Heavy-Light flavour Physics: D^* and B^* mesons decay constants in lattice QCD

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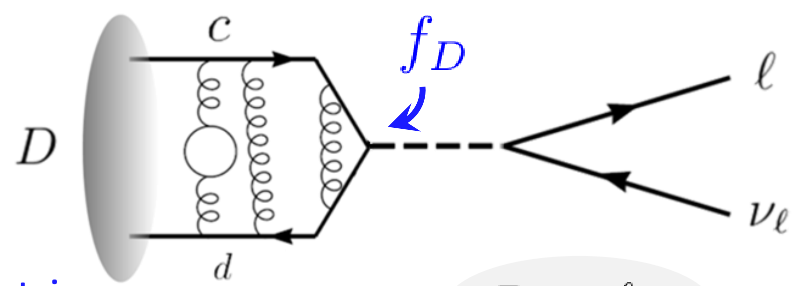
Decay Constants

Why are they interesting parameters?

DCs parametrize the matrix element of a weak current between the vacuum and the meson of interest. They characterize a meson as much as its mass.

$$\mathcal{Z}_V := \langle 0|\bar{c}\gamma_\mu \ell|V(p,\lambda)\rangle = M_V f_V \varepsilon_\mu^\lambda$$

$$\mathcal{Z}_A := \langle 0|\bar{c}\gamma_\mu\gamma_5 \ell|P(p)\rangle = p_\mu f_P$$



$D \to \ell\nu_\ell$

Matrix Elements

$$\Gamma(P \to \ell\overline{\nu}_\ell) = \frac{G_F^2 |V_{ij}|^2}{8\pi} f_P M_P m_\ell^2 \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2$$

SM parameters: CKM matrix elements

Kinematic factors

D^*

	Mode	Fraction (Γ_i/Γ)
Γ_1	$D^0\pi^+$	$(67.7 \pm 0.5)\%$
Γ_2	$D^+\pi^0$	$(30.7 \pm 0.5)\%$
Γ_3	$D^+\gamma$	$(1.6 \pm 0.4)\%$

Vector meson decays are dominated by the strong and electromagnetic decays. → f_V is **not** directly measurable.

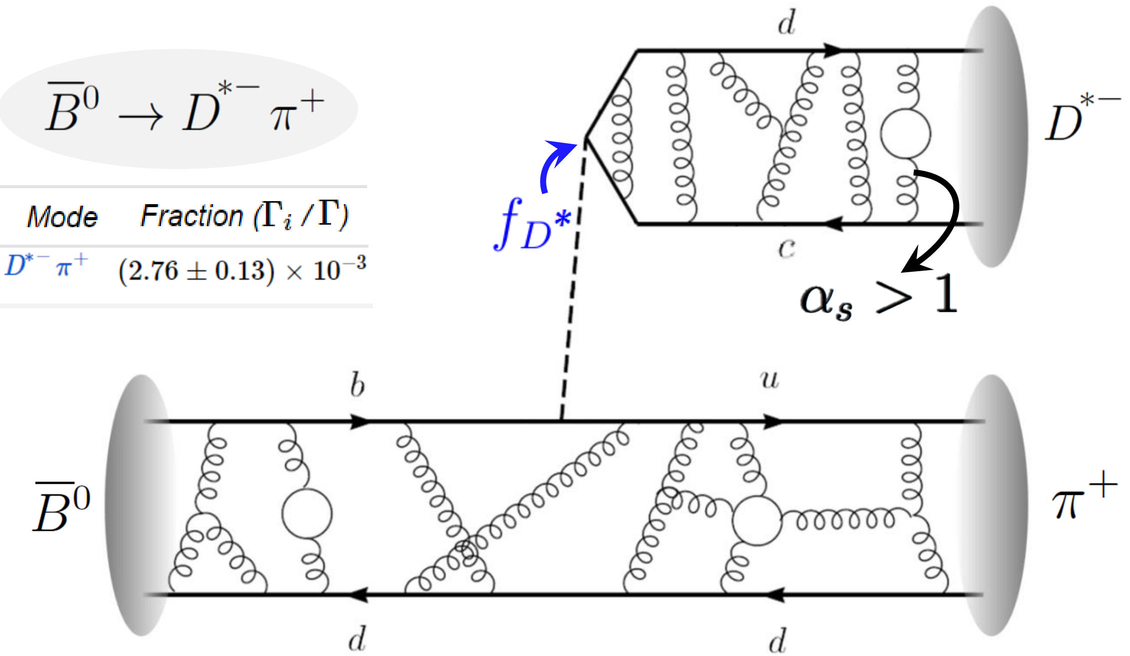
Decay Constants

Why are they interesting parameters?

Vector DCs are involved in the description of semileptonic form factors and non-leptonic decays of hadrons through *the factorization approximation :*

$$\overline{B}^0 \to D^{*-} \pi^+$$

	Mode	Fraction (Γ_i/Γ)
Γ_{41}	$D^{*-}\pi^+$	$(2.76 \pm 0.13) \times 10^{-3}$



$$A_{\text{fact}} = -\frac{G_F}{\sqrt{2}} V_{ub} V_{cd} \left[C_2(m_b) + \frac{C_1(m_b)}{N_c} \right] \overbrace{\langle D^{*+} | \bar{c} \gamma^{\mu L} d | 0 \rangle}^{f_{D^*}} \langle \pi^- | \bar{b} \gamma^{\mu L} u | B^0 \rangle$$

Data Ensambles Nf = 2+1+1

	β	L^3xT	$am_{u/d}$	am_h	
A.40.24	1.90 ($a^{-1} \sim 2.19$ GeV)	$24^3 \times 48$	0.0040	0.01800 0.02200 0.02600	s
A.60.24	$a \sim 0.0885\,\text{fm}$		0.0060	0.21256 0.2500 0.29404	c
A.80.24			0.0080	0.34583 0.40675 0.47480	h
A.100.24			0.0100	0.56267 0.66178 0.77836	
A.30.32		$32^3 \times 64$	0.0030	0.91546	
A.40.32			0.0040		
A.50.32			0.0050		
B.85.24	1.95 ($a^{-1} \sim 2.50$ GeV)	$24^3 \times 48$	0.0085	0.01550 0.01900 0.02250	
B.25.32	$a \sim 0.0815\,\text{fm}$	$32^3 \times 64$	0.0025	0.18705 0.22000 0.25875	
B.35.32			0.0035	0.30433 0.35794 0.42099	
B.55.32			0.0055	0.49515 0.58237 0.68495	
B.75.32			0.0075	0.80561	
D.15.48	2.10 ($a^{-1} \sim 3.23$ GeV)	$48^3 \times 96$	0.0015	0.01230 0.01500 0.01770	
D.20.48	$a \sim 0.0619\,\text{fm}$		0.0020	0.14454 0.17000 0.19995	
D.30.48			0.0030	0.23517 0.27659 0.32531	
				0.38262 0.45001 0.52928	
				0.62252	

Interpolation

Continuum extrapolation

Chiral extrapolation

Data Ensambles Nf = 2+1+1

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$$m_q a \ll 1$$

$$m_b^{phys} = 4.26(9)\ \text{GeV} > a^{-1}$$

Heavy quark extrapolation

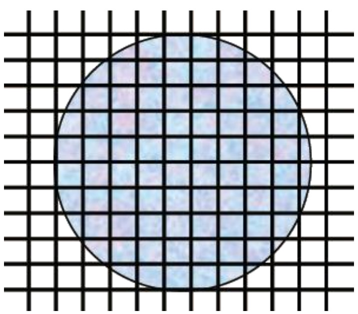
Correlation functions

f_V and f_P can be extracted from the asyptotic behaviour in time of the vector and axial 2-point correlation functions $C_V(t), C_A(t)$:

Sink Source

$$C_i(x) = \langle 0 | T(O_i(x) O_i^\dagger(0)) | 0 \rangle \qquad (O_i = \bar{c}\gamma^\mu \ell \, , \, \bar{c}\gamma^\mu\gamma_5 \ell \ )$$

$$C_i(t) = \underbrace{\frac{1}{2M} \mathcal{Z}_i{}^2 e^{-Mt}}_{\text{Foundamental state}} + \underbrace{\sum_{n>1} \frac{1}{2M_n} \mathcal{Z}_n^2 e^{-M_n t}}_{\text{Excited states}}$$



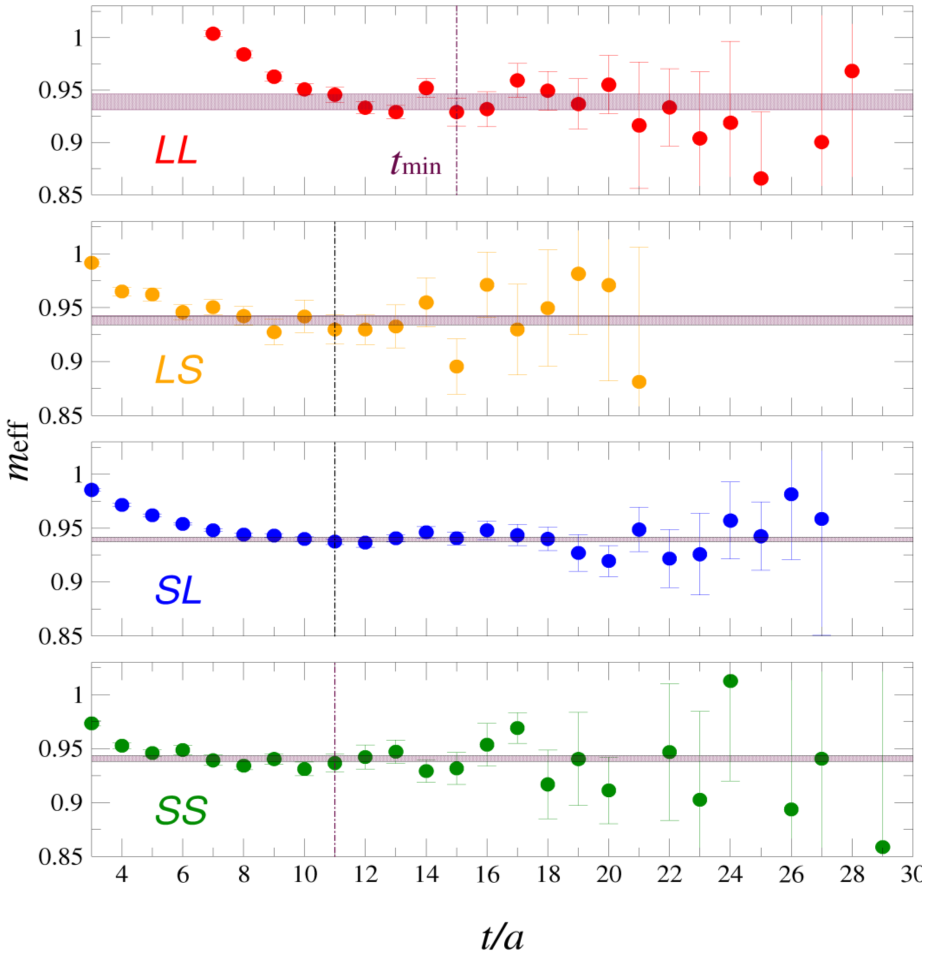
For heavy mesons, we can construct ***smeared S*** operators which have a better superposition with the foundamental state rather than the excited states:

$$\mathcal{Z}^S = \langle 0 | O_i^S | M \rangle \gg \mathcal{Z}_n = \langle 0 | O_i | M_n \rangle$$

We have a matrix of correlation functions made up of the possible combinations of ***local L*** and ***smeared S*** operators: *LL, LS, SL, SS.*

Effective mass

$$m_{\text{eff}}(t) = \operatorname{arccosh}\left[\frac{C(t)+C(t+2)}{2C(t+1)}\right] \xrightarrow[t\gg 0]{} M$$



The meson mass is extracted from a constant fit of the effective mass curve in the **plateau interval [t_{min}:t_{MAX}]** .

$$C^{LL}(t) \xrightarrow[t\gg 0]{} \frac{1}{M}(\mathcal{Z}^L)^2 \mathrm{e}^{-M\frac{T}{2}} \cosh\left[M\left(\frac{T}{2}-t\right)\right]$$

$$C^{LS}(t) \xrightarrow[t\gg 0]{} \frac{1}{M}\mathcal{Z}^L \mathcal{Z}^S \mathrm{e}^{-M\frac{T}{2}} \cosh\left[M\left(\frac{T}{2}-t\right)\right]$$

$$C^{SL}(t) \xrightarrow[t\gg 0]{} \frac{1}{M}\mathcal{Z}^S \mathcal{Z}^L \mathrm{e}^{-M\frac{T}{2}} \cosh\left[M\left(\frac{T}{2}-t\right)\right]$$

$$C^{SS}(t) \xrightarrow[t\gg 0]{} \frac{1}{M}(\mathcal{Z}^S)^2 \mathrm{e}^{-M\frac{T}{2}} \cosh\left[M\left(\frac{T}{2}-t\right)\right]$$

M : are calculated out of ***SL*** effective mass curves.

f_V and f_P

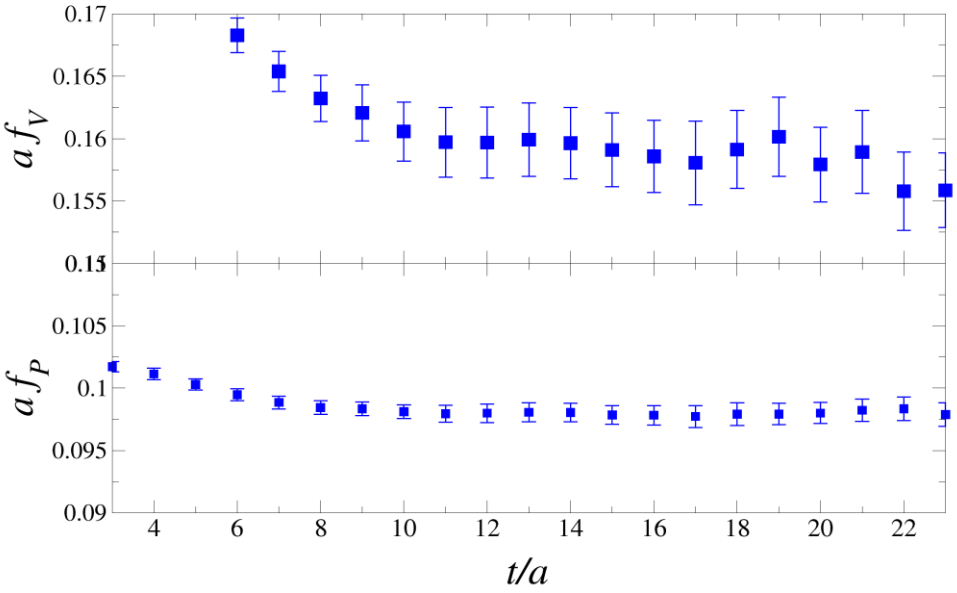
$$f_V = \frac{1}{Z_A M_V}\mathcal{Z}_V^L$$

It requires renormalization

$$f_P = \frac{(m_\ell + m_c)}{M_P \sinh M_P}\mathcal{Z}_P^L$$

$\partial_\mu A^\mu(x) = (m_\ell + m_c)P_5(x)$

Applaying the PCAC ,
we don't need any RC!



$$C^{SL}(t) \xrightarrow[t\gg 0]{} \frac{1}{M}\mathcal{Z}^S\mathcal{Z}^L e^{-M\frac{T}{2}} \cosh\left[M\left(\frac{T}{2}-t\right)\right]$$

$$C^{SS}(t) \xrightarrow[t\gg 0]{} \frac{1}{M}(\mathcal{Z}^S)^2 e^{-M\frac{T}{2}} \cosh\left[M\left(\frac{T}{2}-t\right)\right]$$

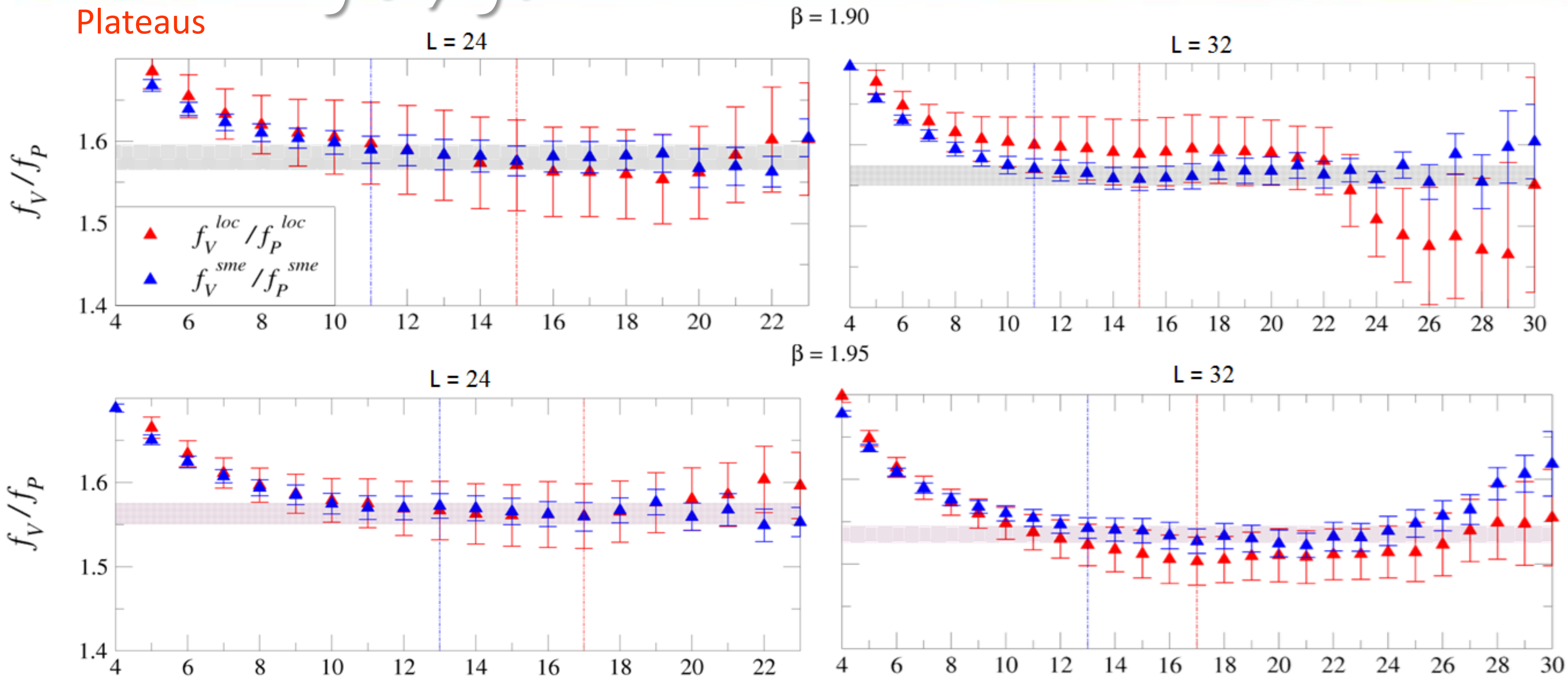
$\mathcal{Z}^L$: can be calculated from a combination of the ***SL*** and ***SS*** correlation functions.

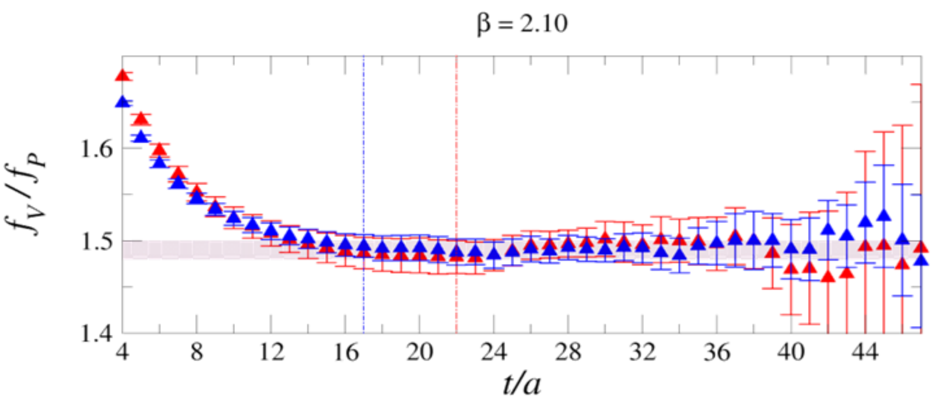
$$f_V(t) = \frac{1}{Z_A}\frac{1}{\sqrt{M_V}}\frac{e^{-M_V\frac{T}{4}}}{\sqrt{\cosh\left[M_V\left(\frac{T}{2}-t\right)\right]}}\frac{C^{SL}_{V_iV_i}(t)}{\sqrt{C^{SS}_{V_iV_i}(t)}}$$

$$f_P(t) = \frac{(m_\ell + m_c)}{\sqrt{M_P}\sinh(M_P)}\frac{e^{-M_P\frac{T}{4}}}{\sqrt{\cosh\left[M_P\left(\frac{T}{2}-t\right)\right]}}\frac{C^{SL}_{PP}(t)}{\sqrt{C^{SS}_{PP}(t)}}$$

Ratios f_V / f_P

Plateaus

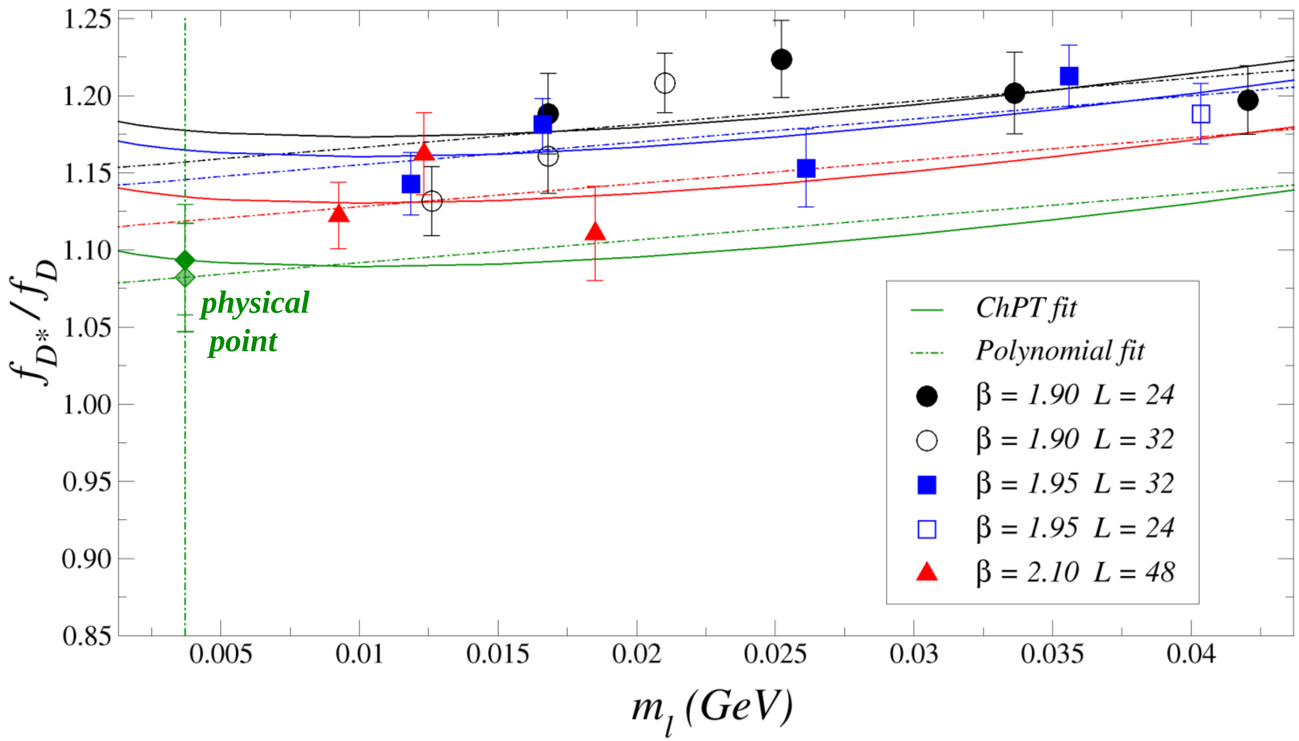




The red and blu curves are constructed from the ***LL*** correlations and the ***SL-SS*** combination respectively.

f_{D^*}

Chiral and continuum extrapolation



- ***Polyn. Fit :*** $\frac{f_V}{f_P}^{fit(1)} = \mathbf{a}^{(1)} + \mathbf{b}^{(1)}\xi_\ell + \mathbf{c}^{(1)} a^2$

$$\xi_\ell = \frac{2B_0 m_\ell}{(4\pi f_0)^2} \qquad B_0 \simeq 5.25\text{GeV} \quad f_0 \simeq 0.12\text{GeV} \qquad \hat{g} = 0.61$$

ref: [arXiv:1403.4504v3 [hep-lat] **(ETMC)**]

- ***ChPT fit :*** $\frac{f_V}{f_P}^{fit(2)} = \mathbf{a}^{(2)} \left[1 + \mathbf{b}^{(2)}\,\xi_\ell + \left(\frac{3(1+3\hat{g}^2)}{4} - \frac{5}{4} \right) \xi_\ell \log\left(\xi_\ell\right) \right] + \mathbf{c}^{(2)} a^2$

f_{D^*} and $f_{D_s^*}$

Results

D^* meson decay constants with $N_f = 2+1+1$:

$$\frac{f_{D^*}}{f_D} = 1.088 \pm 0.037 \qquad f_{D^*} = 232.2 \pm 0.90\,\text{MeV}$$

$$\frac{f_{D_s^*}}{f_{D_s}} = 1.091 \pm 0.018 \qquad f_{D_s^*} = 273.4 \pm 0.65\,\text{MeV}$$

For fD and fDs see ref: [arXiv:1411.7908 [hep-lat] **(ETMC)**]

D^* meson masses :

$$\frac{M_{D^*}}{M_D} = 1.0780 \pm 0.0077 \qquad M_{D^*} = 2012 \pm 14\,\text{MeV}$$

$$M^{exp}_{D^{*\pm}} = (2010.27 \pm 0.05)\text{MeV}$$

$$\frac{M_{D_s^*}}{M_{D_s}} = 1.0751 \pm 0.0056 \qquad M_{D_s^*} = 2117 \pm 11\,\text{MeV}$$

$$M^{exp}_{D_s^{*\pm}} = (2112.1 \pm 0.4)\text{MeV}$$

f_{D^*} and $f_{D_s^*}$

Results

D^* meson decay constants with $N_f = 2 + 1 + 1$:

$$\frac{f_{D^*}}{f_D} = 1.088 \pm (0.030_{stat} \pm 0.020_{tmin} \pm 0.006_{input} \pm 0.005_{fit})$$

$$\frac{f_{D_s^*}}{f_{D_s}} = 1.091 \pm (0.015_{stat} \pm 0.009_{tmin} \pm 0.006_{input})$$

For the input parameters see ref: [arXiv:1403.4504v3 [hep-lat] **(ETMC)**]

Previous values

- $N_f = 2$:

$$f_{D^*}/f_D = 1.208 \pm 0.027$$

ref: [arXiv:1407.1019 [hep-ph] **(ETMC)**]

$$f_{D_s^*}/f_{D_s} = 1.26 \pm 0.03$$

ref: [arXiv:1201.4039 [hep-lat] **(ETMC)**]

- $N_f = 2 + 1$:

-

$$f_{D_s^*}/f_{D_s} = 1.10 \pm 0.02$$

ref: [arXiv:1312.5264 [hep-lat] **(HPQCD)**]

ETMC Ratio Method

$$\mathcal{R}_\ell(\overline{m}_h^{(n)}) = \frac{\mathcal{F}_\ell(\overline{m}_h^{(n)})}{\mathcal{F}_\ell(\overline{m}_h^{(n-1)})} \frac{C_W(\overline{m}_h^{(n-1)}, \mu)}{C_W(\overline{m}_h^{(n)}, \mu)}$$

where

- $\mathcal{F}_\ell = f_V/f_P \text{ or } M_V/M_P$
- $C_W(m) = 1 - \frac{2}{3}\frac{\alpha_s(m)}{\pi} - \left[-\frac{1}{9}\zeta(3) + \frac{2}{27}\pi^2 \log 2 + \frac{4}{81}\pi^2 + \frac{115}{36}\right]\left(\frac{\alpha_s(m)}{\pi}\right)^2$

ref: [arXiv: hep-ph/0303052v4]

We can construct ratios that go to 1 in the static limit (1/mh → 0) :

$$\lim_{m_h \to \infty} \mathcal{R}_\ell(m_h) = 1$$

List of reference masses:

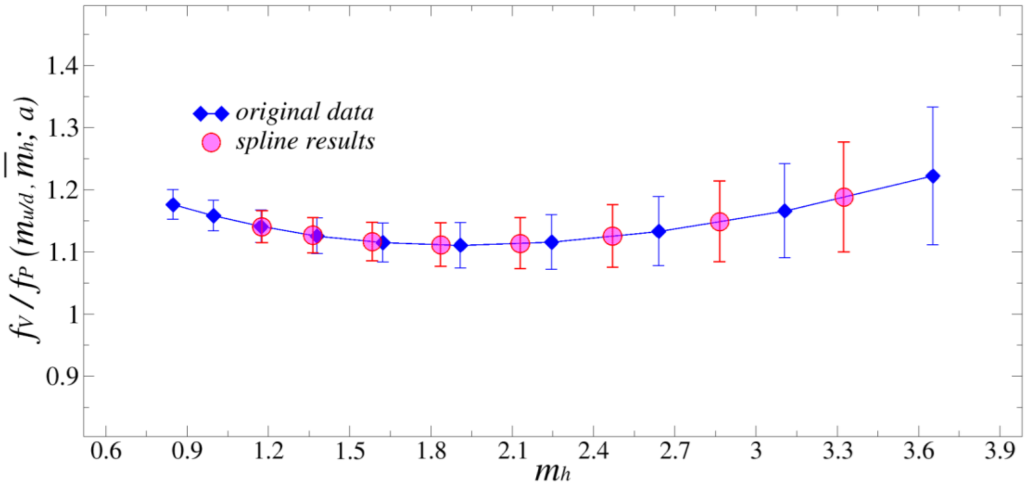
$$\{\overline{m}_h^{(n)}\} = \{\overline{m}_h^{(0)}, \overline{m}_h^{(1)}, \ldots, \overline{m}_b^{phys}\}$$

$$\lambda = \frac{\overline{m}_h^{(n)}}{\overline{m}_h^{(n-1)}} = 1.1600 \qquad N = 10$$

$$\overline{m}_h^{(0)}(\overline{\mathrm{MS}}, 2\mathrm{GeV}) = 1.175\ \mathrm{GeV}$$

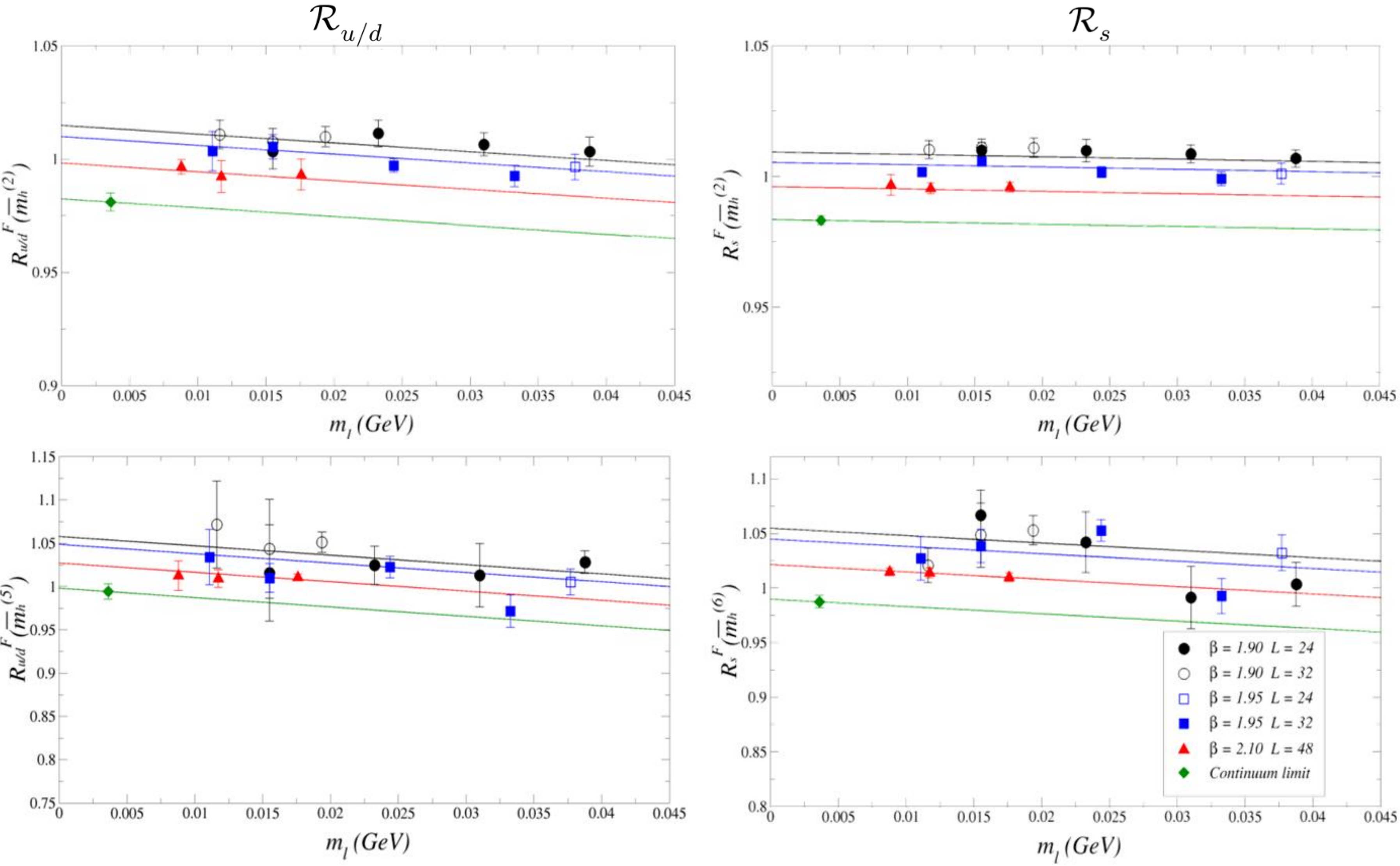
$$m_b^{phys}(\overline{\mathrm{MS}}, m_b) = 4.26(9)\ \mathrm{GeV}$$

ref: [*Petros talk*]



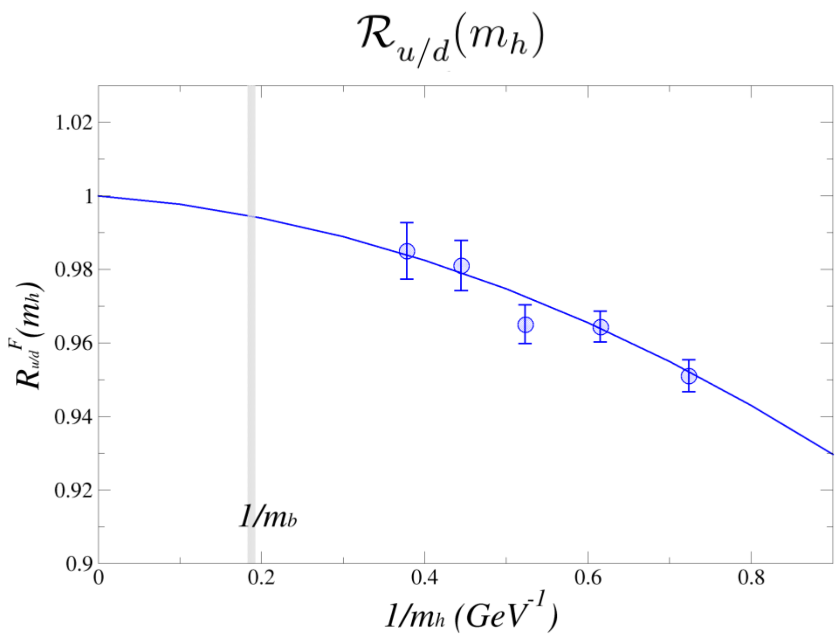
ETMC Ratio Method

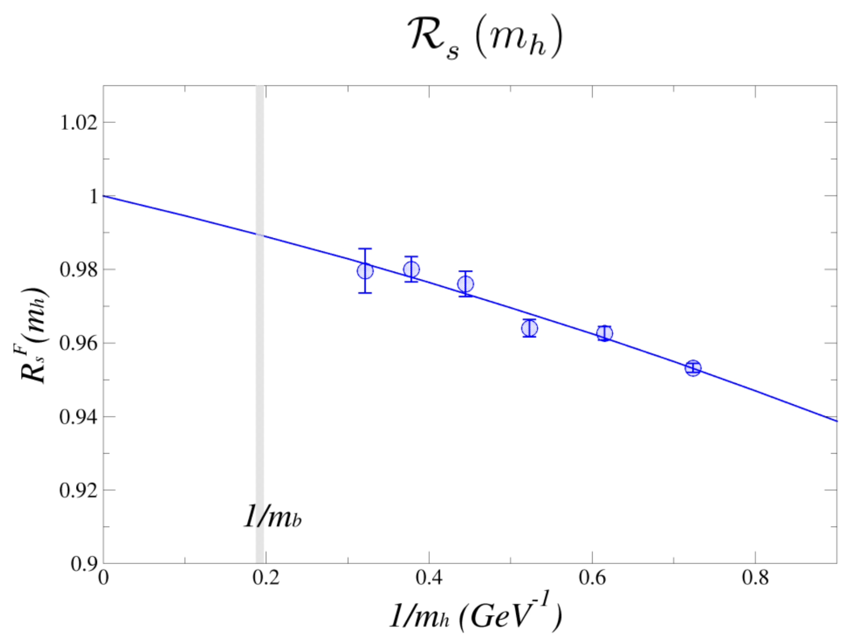
Chiral and continuum extrapolation



ETMC Ratio Method

Heavy quark extrapolation





$$\mathcal{R}_\ell^{(fit)}(m_h) = 1 + \frac{\mathbf{b}_\ell^{(1)}}{m_h} + \frac{\mathbf{b}_\ell^{(2)}}{m_h^2}$$

The result, corresponding to the b quark mass, is reached multiplying the $N = 10$ ratios obtained from the fit .

$$\mathcal{F}_\ell(m_b^{phys}) = \frac{\mathcal{F}_\ell(\overline{m}_h^{(0)})}{C_W(\overline{m}_h^{(0)}, \mu)} \prod_{i=1}^{N} \left[\mathcal{R}_\ell^{(fit)}(\overline{m}_h^{(n)})\right] C_W(\overline{m}_h^{(N)}, \mu)$$

fB* and fBs*

Results

PRELIMINARY

B* meson decay constants with Nf = 2 + 1 + 1 :

$$\frac{f_{B^*}}{f_B} = 0.945 \pm 0.032 \qquad f_{B^*} = 183.3 \pm 7.8\,\mathrm{MeV}$$

$$\frac{f_{B_s^*}}{f_{B_s}} = 0.974 \pm 0.010 \qquad f_{B_s^*} = 223.1 \pm 6.3\,\mathrm{MeV}$$

For fB and fBs see ref: [*Petros talk*]

B* meson masses :

$$\frac{M_{B^*}}{M_B} = 1.0049 \pm 0.0057 \qquad M_{B^*} = 5304 \pm 30\,\mathrm{MeV}$$

$$M^{exp}_{B^{*\pm}} = (5324.83 \pm 0.32)\ \mathrm{MeV}$$

$$\frac{M_{B_s^*}}{M_{B_s}} = 1.0070 \pm 0.0018 \qquad M_{B_s^*} = 5404.0 \pm 9.7\,\mathrm{MeV}$$

$$M^{exp}_{B_s^*} = (5415.4 \pm 1.6)\ \mathrm{MeV}$$

f_{B^*} and $f_{B_s^*}$

Results

PRELIMINARY

B^* meson decay constants with $N_f = 2 + 1 + 1$:

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For fB and fBs see ref: [*Petros talk*]

Previous values

- $N_f = 2$:

$$f_{B^*}/f_B = 1.051 \pm 0.017$$

ref: [arXiv:1407.1019 [hep-ph] **(ETMC)**]

-

- $N_f = 2 + 1 + 1$:

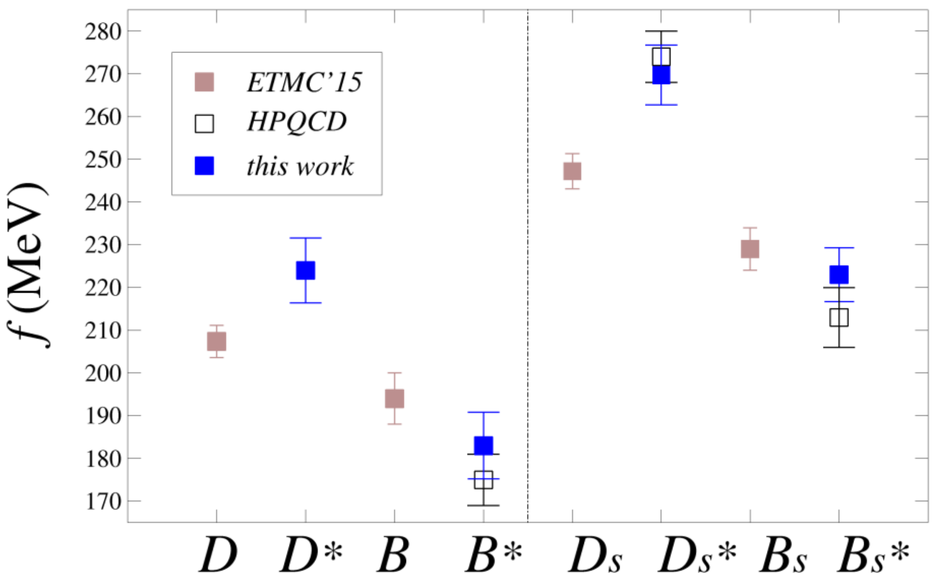
$$f_{B^*}/f_B = 0.941 \pm 0.026$$

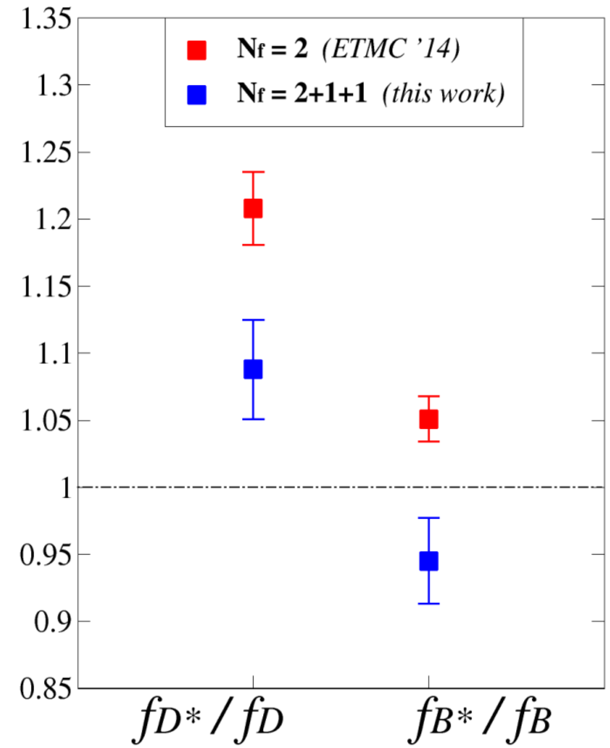
ref: [arXiv:1503.05762 [hep-ph] **(HPQCD)**]

$$f_{B_s^*}/f_{B_s} = 0.953 \pm 0.023$$

ref: [arXiv:1503.05762 [hep-ph] **(HPQCD)**]

Summary





- ***fD*(s) ≠ fD(s)*** and ***fB*(s) ≠ fB(s) :***
  we can observe an heavy quark symmetry breaking effect both in the charm and in the beauty sector.

- ***fD*> fD*** while ***fB*< fB :***
  the breaking effect has an opposite sign for the charm and beauty sector.

- The entity of this breaking is lower than it was found in the analysis of Nf = 2 correlation functions.
  Quenching effect of the strange quark?