Newton-Cartan trace anomalies

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with G. Nardelli, S. Baiguera and F. Filippini

Introduction: c and a theorem

Zamolodchikov's c-theorem $d = 2$:

There exists of a $c-$function which decreases along the RG flow, and it agrees with the central charge at the conformal fixed point $\langle T^\mu_\mu \rangle = \frac{c}{24\pi} R$

a-theorem in $d = 4$:

$$\langle T^\mu_\mu \rangle = c(\mathrm{Weyl})^2 - aE_4 + bR^2 + \tilde{b}\nabla^2 R + d\epsilon^{\mu\nu\rho\sigma} R^{\alpha\beta}_{\mu\nu} R_{\alpha\beta\rho\sigma} \ ,$$

$$E_4 = R^2_{\mu\nu\rho\sigma} - 2R^2_{\mu\nu} + \frac{1}{3} R^2$$

a is a candidate decreasing function !! (Cardy)

a-theorem $d = 4$

$$d = 4\,, \qquad T^\mu_\mu = cW^2 - aE_4\,, \qquad a_{UV} > a_{IR}\,,$$

Free theory: $a = \frac{1}{90(8\pi)^2}\left(n_S + \frac{11}{2}n_{F,W} + 62 n_V\right)$

Tools to study irreversibility properties of RG flows:

- Local Renormalization Group equations
  (perturbative proof of a-theorem by Osborn, 1991)
- Holography
- SUSY, a-maximization
- Dispersion Relations
  Dilaton scattering amplitudes
  (Komargodski, Schwimmer 2011)

Trace anomalies

Anomalies must satisfy Wess-Zumino consistency conditions:

$$[\Delta_{\sigma}^{W}, \Delta_{\sigma'}^{W}]W = 0$$

Eliminates R^2 term at conformal fixed point

Moreover, some of them can be eliminated by local counterterms and are not genuine anomalies, like $\nabla^2 R$

- **type B** anomaly: vanish under Weyl transformation (like σW^2)
- **type A** anomaly: does not trivially vanish under Weyl, but it is still WZ consistent (like σE_4)

Is there a non-relativistic version?

In order to inspect these issues, we should couple the non-relativistic theory to a non-relativistic version of gravity

Gravity here is just a source for

energy-momentum tensor:

$$\langle T_{\mu\nu} \rangle = \frac{1}{\sqrt{-g}} \frac{\delta W}{\delta g^{\mu\nu}}$$

The kind of non-relativistic gravity depends on the symmetry of the non-relativistic theory: **Lifshitz** or **Schrödinger**

With or without boost

$$t \longrightarrow \lambda^z t\,, \qquad x \longrightarrow \lambda x\,.$$

- Without boost:
  studied in detail for various d, z by
  Arav, Chapman, Oz: 1410.1831
  All anomalies are type B (vanish under Weyl transformation)
- With boost:
  Jensen1412.7750, found a type A anomaly
  Basic tool to classify boost invariant terms in the anomaly:
  Newton Cartan gravity

Newton-Cartan gravity

- "Spatial metric" $h^{\mu\nu}$, $h_{\mu\nu}$
- 1-form n_μ (local time direction),
- background gauge field for particle number symmetry A_μ
- vector field v^μ
- Conditions: $n_\mu h^{\mu\alpha} = 0$, $n_\mu v^\mu = 1$ $h^{\mu\alpha} h_{\alpha\nu} = \delta^\mu_\nu - v^\mu n_\nu = P^\mu_\nu$

Nearby flat space: introduce δu^μ with $\delta u^\mu n_\mu = 0$ and a transverse metric perturbation $\delta \tilde{h}^{\alpha\beta} n_\beta = 0$.

$$\delta W = \int d^3x \sqrt{-g} \left(\frac{1}{2} T_{ij} \delta \tilde{h}_{ij} + j^\mu \delta A_\mu - \epsilon^\mu \delta n_\mu - p_i \delta u_i \right) ,$$

Sources for stress tensor, number current, energy current, momentum density

Newton-Cartan from null reduction

Invariance under local non-relativistic boost transforms v^μ, $h_{\mu\nu}$, A_μ non-trivially

Null reduction is an instrument to deal efficiently with these symmetries:

$$G_{MN} = \begin{pmatrix} 0 & n_\mu \\ n_\nu & n_\mu A_\nu + n_\nu A_\mu + h_{\mu\nu} \end{pmatrix} .$$

The anomaly can be written in terms of the extra dimensional curvatures

Causality

The form $n = n_\mu dx^\mu$ gives the local time direction

Frobenius condition

$$dn \wedge n = 0\,,$$

it is equivalent integrability of time slices

If it not satisfied,
no absolute notion of future and past,
causality is lost

No Frobenius, the anomaly has infinite number of terms

$d = 2$, $z = 2$ case, [Jensen, 1412.7750]:

$$\mathcal{A} = T^i_i - 2\epsilon^0 - = c(\mathrm{Weyl})^2 - aE_4 + bR^2 + \tilde{b}\nabla^2 R + \dots$$

the same expression as for the relativistic $3+1$ anomaly, one should just replace null reduction curvatures+

$+\dots$ which stands for an **infinite number** of terms, such as:

$$W_{ABCP} W^{ABCQ} W^P{}_{MQN} n^M n^N \,,$$

(for n^A, we buy 2 more derivatives)

Extra terms have not been yet classified [**work in progress**]

With Frobenius, type B

$$dn \wedge n = 0\,,$$

The anomaly has just a finite number of terms

$$\mathcal{A} = 2\epsilon^0 - T^i_i = c(\mathrm{Weyl})^2 + \mathrm{local\ counterterms}$$

Only one type B anomaly

R.A., S. Baiguera, G. Nardelli: arXiv:1511.08150
I. Arav, S. Chapman, Y. Oz arXiv:1601.06795

A free scalar in curved NC space

$$\int d^{d+1}x\sqrt{g}\left\{imv^{\mu}\left(\phi^{\dagger}D_{\mu}\phi - D_{\mu}\phi^{\dagger}\phi\right) - h^{\mu\nu}D_{\mu}\phi^{\dagger}D_{\nu}\phi - \xi R\phi^{\dagger}\phi\right\}\,,$$

$$D_{\mu}\phi = \partial_{\mu}\phi - imA_{\mu}\phi\,.$$

Conformal coupling:

$$\xi = \frac{1}{6}$$

Heat kernel method

Heat kernel method to compute anomaly:

$$\triangle = \triangle_t + \partial_i^2 \,, \qquad \triangle_t = -2m\sqrt{-\partial_t^2} \,,$$

In flat spacetime:

$$K_\triangle(s) = \langle x, t | e^{s\triangle} | y, t' \rangle =$$

$$= \frac{1}{2\pi} \frac{ms}{m^2 s^2 + \frac{(t-t')^2}{4}} \frac{1}{(4\pi s)^{d/2}} \exp\left(-\frac{(x-y)^2}{4s} \right) .$$

$$\tilde{K}_\triangle(s) = \langle x, t | e^{s\triangle} | x, t \rangle = \frac{2}{m(4\pi s)^{1+d/2}} .$$

The same spectral dimension as relativistic scalar in $d+2$ dimensions !!

Heat kernel in curved spacetime

Seeley-DeWitt expansion:

$$\tilde{K}_M(s) = \frac{1}{s^{d/2+1}} \left(a_0 + a_2 s + a_4 s^2 + \ldots \right) \, .$$

a_4 is proportional to the Weyl anomaly

Result:

$$T^i_i - 2\epsilon^0 = \frac{1}{8m\pi^2} \left(-\frac{1}{360} E_4 + \frac{3}{360} W^2 + \right.$$

$$\left. + \frac{1}{2} \left(\xi - \frac{1}{6} \right)^2 R^2 + \frac{1 - 5\xi}{30} D_A D^A R \right) + \ldots ,$$

A non-relativistic a-theorem ?

A free scalar contributes to the non-relativistic a anomaly:

$$\frac{1}{8m\pi^2}\frac{1}{360}$$

If the UV and IR fixed points contain just scalars, the (conjectured) non relativistic a-theorem would tell that:

$$\sum_k^{UV}\frac{1}{m_k} \geq \sum_k^{IR}\frac{1}{m_k}$$

Fermions: **Work in progress**

Future directions

- Heat kernel calculation for fermion, vector and Chern-Simon field
- Try to check the conjecture in some examples
- Can we apply Osborn's local RG formalism for a perturbative proof?
- Dispersion relations a la Komargodski and Schwimmer?
- Is there any interesting SUSY story, as in a-maximization ?
- Full classification of anomalies with more derivatives

Thank you