Can one hear the shape of a black hole?

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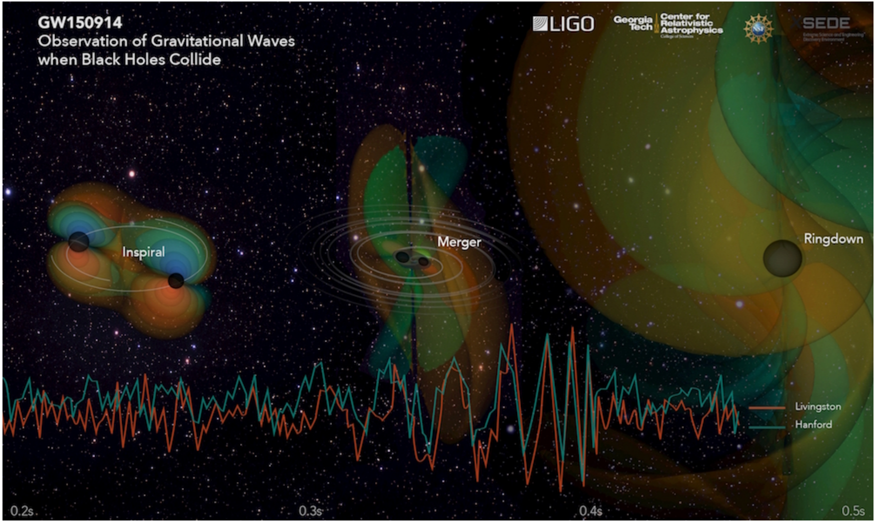
The first observation of gravitational waves: GW150914 LIGO/Virgo collab. (2016)



Credits: Georgia Tech Visualizations

Event Horizons

This detection enormously strengthens the evidence for stellar-mass black holes, whose existence is already supported by various indirect observations in the electromagnetic band.

But, is there any evidence for horizons?

Electromagnetic observations cannot probe the existence of event horizons, they may probe only the existence of light rings. Abramowicz, Kluźniak & Lasota (2002)

The gravitational wave ringdown signal might arguably provide the only conclusive proof of the existence of an event horizon in dark, compact objects.

Is there any evidence for *horizonless* compact objects? Can they be stable?

Quasi-Normal Modes

Quasi-normal modes (QNM) arise in general relativity in the study of perturbations of stellar or black hole spacetimes. Kokkotas & Schmidt (1999); Berti, Cardoso & Starinets (2009)

It is commonly believed that the ringdown is dominated by the QNMs of the final object.

For a rotating black hole, the entire QNM spectrum is characterized only by the black hole mass and angular momentum. Thus, the detection of a few modes from the ringdown signal can allow for precision measurements of the black hole mass and spin. Berti, Cardoso & Will (2006)

For a horizonless object, the boundary conditions change completely, and so the QNM structure.

Light Rings and Ringdown

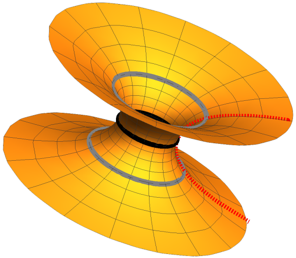
Light rings are boundaries within which photons can be trapped in circular orbits.

In principle, the ringdown phase should not depend on the presence of a horizon as long as the final object has a light ring.

For a black hole, the ingoing condition at the horizon simply takes the ringdown waves and 'carries' them inside the black hole. In this case, the black hole QNMs *incidentally* describe also the ringdown phase.

For another horizonless object, its relaxation should consist on the usual light ring ringdown modes (which are no longer QNMs), followed by the proper modes of vibration of the object itself.

We study the gravitational radiation emitted by a point particle in radial motion towards a traversable wormhole. Morris, Thorne & Yurtsever (1988); Visser (1996)



Two copies of the Schwarzschild spacetime with the same mass M,

$$ds^2 = -F\,dt^2 + F^{-1}\,dr^2 + r^2\,d\Omega^2, \quad \text{where } F = 1 - 2M/r,$$

glued together at the throat $r = r_0 > 2M$.

Consider an infalling particle with mass $\mu_p \ll M$ and energy E.

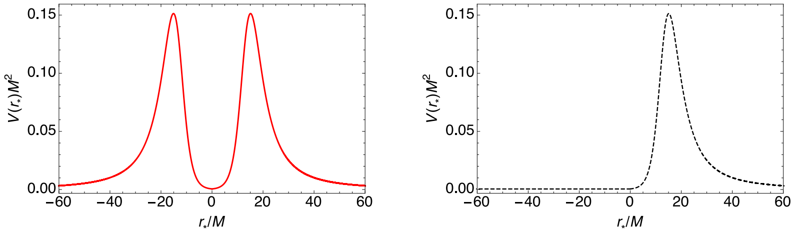
In the point-particle limit, the Einstein equations reduce to a pair of Zerilli equations, [Zerilli (1970)]

$$\frac{d^2\psi_\ell(\omega, r)}{dr_*^2} + \left(\omega^2 - V_\ell(r)\right)\psi_\ell(\omega, r) = S_\ell\,.$$

The junction conditions for ψ_ℓ at the throat depend on the properties of the matter confined in the thin shell. [Pani et al. (2009)] Here we assume that the microscopic properties of the shell are such that ψ_ℓ and $d\psi_\ell/dr_*$ are continuous at the throat.

Traversable Wormholes: Gravitational Potential

Effective $\ell = 2$ potential for a static traversable wormhole with $r_0 = 2.001M$ and for a black hole



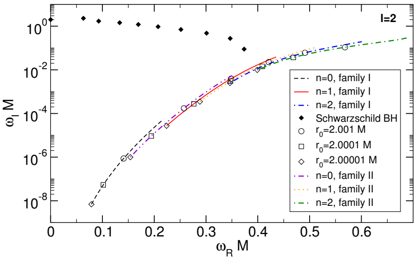
The effective potential is Z_2 symmetric and develops another barrier at $r_* < 0$.

For any $r_0 \lesssim 3M$, wormholes can support long-lived modes trapped between the two potential wells near the light rings.

The QNMs of the wormhole are defined by the eigenvalue problem associated with the Zerilli equation with $S_\ell = 0$ supplemented by regularity boundary conditions.

- At the asymptotic boundaries of both universes we require $\psi_\ell \sim e^{\pm i\omega r_*}$.
- At the throat we impose continuity of $d\psi_\ell/dr_*$, i.e., either $d\psi_\ell(0)/dr_* = 0$ or $\psi_\ell(0) = 0$.

We find two families of QNMs, $\omega = \omega_R + i\,\omega_I$, for different values of the throat location r_0.
In the black hole limit $r_0 \to 2M$, the spectrum is dramatically different from that of Schwarzschild. The QNMs of the wormhole approach the real axis and become long lived.

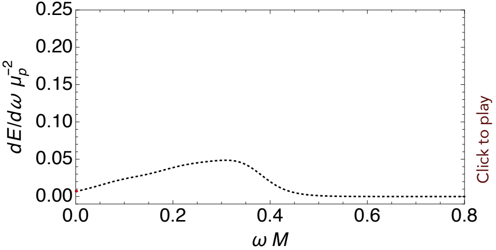


Quadrupolar Gravitational Waves Energy Spectra

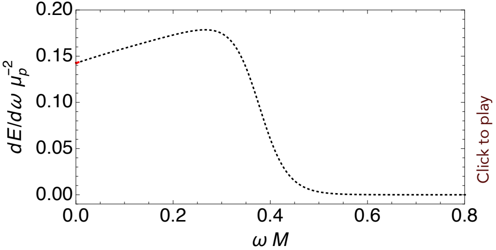
The energy flux emitted in gravitational waves is

$$\frac{\mathrm{d}E}{\mathrm{d}\omega} = \frac{1}{32\pi} \sum_{\ell \geqslant 2} \frac{(\ell+2)!}{(\ell-2)!} \omega^2 \left|\psi_\ell(\omega, r \to \infty)\right|^2 .$$

$r_0 = 2.1M,\ E = 1.1$



$r_0 = 2.001M,\ E = 1.5$



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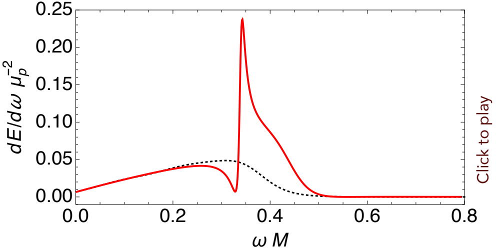

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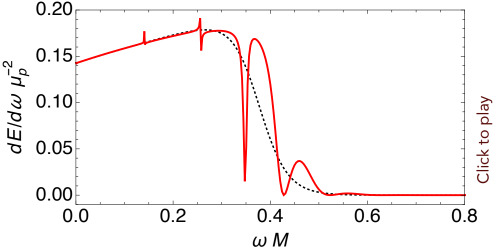
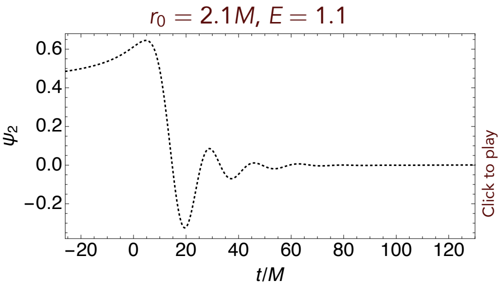


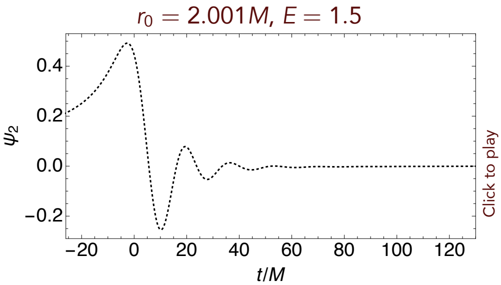
The spectra coincide only at low frequencies, but are generically very different.

In the black hole limit, the long-lived QNMs of the wormhole can be excited and correspond to narrow, Breit-Wigner resonances in the spectrum. Pons et al. (2002); Berti, Cardoso & Pani (2009)

Gravitational Wave Extraction

Given the drastically differences between the QNM and energy spectra of a wormhole and those of a black hole, one might be tempted to expect a completely different ringdown signal.





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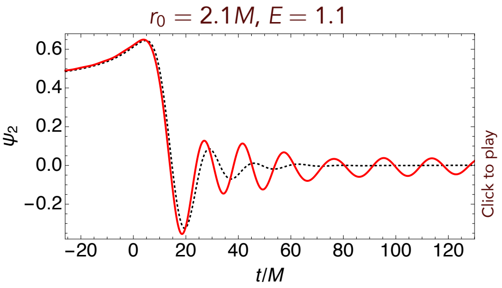

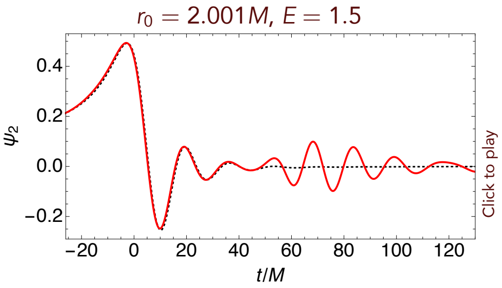

The differences in the energy spectra do not leave any trace in the initial ringdown waveform.

The QNMs of the wormhole contain low energy and get excited only at late times.

Summary

A highly counterintuitive phenomenon: the initial ringdown signal chiefly depends on the properties of the light ring of the final object.

The actual QNMs of the object are excited only at late times and typically do not contain a significant amount of energy.

Our results suggest that future gravitational waves detections by aLIGO, aVIRGO and KAGRA should focus on extracting the late-time ringdown signal, where the actual QNMs of the final object are eventually excited. Such detections might rule out exotic alternatives to black holes and to test quantum effects at the horizon scale. [Giddings (2016)] The post-merger signal leaves room for alternative theories of gravity and exotic compact objects. [Chirenti & Rezzolla (2016); Yunes, Yagi & Pretorius (2016)]

As it stands, the single event GW150914 does not provide the final evidence for horizons, but strongly supports the existence of light rings.

The title is a tribute to a famous work by Marc Kac,

M. Kac. ‘Can one hear the shape of a drum?’ *Am. Math. Mon.* **73**, 1 (1966)

where he discusses an old conjecture that the shape of a drum can be inferred solely from its sound. See also https://en.wikipedia.org/wiki/Hearing_the_shape_of_a_drum

Note: To play animations you might need Adobe Acrobat Reader.