1 + \frac{\rho^2 u_H^2}{\alpha^2 m^2 V'(\alpha^2 u_H^2)}$$

Quantum Pairs Creation
CFT
Probe Result

Dissipative Term
"à la Drude"

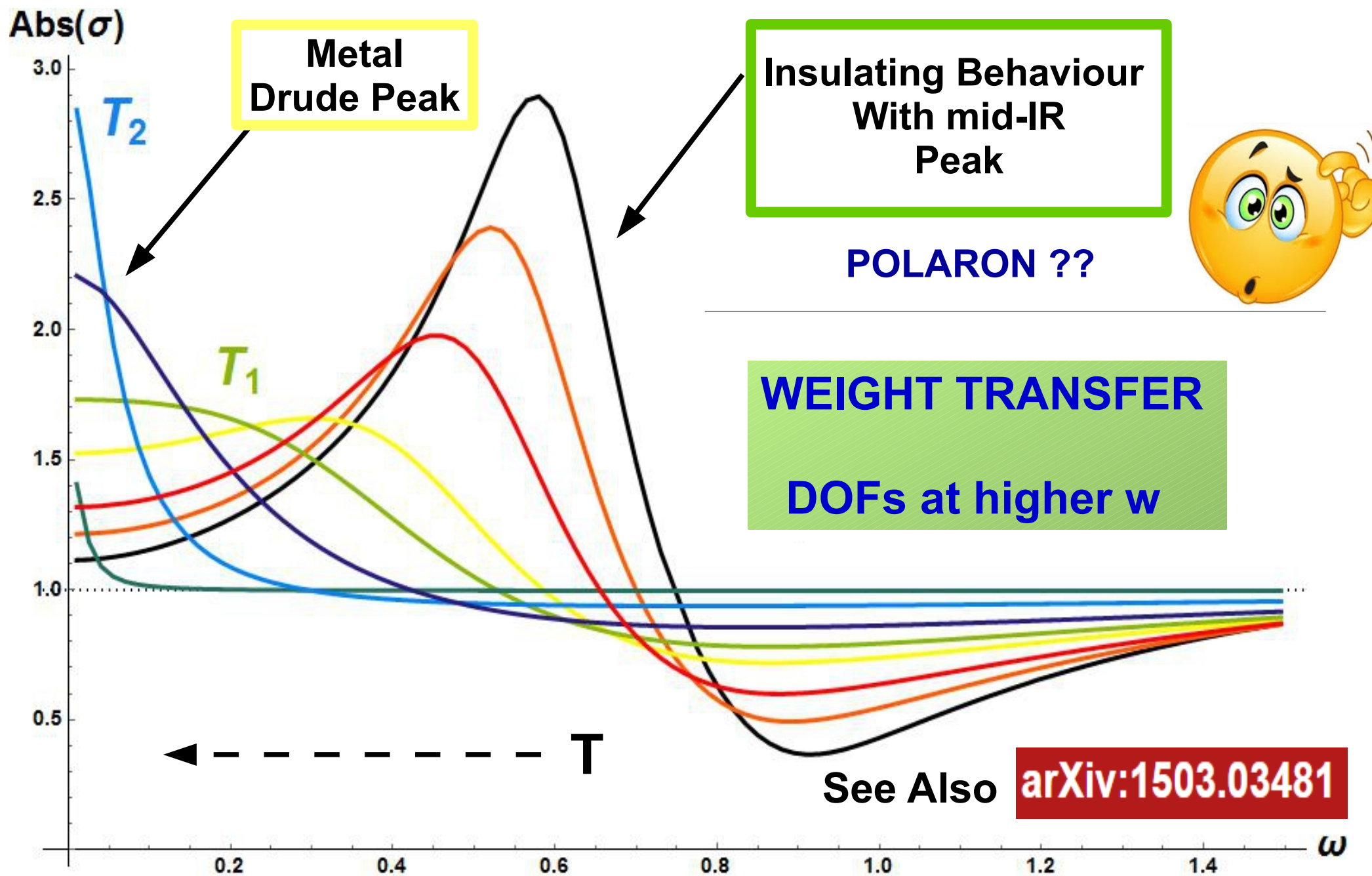
Possible ways out ...



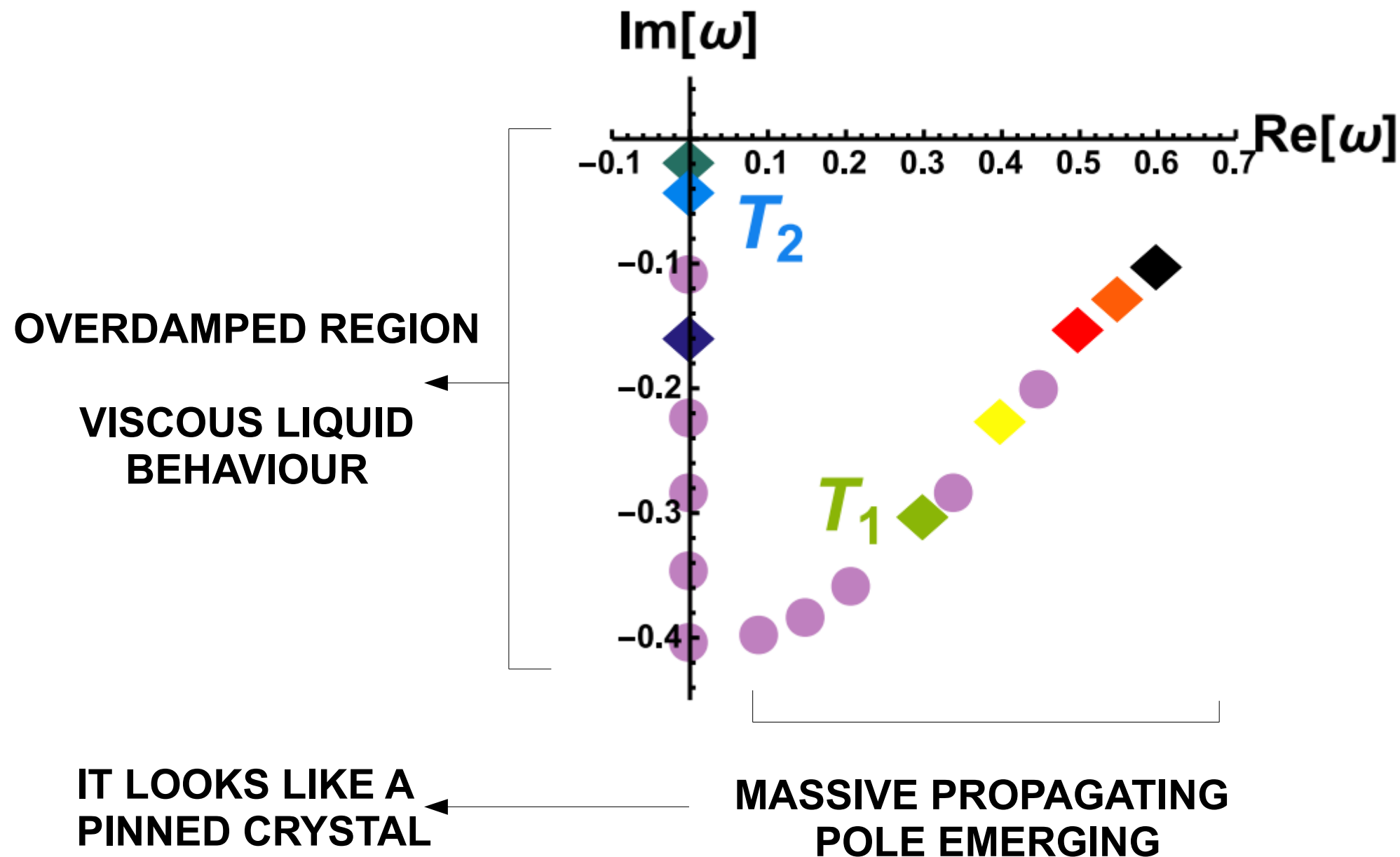
NEWS:
V GENERIC !!

Temperature Dependence

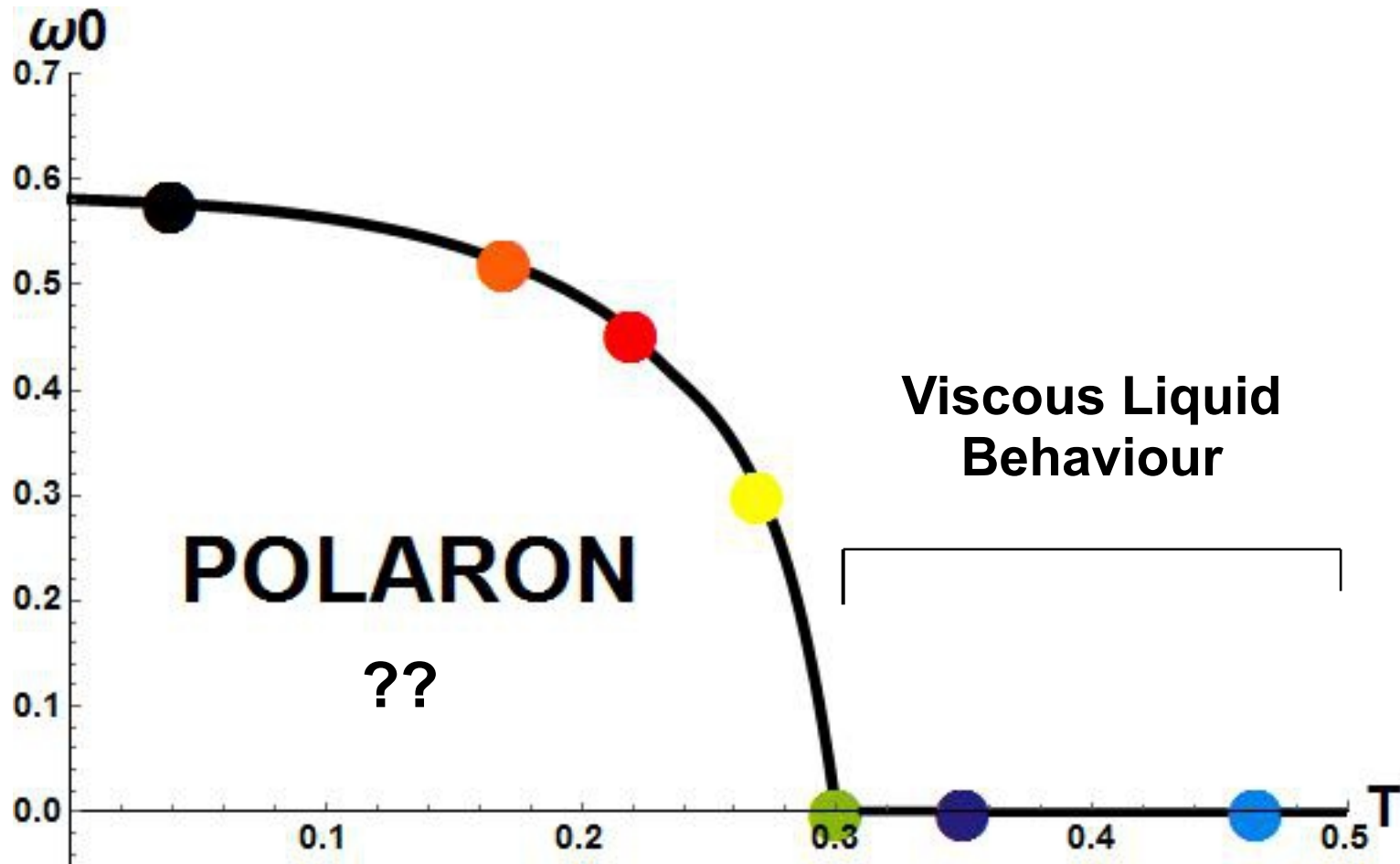
AC conductivity



QNMs and their physics



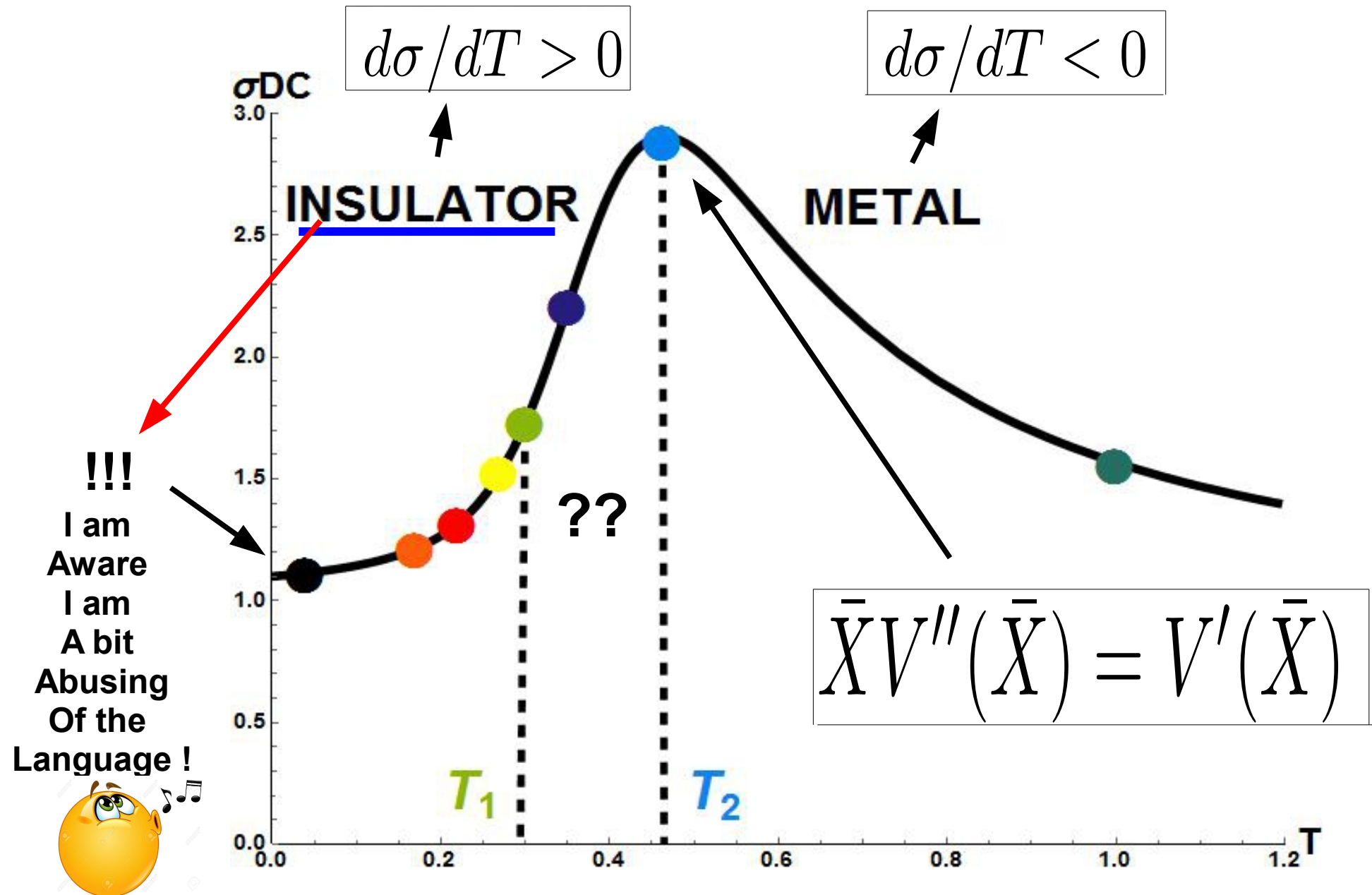
Localized peak formation



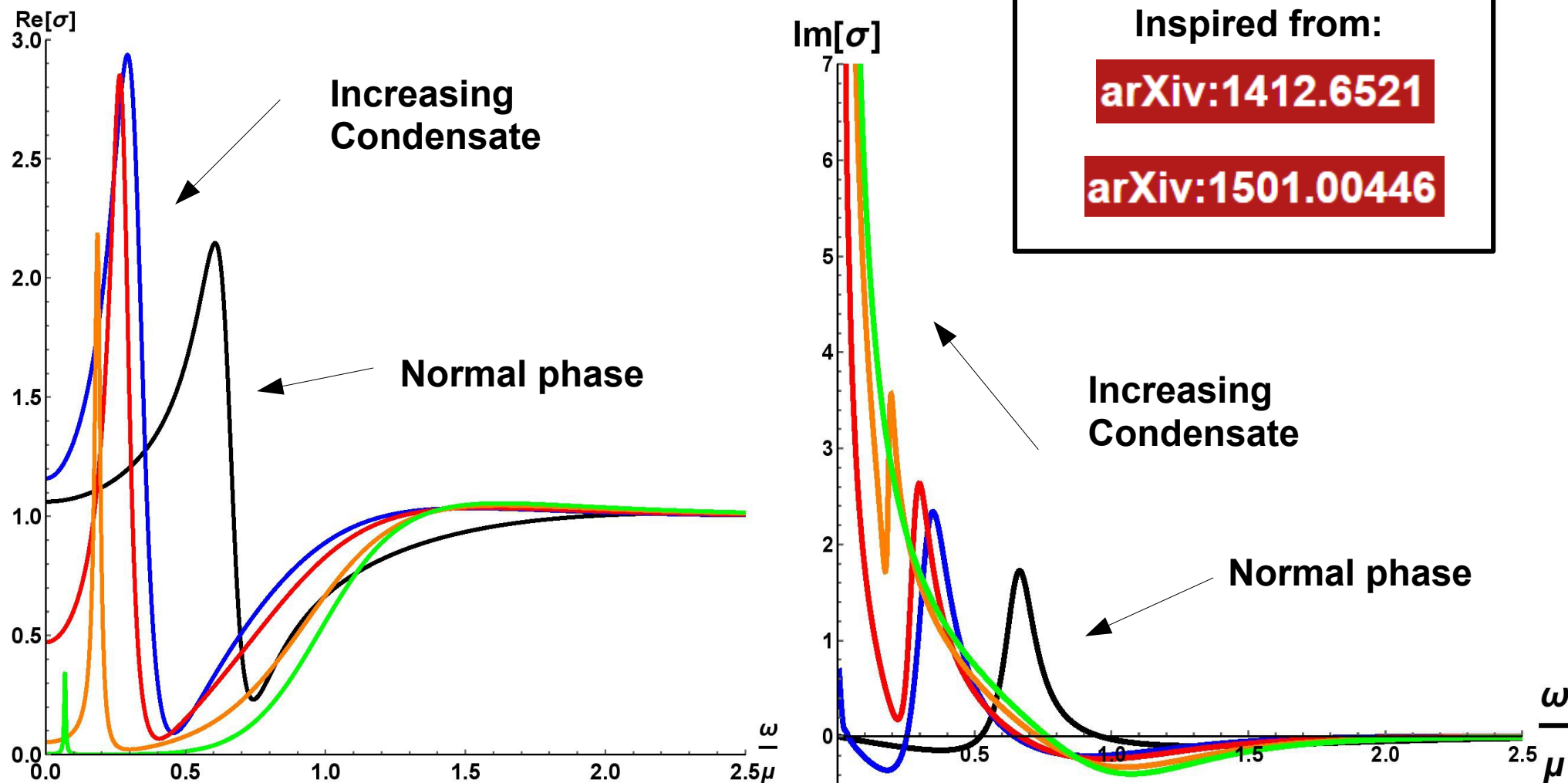
Looks like 2nd order Phase transition (Holographic Superconductors)

BUT : No Spontaneous Symmetry Breaking , No Condensate

“MIT” : from a viscous liquid to a pinned crystal



Holographic Superconductors



With M. Goykhman



- ✓ Additional Drag and Momentum Dissipation
- ✓ Non zero entropy at zero temperature
- ✓ Decreased mobility
- ✓ Heat Capacity
- ✓ Temperature Dependent Damping

PRE-CONCLUSIONS

~~AXIONS~~



Thanks ...

Conclusions

**MASSIVE GRAVITY PROVIDES AN EFFECTIVE
HOLOGRAPHIC THEORY FOR PHONONS IN SOLIDS**

**PHONONS ARE ENCODED IN THE FLUCTUATIONS OF THE STUCKELBERG
FIELDS WHICH ARE NOTHING ELSE THAN THE GOLDSTONES**

**SCALARS VEVs PROVIDES ALSO AN EXPLICIT BREAKING
WHICH CAN MIMICK IMPURITIES EFFECTS**

**MASSIVE GRAVITY PROVIDES
INTERESTING HOLOGRAPHIC FEATURES
LORENTZ VIOLATING MASSIVE GRAVITIES
ARE WAY MORE GENERAL THAN dRGT AND
HAVE STILL A LOT OF POWER TO BE EXPLOITED**

IDEA: TEST AND IMPROVE THE QUALITY OF THE ADS-CMT TOOL

To Do

Thermoelectric and Elastic Properties
To understand more the Phonons like
nature of the theory

PHONONS DRAG ??

ELASTIC MODULI
HOLOGRAPHIC THEORY
OF ELASTICITY ?

HOLOGRAPHIC
THEORY
FOR SOLIDS



**THANK YOU FOR YOUR
ATTENTION**



YESSSS, FINALLY OVER!

BACKGROUND

$$T = -\frac{f'(u_H)}{4\pi} = \frac{6 - \mu^2 u_H^2 - 2m^2 V(\alpha^2 u_H^2)}{8\pi u_H}$$

$$f(u) = u^3 \int_u^{u_H} dv \left[\frac{3}{v^4} - \frac{\rho^2}{2\ell^2} - \frac{(m\ell)^2}{v^4} V\left(\frac{\alpha^2 v^2}{\ell^2}\right) \right]$$

GOLDSTONE SECTOR AND INSTABILITY

$$V'(\bar{X})\partial_\mu\phi^i\partial^\mu\phi^i + \bar{X}V''(\bar{X})(\partial_i\phi^i)^2$$

$$M_\Phi^2(u) = (\partial\bar{a})^2 + \square\bar{a} = \frac{f(u\partial_u\bar{a})^2}{\ell^2} + \frac{u^2\partial_u(f\partial_u\bar{a})}{\ell^2}$$

$$\sqrt{V'(\bar{X})}\phi^i \qquad \bar{a}(u) = \frac{1}{2} \log V'(\bar{X})$$

$$M_\Phi^2 \sim -|f'(u)| \frac{V''(\bar{X})}{V'(\bar{X})} \Big|_{\text{horizon}} < 0$$

EOM

$$T_i \equiv u^2 h_{ti} - \frac{\partial_t \phi_i}{\alpha} \ , \ U_i \equiv f(u) \left[h_{ui} - \frac{\partial_u \phi_i}{\alpha u^2} \right] \ , \ B_i \equiv b_i - \frac{\phi_i}{\alpha}$$

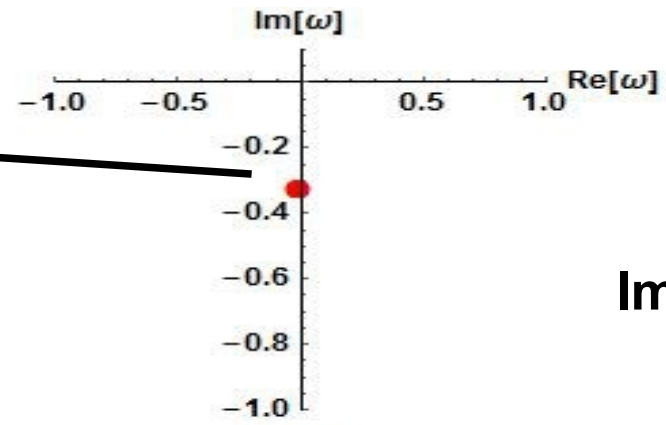
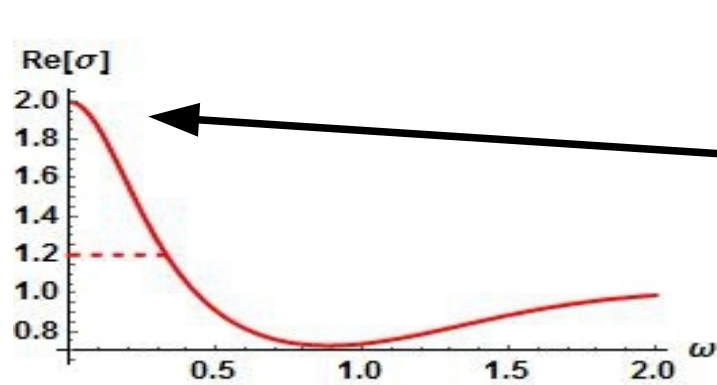
$$\partial_u (f \partial_u a_i) + \left[\frac{\omega^2}{f} - k^2 - 2u^2 \rho^2 \right] a_i = \frac{i \rho u^2 (2\bar{m}^2 + k^2)}{\omega} U_i - \frac{i f \rho k^2}{\omega} \partial_u B_i$$

$$\frac{1}{u^2} \partial_u \left[\frac{f u^2}{\bar{m}^2} \partial_u (\bar{m}^2 U_i) \right] + \left[\frac{\omega^2}{f} - k^2 - 2\bar{m}^2 \right] U_i = -2i \rho \omega a_i + \frac{f' k^2}{u^2} B_i$$

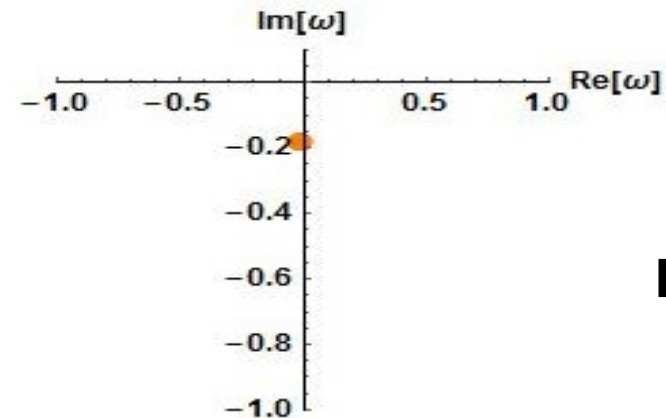
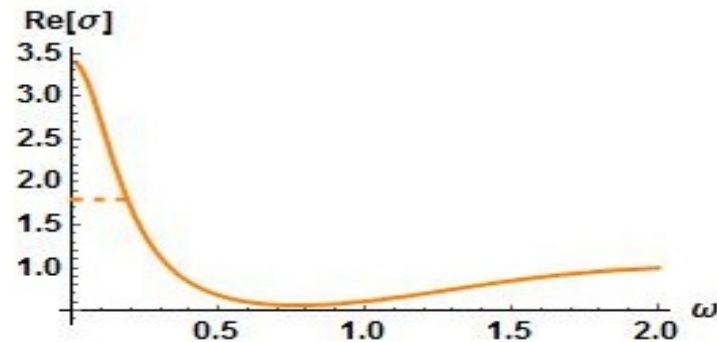
$$k \left\{ u^2 \partial_u \left(\frac{f}{u^2} \partial_u B_i \right) + \left[\frac{\omega^2}{f} - k^2 - 2\bar{m}^2 \right] B_i = -2 \frac{\bar{m}'}{\bar{m}} U_i \right\}$$

$$\bar{m}^2(u) = \alpha^2 m^2 V'(\alpha^2 u^2) \qquad \sigma(\omega) = \frac{\partial_u a_j}{i \omega a_j} \Big|_{u \rightarrow 0}$$

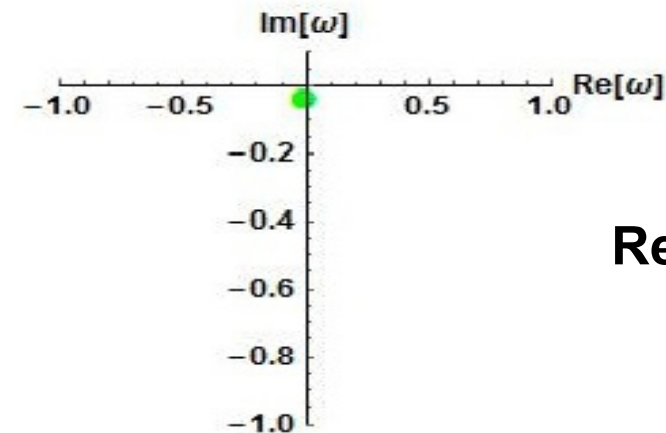
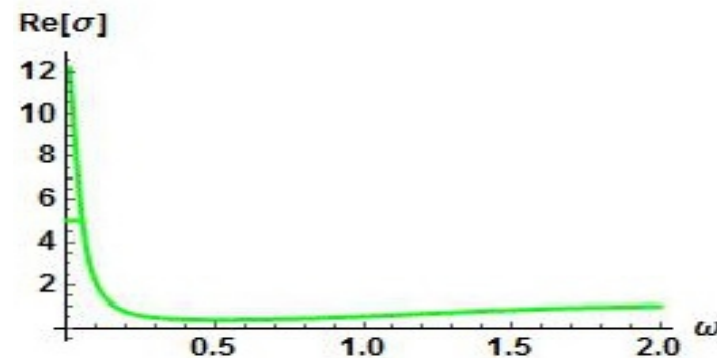
Quasinormal Modes I



Imaginary Part

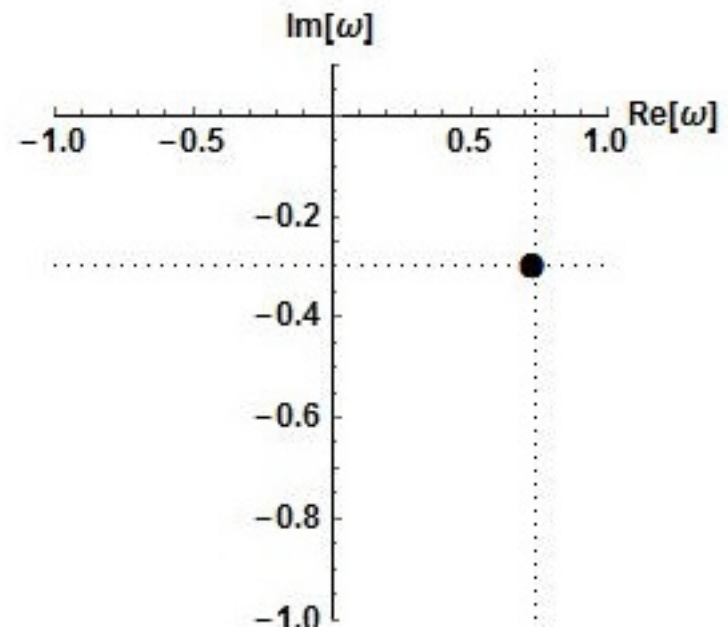
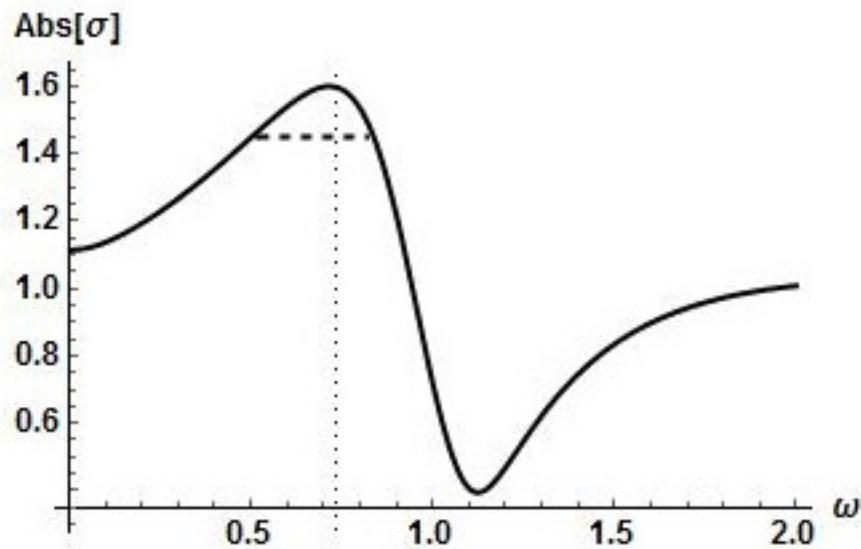
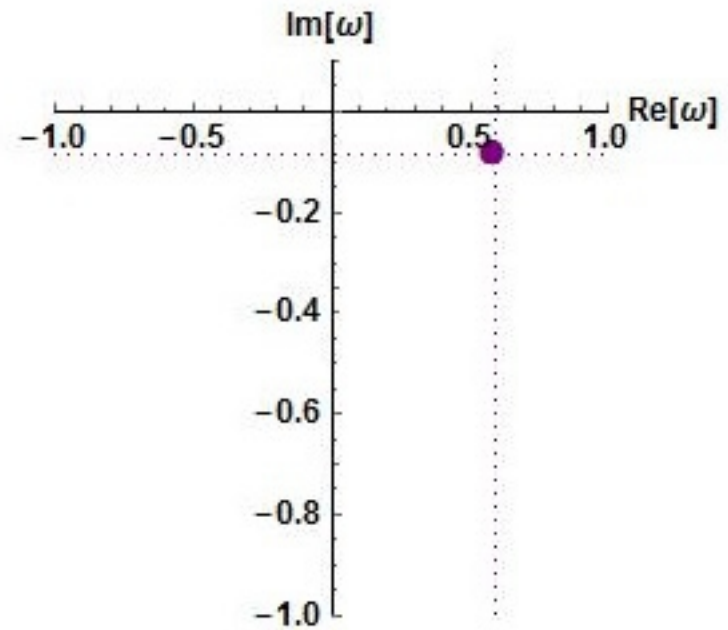
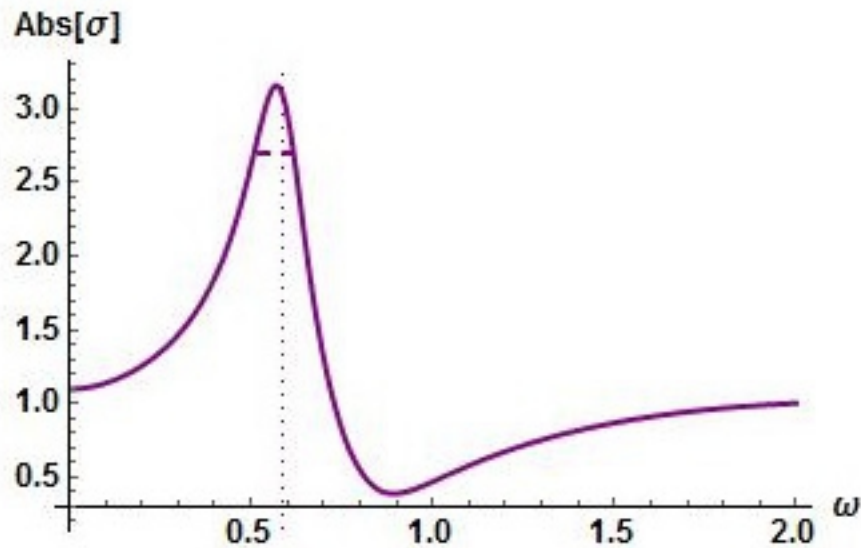


Decay width

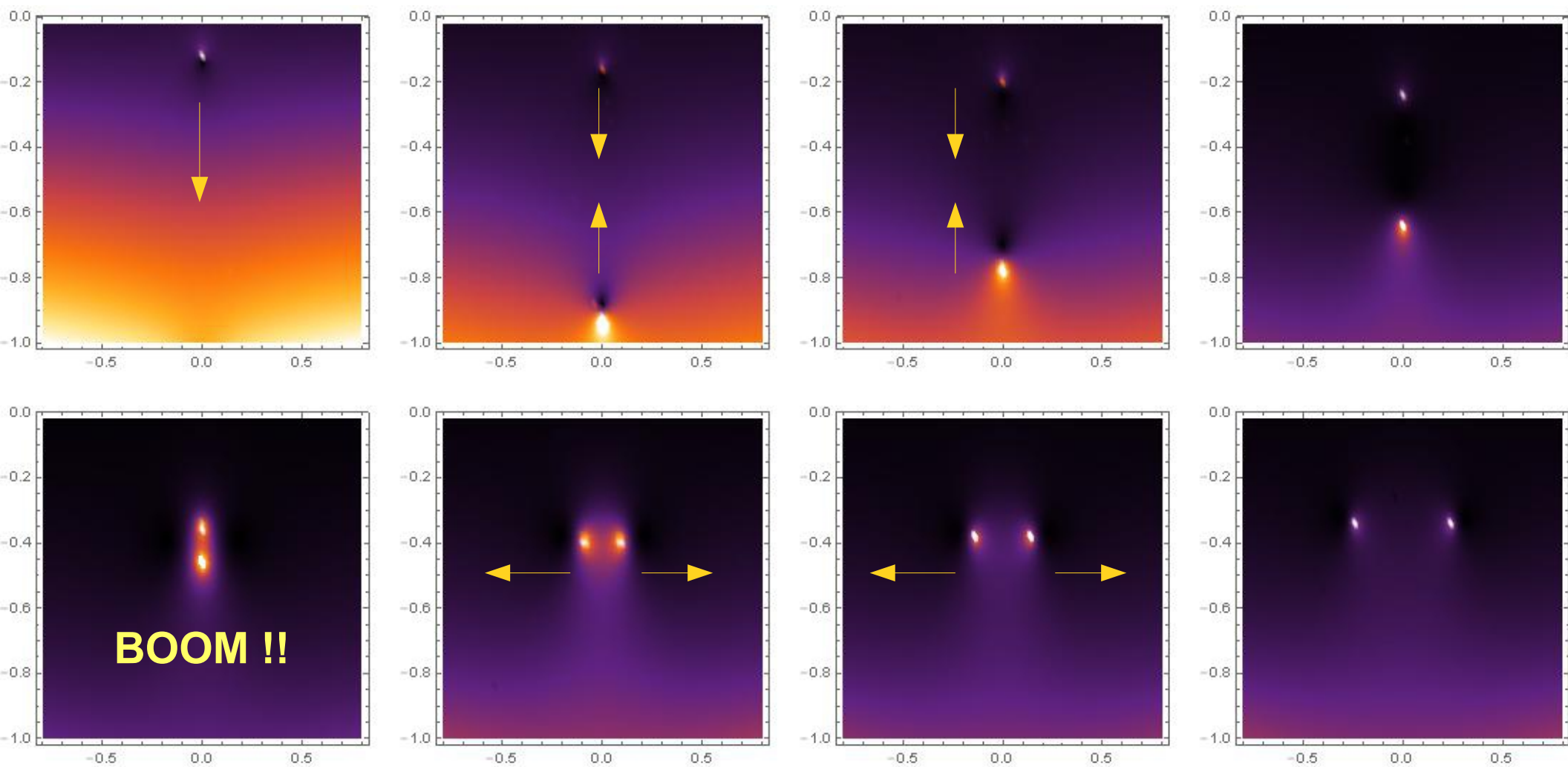


**Relaxation Time
Inverse**

Quasinormal Modes II



Quasinormal Modes IV



COHERENT – INCOHERENT TRANSITION

&

LOCALIZED PROPAGATING EXCITATION

arXiv:1411.1062

NEW

Non-Relativistic Effective field theories

arXiv:hep-ph/9311264

arXiv:hep-ph/9609466

SOLIDS :

relevant effective fields are the components of the vector $\vec{\xi}(x) = (\xi^1(x), \xi^2(x), \xi^3(x))$, which specifies the displacement of the material from the position in the ground state.

$$\mathcal{L}_2 = \frac{1}{2}\rho_0 \dot{\xi}_a \dot{\xi}_a - \frac{1}{2}\mu \partial_a \xi_b \partial_a \xi_b - \frac{1}{6}(\mu + 3K) \partial_a \xi_a \partial_b \xi_b + l_0 \xi_a \xi_a + O(p^4)$$

Theory of Sound Waves in Solids
Phonons as goldstone fields

$$l_0 = 0, \quad k_1 = -\mu, \quad k_2 = \frac{2}{3}\mu - K.$$

COVARIANT FORMULATION :

$$z_a(t, \vec{x}) = x_a - \xi_a(t, \vec{x})$$

$$\mathcal{L}_{eff} = \sqrt{|g|} F(\partial z, \nabla \partial z, \dots)_g.$$

If only 1st derivatives :

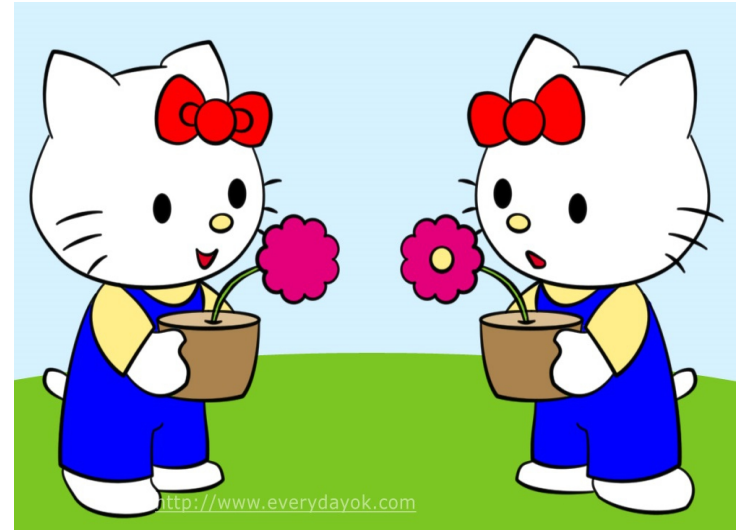
$$H_{ab} = g^{\mu\nu} \partial_\mu z_a \partial_\nu z_b$$

$$\mathcal{L}_{eff} = \sqrt{|g|} c^2 f(H)$$

**Back
Soon!**

Find the differences

Effective theories for phonons in solids :



COVARIANT FORMULATION :

$$z_a(t, \vec{x}) = x_a - \xi_a(t, \vec{x})$$

$$\mathcal{L}_{eff} = \sqrt{|g|} F(\partial z, \nabla \partial z, \dots)_g .$$

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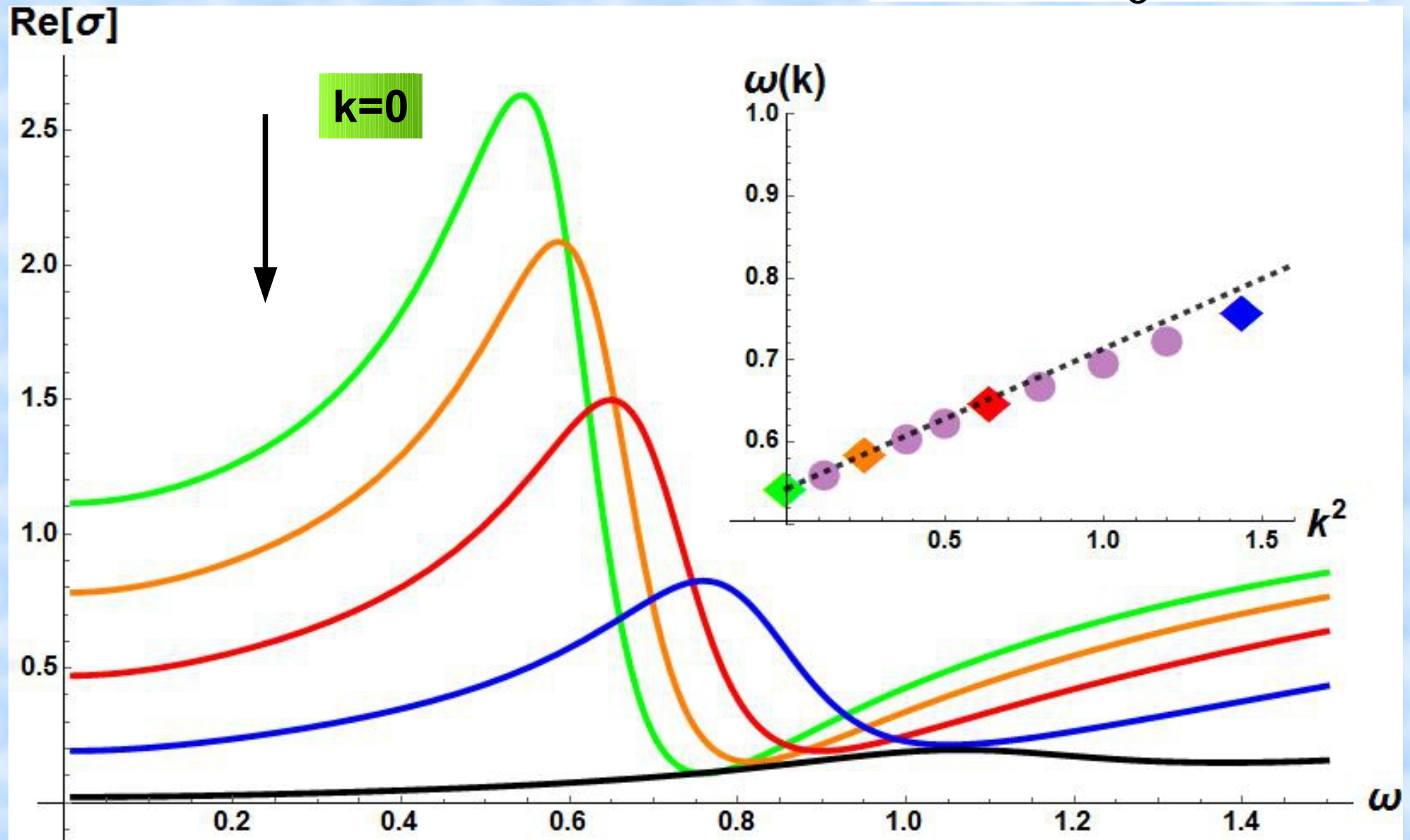
Back
Soon!

PREVIEW – RECIPE :

WE ARE JUST GOING TO GAUGE IT
AND PUT IT IN ANTI DE SITTER !

EMERGED PROPAGATING

“(quasi) PARTICLE” $\omega = \omega_0 + i\Gamma$



Quasinormal Modes Imprints

