Holographic computation of the Neutron Electric Dipole Moment

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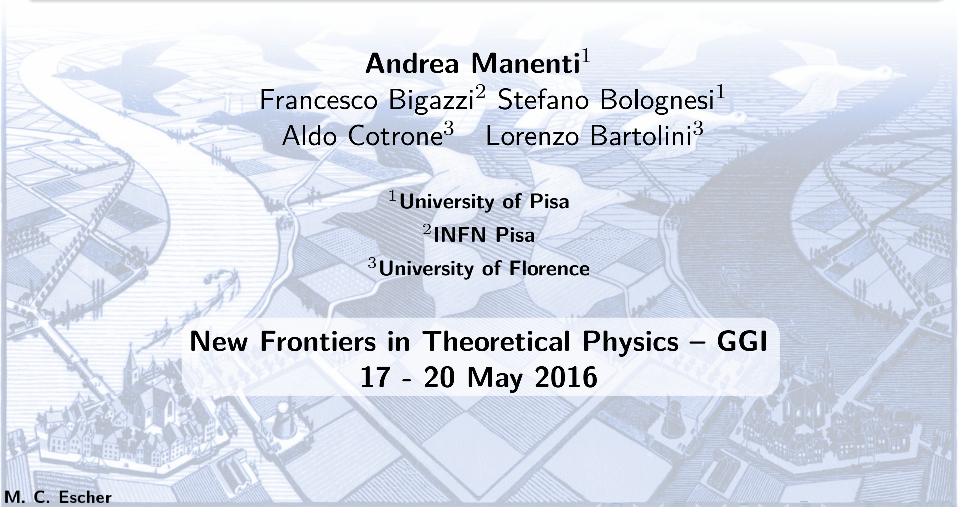

M. C. Escher

Introduction

Objectives

- We are interested in a possible P, T violation of strong interactions. The so called θ *parameter*.
- In order to make contact with experiments we need to find observables that could be affected: the NEDM
- Neutron = Baryon, bound state of QCD $\Longrightarrow$ A non perturbative description is unavoidable
- Make use of the *AdS*/CFT correspondence

Motivations

- The Lattice approach is Euclidean, it's difficult to handle θ. Moreover hard to describe nucleons
- The holographic model can improve standard effective models (Skyrme model). Has the whole tower of massive mesons.

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Outline

Remarks on non perturbative QCD

The Lagrangian of QCD

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2g^2}\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} + \sum_{f=1}^{N_f}\overline{q}_f(i\not{D} - m_f)q_f$$

We have instantonic configurations: Euclidean space–time localized solutions of ${}^\star F = \pm F$. Those are topological solutions

$$\frac{1}{32\pi^2}\int \mathrm{d}^4x\,\mathrm{Tr}\,\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \equiv \int \mathrm{d}^4x\,\mathcal{Q}[A] = \nu_{\mathrm{inst}}$$

The ν_{inst} is a "S^1 direction" in the space of configurations $\Longrightarrow$ add a θ term in the Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} \to \mathcal{L}_{\mathrm{QCD}} + \theta\mathcal{Q}[A]$$

θ is a super selected, universal constant. The term $\mathcal{Q}[A]$ is an explicit violation of discrete symmetries P and T (because of ε tensor).

Neutron electric dipole moment

- The Neutron electric dipole moment (NEDM), call it $\vec{D}_n$, is a P and T violating quantity ($\vec{D}_n \propto$ spin, which is a pseudovector and $\vec{E}$ is a regular vector).
- It allows us to estimate θ, in fact at small θ:

$$\vec{D}_n = \theta\, d_n \vec{\sigma}\ , \quad \vec{D}_n(\exp) \leq (2.9 \cdot 10^{-26}\, e \cdot \mathrm{cm})\, \vec{\sigma}$$

- Our final objective is the computation of d_n
- **Strong CP problem**: All estimates of d_n (ours included) give

$$d_n \sim 10^{-16} e \cdot \mathrm{cm} \implies \theta \lesssim 10^{-10}$$

The smallness of θ is quite puzzling.

Previous estimates

Year	Approach/model	$c_n = d_n/(\theta \cdot 10^{-16} e \cdot \mathrm{cm})$
1979	bag model	2.7
1980	ChPT	3.6
1981	ChPT	1
1981	ChPT	5.5
1982	ChPT	20
1984	chiral bag model	3.0
1984	soft pion Skyrme model	1.2
1984	single nucleon contribution	11
1990	Skyrme model $N_f = 3$	2
1991	Skyrme model $N_f = 2$	1.4
1991	ChPT	3.3(1.8)
1991	ChPT	4.8
1992	ChPT	-7.2, -3.9
1999	sum rules	2.4(1.0)
2000	heavy baryon ChPT	7.5(3.2)
2004	instanton liquid	10(4)
2007	holographic QCD "hard–wall"	1.08
2015	Lattice QCD	-3.9(2)(9)

Outline

Brief introduction to *AdS*/CFT correspondence I

The *AdS*/CFT correspondence establishes a duality

Superstring theory
D–branes | Closed strings

↙ ↘

$SU(N)$ gauge theory ⟷ Supergravity theory

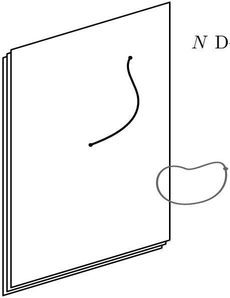
↓ ↓

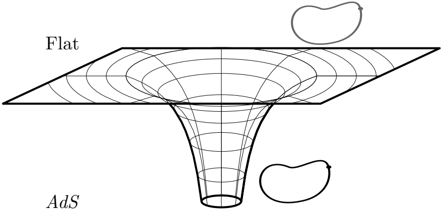
1. Large N
2. strong coupling limit

≡

1. non interacting strings limit
2. Small curvature

Brief introduction to *AdS*/CFT correspondence II





Gauge theory is conformal $\Longleftrightarrow$ Gravity is Anti–deSitter

We need to break conformality hence the space won't be *AdS*.

Witten's background

- We need to describe YM $\Rightarrow$ need to break supersymmetry
- Go in an higher number of dimensions and compactify with antiperiodic b.c.

D4 branes wrapped on S^1 $\equiv$ $SU(N_c)$ Yang–Mills in $4d$ + Kaluza Klein adjoint fields	$\longleftrightarrow$	Type IIA SUGRA on Witten's metric

- Witten's metric depends on only one coordinate: U. Boundary = $U \to \infty$
- The compactified coordinate τ sets the energy scale

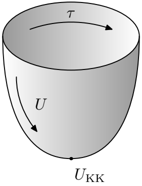
[Witten, 1998]

The “cigar”

The coordinate τ runs on a circle whose size shrinks when we go from the boundary to the bulk (the coordinate U decreases)

$$\mathrm{d}s^2 = \left(\frac{U}{R}\right)^{3/2} \left(\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + f(U)\mathrm{d}\tau^2\right) + \\ + \left(\frac{R}{U}\right)^{3/2} \left(\frac{\mathrm{d}U^2}{f(U)} + U^2 \mathrm{d}\Omega_4{}^2\right)$$

$$f(U) = 1 - \frac{U_{\mathrm{KK}}^3}{U^3}$$



It describes

1. Breaking of conformal invariance (U_{KK} = energy scale)
2. Confinement ($g_{00}(U_{\mathrm{KK}}) \neq 0$)
3. Mass gap ($\sim M_{\mathrm{KK}} = 1/$radius of S^1)

Outline

Branes configuration I

- We want to introduce quarks [Sakai and Sugimoto, 2005]
- Insert defects in the theory: N_f D8 ($\overline{\text{D8}}$) branes as **probes** (corresponds to quenched quarks)

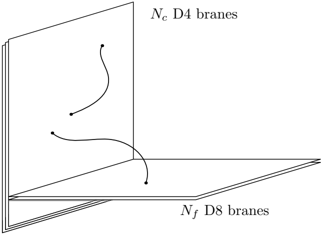
Strings from D8($\overline{\text{D8}}$) to D4 $\equiv$ quarks ($\overline{\text{quarks}}$)	$\longleftrightarrow$	D8 effective action on Witten's background $\equiv$ YM + CS action

- The transverse coordinate is τ

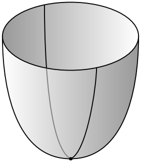
Things to know about branes

- They are dynamical objects, their embedding warps spacetime
- A field theory lives on them: DBI + CS. The DBI reduces to YM
- A stack of N branes realizes a $U(N)$ gauge theory

Branes configuration II



(a) Depiction of the branes configuration and strings



(b) Antipodal embedding

The two stacks of D8 branes join in the cigar. The symmetry $SU(N_f) \times SU(N_f)$ breaks to $SU(N_f)_V \implies$ chiral symmetry breaking

Outline

Baryons as instantons

We have a $U(N_f)$ **Yang–Mills + Chern–Simons** theory living on the D8 branes. The spacetime is curved by Witten's background.

$$S_{D8} = -\frac{\lambda N_c}{216\pi^3}\int \mathrm{d}^4x\mathrm{d}z \left(\frac{1}{2}h(z)\,\mathrm{Tr}\,\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + k(z)\mathrm{Tr}\,\mathcal{F}_{\mu z}\mathcal{F}^{\mu}{}_{z}\right) +$$

$$+\frac{N_c}{24\pi^2}\int \mathrm{Tr}\left(\mathcal{A}\wedge\mathcal{F}^2 - \frac{i}{2}\mathcal{A}^3\wedge\mathcal{F} - \frac{1}{10}\mathcal{A}^5\right)$$

$$k(z) = 1+z^2\,, \qquad h(z) = (1+z^2)^{-1/3}$$

We look for instantonic solutions in this system, living in the space $x^1, x^2, x^3, z \equiv x^M$ (euclidean signature) [Hata et al., 2007].

- We have, in the simpler case $N_f = 2$, a BPST instanton

$$A^a_M(x^M) = -i\frac{x^2}{x^2+\rho^2}g\partial_M g^{-1}$$

- The presence of the CS term prevents the size of the instanton to shrink to zero, but it's still very small

$$\rho \propto \frac{1}{\sqrt{\lambda}}\,, \qquad \lambda = g^2_{\mathrm{YM}}N_c$$

- The instanton number $\int \mathcal{Q}$ is interpreted as the *baryon number*

Outline

Mass and θ deformation

- To have a non zero NEDM we need to break CP $\Longrightarrow$ switch on a non zero θ term
- A $C_{(1)}$ R.R. form is dual to a θ term.
- $U(1)_A$ anomaly makes the θ disappear in the chiral limit $\Longrightarrow$ we need non zero masses

Axial anomaly in QCD	$\longleftrightarrow$	CS anomalous couplings

- Explicitly break the chiral symmetry group to introduce quark masses: string between the two D8 stacks [Aharony and Kutasov, 2008]

$$\delta S = c \int \mathrm{Tr}\left[M e^{i \int \mathrm{d}z \mathcal{A}_z} + \mathrm{c.c.} \right]$$

Outline

Ansatz

We have a system of coupled **Yang–Mills** + **Chern–Simons** equations. For simplicity restrict to the case

$$N_f = 2\,, \qquad M = \begin{pmatrix} m_q & 0 \\ 0 & m_q \end{pmatrix}$$

Solve the equations perturbatively around the instantonic solution

$$\mathcal{A} = \mathcal{A}^{\text{inst}} + \varepsilon \mathcal{A}^{\text{mass}}\,, \qquad \varepsilon \in \mathcal{O}(m_q/M_{\text{KK}})$$

The relevant fields are (the others being zero or not contributing to the NEDM), decompose $\mathcal{A} = \widehat{A}\frac{\mathbb{1}}{2} + A^a\frac{\tau^a}{2}$

$$\widehat{A}_z^{\text{mass}} = \frac{u(r)}{1+z^2}\,, \qquad A_0^{\text{mass}} = W(r,z)\,\vec{x}\cdot\vec{\tau}$$

Where $r = \sqrt{x^i x^i}$ and τ^i the Pauli matrix i.

Quantizing the solution

The solution represents the vacuum state of a baryon in its rest frame

Excite spin and isospin $\Longleftrightarrow$ Perform an (iso)spin rotation

This is achieved by a "*wrong*" gauge transformation $V \equiv V(x^M, t)$

$$
\begin{aligned}
A_0 &\longmapsto & A'_0 &= V A_0 V^{-1} \\
A_M &\longmapsto & A'_M &= V A_M V^{-1} - iV\partial_M V^{-1}
\end{aligned}
$$

The parameters of this (iso)rotation are called *moduli* and they live in the *moduli space*.

$$
\lim_{z\to\infty} V = \boldsymbol{a}(t) \,, \qquad \boldsymbol{a}(t) = a^a(t)\tau^a
$$

Quantize the moduli $\boldsymbol{a}$ and regard the baryons as the energy eigenstates of the resulting Hamiltonian. (The corrections to the Hamiltonian are $\mathcal{O}(\theta^2)$)

$$
\boldsymbol{a} \mapsto \hat{\boldsymbol{a}} \,, \quad \dot{\boldsymbol{a}} \mapsto -i\frac{\partial}{\partial \boldsymbol{a}} \,, \qquad |B\rangle = \Psi(\boldsymbol{a})
$$

The chiral current and the NEDM

We can define *Noether currents* associated to the Chiral symmetry

$$\mathcal{J}_{\mu\,V} = -\frac{N_c \lambda}{216\pi^3}\left[k(z)\mathcal{F}_{\mu z}\right]_{z\to-\infty}^{z\to\infty}$$

The electromagnetic current is

$$J_\mu^{\text{em}} = J_V^{0,\ a=3} + \frac{1}{N_c}\widehat{J}_V^0$$

The dipole is defined as usual (B is a baryon with spin s and isospin $I = s$)

$$\vec{D}_{B,s} = \int \mathrm{d}^3x\,\vec{x}\,\langle B, s|J_0^{\text{em}}|B, s\rangle \propto$$

$$\propto \theta\,(\vec{\sigma})_{ss'}(\vec{\tau})_{I_3 I_3'}$$

Outline

Numerical results

We use the parameters fitted by meson physics m_ρ, m_π, f_π

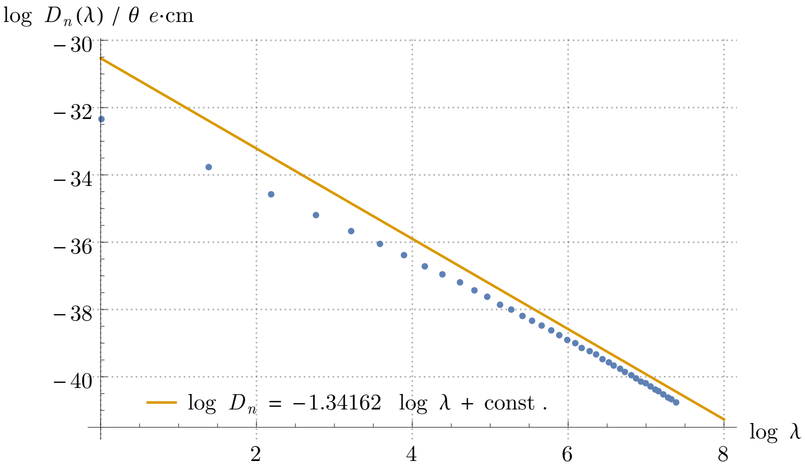
$$\lambda = 16.632\ , \quad M_{\rm KK} = 949\,{\rm MeV}\ , \quad m_q = 2.92\,{\rm MeV}$$

Where $\lambda = g_{\rm YM}^2 N_c$, $M_{\rm KK}$ is the inverse of the radius of S^1 and m_q the quark mass.
We get the numerical value

$$d_n = 4.97 \cdot 10^{-16}\,\theta\, e \cdot {\rm cm}$$

Numerical analysis for various $\lambda \to$ Power law

Logarithmic plot



Conclusions

Achievements

- We computed d_n holographically adding a new estimate to the previous ones coming from effective theories and lattice QCD
- Our computation confirms previous estimates
- We studied the mass and θ deformation of the Sakai–Sugimoto model

Follow ups

- Use the perturbed solution $\mathcal{A}^{\text{mass}}$ to compute next order corrections to the Baryon mass spectrum
- Consider $m_u - m_d$ corrections to our computations
- Consider the case with 2+1 flavours $m_u = m_d = m$, $m_s > m$.

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Thank you!

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Formulae I

Gravity action (bulk + D8)

$$S_{C_7} = -\frac{1}{4\pi}(2\pi l_s)^6 \int \mathrm{d}C_{(7)} \wedge {}^\star \mathrm{d}C_{(7)} + \frac{1}{2\pi}\int C_{(7)} \wedge \mathrm{Tr}\,\mathcal{F} \wedge \omega_y \tag{5.1}$$

$$\begin{aligned} S_{\mathrm{bulk+D8}} = -\kappa \int \mathrm{d}^4x\mathrm{d}z \left(\frac{1}{2}h(z)\,\mathrm{Tr}\,\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + k(z)\mathrm{Tr}\,\mathcal{F}_{\mu z}\mathcal{F}^{\mu}{}_{z}\right) + \\ + S_{C_7} + \frac{N_c}{24\pi^2}\int \mathrm{Tr}\left(\mathcal{A}\wedge\mathcal{F}^2 - \frac{i}{2}\mathcal{A}^3\wedge\mathcal{F} - \frac{1}{10}\mathcal{A}^5\right) \end{aligned} \tag{5.2}$$

BPST Instanton near core limit

$$A^{\mathrm{cl}}_M = -if(\xi)g\partial_M g^{-1} \tag{5.3}$$

$$f(\xi) = \frac{\xi^2}{\xi^2+\rho^2}\,, \quad g(x) = \frac{(z-Z)\mathbb{1} - i(\vec{x}-\vec{X})\cdot\vec{\tau}}{\xi} \tag{5.4}$$

$$\widehat{A}^{\mathrm{cl}}_0 = \frac{N_c}{8\pi^2\kappa}\frac{1}{\xi^2}\left[1 - \frac{\rho^4}{(\rho^2+\xi^2)^2}\right]\,, \quad A^{\mathrm{cl}}_0 = \widehat{A}_M = 0 \tag{5.5}$$

Formulae II

Mass perturbed equations

$$-\kappa\left(h(z)\partial_\nu \widehat{F}^{\mu\nu}+\partial_z(k(z)\widehat{F}^{\mu z})\right)+\frac{N_c}{128\pi^2}\varepsilon^{\mu\alpha\beta\gamma\delta}\left(F^a_{\alpha\beta}F^a_{\gamma\delta}+\widehat{F}_{\alpha\beta}\widehat{F}_{\gamma\delta}\right)=0 \tag{5.6}$$

$$-\kappa\left(h(z)D_\nu F^{\mu\nu}+D_z(k(z)F^{\mu z})\right)^a+\frac{N_c}{64\pi^2}\varepsilon^{\mu\alpha\beta\gamma\delta}F^a_{\alpha\beta}\widehat{F}_{\gamma\delta}=0 \tag{5.7}$$

$$\begin{aligned}&-\kappa\, k(z)\partial_\nu \widehat{F}^{z\nu}+\frac{N_c}{128\pi^2}\varepsilon^{z\mu\nu\rho\sigma}\left(F^a_{\mu\nu}F^a_{\rho\sigma}+\widehat{F}_{\mu\nu}\widehat{F}_{\rho\sigma}\right)=\\ &=-\frac{4\pi}{3}\sqrt{\frac{N_f}{2}}[\mathrm{d}C_{(7)}]_{0123}+ic\,\mathrm{Tr}\left[\frac{M}{\sqrt{2N_f}}\left(\mathcal{P}e^{i\frac{\theta}{N_f}}e^{-i\int_{-\infty}^{\infty}\mathcal{A}_z\mathrm{d}z}-\mathrm{c.c.}\right)\right]\end{aligned} \tag{5.8}$$

$$\begin{aligned}&-\kappa\, k(z)(D_\nu F^{z\nu})^a+\frac{N_c}{64\pi^2}\varepsilon^{z\mu\nu\rho\sigma}F^a_{\mu\nu}\widehat{F}_{\rho\sigma}=\\ &=ic\,\mathrm{Tr}\,\mathcal{P}\left[M\frac{\tau^a}{2}\left(e^{i\frac{\theta}{N_f}}e^{-i\int_{-\infty}^{\infty}\mathcal{A}_z\mathrm{d}z}-\mathrm{c.c.}\right)\right]\end{aligned} \tag{5.9}$$

Formulae III

Equations for the two relevant components

$$\kappa\, k(z)\partial_i\partial^i \widehat{A}_z^{\mathrm{mass}} = 2cm_q \sin\frac{\theta}{2}\left[\cos\left(\frac{\pi}{\sqrt{1+\rho^2/r^2}}\right)+1\right] \quad (5.10)$$

$$h(z)\left(\partial_r^2 W(r,z) + \frac{4}{r}\partial_r W(r,z) + \frac{8\rho^2}{(\xi^2+\rho^2)^2}W(r,z)\right) + \partial_z(k(z)\partial_z W(r,z)) = \\ = \frac{27\pi}{\lambda}\frac{\rho^2}{(\xi^2+\rho^2)^2}\frac{1}{r}\frac{u'(r)}{1+z^2} \equiv \mathscr{F}(r,z) \quad (5.11)$$