# Hawking-like radiation from gravitational Bremsstrahlung beyond the Planck scale

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In collaboration with M. Ciafaloni, F. Coradeschi and G. Veneziano

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# Outline

- Introduction
  - Motivation
    - transpl. collisions $\rightarrow$ gravity at quantum level
- Method
  - based on emission amplitude unifying
    - central region (Regge limit)
    - fragmentation region
  - resum infinite diagrams
- Result
  - metric
  - energy spectrum
    - small deflection angles
    - large deflection angles
  - Characteristic frequency typical of Hawking radiation

# Motivation

Why particle collisions at transplanckian energies $E \gg M_P$?

To find a solution to the information paradox

- Quantum collision in QFT or ST $\rightsquigarrow$ unitary $S$-matrix
- At large energy, gravitational collapse expected from GR (+ Hawking radiation) $\to$ loss of information of initial state

Does a semiclassical picture of collapse emerge from a Quantum Theory?

# Introduction

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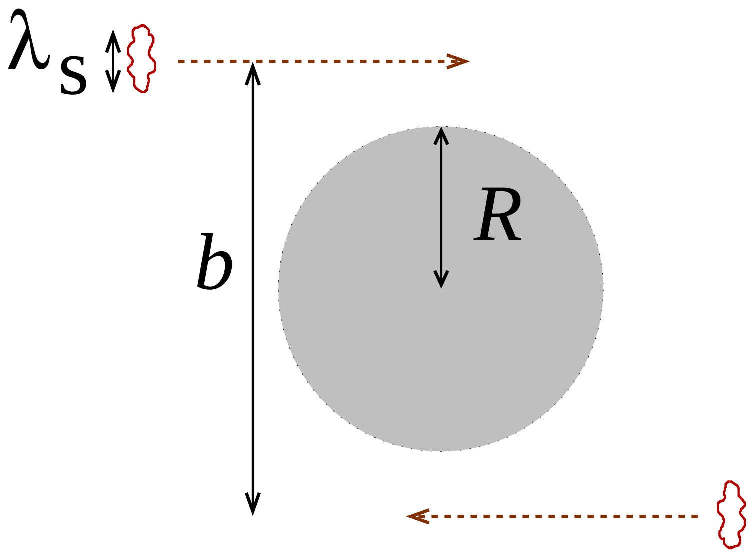
Scattering of 2 strings at trans-planckian energies $2E = \sqrt{s} \gg M_P \equiv \hbar/G$;

$\lambda_P \equiv \sqrt{\hbar G}$ $\lambda_s \equiv \sqrt{\hbar\alpha'}$ $R \equiv 2G\sqrt{s}$ $b$



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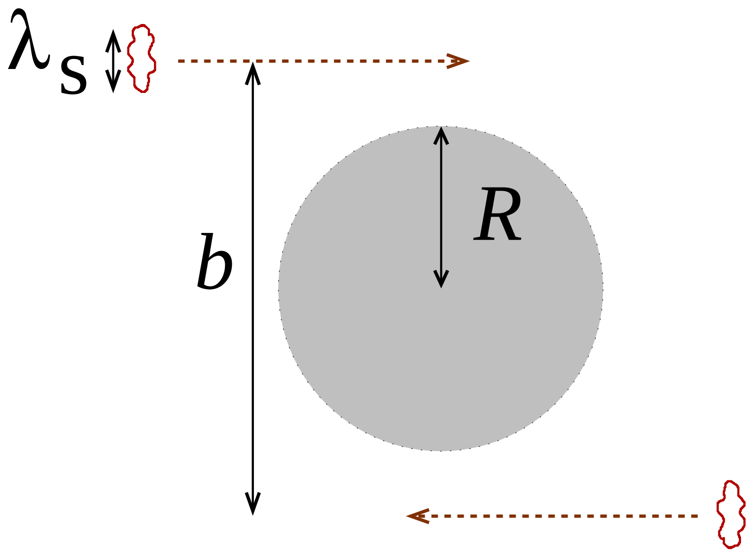
Semiclassical regime:

- $\lambda_P \ll R \iff \sqrt{s} \gg M_P$
  action $\sim Gs/\hbar \gg 1$
- $\lambda_s \ll R$ string doesn't mask $R$
- $R \ll b$ eikonal scattering:
  $\theta_E = 2R/b \ll 1$
- $R \sim b$ large deflections:
  $\theta_E \sim 1$ (strong coupling)



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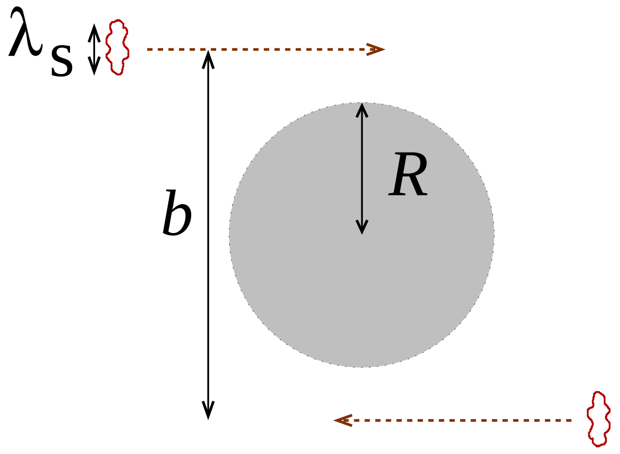

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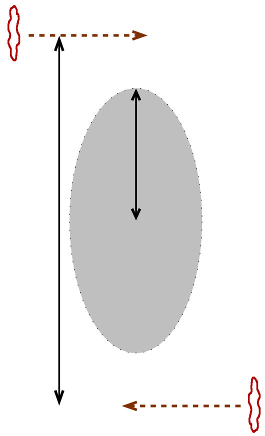

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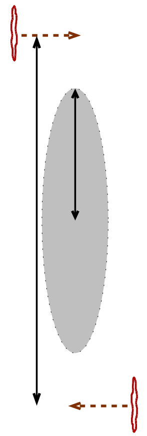

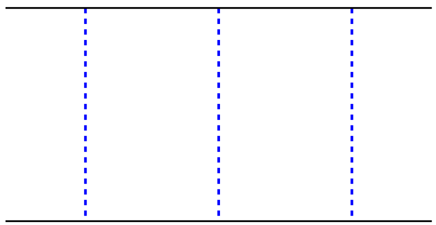

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  $\to$ effective 2D lagrangian
- Leading level: effective ladder diagrams for elastic scattering

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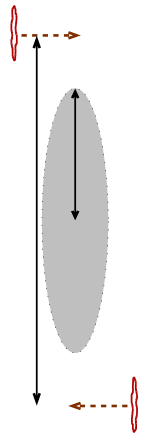

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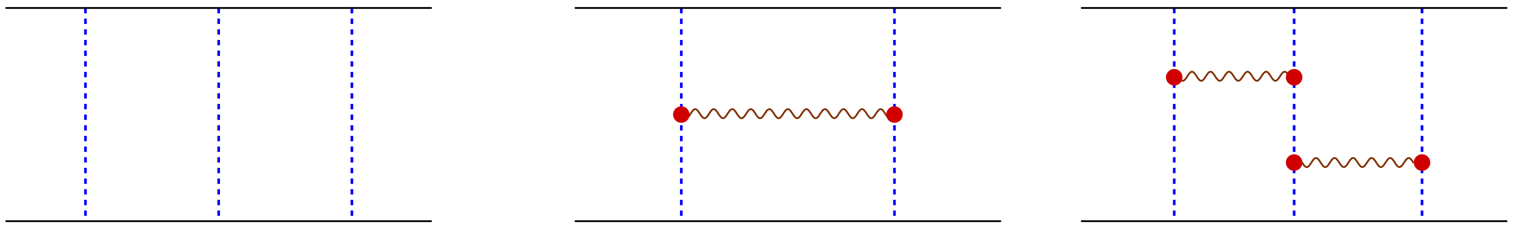

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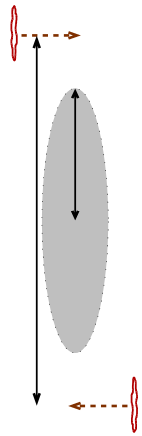

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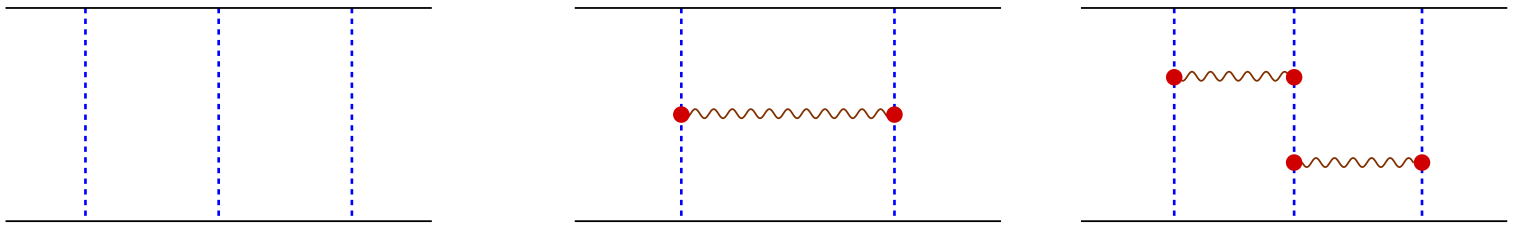

- Convolutions in transverse momentum $\rightsquigarrow$ diagonalized in $b$-space
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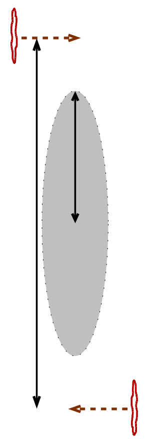

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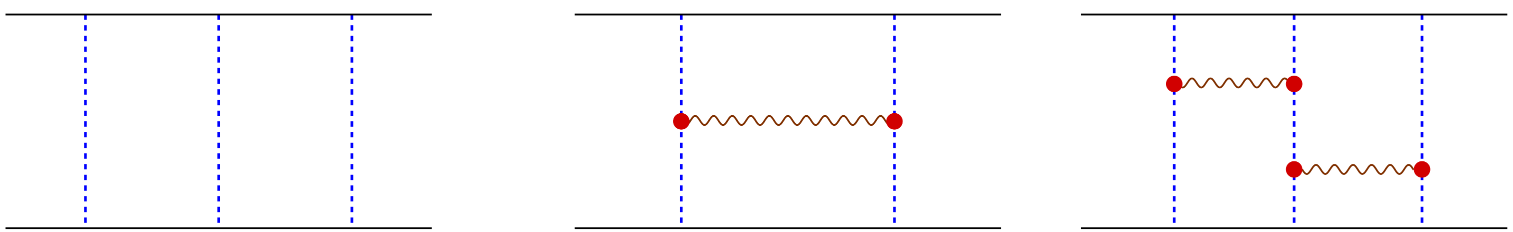

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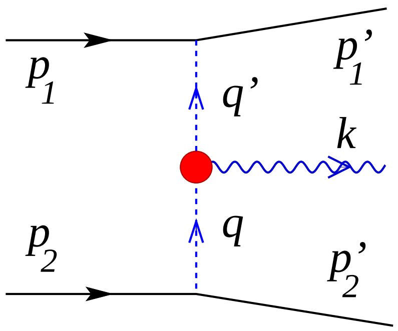
- Corrections $\mathcal{O}\left(\frac{R^2}{b^2}\right) \Rightarrow$ critical $b_c = 1.6R$ $\qquad \begin{cases} \Im\delta_0(b) > 0 \\ |S_{\rm el}| < 1 \end{cases} \qquad (b < b_c)$

# Bremsstrahlung: Regge amplitude

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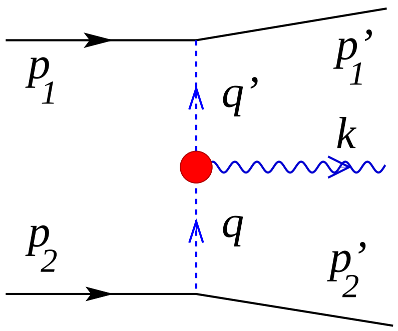


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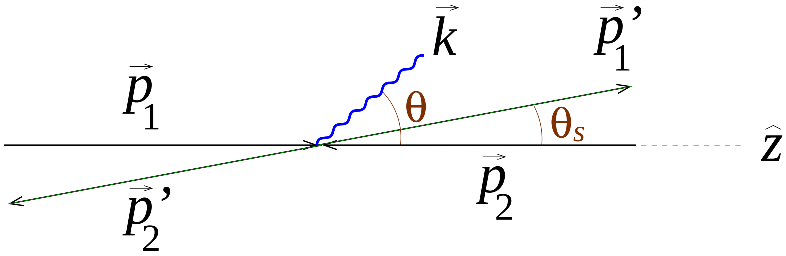

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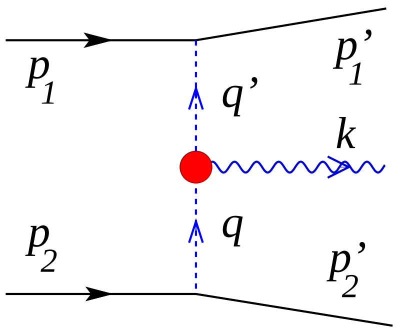


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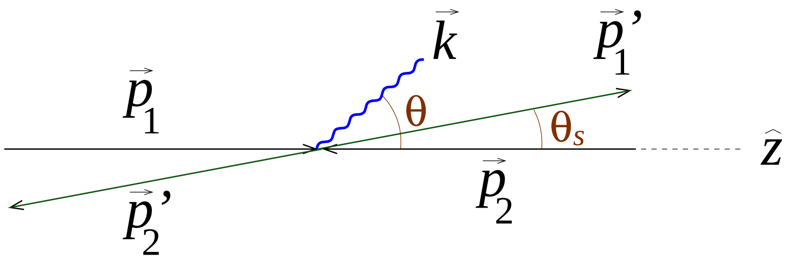
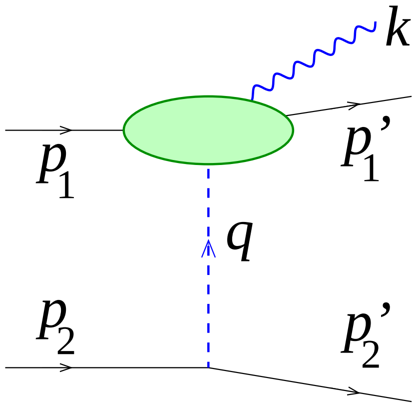



$$M_{\mathrm{Regge}} \simeq M_{\mathrm{el}} J_L^{\mu\nu}(\boldsymbol{q},\boldsymbol{q}')\epsilon^{(+)}_{\mu\nu} = \kappa^3 s \frac{\mathrm{e}^{2\mathrm{i}(\phi_{\boldsymbol{q}}-\phi_{\boldsymbol{q}'})}-1}{\boldsymbol{k}^2}$$

$$= \kappa^3 s \frac{\boldsymbol{k}^*\boldsymbol{q}-\boldsymbol{k}\boldsymbol{q}^*}{\boldsymbol{k}\boldsymbol{k}^*\boldsymbol{q}\boldsymbol{q}'^*} \quad (\boldsymbol{k} \equiv k_x + \mathrm{i}k_y \in \mathbb{C})$$

Helicity amplitude has an unphysical collinear singularity at $\boldsymbol{k} = 0$ i.e. $\theta = 0$

# Bremsstrahlung: Unified amplitude




For $\omega \ll E$ *[Weinberg '65]* theorem holds:

$$M_{\text{soft}} \simeq M_{\text{el}} \times J_W^{\mu\nu}(\boldsymbol{k})\epsilon_{\mu\nu}^{(+)} = \kappa^3 s \frac{1 - \mathrm{e}^{2\mathrm{i}(\phi_{\boldsymbol{\theta}} - \phi_{\boldsymbol{\theta}-\boldsymbol{\theta}_s})}}{E^2 \boldsymbol{\theta}_s^2}$$

$$= \kappa^3 s \frac{\boldsymbol{k}^* \boldsymbol{q} - \boldsymbol{k} \boldsymbol{q}^*}{(\boldsymbol{k} - \frac{\omega}{E}\boldsymbol{q})\boldsymbol{k}^* \boldsymbol{q} \boldsymbol{q}^*} \qquad \begin{cases} \boldsymbol{k} & = \omega \boldsymbol{\theta} \\ \boldsymbol{q} & = \boldsymbol{p}_1' = E\boldsymbol{\theta}_s \end{cases}$$

valid if $|\boldsymbol{k}| \ll |\boldsymbol{q}| \quad \Longleftrightarrow \quad \omega\theta \ll E\theta_s$

No collinear singularity at $\boldsymbol{\theta} = 0$ ($\boldsymbol{k} = 0$) nor at $\boldsymbol{\theta} = \boldsymbol{\theta}_s$ ($\boldsymbol{k} = \frac{\omega}{E}\boldsymbol{q}$)

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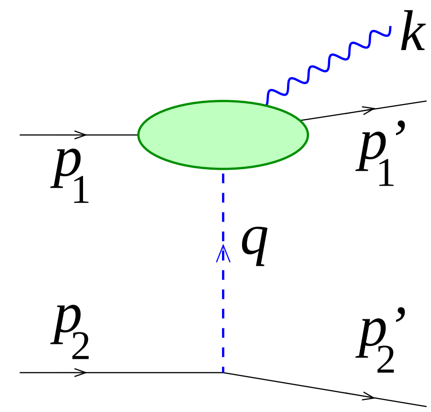


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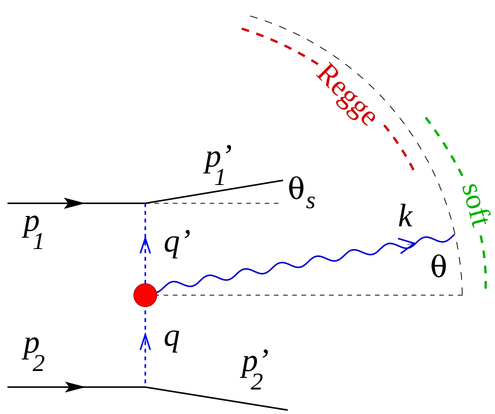


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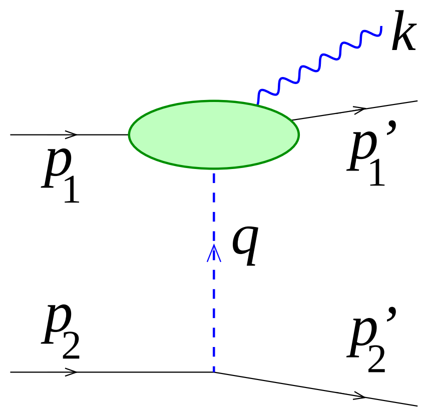


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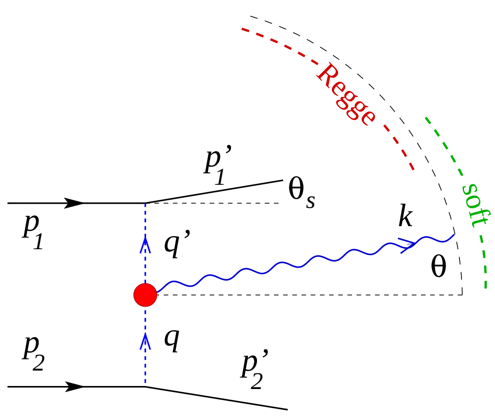


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$$\frac{M_{\rm soft}}{\kappa^3 s} = \frac{\boldsymbol{k}^*\boldsymbol{q} - \boldsymbol{k}\boldsymbol{q}^*}{(\boldsymbol{k} - \frac{\omega}{E}\boldsymbol{q})\boldsymbol{k}^*\boldsymbol{q}\boldsymbol{q}^*}$$

# Metric fields (part 1)

From $M_{\mathrm{match}}$ define 2 fields: $h_{\mathrm{Regge}}$ and $h_{\mathrm{soft}}$

$$M_{\mathrm{match}}(\boldsymbol{q}, \boldsymbol{k}) \quad \xrightarrow{IFT^2} \quad M_{\mathrm{match}}(\boldsymbol{b}, \boldsymbol{x})$$

probability amplitude
of finding a graviton
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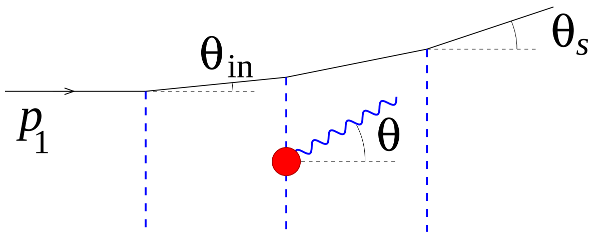
In the classical limit $\omega/E \to 0$ it is related to the perturbation to the metric at $\boldsymbol{x}$:
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\begin{aligned} h_{11} + h_{22} = h_{\boldsymbol{x}\boldsymbol{x}^*} &= \Re \lim_{\omega/E\to 0} \kappa M_{\text{match}}(\boldsymbol{b},\boldsymbol{x}) \\ &= \frac{R^2}{|\boldsymbol{b}|^2} \Re \frac{\boldsymbol{x}\boldsymbol{b}^* - \boldsymbol{x}^*\boldsymbol{b}}{\boldsymbol{x}(\boldsymbol{x}-\boldsymbol{b})^*} \equiv \Re\, \kappa M_{\text{Regge}}(\boldsymbol{b},\boldsymbol{x})!! \\ &= h_{\text{Regge}}\big|_{ACV} \end{aligned}$$

# Resummation

However, $h_{\rm Regge}$ is not the field needed for the resummation of




- Incoming particle has generic direction $\boldsymbol{\theta}_{\rm in} \neq 0$
- Corresponding amplitude related by a rotation

$$M(\boldsymbol{\theta}_{\rm in}, \boldsymbol{\theta}) = \mathrm{e}^{2\mathrm{i}\phi_{\boldsymbol{\theta}}} M(\mathbf{0}, \boldsymbol{\theta} - \boldsymbol{\theta}_{\rm in}) \mathrm{e}^{-2\mathrm{i}\phi_{\boldsymbol{\theta}-\boldsymbol{\theta}_{\rm in}}}$$

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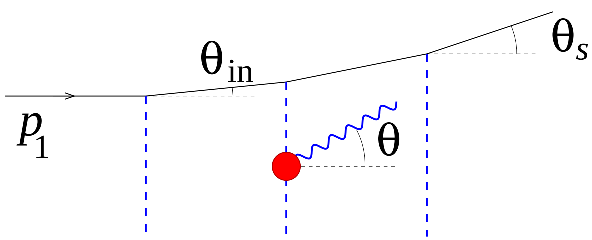


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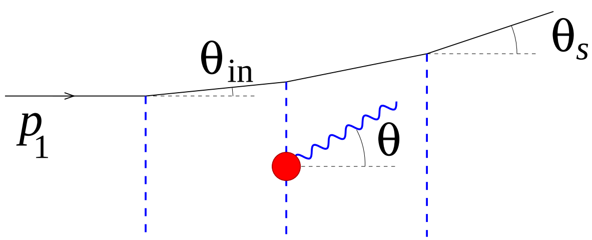


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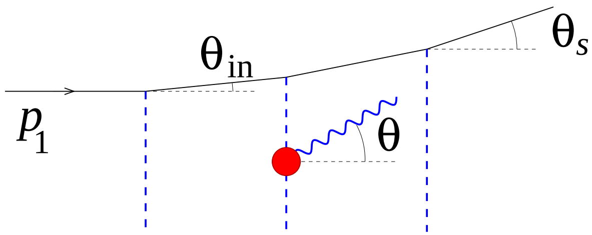


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$$M_{\rm tot}(\boldsymbol{b}, \boldsymbol{k}) = e^{2i\delta_0(b)} \mathfrak{M}(\boldsymbol{b}, \boldsymbol{k}) \qquad h_{\rm soft}(\boldsymbol{b}, \boldsymbol{x}) = \frac{\partial_{\boldsymbol{x}}}{\partial_{\boldsymbol{x}^*}} M_{\rm match}$$

$$\mathfrak{M}(\boldsymbol{b}, \boldsymbol{k}) = e^{2i\phi_{\boldsymbol{\theta}}} \kappa^3 s \int d^2\boldsymbol{x} \int_0^1 d\xi \; e^{i\omega \boldsymbol{x} \cdot (\boldsymbol{\theta} - \xi \boldsymbol{\Theta}_E)} h_{\rm soft}(\boldsymbol{b}, \boldsymbol{x})$$

$$h_{\rm soft} = \frac{1}{\boldsymbol{x}^2} \left[ \frac{E}{\omega} \ln \left| \frac{\boldsymbol{b} - \frac{\omega}{E}\boldsymbol{x}}{\boldsymbol{b}} \right| - (E \to \omega) \right] \equiv M_{\rm soft}(\boldsymbol{b}, \boldsymbol{x}; E) - (E \to \omega)$$

# Metric fields (part 2)

Recall that $\lim_{\omega/E\to 0} M_{\mathrm{match}}(\boldsymbol{b},\boldsymbol{x}) = h_{\mathrm{Regge}} = h_{\boldsymbol{x}\boldsymbol{x}^*}$

From $$h_{\mathrm{soft}}(\boldsymbol{q},\boldsymbol{k}) \equiv \frac{\boldsymbol{k}^*}{\boldsymbol{k}} M_{\mathrm{match}}$$

$\Rightarrow$ $$\partial_{\boldsymbol{x}} h_{\mathrm{soft}}(\boldsymbol{b},\boldsymbol{x}) = \partial_{\boldsymbol{x}^*} M_{\mathrm{match}}$$

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Yes: $\lim_{\omega/E\to 0} h_{\rm soft}(\boldsymbol{b},\boldsymbol{x}) = h_{\boldsymbol{x}\boldsymbol{x}} = h_{11} - h_{22} - 2\mathrm{i}h_{12} = \frac{\partial_{\boldsymbol{x}^*}}{\partial_{\boldsymbol{x}}} h_{\rm Regge}$

equivalent to the transversality condition $\frac{\partial}{\partial x_i} h_{ij} = 0$ , $(i, j = 1, 2)$

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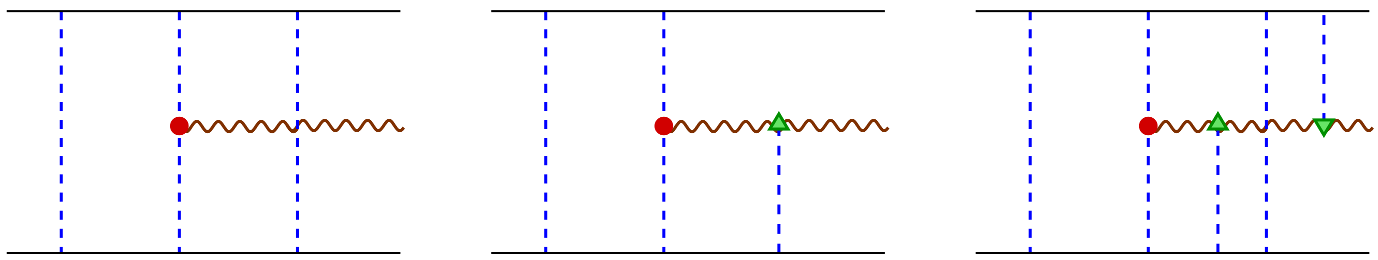
equivalent to the transversality condition $\frac{\partial}{\partial x_i} h_{ij} = 0$ , $\qquad (i,j = 1,2)$

- In gravity the Regge field and the soft field are intimately linked together
- The spin-2 nature of the graviton allows us to find a unique limiting form describing both the Regge kinematics and the soft limit

# Rescattering

New surprise: if one resums also rescattering diagrams



the soft field exponentiates:

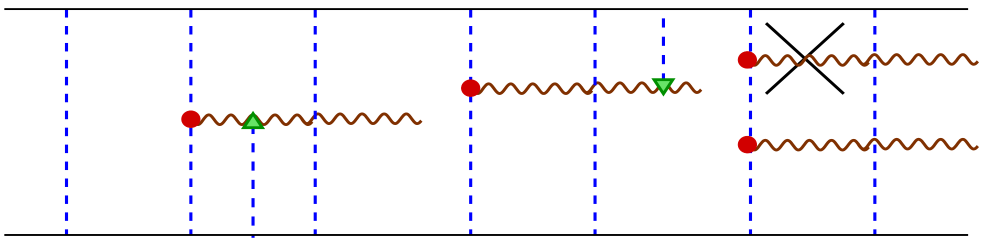
$$\frac{M_{\text{tot}}}{e^{2i\delta_0(b)}} \equiv \mathfrak{M} = \frac{e^{2i\phi_{\boldsymbol{\theta}}}}{i2\omega R} \int d^2\boldsymbol{x}\; e^{i\omega(\boldsymbol{\theta}-\boldsymbol{\Theta}_E)\cdot\boldsymbol{x}}\; \frac{e^{i2\omega R\Phi}-1}{\boldsymbol{x}^2}$$

$$h_{\text{soft}} = \frac{\frac{\boldsymbol{b}\cdot\boldsymbol{x}}{|\boldsymbol{b}|^2} - \ln\left|\frac{\boldsymbol{b}-\boldsymbol{x}}{\boldsymbol{b}}\right|}{\boldsymbol{x}^2} \equiv \frac{\Phi(\boldsymbol{b},\boldsymbol{x})}{\boldsymbol{x}^2}$$

- The denominator $1/\boldsymbol{x}^2$ corresponds to the Riemann tensor of the *[Aichelburg-Sexl '71]* gravitational shock wave generated by one of the incoming (massless) particle
- The metric fields are generated by the collisions of the two shock waves

# Multiple Emissions

Multi-graviton emissions (without correlations) can be resummed in closed form:



$$M_{\rm tot}(2 \to 2 + \boldsymbol{k}_1 \cdots \boldsymbol{k}_n) \equiv \mathrm{e}^{2\mathrm{i}\delta_0(b)} \frac{1}{n!} \mathfrak{M}(\boldsymbol{k}_1) \cdots \mathfrak{M}(\boldsymbol{k}_n)$$

$$|\text{gravitons}\rangle = \mathrm{e}^{2\mathrm{i}\delta_0(b)} \exp\left\{ \mathrm{i} \sum_{\lambda=\pm} \int \frac{\mathrm{d}^3 k}{2\omega_k} \mathfrak{M}^{(\lambda)}(k) a^{(\lambda)\dagger}(k) + \text{h.c.} \right\} |0\rangle$$

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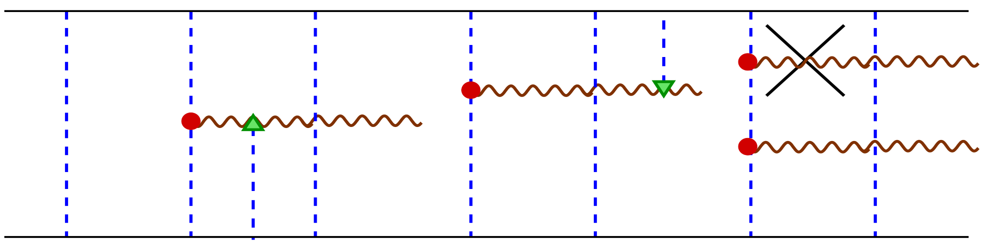

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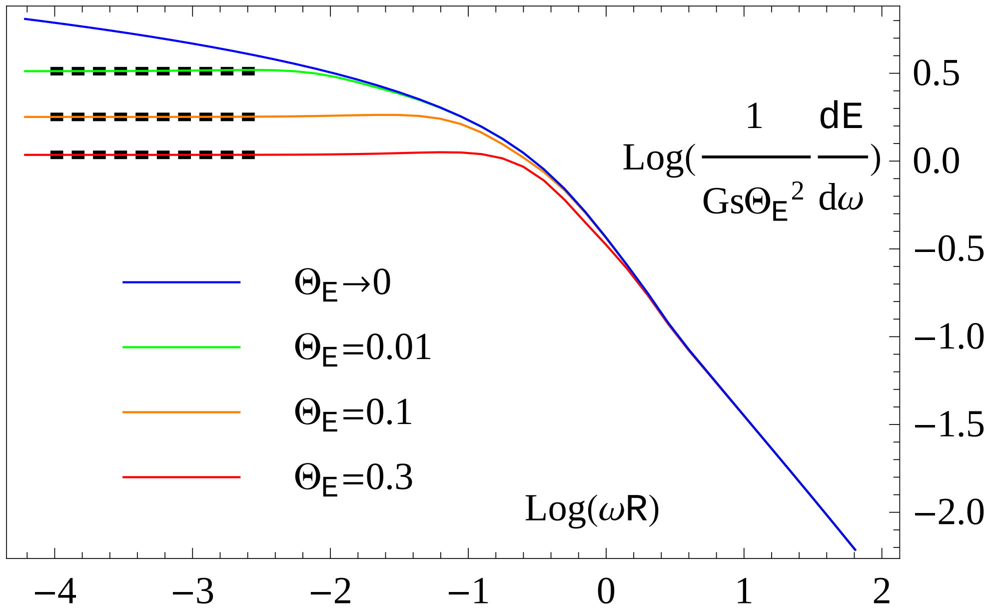
The spectrum and other observables can be explicitly computed

$$\frac{\mathrm{d}E^{\mathrm{GW}}}{\mathrm{d}\omega} = \frac{2Gs\,\boldsymbol{\Theta}_E^2\, b^2}{\pi^2} \int \frac{\mathrm{d}^2\boldsymbol{x}}{|\boldsymbol{x}|^4} \left( \frac{\sin \omega R \Phi(\boldsymbol{x})}{\omega R} \right)^2$$

and agrees with *[Gruzinov,Veneziano 2015]* for $\omega/E \to 0$

# Spectrum

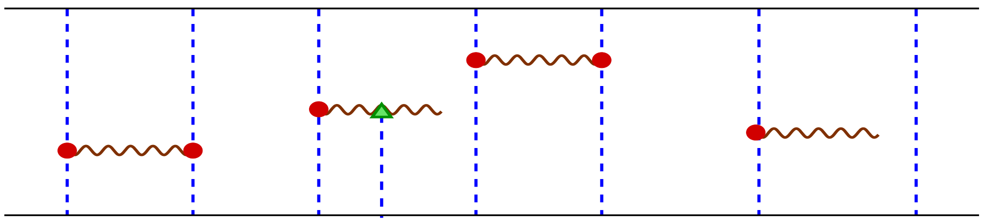
$$\frac{\mathrm{d}E^{\mathrm{GW}}}{\mathrm{d}\omega} = \frac{2Gs\,\boldsymbol{\Theta}_E^2\,b^2}{\pi^2}\int\frac{\mathrm{d}^2\boldsymbol{x}}{|\boldsymbol{x}|^4}\left(\frac{\sin\omega R\Phi(\boldsymbol{x})}{\omega R}\right)^2 \xrightarrow[\omega R\gg 1]{} Gs\,\boldsymbol{\Theta}_E^2\,\frac{4}{3}(1-\log 2)\frac{1}{\omega R}$$



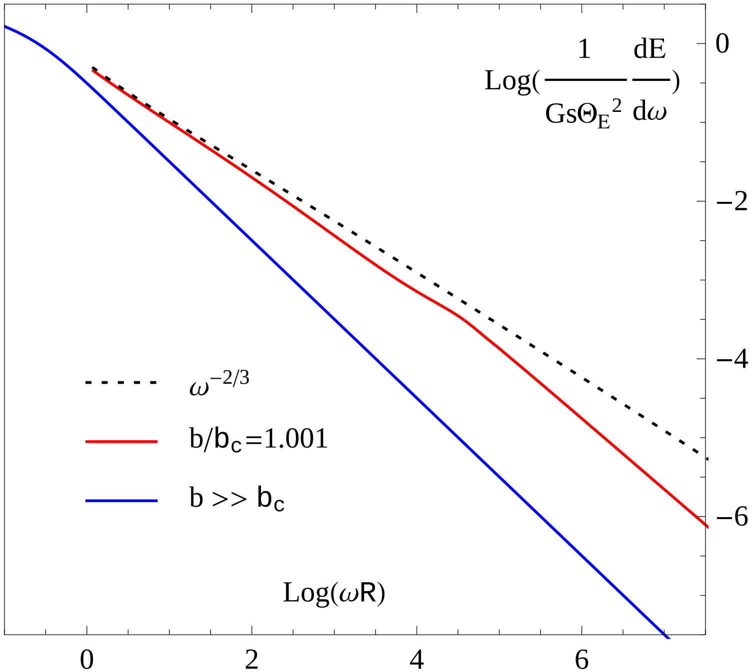

- Agrees with Zero Frequency Limit ($\omega \to 0$)
- Characteristic frequency $\langle\omega\rangle \sim 1/R$ almost independent of $b$, i.e., of $\boldsymbol{\Theta}_E$
- $\langle\omega\rangle$ decreases as $E$ increases, like Hawking radiation

# Towards $b \to R$

Previous results are correct for $b \gg R$, i.e., $\mathbf{\Theta}_E \ll 1$
Investigating emission for $b \sim R$ requires taking into account multi-H diagrams into the ladder



- We have shown that a similar factorization formula holds, by replacing $\delta_0(b) \to \delta(b)$, the subleading phase shift having the branch-cut at $b_c = 1.6R$
- Non-analytic behaviour of $\delta(b \to b_c^+)$ severely affects the energy spectrum
- Increasingly large intermediate region with enhanced emission $\omega^{-2/3}$ instead of $\omega^{-1}$



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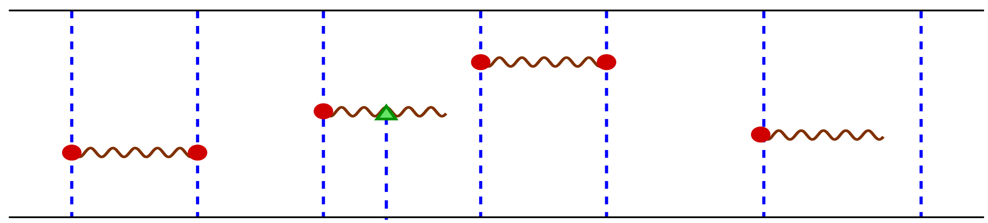

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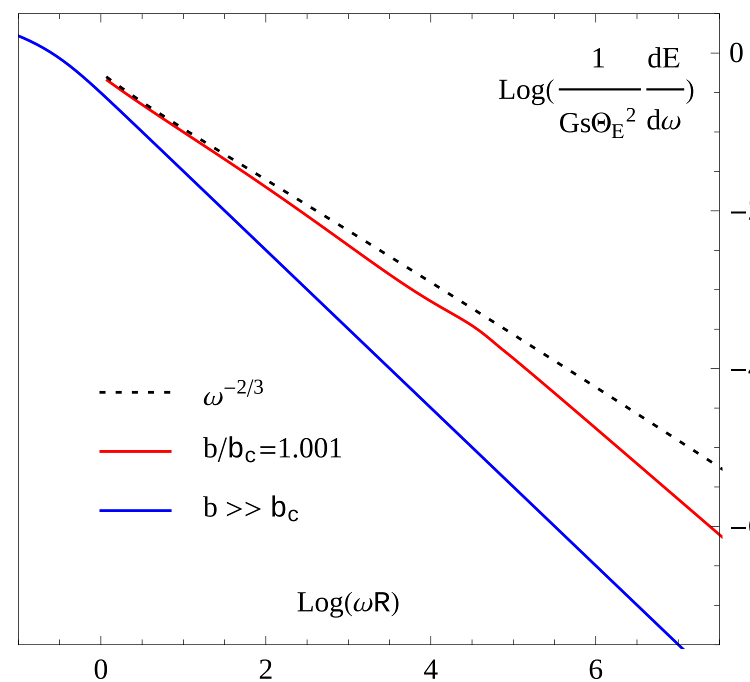

# Conclusions

- We are able to compute graviton radiation in transplanckian collisions
  - Unified limiting form of graviton emission amplitudes
  - Resolution of energy crisis
  - Finally we see the role of $R = \langle \omega \rangle$, like Hawking's
- Enhanced emission for $b \simeq R$
  - Too early to establish whether
    such enhancement is the signal of recovery of information
  - However, this feature is robust and interesting
  - Further study might help us in clarifying the flow of information