Totally asymptotically free Trinification

Giulio Maria Pelaggi

Università di Pisa & INFN Pisa

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Based on arXiv:1507.06848 by G.M.P., A. Strumia, S. Vignali
and arXiv:1512.07225 by G.M.P., A. Strumia, E. Vigiani

The hierarchy problem and the Naturalness

How can we deal with the quadratically divergent corrections to the mass of the scalar boson?

Guide line for Beyond the SM physics $\rightarrow$ Naturalness: divergences are canceled by new physics at some energy scale $\Lambda_{\rm nat}$.

$$\delta m_h^2(\Lambda_{\rm nat}^2) \lesssim m_h^2.$$

Common solutions: SUSY and composite Higgs models.

Naturalness suggests new physics below the TeV but LHC didn't see anything (significant)... Maybe it doesn't work in this way.

What is TAF

A model that can be extrapolated using the RGEs up to infinite energy is

Totally Asymptotically Free

TAF models can bypass the hierarchy-naturalness problem: there are no cut-off scales, so power divergent corrections have no physical meaning.

[Farina, Pappadopulo, Strumia, arXiv:1303.7244;
Giudice, Isidori, Salvio, Strumia, arXiv:1412.2769]

We suppose that gravity does not spoil this behavior, so this property holds also over M_{Pl}.

How to find a TAF model

The general form of the RGEs for all the couplings is known [Machacek & Vaughn, Nuc. Phys. B **222** (1983), 83 and subsequent].

One can solve them, finding the asymptotic behaviour. Simple.

BUT

If we have tens of couplings, the system of differential equations is difficult to solve.

Is there a general way to deal with it?

RGEs and fixed points

We rescale the couplings by their leading asymptotic behavior

$$g_i^2(t) = \frac{\tilde{g}_i^2(t)}{t} \qquad y_a^2(t) = \frac{\tilde{y}_a^2(t)}{t} \qquad \lambda_m(t) = \frac{\tilde{\lambda}_m(t)}{t},$$

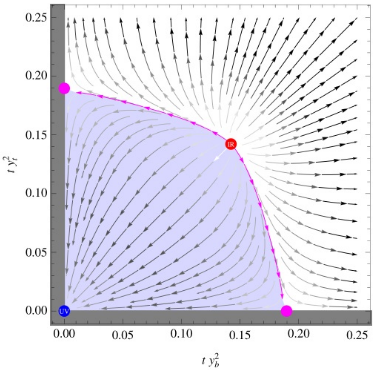
where $t = \ln(\mu^2/\mu_0^2)/(4\pi^2)$. The one-loop RGEs become

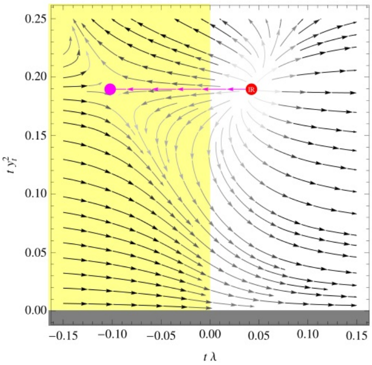
$$\frac{dx_I}{d\ln t} = V_I(x) \qquad \text{where} \qquad x_I = \{\tilde{g}_i, \tilde{y}_a, \tilde{\lambda}_m\}$$

Solving the **algebraic** system $\frac{dx_I}{d\ln t} = V_I = 0$, we find the fixed points x_∞: the couplings flow to zero with fixed ratios.

If there is at least one fixed point the model is TAF.

SM Flows



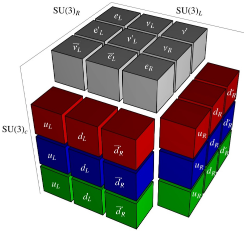


	$\tilde{y}^2_{t\infty}$	$\tilde{y}^2_{b\infty}$	$\tilde{y}^2_{\tau\infty}$	$\tilde{y}^2_{\nu\infty}$	Eigenvalues
Solution 1	227/1197	0	0	0	$+-++$
Solution 2	0	227/1197	0	0	$-+++$
Solution 3	227/1596	227/1596	0	0	$++++$
Solution 4	0	0	0	0	$--++$

	$\tilde{\lambda}_\infty$	Eig
Sol. 1	$\frac{-143+\sqrt{119402}}{4788} \approx +0.0423$	$+$
Sol. 2	$\frac{-143-\sqrt{119402}}{4788} \approx -0.1020$	$-$

[Giudice, Isidori, Salvio, Strumia, arXiv:1412.2769]

Minimal Trinification



Is the SM TAF? No.

The coupling g_Y hits a Landau pole (like all the abelian groups).

We embed it in a non-abelian group

$$G_{\text{Trin}} = \text{SU}(3)_L \otimes \text{SU}(3)_R \otimes \text{SU}(3)_c$$

Minimal Trinification				
Field	spin	$\text{SU}(3)_L$	$\text{SU}(3)_R$	$\text{SU}(3)_c$
$Q_R = \begin{pmatrix} u_R^1 & u_R^2 & u_R^3 \\ d_R^1 & d_R^2 & d_R^3 \\ d_R'^1 & d_R'^2 & d_R'^3 \end{pmatrix}$	$1/2$	1	3	$\bar{3}$
$Q_L = \begin{pmatrix} u_L^1 & d_L^1 & \bar{d}_R'^1 \\ u_L^2 & d_L^2 & \bar{d}_R'^2 \\ u_L^3 & d_L^3 & \bar{d}_R'^3 \end{pmatrix}$	$1/2$	3	1	$\bar{3}$
$L = \begin{pmatrix} \bar{\nu}_L' & e_L' & e_L \\ \bar{e}_L' & \nu_L' & \nu_L \\ e_R & \nu_R & \nu' \end{pmatrix}$	$1/2$	3	$\bar{3}$	1
H	0	3	$\bar{3}$	1

Features of Trinification models

- To reproduce the right gauge couplings for the SM it predicts

$$g_L = g_2 \qquad g_R = \frac{2 g_2 g_Y}{\sqrt{3 g_2^2 - g_Y^2}} \qquad g_c = g_3$$

- The Higgses have a non-zero VEV:

$$\langle H \rangle = \begin{pmatrix} v_u & 0 & 0 \\ 0 & v_d & v_L \\ 0 & V_R & V \end{pmatrix}$$

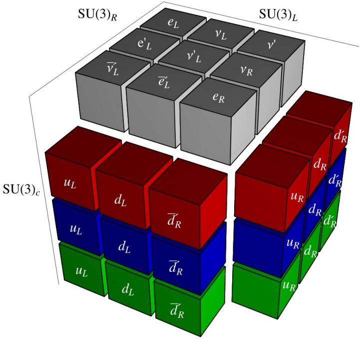
so G_{Trin} is spontaneously broken:

$$G_{\text{Trin}} \xrightarrow{V} G_{LRSM} \xrightarrow{V_R} G_{SM} \xrightarrow{v} \text{SU}(3)_c \otimes \text{U}(1)_{em}$$

- The lightest new vectors are the right-handed $W_R^{\pm}$, with mass $\sim g_R V$

Features of Trinification models

- Each generation of $Q_R \oplus Q_L \oplus L$ contains 27 fermions:
  - the 15 SM chiral fermions,
  - a vector-like lepton doublet $L' \oplus \bar{L}'$,
  - a vector-like right-handed down quark $d'_R \oplus \bar{d}'_R$,
  - two neutral singlets, ν_R and ν'.



- 3 Higgses are needed to give a mass $\sim yV$ to the new heavy fermions without fine-tuning.

The phenomenology seems promising, but
Minimal Trinification with 3 Higgses is not TAF.

Expanding the minimal model

Adding extra particles modifies the UV behavior of the couplings:

	name	representation	Δb_i			Yukawas	
unstable	1	$(1,1,1)$	0	0	0	$1LH^*$	–
	8_L	$(8,1,1)$	2	0	0	$8_L LH^*$	–
	8_R	$(1,8,1)$	0	2	0	$8_R LH^*$	–
	$L' \oplus \bar{L}'$	$(3,\bar{3},1) \oplus (\bar{3},3,1)$	2	2	0	$L'LH$	$L'L'H + \bar{L}'\bar{L}'H^*$
	$Q'_L \oplus \bar{Q}'_L$	$(\bar{3},1,3) \oplus (3,1,\bar{3})$	2	0	2	$Q'_L Q_R H$	–
	$Q'_R \oplus \bar{Q}'_R$	$(1,3,\bar{3}) \oplus (1,\bar{3},3)$	0	2	2	$Q'_R Q_L H$	–
stable	$3_L \oplus \bar{3}_L$	$(3,1,1) \oplus (\bar{3},1,1)$	$\frac{2}{3}$	0	0	–	–
	$3_R \oplus \bar{3}_R$	$(1,3,1) \oplus (1,\bar{3},1)$	0	$\frac{2}{3}$	0	–	–
	$3_c \oplus \bar{3}_c$	$(1,1,3) \oplus (1,1,\bar{3})$	0	0	$\frac{2}{3}$	–	–
	8_c	$(1,1,8)$	0	0	2	–	–
	$6_L \oplus \bar{6}_L$	$(6,1,1) \oplus (\bar{6},1,1)$	$\frac{10}{3}$	0	0	–	–
	$6_R \oplus \bar{6}_R$	$(1,6,1) \oplus (1,\bar{6},1)$	0	$\frac{10}{3}$	0	–	–
	$6_c \oplus \bar{6}_c$	$(1,1,6) \oplus (1,1,\bar{6})$	0	0	$\frac{10}{3}$	–	–
	$\tilde{L} \oplus \bar{\tilde{L}}$	$(3,3,1) \oplus (\bar{3},\bar{3},1)$	2	2	0	–	–
	$\tilde{Q}_L \oplus \bar{\tilde{Q}}_L$	$(3,1,3) \oplus (\bar{3},1,\bar{3})$	2	0	2	–	–
	$\tilde{Q}_R \oplus \bar{\tilde{Q}}_R$	$(1,3,3) \oplus (1,\bar{3},\bar{3})$	0	2	2	–	–

A phenomenologically interesting TAF model:
Minimal Trinification (with Q_L, Q_R, L and 3 Higgses)
plus a vector-like quark family $\tilde{Q}_L \oplus \bar{\tilde{Q}}_L$ and $\tilde{Q}_R \oplus \bar{\tilde{Q}}_R$.

Diboson excess

In Run 1 of LHC there were excesses ($\sim 3\sigma$) in some channels at an energy $\simeq 2$ TeV.

These anomalies can be fitted with the processes

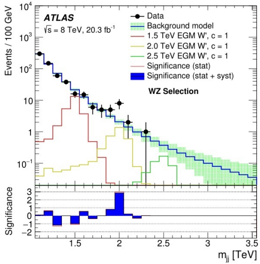
$$pp \to W_R^\pm \to W^\pm Z$$

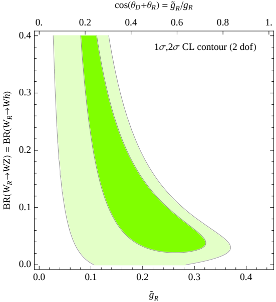
and

$$pp \to W_R^\pm \to W^\pm H.$$

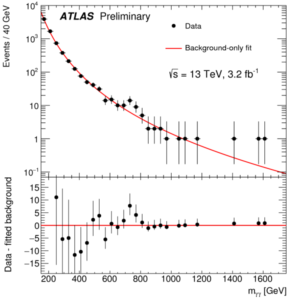
The predicted value $g_R \simeq 0.444$ is compatible with the excess.

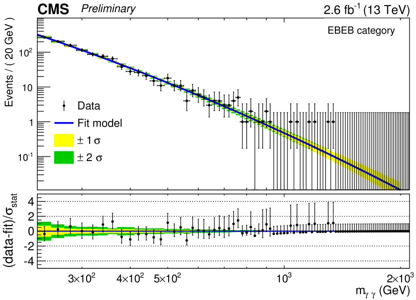
[ATLAS Coll. arXiv:1506.00962]





Diphoton excess





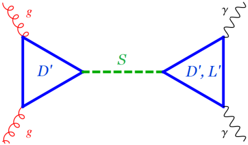
ATLAS and CMS observed an excess, with a statistical significance $\sim 3\sigma$, in the di-photon channel, at an energy $\simeq 750$ GeV. [ATLAS-CONF-2015-081, CMS-PAS-EXO-15-004]

In Trinification models it can be interpreted as one of the extra scalars (singlet or doublet) in the Higgs multiplets.

Diphoton excess

We call this new scalar S: it is produced by gluon fusion and decays in $\gamma\gamma$ through loops of the extra heavy fermions D' and L'.

$$\begin{aligned}\mathscr{L} \ni \; & S\bar{\mathcal{Q}}(y_f + y_{5f}\gamma_5)\mathcal{Q} \\ & - SA_s|\tilde{\mathcal{Q}}_s|^2 + g_V M_V S|V_\mu|^2\end{aligned}$$



$$\frac{\Gamma(S \to gg)}{M} \approx 0.6\ 10^{-4} \left(y_{5D}^2 + \frac{4}{9} y_D^2\right) \left(\frac{1\text{TeV}}{M_{D'}} \frac{N_{D'}}{3}\right)^2$$

$$\frac{\Gamma(S \to \gamma\gamma)}{M} \approx 10^{-6} \left[6.7 \left(y_{5E} \frac{N_{L'}}{3}\right)^2 + \left(1.05 y_E \frac{N_{L'}}{3} + 1.15 \frac{A_s}{M_s} \frac{N_s}{9}\right)^2\right]$$

No signal has been seen in other channels ($S \longrightarrow WW$, ZZ, $Z\gamma$). The predictions of this model are under the experimental bounds.

Conclusions

We performed a systematic search of TAF models such that:

- the theory holds up to infinite energy
- the phenomenology is consistent with the data

Among Trinification models we found that:

- minimal Trinification has no TAF solutions
- to get the right fermion masses, 3 Higgs are needed.
- the most interesting Trinification TAF model includes $\tilde{Q}_L \oplus \bar{\tilde{Q}}_L$ and $\tilde{Q}_R \oplus \bar{\tilde{Q}}_R$
- Trinification can explain the diboson and diphoton excesses.

Backup slides

Quark masses

Up quarks:

$$\begin{array}{c} \\ u_{Li} \\ U_L \\ \bar{U}_R \end{array} \begin{array}{c} \begin{array}{ccc} u_{Rj} & U_R & \bar{U}_L \end{array} \\ \begin{pmatrix} v_{un} y_Q^{nij} & v_{un} y_Q^{ni4} & 0 \\ v_{un} y_Q^{n4j} & v_{un} y_Q^{n44} & M_L \\ 0 & M_R & v_{un} y_{\bar{Q}}^{n} \end{pmatrix} \end{array}$$

Down quarks:

$$\begin{array}{c} \\ d_L^i \\ \bar{d}_R^{i'} \\ \bar{D}'_R \\ D_L \\ D'_L \\ \bar{D}_R \end{array} \begin{array}{c} \begin{array}{cccccc} d_R^j & d_R^{j'} & D'_R & D_R & \bar{D}'_L & \bar{D}_L \end{array} \\ \begin{pmatrix} v_{dn} y_Q^{nij} & v_L y_Q^{2ij} & v_L y_Q^{2i4} & v_{dn} y_Q^{ni4} & 0 & 0 \\ V_R y_Q^{2ij} & V_n y_Q^{nij} & V_n y_Q^{ni4} & V_R y_Q^{2i4} & 0 & 0 \\ V_R y_Q^{24j} & V_n y_Q^{n4j} & V_n y_Q^{n44} & V_R y_Q^{244} & M_L & 0 \\ v_{dn} y_Q^{n4j} & v_L y_Q^{24j} & v_L y_Q^{24j} & v_{dn} y_Q^{n44} & 0 & M_L \\ 0 & 0 & M_R & 0 & v_{un} y_{\bar{Q}}^{n} & 0 \\ 0 & 0 & 0 & M_R & 0 & v_{un} y_{\bar{Q}}^{n} \end{pmatrix} \end{array}$$

Quark masses with 3 Higgses

Let's assume that H_1 breaks G_{333} but preserves $G_{\rm SM}$ (i.e. $V_1 \neq 0$ and $v_{d1} = v_{u1} = v_{L1} = 0$).
The Yukawa couplings y_{Q1} and y_{L1} allow to give large enough masses $M_{d'_R} = V_1 y_{Q1} \gtrsim 700\text{GeV}$ and $M_{e'_R} = V_1 y_{L1} \gtrsim 200\text{GeV}$ to the extra primed fermions, without also giving too large masses to the SM fermions.
H_2 and H_3 can have the small Yukawa couplings needed to reproduce the light SM fermion masses,

$$m_e \sim \sum_{n=2}^{3} v_{dn} y_{Ln}, \qquad m_u \sim \sum_{n=2}^{3} v_{un} y_{Qn}, \qquad m_d \sim \sum_{n=2}^{3} v_{dn} y_{Qn}.$$

Lepton masses

Charged leptons:

$$\begin{array}{c} \\ e_L \\ e'_L \end{array} \begin{array}{c} \begin{array}{cc} \quad e_R \quad & \quad \bar{e}'_L \quad \end{array} \\ \begin{pmatrix} -v_{dn} y_{Ln} & V_{Rn} y_{Ln} \\ v_{Ln} y_{Ln} & -V_n y_{Ln} \end{pmatrix} \end{array}$$

Neutral leptons:

$$\begin{array}{c} \\ \nu_L \\ \nu_R \\ \nu'_L \\ \bar{\nu}'_L \\ \nu' \end{array} \begin{array}{c} \begin{array}{ccccc} \nu_L & \nu_R & \nu'_L & \bar{\nu}'_L & \nu' \end{array} \\ \begin{pmatrix} 0 & -v_{un} y_{Ln} & 0 & -V_{Rn} y_{Ln} & 0 \\ & 0 & 0 & -v_{Ln} y_{Nn} & 0 \\ & & 0 & V_n y_{Ln} & v_{un} y_{Ln} \\ & & & 0 & v_{dn} y_{Ln} \\ & & & & 0 \end{pmatrix} \end{array}$$

Vector masses

The vev V_1 alone breaks
$G_{333} \to \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L} \otimes \mathrm{SU}(3)_c$.
A $\mathrm{SU}(2)_L$ doublet H_L, a $\mathrm{SU}(2)_R$ doublet H_R and a Z' singlet acquire mass:

$$M_{H_L} = \frac{g_L}{\sqrt{2}} V_1, \qquad M_{H_R} = \frac{g_R}{\sqrt{2}} V_1, \qquad M_{Z'} = V_1 \sqrt{\frac{2}{3}(g_L^2 + g_R^2)}.$$

Vector masses

Let's take only V_n and V_{Rn}: SM group still unbroken.

Defining $V^2 \equiv \sum_n (V_n^2 + V_{Rn}^2)$, $\alpha \equiv \sum_n V_{Rn}^2 / V^2$ and $\beta \equiv \sum_n V_n V_{Rn} / V^2$, the gauge bosons are:

- a $SU(2)_L$ doublet with 4 components: $M_{H_L} = g_L V / \sqrt{2}$;
- two charged fields $H_R^\pm$ with mass $M_{H_R^\pm} = g_R V / \sqrt{2}$
- two neutral fields H_R^0 with mass

$$M_{H_R^0}^2 = \frac{g_R^2 V^2}{4} \left[1 + \sqrt{(1 - 2\alpha)^2 + 4\beta^2} \right].$$

Vector masses

- the right-handed $W_R^{\pm}$ vectors (the lightest) with mass

$$M^2_{W_R^{\pm}} = \frac{g_R^2 V^2}{4}\left[1 - \sqrt{(1-2\alpha)^2 + 4\beta^2}\right]$$

- the Z_R and the Z_{B-L} vectors, that mix together. In the limit $V_{Rn} \ll V_n$ the mass eigenvalues are

$$M_{Z'} = V\sqrt{\frac{2}{3}(g_L^2 + g_R^2)}, \quad M_{Z''} \simeq |\beta| g_R V_R \sqrt{\frac{g_R^2/2 + 2g_L^2}{g_R^2 + g_L^2}}.$$

- The 12 SM vectors remain massless.