Holographic thermo-electric transport properties and strange metals

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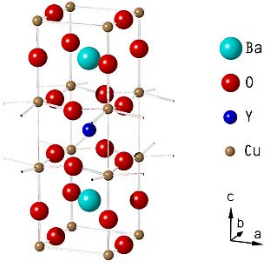
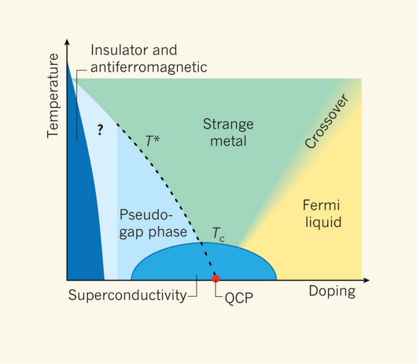
based on: arXiv:1603,03029, 1502.02631, 1411.6631, 1406.4134

with Matteo Baggioli, Alessandro Braggio, Nicodemo Magnoli, Daniele Musso

Motivations

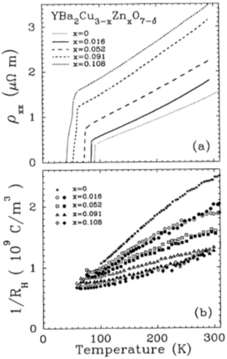
Understand the properties of the cuprates: strongly coupled materials with exotic transport properties





Motivations

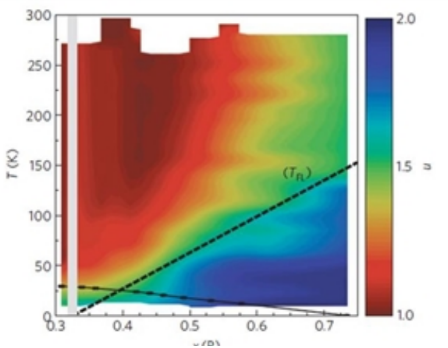
- The Strange Metal phase presents exotic transport properties: the most famous is the linear in T resistivity up to 700 K



Chien et al. PRL 91

Motivations

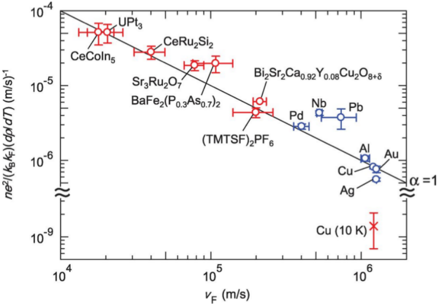
- Linear resistivity related to a critical point. Temperature is the only scale.



Analytis et al. Nature 15

Motivations

- Scattering rates of metals with T-linear resistivity.
  $(\tau T)^{-1} \propto k_B/\hbar$

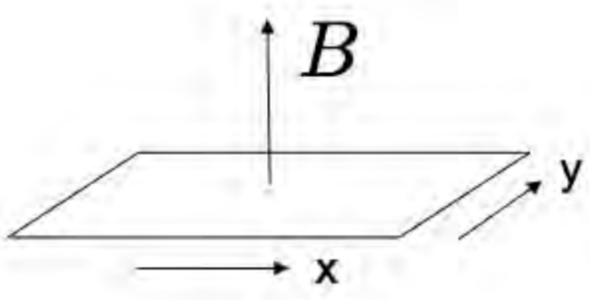


Bruin et al. Science 13

- Naively using the Drude formula: $\rho = \frac{m}{ne^2\tau} \propto T$

Motivations

The resistivity is not the end of the story: measurements on strange metals are commonly performed at non-zero magnetic field to suppress T_c (and phonons)



Motivations

- Response to an external electric field E_i and thermal gradient $\nabla_i T$

$$\begin{pmatrix} J_i \\ Q_i \end{pmatrix} = \begin{pmatrix} \sigma_{ij} & \alpha_{ij} \\ T\alpha_{ij} & \bar{\kappa}_{ij} \end{pmatrix} \begin{pmatrix} E_j \\ -\nabla_j T \end{pmatrix}$$

- transport coefficients are now matrices

$$\sigma_{xx} = \sigma_{yy} \;, \qquad \sigma_{xy} = \sigma_{yx}$$

- There are six independent transport coefficients

Motivations

- Almost all the transport properties deviate from the Fermi liquid behaviour

	Fermi Liquid	Strange Metals
ρ	T^2	T e.g. Hussey review, '08
$s \equiv \frac{\alpha_{xy}}{\alpha_{xx}}$	T	$s \sim A - BT$ Orbetelli et al. '92
$\tan\theta_H \equiv \frac{\sigma_{xy}}{\sigma_{xx}}$	$\frac{1}{T^2}$	$\frac{1}{T^2}$ e.g. Hussey review, '08
Kohler's rule	$\frac{\Delta\rho}{\rho} \sim \frac{B^2}{\rho^2}$	$\frac{\Delta\rho}{\rho} \sim \tan^2\theta_H$ Harris '92

Motivations

??

What can be said using gauge/gravity duality?

What is Gauge/Gravity duality?

$d+1$-dimensional classical gravity theories on AdS_{d+1} vacuum are equivalent to the large N (degrees of freedom per site) limit of strongly coupled d-dimensional CFTs in flat space

- It is an equivalence between partition functions:

$$Z_{CFT} = e^{i\left(S_{\text{gravity}}\right)_{\text{on-shell}}}$$

- At present it is a still a conjecture but we have many successful tests

What is Gauge/Gravity duality?

At the phenomenological level we can construct strongly coupled toy models by means of the holographic dictionary:

- asymptotic value of the gravitational fields $\leftrightarrow$ sources of operators in the CFT
- local gravitational symmetries $\leftrightarrow$ global CFT symmetries
- black holes $\leftrightarrow$ finite temperature field theories

Massive gravity and momentum dissipation

- Breaking diffeomorphisms in the bulk by adding a mass term for the graviton

$$S = \int d^4x\sqrt{-g}\left[R - \Lambda - \frac{1}{4}F^2 + \beta\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)\right]$$

where $\mathcal{K}^{2\ \nu}_{\mu} \equiv f_{\mu\rho}g^{\rho\nu}$, $\mathcal{K} \equiv \sqrt{\mathcal{K}^2}$

- the fixed metric $f_{\mu\nu}$ controls how diffeomorphisms are broken
- Holographic dictionary $\Rightarrow \ \ \partial_\mu T^{\mu\nu} \neq 0$
- we want to dissipate momentum but to conserve energy (elastic processes)

$$f_{xx} = f_{yy} = 1\ , \ \text{and zero otherwise}$$

$B = 0$ and momentum dissipation

- In the hydrodynamic regime ($|\beta| \ll T^2$) a dissipation rate τ^{-1} can be defined (at $\mathcal{O}(\beta^2)$) Davison, '13, Davison & Gouteraux '15

$$\partial_t T^{tt} = 0, \quad \partial_t T^{ti} = \tau^{-1} T^{ti}$$
$$\tau^{-1} \equiv -\frac{\mathcal{S}\beta}{2\pi(\mathcal{E} + P)}$$

- At sufficiently low $|\beta|$ there is a Drude peak in the electric conductivity $\sigma(\omega)$ Vegh, '13

$B = 0$ and momentum dissipation

- The DC electric conductivity σ_{DC} splits into two parts Blake & Tong, '13

$$\sigma_{DC} = \sigma_{Q0} + \frac{\rho^2 \tau}{\mathcal{E} + P}$$

- The thermal $\bar{\kappa}_{DC}$ and thermoelectric α_{DC} DC conductivities are affected only by the Drude part A.A. et al., '14

$$\alpha_{DC} = \frac{\mathcal{S} \rho \tau}{\mathcal{E} + P} \qquad \bar{\kappa}_{DC} = \frac{\mathcal{S}^2 T \tau}{\mathcal{E} + P}$$

Switch on B

- modify the gauge field A in order to introduce a magnetic field perpendicular to the xy plane

$$A = (\mu - \rho z)\, dt + Bx\, dy$$

- a background black-brane solution can be found and consequently the thermodynamics can be defined in terms of the horizon radius z_h ($g_{tt}(z_h) = 0$):

$$T = -\frac{z_h^2\left(B^2 z_h^2 + \mu^2\right) - 2\left(\beta z_h^2 + 3\right)}{8\pi z_h}\,, \qquad \mathcal{S} = \frac{2\pi}{z_h^2}$$

$$\rho = \frac{\mu}{z_h}\,, \qquad \mathcal{E} + P = T\mathcal{S} + \mu\rho$$

DC thermo-electric response

- having $J^i(z_h)$ and $Q^i(z_h)$ we can compute the DC transport

$$\begin{pmatrix} J_i \\ Q_i \end{pmatrix} = \begin{pmatrix} \sigma_{ij} & \alpha_{ij} \\ T\alpha_{ij} & \bar{\kappa}_{ij} \end{pmatrix} \begin{pmatrix} E_j \\ -\nabla_j T \end{pmatrix}$$

four quantities determine the six transport coefficients

$$\sigma_{Q0}\,, \qquad \rho \qquad \frac{\tau}{\mathcal{E}+P}\,, \qquad \mathcal{S}$$

$$\sigma_{xx} = \frac{\mathcal{E}+P}{\tau}\,\frac{\rho^2+\sigma_{Q0}\left(B^2\sigma_{Q0}+\frac{\mathcal{E}+P}{\tau}\right)}{B^2\rho^2+\left(B^2\sigma_{Q0}+\frac{\mathcal{E}+P}{\tau}\right)^2}$$

$$\sigma_{xy} = \rho B\,\frac{\rho^2+\sigma_{Q0}\left(B^2\sigma_{Q0}+2\,\frac{\mathcal{E}+P}{\tau}\right)}{B^2\rho^2+\left(B^2\sigma_{Q0}+\frac{\mathcal{E}+P}{\tau}\right)^2}$$

$$\alpha_{xx} = \rho\,\mathcal{S}\,\frac{\mathcal{E}+P}{\tau}\,\frac{1}{B^2\rho^2+\left(B^2\sigma_{Q0}+\frac{\mathcal{E}+P}{\tau}\right)^2}$$

$$\alpha_{xy} = \mathcal{S}B\,\frac{\rho^2+\sigma_{Q0}\left(B^2\sigma_{Q0}+\frac{\mathcal{E}+P}{\tau}\right)}{B^2\rho^2+\left(B^2\sigma_{Q0}+\frac{\mathcal{E}+P}{\tau}\right)^2}$$

$$\bar{\kappa}_{xx} = \frac{\mathcal{S}^2 T\left(B^2\sigma_{Q0}+\frac{\mathcal{E}+P}{\tau}\right)}{B^2\rho^2+\left(B^2\sigma_{Q0}+\frac{\mathcal{E}+P}{\tau}\right)^2}$$

$$\bar{\kappa}_{xy} = \frac{B\rho\,\mathcal{S}^2 T}{B^2\rho^2+\left(B^2\sigma_{Q0}+\frac{\mathcal{E}+P}{\tau}\right)^2}$$

Holographic strange metals phenomenology

Phenomenological temperature scalings in strange metals

We need 4 phenomenological inputs to predict the scalings of all the 6 transport coefficients

- Blake & Donos, '14:

$$\sigma_{Q0} \sim \frac{\sigma^0_{Q0}}{T}, \qquad \sigma_D \equiv \frac{\rho^2 \tau}{\mathcal{E} + P} \sim \frac{\sigma^0_D}{T^2}$$

and $\sigma^0_D \ll \sigma^0_{Q0}$, reproduces the correct scaling for the resistivity and the hall angle:

$$\rho_{xx} \sim T \;, \qquad \tan\theta_H \equiv \frac{\sigma_{xy}}{\sigma_{xx}} \sim \frac{1}{T^2}$$

Holographic strange metals phenomenology

Proposal

$$\sigma_{Q0} \sim \frac{\sigma^0_{Q0}}{T}, \quad \sigma_D \sim \frac{\sigma^0_D}{T^2}, \quad \rho \sim \rho_0, \quad \sigma^0_D \ll \sigma^0_{Q0}, \quad \mathcal{S} \sim \mathcal{S}_0 T^\delta$$

To fix the scaling exponent δ we need phenomenological inputs which are free form spurious interactions (phonons effects): transverse conductivities do the game!

- $\kappa_{xy} \sim \frac{1}{T}$ Zhang et al., '00, Matusiak et al., '09

$$\Rightarrow \mathcal{S} \sim \mathcal{S}_0 T, \text{ and } L_{xy} \equiv \frac{\sigma_{xy}}{T\kappa_{xy}} \sim T$$

in accordance with Loram et al., '93 and with Zhang, '00

Holographic strange metals phenomenology

Magneto-resistance

$$\frac{\Delta\rho}{\rho} \equiv\sim \sigma^0_{Q0}\,\sigma^0_D \left(\frac{B}{\rho_0}\right)^2 \left(\frac{\sqrt{\rho_0}}{T}\right)^3 - 2\sigma^{0\,2}_D \left(\frac{B}{\rho_0}\right)^2 \left(\frac{\sqrt{\rho_0}}{T}\right)^4$$

Experiments: T^{-n} with $n \sim 3.5 - 3.9$ Harris '92

Seebeck coefficient

$$s \equiv \frac{\alpha_{xy}}{\alpha_{xx}} \sim \frac{\mathcal{S}_0\,\sigma^0_D}{\rho_0\,\sigma^0_{Q0}} - \frac{\mathcal{S}_0\sigma^{0\,2}_D}{\rho_0\sigma^{0\,2}_{Q0}}\,\frac{\sqrt{\rho_0}}{T}$$

Experiments: $A - BT$ Orbetelli et al., '92
Possible $1/T$ correction at high-T? Kim et al., '04 What about phonon drag?

Some more technical outcomes

- Analysing real black-brane geometries to achieve cuprates phenomenology: need of hyperscaling violating geometries plus additional time scales! arXiv:1603.03029
- Possible existence on bounds on the diffusion constants related to minimal dissipating time scales arXiv:1411.6631 (see also Blake '16)

Conclusions

- The six thermoelectric transport coefficients are functions of four quantities: possibility to be predictive!
- At finite density thermodynamics and transport are intimately related
- Does the magnetic field play a role in criticality?
- Holography seems to be a very promising framework which provides new ideas for the understanding of transport properties in the cuprates
- To get phenomenological insight we need data clean from spurious effects: working directly with experimentalists!


Thank you