Quark number fluctuations in the strongly interacting medium

Giuseppe Gagliardi

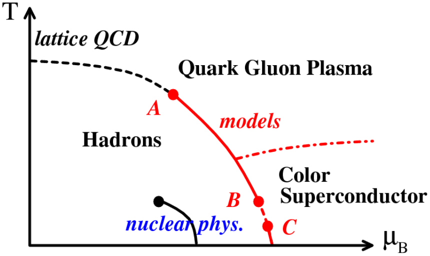
In collaboration with Prof. Massimo D'Elia

University of Pisa

18 May 2016

The QCD phase diagram

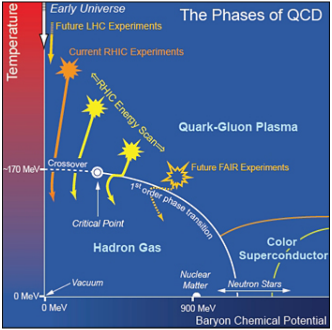
- We are interested in the study of the QCD phase diagram in the (T, μ_B) plane.
- It is supposed that the zero density crossover ends up in a second order critical point for some $\mu_B = \mu_B^{crit}$.
- This claim is supported by many phenomenological models (NJL, random matrix model)...
- ...but it has never been experimentally observed or predicted from *ab-initio* calulations.



- At these energy scales ($\approx \Lambda_{QCD}$) the problem requires the treatment of QCD at non perturbative level.

Ultrarelativistic heavy ion collisions

- It is possible to investigate the properties of the strongly interacting medium by means of ultrarelativistic heavy ion collisions.
- The hadrons produced by the collision recall the hadron distributions at the time of the so called *chemical freeze-out*, i.e. the time of last inelastic scattering.
- The detectors are able to measure the relative abundances of many hadrons produced ($\pi^{\pm}, p, \bar{p}, K^{\pm}, K_s^0, \Lambda, \bar{\Lambda}$,..)
- From the hadron abundances, they can determine the value of conserved charges Q, B, S.
- The event-by-event analysis also allows to evaluate their local fluctuations.



If freeze out takes place slight below the critical line some critical behavior in the conserved charge fluctuations is expected as the second order critical point is approached.

Comparison of theory and experiment

- It is possible to compare the result on heavy ion collisions with predictions from theoretical models.
- The conserved charge fluctuations are related to the so called *generalized susceptibilities* (via the fluctuation-dissipation theorem):

$$\chi^{B,Q,S}_{i,j,k} = \frac{1}{VT^4} \frac{\partial^{(i+j+k)} \mathcal{F}(T,V,\mu)}{\partial \hat{\mu}^i_B \partial \hat{\mu}^j_Q \partial \hat{\mu}^k_S}$$

  $\hat{\mu}_i = \mu_i/T$ and the μ_i $\{i = Q, B, S\}$ are the chemical potentials coupled with the conserved charge operators $\{\hat{N}_Q, \hat{N}_B, \hat{N}_S\}$.

  Ex: $\chi_{1,0,0} = \langle N_B \rangle$ $\chi_{2,0,0} = \sigma^2_B$ $\frac{\chi_{3,0,0}}{\chi_{1,0,0}} = \frac{\sigma^3_B S_B}{N_B}$.

- To this purpose we need some estimation of the QCD equation of state (EOS).
- The comparison is mainly done using the Hadron Resonances Gas Model .
- It fits well the low-T QCD but does not predict any critical behavior .
- Needs of calculations using QCD. At present the only fully non-perturbative approach is given by its lattice formulation.

Thermodynamic of lattice QCD: Highlights

- The starting point is the QCD partition function. It can be written as euclidean Path-Integral with a compactified temporal direction:

$$\mathcal{Z} = \mathrm{Tr}[e^{-\beta H}] = \int [dA][d\bar{\psi}\psi] e^{-\int_0^\beta d\tau \int d^3x \mathcal{L}_{(\mathbf{eucl.})}}$$

- Bosonic(fermionic) fields satisfy periodic(antiperiodic) temporal boundary condition.
- **Discretization**: The Path-Integral is discretized over a space-time lattice introducing a finite lattice spacing **a** ($x_\mu \to a n_\mu$). Fermionic and bosonic fields take values only on the lattice sites. The functional integral over the gauge fields $A_\mu(x)$ is switched on the link variables $U_\mu(n)$ one.

- $\mathbf{S}_{(eucl.)} \Rightarrow \mathbf{S}_{lat} = S_G[U] + \bar{\psi} M[U] \psi$; $\mathcal{Z} = \mathrm{Tr}\left[e^{-\beta H}\right] = \int D\mathcal{U} e^{-S_G[U]} \det M[U]$
- If $e^{-S_G[U]} \det M[U]$ is real and positive the Path-Integral can be evaluated by means of Montecarlo algorithms.

Lattice QCD at finite density

- It is possible to add three chemical potentials μ_i coupled to the quark number operators $\hat{N}_i$ $(i = u, d, s)$ as well as to the three conserved charges $\{Q, B, S\}$.
- $Z(T, \mu_{(u,d,s)}) = \mathrm{Tr}\left[e^{-\beta(H-\mu_i N_i)}\right] = \int D\mathcal{U} e^{-S_G[U]} \det M\left[U, \mu_{(u,d,s)}\right]$
- $\det M\left[U, \mu_{(u,d,s)}\right]$ acquires a non zero imaginary part (*sign problem*) $\Rightarrow$ Montecarlo simulations no longer feasible.

How to have access to the finite density Equation of State?

- **Taylor method**: $F(T, \mu) = F(T, 0) + VT^4 \sum_k \frac{\chi_{2k}(T)}{2k!}\left(\frac{\mu}{T}\right)^{(2k)}$.
- χ_{2k} corresponds to the zero density susceptibilities $\Rightarrow$ no sign problem .
- The direct sampling of $\chi_{2k}(T)$ suffers of at least two problems:
  - Too computationally expensive at higher order (i.e. with increasing k)
  - Problems with *lack-of-self-averaging* $\Rightarrow$ Higher order fluctuations very noisy as the infinite volume limit is approached
- Current state of the art: 8th order susceptibilities for $N_f = 2$ QCD (Gavai-Gupta) and 4th order for $N_f = 2 + 1$ QCD (Fodor et al.).

Moving to imaginary chemical potential

Main idea

- det $M[U, i\mu_I] > 0$
- The quark number susceptibilities at imaginary chemical potentials can be expanded as ($N_f = 2+1$ QCD is considered):

$$\chi_{ijk}(T,\mu) = \sum_{(l=i,m=j,n=k)}^{\infty} \frac{\chi_{lmn}(T)}{(l-i)!(m-j)!(n-k)!} \left(\frac{\mu_u}{T}\right)^{(l-i)} \left(\frac{\mu_d}{T}\right)^{(m-j)} \left(\frac{\mu_s}{T}\right)^{(n-k)}$$

- In this way we can have access to higher order zero density fluctuations through the measure of a relative small number of $\chi_{ijk}(T,\mu)$.

Principal sources of error:

- Statistical: the error in a Montecarlo estimate scales like: $1/\sqrt{(\text{sampled config.})}$
- Systematic: ultraviolet (UV) cut-off and finite volume effects, errors from series truncation.

Main goals of the work

- Determination of the coefficients in the free energy expansion around the zero density point in $N_f = 2+1$ QCD.
- Comparison of the efficiency of this method with respect to the direct calculus.
- Freeze-out parameters estimate and location of the second order critical point.

Numerical Setup

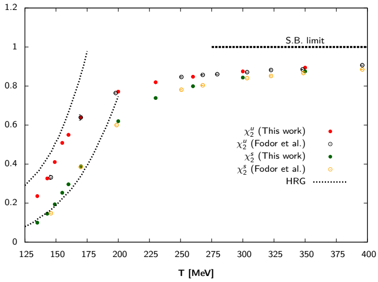
- Runs mostly done on $N_t = 8$ lattice with aspect ratio N_s/N_t fixed to 4.
- Temperature range(MeV): $[135, 350]$
- Temperature changed according to $T = \frac{1}{N_t a}$ moving on a line of constant physics.

Numerical results

Explored region in the complex μ plane

- For $T < T_c$ analytic continuation quite safe (no singularity in the $Im\mu$ region) $\Rightarrow$ $\mu_{max}/T = 0.8\pi$.
- For $T > T_c$ it is possible to meet the continuation of the pseudo-critical line (principal problems slight above T_c) $\Rightarrow$ $\mu_{max}/T = 0.3\pi$.

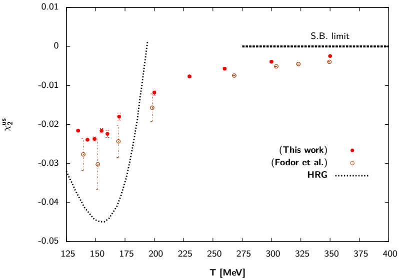
$$\chi_2^{\mathbf{X}} = \frac{1}{VT^3}\langle N_X^2 \rangle$$

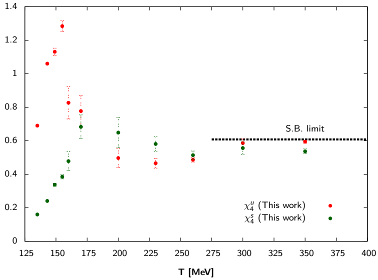


- 2nd order susceptibilities agree very well with previous determination.
- Quark-Gluon-Plasma limit reasonably achieved at 350Mev.

- Discrepancy with HRG calculations even at low T.
- Probably disappears after continuum limit is taken.

$$\chi_2^{\mathbf{XY}} = \frac{1}{VT^3}\langle N_X N_Y \rangle$$





- $\lim_{T\to\infty} \chi_4^u = \lim_{T\to\infty} \chi_4^s = \frac{6}{\pi^2}$
- Diagonal 4th order susceptibilities reach their asymptotic value .

Efficiency test

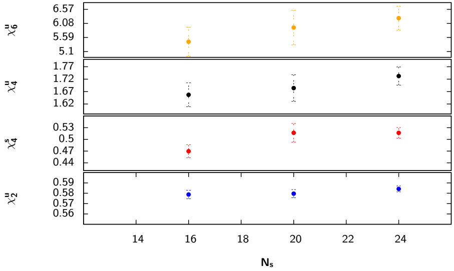
$T/T_c \approx 0.95$	$\chi_{2,0,0}$	$\chi_{0,0,2}$	$\chi_{1,1,0}$	$\chi_{1,0,1}$
$\mu_u/T = 0.025\pi$	0.420(38)	0.2012(65)	-0.057(36)	-0.05(11)
Im μ	0.4147(39)	0.1881(15)	-0.0304(11)	-0.0669(30)

- We compare the fitted zero density 2th order susceptibilities with the one obtained from direct sampling (and with the same sample size n_c).
- To obtain the same statistical accuracy in the direct sampling, 100(900)n_c gauge configurations are required for diagonal(non-diagonal) susceptibilities.
- We used a complex grid of 96 points $\Rightarrow$ $Im\mu$ method more efficient.

Finite volume effects and error scaling

Numerical Setup

- Simulations on $N_t = 6$ and $N_s = 16, 20, 24$ lattices with $T/T_c \approx 0.95$.
- We determined all the susceptibilities up to 6th order.



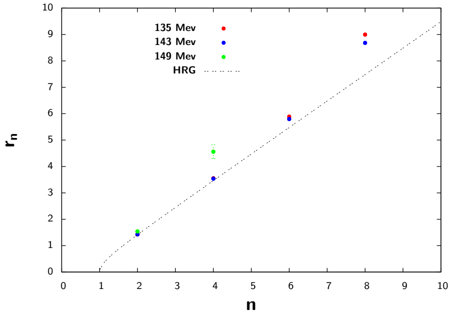
- Finite volume effects are less then our statistical accuracy.
- No effects of lacking of self averaging $\Rightarrow$ Errors decrease with increasing volume.
- Furthermore, we controlled that no finite volume effects arise at higher temperature (spatial volume decreases according to $\frac{1}{T}\frac{N_s}{N_t}$) .

Location of the second order critical point

- In the $\mu_Q = \mu_S = 0$ plane the free energy $\mathcal{F}$ can be expanded as:

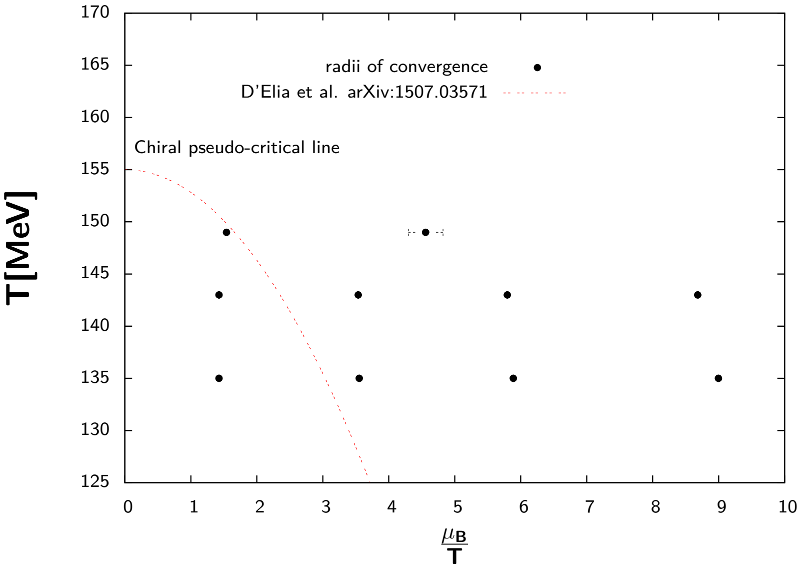
$$\mathcal{F}(T, V, \mu_B) = \mathcal{F}(T, V, \mu_B = 0) + VT^4 \sum_n \frac{\chi_{2n}^B}{(2n)!} \left(\frac{\mu_B}{T}\right)^{2n}$$

- In presence of a second order critical point $\frac{\partial^2 \mathcal{F}}{\partial \mu_B^2}$ exhibits a singularity which can be located by estimating the radius of convergence of the power series.



$$\rho = \lim_{n\to\infty} r_n = \lim_{n\to\infty} \sqrt{\frac{n(n-1)\chi_n^B}{\chi_{n+2}^B}}$$

- To evaluate the radii of convergence, we used all the susceptibilities up to 10th order for $T = 135, 143$ Mev and up to 6th order for $T = 149$Mev .
- Quite good agreement with HRG prediction $\Rightarrow$ no signal of criticality has been found.


radii of convergence
D'Elia et al. arXiv:1507.03571
Chiral pseudo-critical line
T[MeV]
$\frac{\mu_B}{T}$
170
165
160
155
150
145
140
135
130
125
0
1
2
3
4
5
6
7
8
9
10

Freeze out parameters

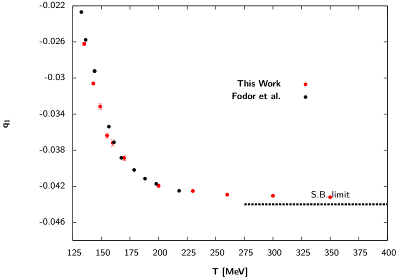
Freeze out setup recostruction

- N_S and $\frac{N_Q}{N_B}$ fixed by the colliding nuclei .
- In $Pb-Pb$ or $Au-Au$ collisions, $\langle N_S\rangle = 0$ e $\frac{\langle N_Q\rangle}{\langle N_B\rangle} = Z/A \approx 0.4$.
- On the lattice we have to tune μ_S, μ_Q, μ_B in order to reproduce this condition.
- In general: $\mu_Q = \mu_Q(\mu_B)$ and $\mu_S = \mu_S(\mu_B)$.
- To leading order $\mu_Q = q_1(T)\mu_B + O(\mu_B^3)$ and $\mu_S = s_1(T)\mu_B + O(\mu_B^3)$.
- Imposing these constraints we obtain (to leading order):

$$q_1 = \frac{r\left(\chi_2^B\chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^Q\chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}\right) - r\left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)} \qquad s_1 = -\frac{\chi_{11}^{QS}}{\chi_2^S}q_1 - \frac{\chi_{11}^{BS}}{\chi_2^S}$$

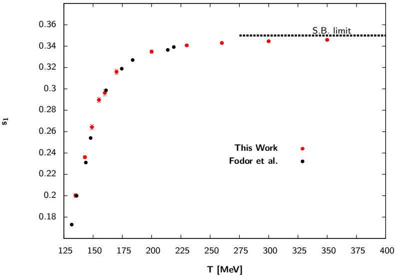
- $s_1 \neq 0$ even if $\langle N_S\rangle = 0$. Different quark flavours are created, due to gluon interactions, when a given μ_i is inserted.
- Ex: $N_s(\mu_{(s,d)} = 0) = \chi_2^{us}\mu_u + O(\mu_u^3)$

Numerical results



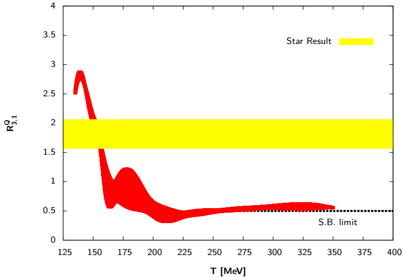
- Good agreement with previous determination.

- For high **T**, $s_1 \approx \frac{1}{3}$. Compatible with the **QGP** limit.
- Higher order corrections turn out to be small (1-10%) Fodor et al., Bazavov et al.



Extraction of $\mathbf{T^f}$ and $\mu_\mathbf{B}^\mathbf{f}$.

- We need two observables sensitive to a variation in T^f and μ_B^f.
- We choose ratios of cumulants of the electric charge Q: $\mathbf{R^Q_{(n,m)}} = \frac{\chi^\mathbf{Q}_\mathbf{n}}{\chi^\mathbf{Q}_\mathbf{m}}$.
  - Independent from the (unknown) value of the freeze-out volume.



- To leading order $R^Q_{3,1}$ independent from $\mu_B \Rightarrow$ Can be used as thermometer.
- Our determination: $\mathbf{143Mev} < \mathbf{T^f} < \mathbf{160Mev}$.
- Star data on $R^Q_{3,1}$ do not show (with the precision achieved) any variation with the center of mass energy. Possibility to use cumulants of net proton number to extract the curvature (Bazavov et al.).

- $R^Q_{1,2}$ can be used as baryometer.
- Our results:

$$\begin{aligned}
\mu_B(\sqrt{s_{NN}} = 27)[\mathbf{Mev}] &= 79.4(2.7)_{\delta T}(1.5)_{(exp.)}(0.4)_{(stat.)}\\
\mu_B(\sqrt{s_{NN}} = 39)[\mathbf{Mev}] &= 62.9(2.1)_{\delta T}(0.5)_{(exp.)}(0.7)_{(stat.)}\\
\mu_B(\sqrt{s_{NN}} = 62.4)[\mathbf{Mev}] &= 36.6(1.3)_{\delta T}(1.1)_{(exp.)}(0.4)_{(stat.)}
\end{aligned}$$