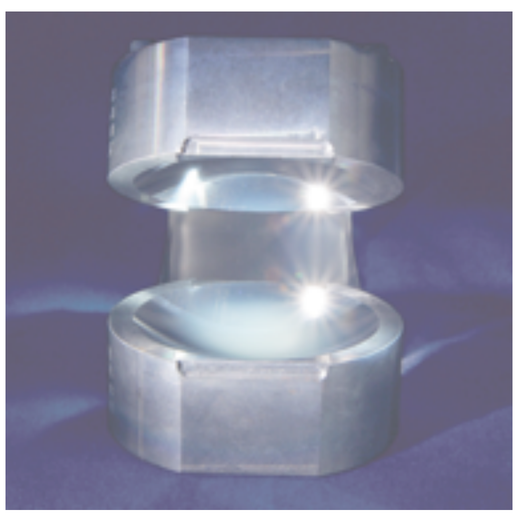
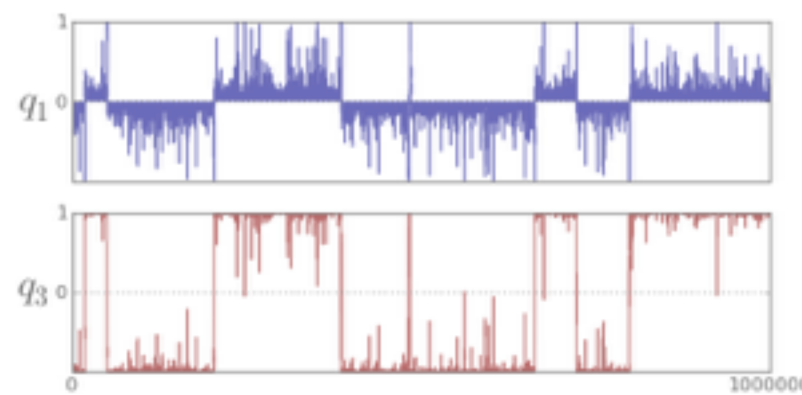
Statistical Aspects of Quantum State Monitoring

(A pinch of quantum mechanics, a drop of probability, ...)



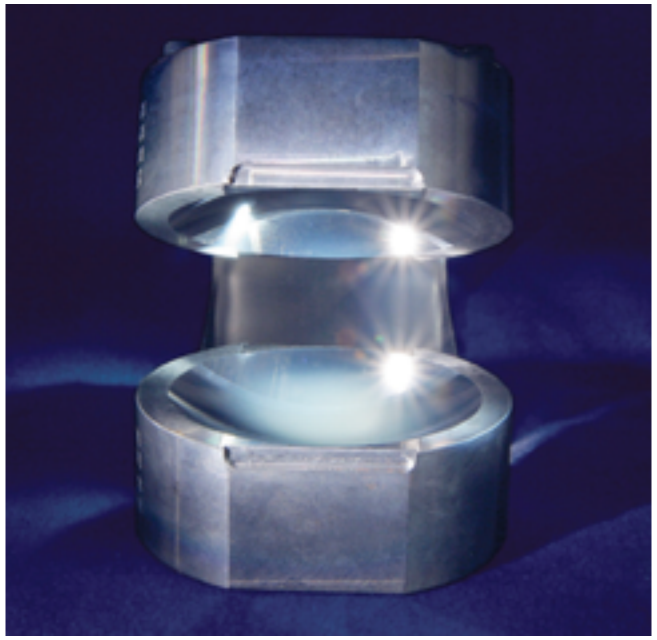


Statistics of « quantum trajectories », quantum jumps and spikes,… with applications.

D.B., with M. Bauer and A. Tilloy

Florence, Mai 2016

Two « typical » experiments (from Haroche's group)



Quantum jumps of light recording the birth and death of a photon in a cavity

Sébastien Gleyzes[1], Stefan Kuhr[1]†, Christine Guerlin[1], Julien Bernu[1], Samuel Deléglise[1], Ulrich Busk Hoff[1], Michel Brune[1], Jean-Michel Raimond[1] & Serge Haroche[1,2]

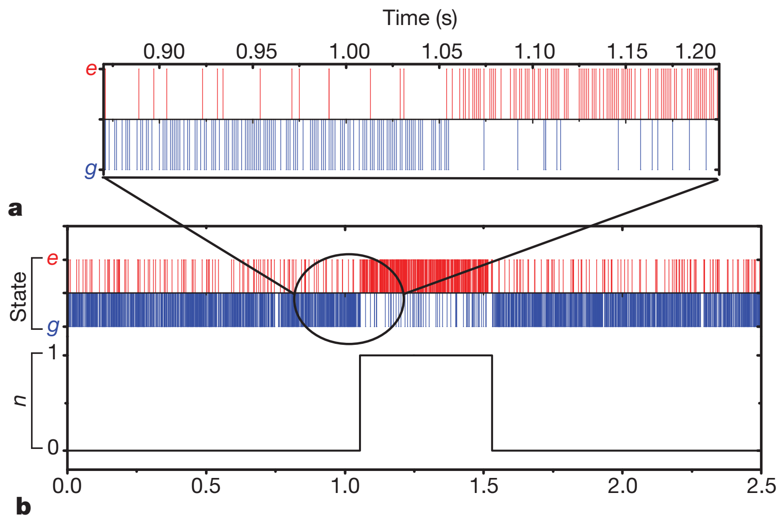


Figure 2 | Birth, life and death of a photon. a, QND detection of a single photon. Red and blue bars show the raw signal, a sequence of atoms detected in *e* or *g*, respectively (upper trace). The inset zooms into the region where the statistics of the detection events suddenly change, revealing the quantum jump from $|0\rangle$ to $|1\rangle$. The photon number inferred by a majority vote over

Courtesy of LKB-ENS.

Progressive field-state collapse and quantum non-demolition photon counting

Christine Guerlin[1], Julien Bernu[1], Samuel Deléglise[1], Clément Sayrin[1], Sébastien Gleyzes[1], Stefan Kuhr[1]†, Michel Brune[1], Jean-Michel Raimond[1] & Serge Haroche[1,2]

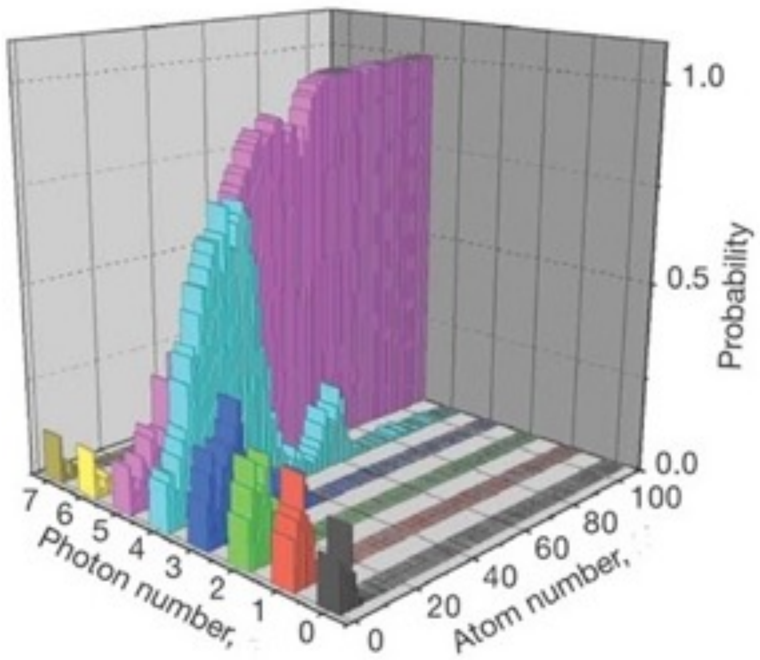
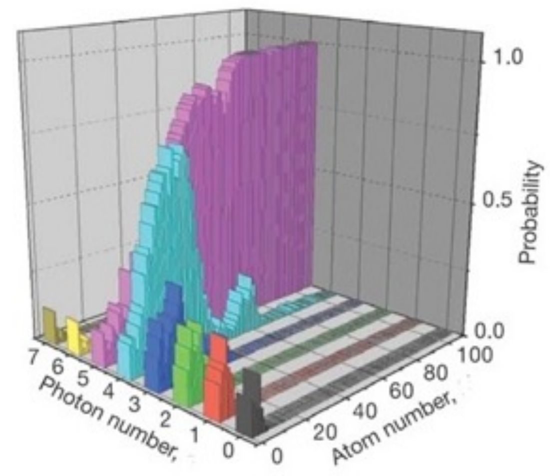


Figure 2 | Progressive collapse of field into photon number state. unity). **c**, Photon number probabilities plotted versus photon and atom numbers n and N. The histograms evolve, as N increases from 0 to 110, from a flat distribution into $n = 5$ and $n = 7$ peaks.

Courtesy of LKB-ENS.

Questions (hopefully motivating...)

-- *How to measure photons without destroying them?*
-- *How to record the cavity states?*
-- *How to extract/read the information?*
-- *What does this p.d.f. represent?*
-- *Why/How does it evolve? What about the quantum Zeno effect?*
-- *What does continuous-in-time quantum measurement mean?*
-- *How to use it? How to describe it? What are his characters?*
-- *etc…*



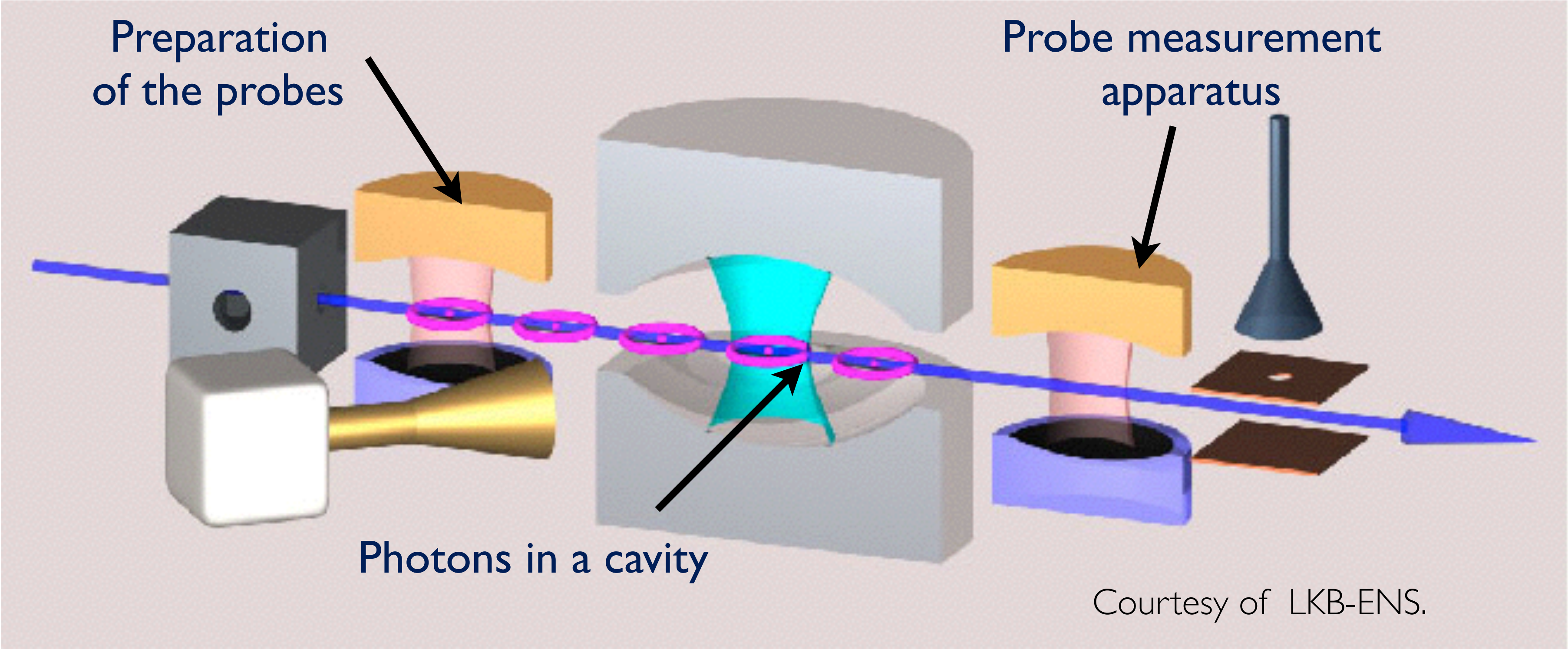
« Esquisse d'un plan »:

— **Quantum non-demolition measurements**
— **Quantum state monitoring**
— **Emergence of quantum jumps**
— **Beyond quantum jumps: quantum spikes.**

Cavity QED « non demolition » experiments.

— **Setup**: Testing light/photon (the quantum system) with matter (the quantum probes).

System (S) = photons in a cavity.
Probes (P) = Rydberg atoms (two state systems)



— **Indirect measurements**: => Direct (von Neumann) measurements on probes (ancillas)

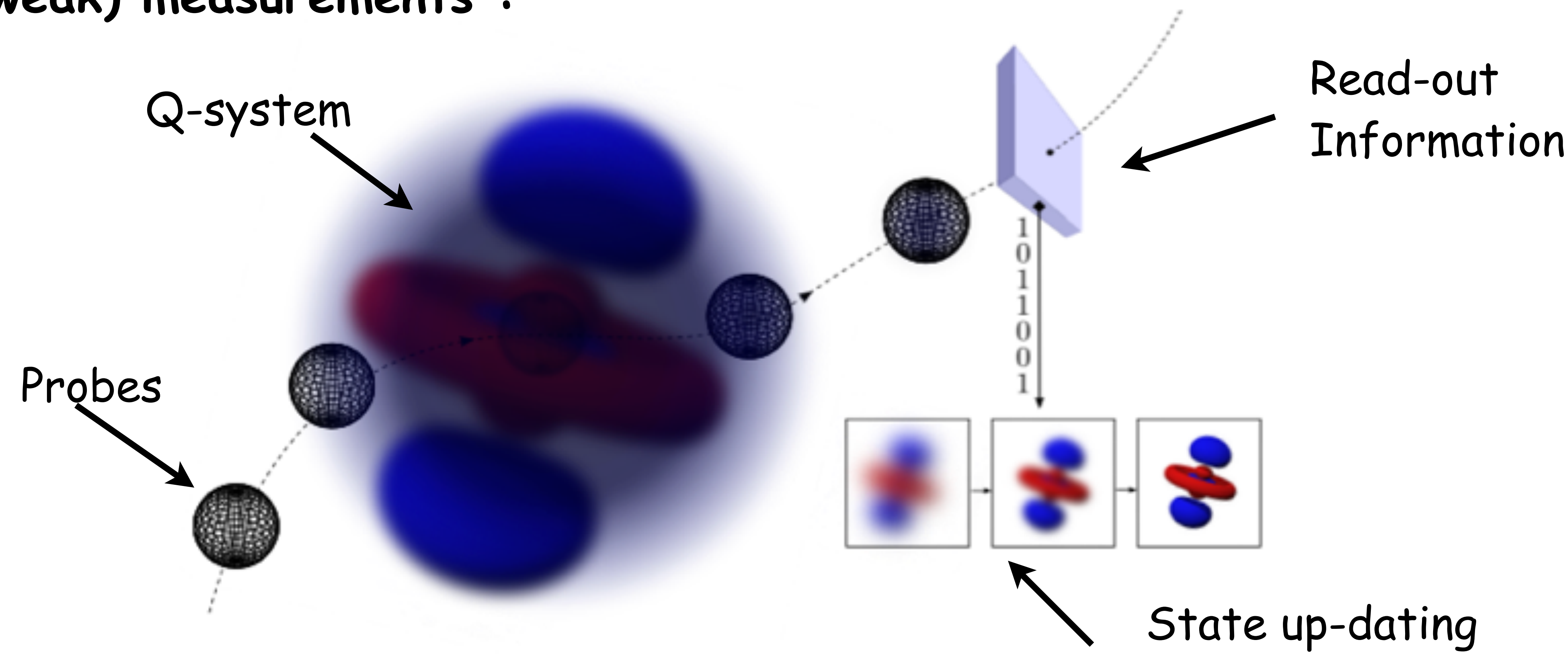
Displacement of the von Neumann cut.

Entanglement: => partial information on the system.

— In this setting: Effective rotation of the probe effective spins. $U = \exp\left[i\,\theta\,N_{\mathrm{photon}}\,\sigma^z\right]$

Indirect measurements and Quantum trajectories.

— **Indirect (weak) measurements :**



— **« Quantum trajectories » :** [Carmichael, Caves-Milburn, Castin-Dalibard-Molner, Zoller,….]

Information from probes measurements : $(s_1, \cdots, s_n, \cdots)$

System state up-dating :

$$\rho \rightarrow \frac{F_s\,\rho\,F_s^\dagger}{\pi(s)}$$ with probability $\pi(s) = \mathrm{Tr}(F_s\,\rho\,F_s^\dagger)$

(also random because of Q.M.) with $F_s := \langle s|U|\varphi\rangle$

Can this help understanding the progressive collapse (I) ?

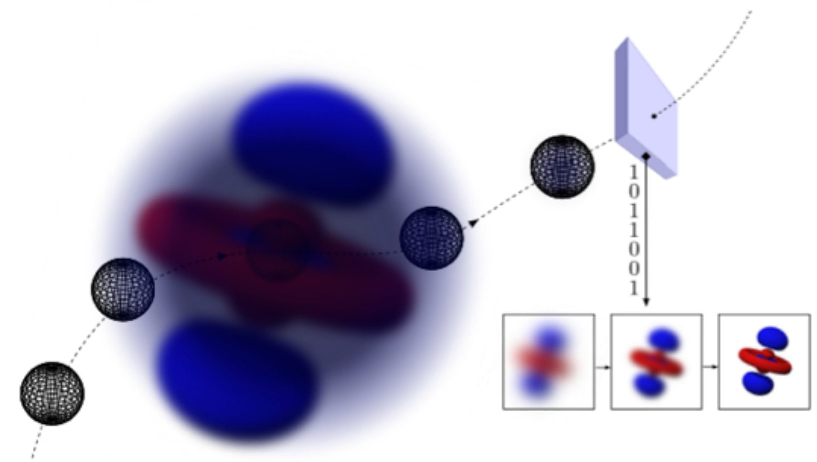
— **Non-demolition measurements:**
Suppose that the probe-system interaction preserves a basis of system states, alias « pointer states »:

$$U = \sum_k |k\rangle\langle k| \otimes U_k$$

pointer states

acting on probes

interaction



— **Evolution of the diagonal matrix elements**
of the system density matrix (in the pointer basis):

$$Q_n(k) := \rho_n(k,k)$$

$$Q_n(k) \to Q_{n+1}(k) = \frac{p(s|k)\,Q_n(k)}{\pi_n(s)}$$ with probability $$\pi_n(s) = \sum_k p(s|k)\,Q_n(k)$$

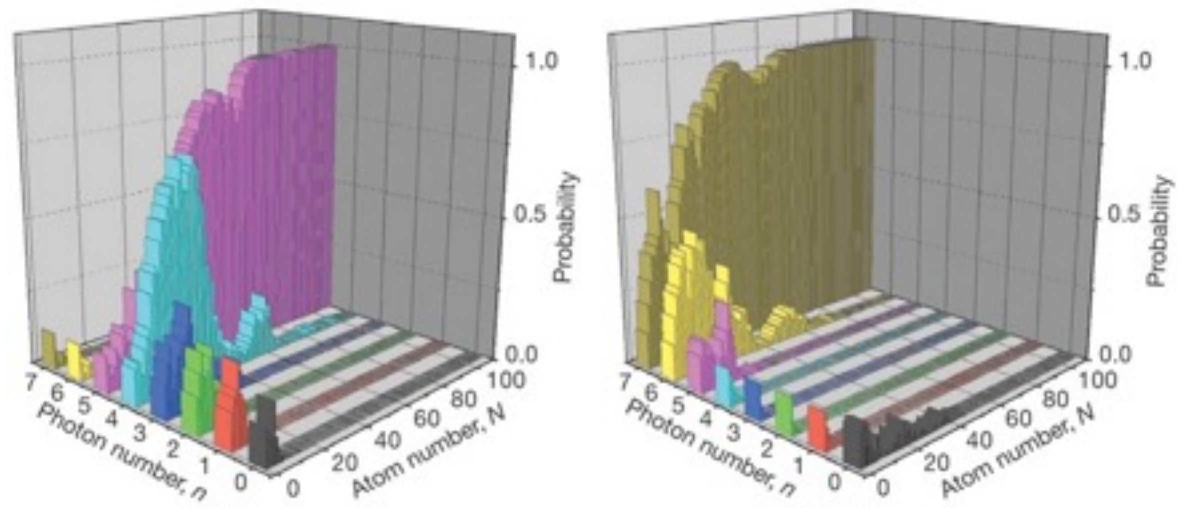
— Evolution (up-dating) deduced from Q.M. only.
It depends on the readout signal « s ».

Can this help understanding the progressive collapse (II) ?

— **Random evolution of the diagonal matrix elements** of the system density matrix (in the pointer basis):

$$Q_n(k) := \rho_n(k,k) \rightarrow Q_{n+1}(k)$$

- Different evolution for different read-outs.
- Convergence towards a peaked distribution.
- With random target (as expected).



— « **Convergence/Progressive collapse** »: [Bauer-Bernard]

- The sequences $Q_n(k)$ converge a.s. and in L1 (for any k).
- The limit distribution is peaked: $Q_\infty(k) = \lim_{n\to\infty} Q_n(k) = \delta_{k=k_\infty}$
- The target is distributed according to the initial distribution: $\mathbb{P}[k_\infty = p] = Q_0(p)$
- The convergence to the target is exponentially fast:

$$Q_n(k)/Q_n(k_\infty) \simeq \exp[-nS(k_\infty|k)]$$

with rates equal the relative entropies $S(k_\infty|k) = -\sum_s p(s|k_\infty) \log[\frac{p(s|k)}{p(s|k_\infty)}]$

— « Mesoscopic measurement apparatus » & Quantum-to-Classical transition.

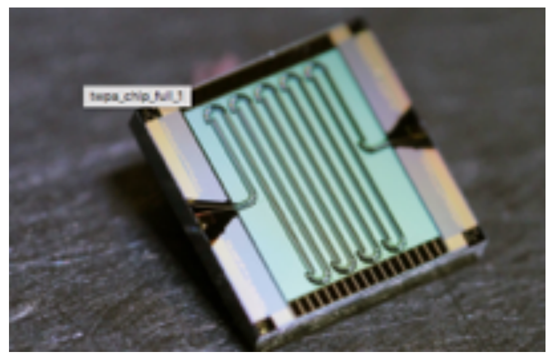
Monitoring Quantum systems… with applications.

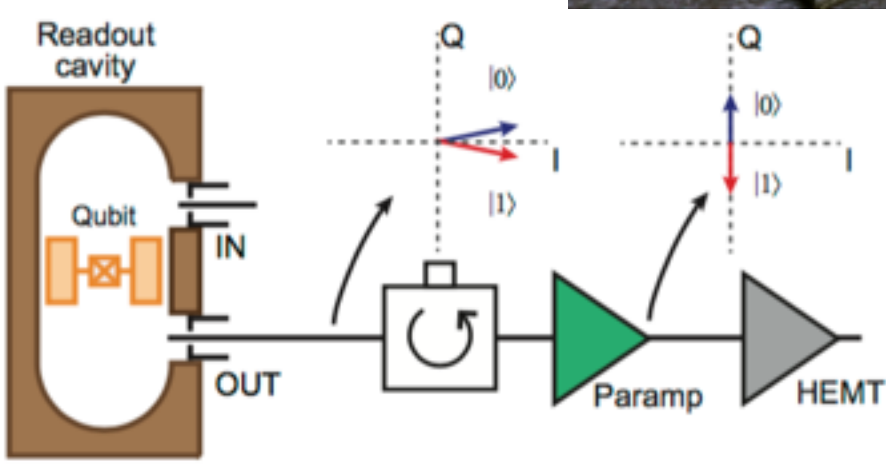
— **Monitoring:** Time continuous indirect (weak) measurements.
— **Ubiquitous** & key to **manipulate** and **control** quantum systems.

— In cavity QED (see above):

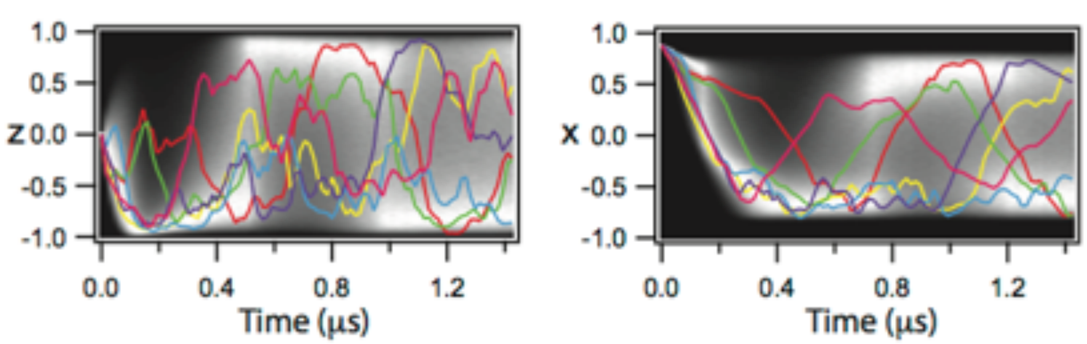
— In circuit QED :

I. Siddiqi's group @Berkeley

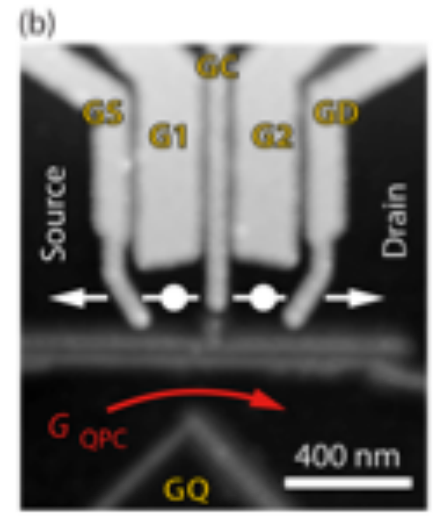




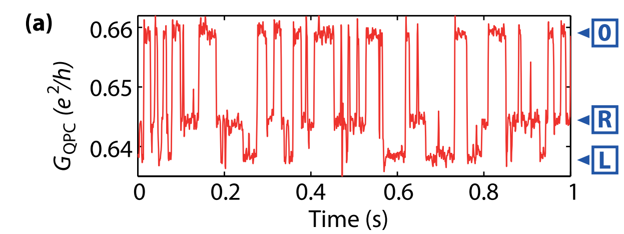
Q-bit QuantumTrajectories



— In quantum dots circuit:



ETH-group

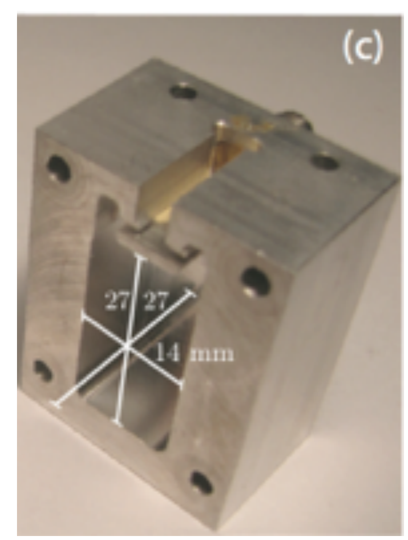


conductance measurement:

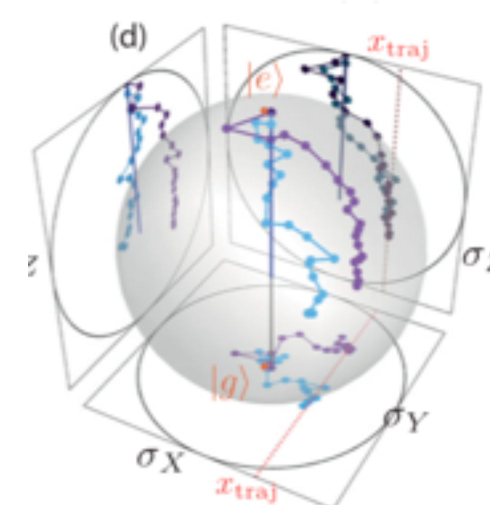
J. Appl. Phys 113, 136507 (2013)

— In circuit QED :

B. Huard's group @ ENS:



Control of
QuantumTrajectories
on the Bloch sphere

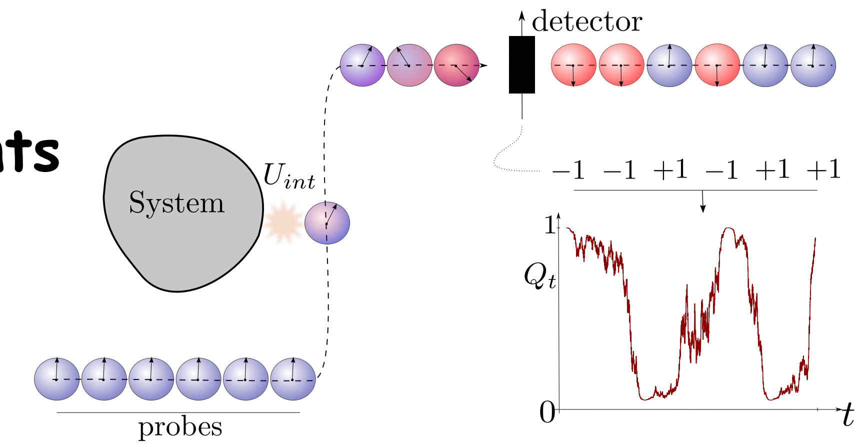


M. Devoret's group @ Yale; etc.

How to model continuous quantum measurements?

— Continuous version of **discrete repeated weak measurements** (with short time interval).

[Belavkin, Barchielli, Diosi, Milburn-Wiseman,…..]



— **Deterministic system evolution plus random evolution due to quantum measurement.**

$$d\rho = (d\rho)_{sys} + (d\rho)_{meas}, \quad \text{during time } dt$$

deterministic random (from Q.M.).

- The **first** term **is** a **deterministic**, unitary or dissipative, system evolution.
- The **second is** the **random** evolution (SDEs) due to the measurement back-action.

Their forms are explicitly known (no mystery).

Say: $$(d\rho)_{\rm meas} = L_{\rm meas}(\rho)\, dt + M_{\rm meas}(\rho)\, dB_t$$

(Brownian motion)

— **Same equation**, but with different interpretation (!), as `objective collapse models' of GRW-type. [Ghirardi-Rimini-Weber, Bassi,…]

Continuous measurement & emergence of quantum jumps.

– **Quantum jumps:** known to the father of quantum mechanics (e.g. N. Bohr) but…

E. Shroedinger: « If we are going to stick to this damned quantum-jumping, then I regret I ever had anything to do with quantum theory ».
N. Bohr's answer: « We are glad you did! ».

– **First observed** in atomic fluorescence in 1986 but any mesoscopic experiment since then.

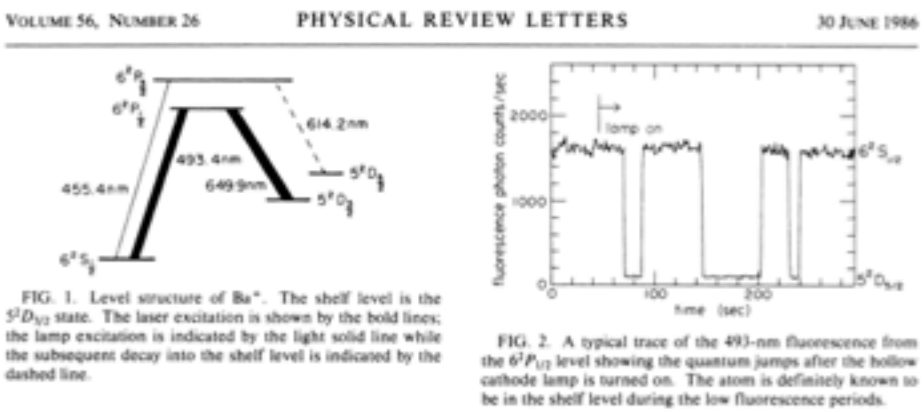

VOLUME 56, NUMBER 26 PHYSICAL REVIEW LETTERS 30 JUNE 1986

FIG. 1. Level structure of Ba^+. The shelf level is the $5^2D_{5/2}$ state. The laser excitation is shown by the bold lines; the lamp excitation is indicated by the light solid line while the subsequent decay into the shelf level is indicated by the dashed line.

FIG. 2. A typical trace of the 493-nm fluorescence from the $6^2P_{1/2}$ level showing the quantum jumps after the hollow cathode lamp is turned on. The atom is definitely known to be in the shelf level during the low fluorescence periods.

– **In continuous measurement:**
They **emerge from the competition** between the deterministic system evolution and the random evolution due to weak quantum measurement.

$$d\rho = (d\rho)_{sys} + (d\rho)_{meas},$$ -> They are generic.

They emerge… (from a diffusive behavior, they are not built in)… because we have moved the von Neumann cut… and they have a finer structure.

Emergence of quantum jumps (I).

— **Monitoring of a coherent Q-bit** (a two-state system) :

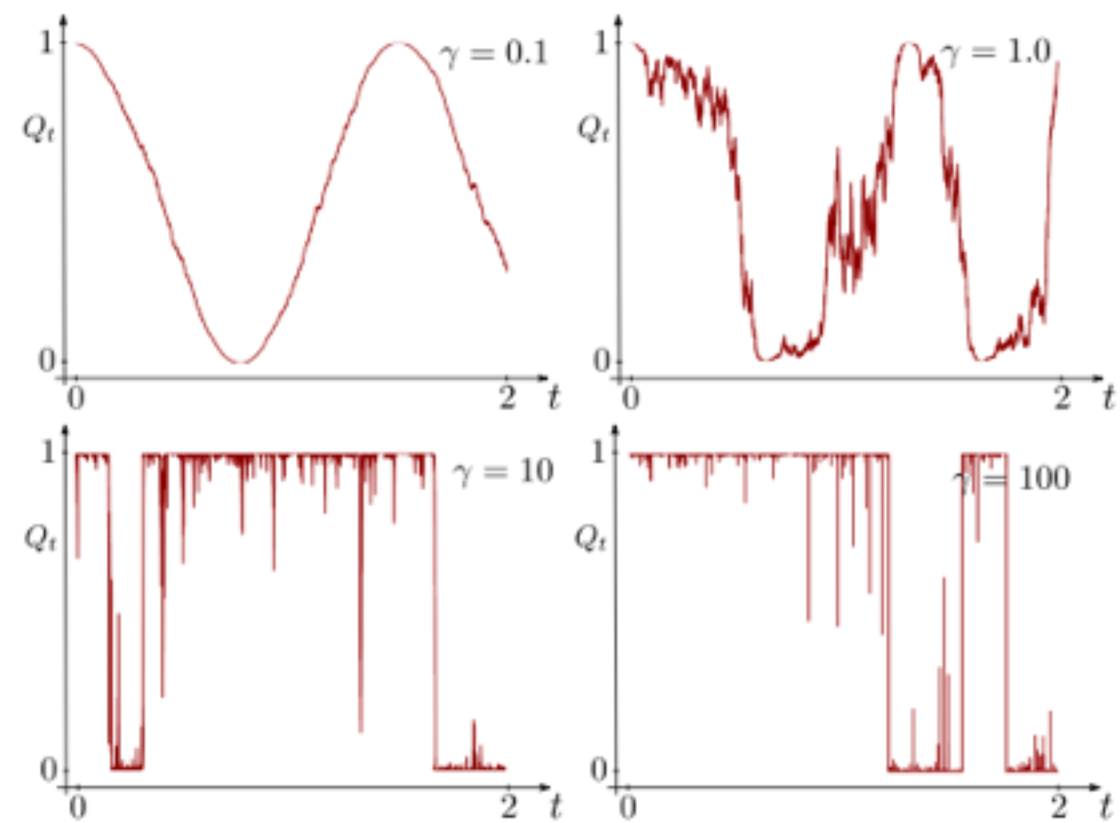
Monitoring an observable **not commuting with the hamiltonian** of a two-state system.

Hamiltonian: $H = \frac{\Omega}{2}\,\sigma_y$ Observable: $\mathcal{O} = \sigma_z$

Two processes are in competition: unitary evolution and measurement

Two time scales: $\tau_{\text{meas}} := \gamma^{-2}$ (Information rate)

$\tau_{\text{evol}} := \Omega^{-1}$ (Rabi frequency)



[... as gamma increases....]

— Evolution of Q with increasing information rate:

Let $Q_t := \langle + |_z\, \rho_t\, | + \rangle_z$

The system state stays pure... but jumps.

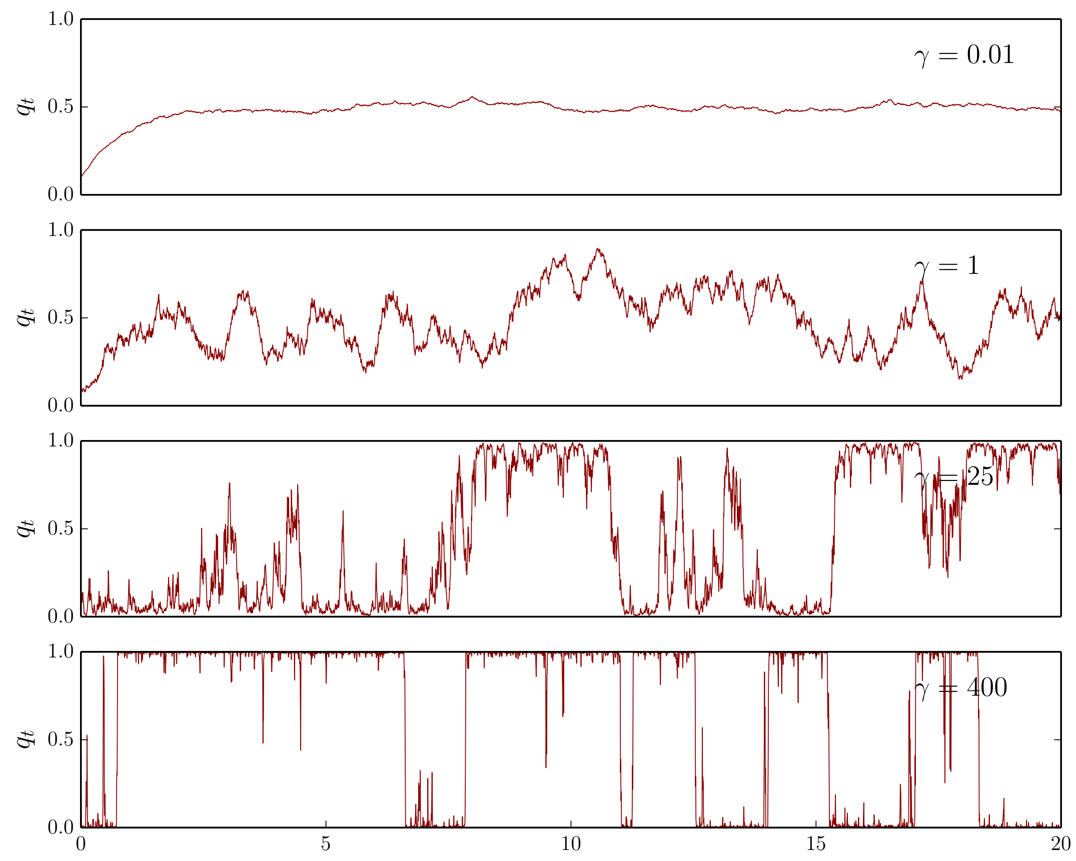
$\tau_{\text{flip}} = \tau_{\text{evol}}^2 / \tau_{\text{meas}} = (\gamma/\Omega)^2$ (Zeno freezing)

— Evolution equation:

$$d\rho_t = -i\frac{\Omega}{2}[\sigma_y, \rho_t]dt - \frac{\gamma^2}{2}[\sigma_z[\sigma_z, \rho_t]]dt + \gamma(\sigma_z\rho_t + \rho_t\sigma_z - 2tr(\sigma_z\rho_t))\, dB_t$$ (Brownian motion)

Emergence of quantum jumps (II).

— A Qu-bit, in contact with a thermal bath, **with its energy continuously monitored**.



[... as gamma increases....]

$$dQ_t = \lambda\,(p_{\rm eq} - Q_t)\,dt + \gamma\,Q_t(1-Q_t)\,dB_t$$

Q = population in lowest energy state.

$$\tau_{\rm flip} = \tau_{\rm thermal\ \ relax}$$

(no Zeno effect)

— **At strong measurement**, these processes converge (weakly) to **Markov chains**:

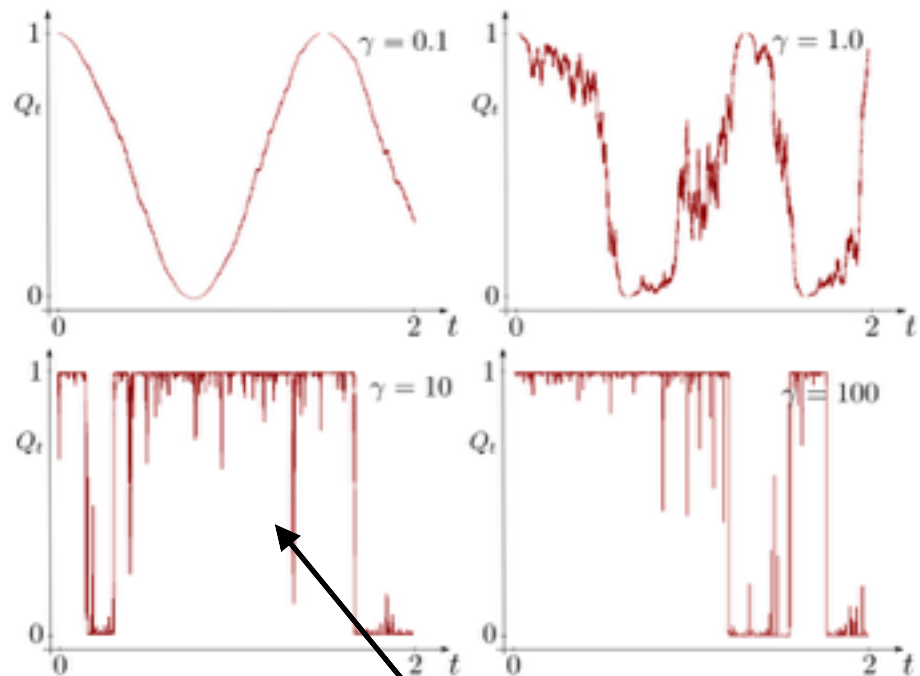
- All N-point functions converge to that of a specified Markov chain on the pointer states.
- The strong measurement limit is a strong noise limit.
- Jump rates are computable form the microscopic data

- If dissipative dynamics: jump rate is independent on the measurement strength.
- If unitary dynamics: jump rate is dependent on measurement strength (via Zeno freezing).

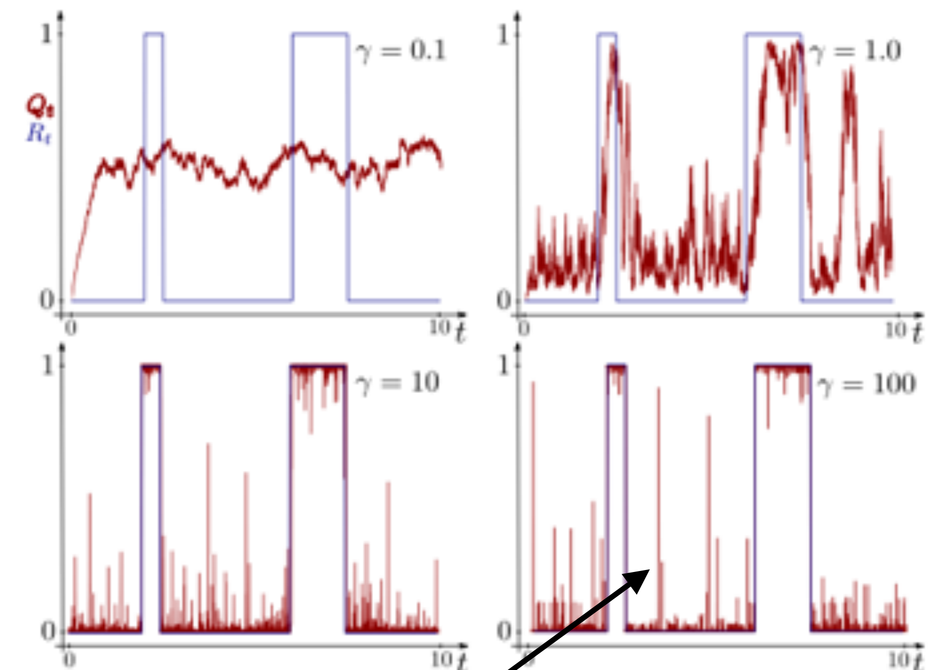
A finer structure: quantum spikes

— Let us look again at the quantum trajectories

For a coherent Qu-bit:



For a thermal Qu-bit:



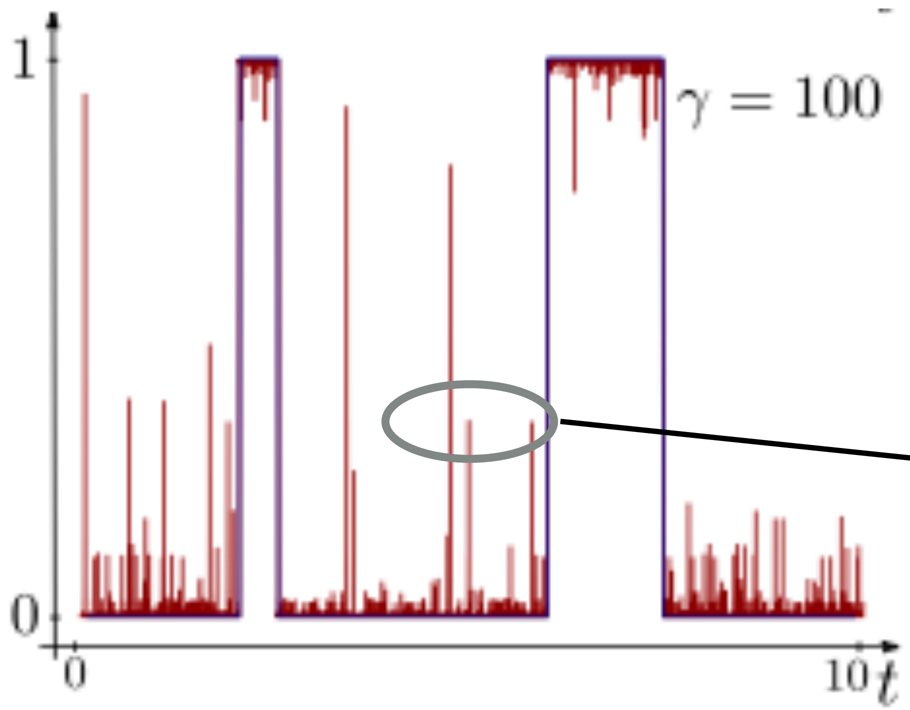
A finer structure survives, besides the jumps: the spikes.

- Spikes of height bigger a than a cutoff are countable.
- They have infinitesimal time duration, of order gamma^{-2}

— They have to be taken into account say when controlling Q-bits (as otherwise the `controller' may trigger to often).

Quantum spikes survive…

— **Spike fluctuations in the monitoring** survive at infinite information rate:
Jumps are always dressed with spikes.



— **Claim**: [Bauer-Bernard-Tilloy]

In the large information rate limit,
- Quantum jumps have a scale invariant statistics;
- They form a **Point Poisson Process** with

$$d\nu = \tilde{\lambda}\, \frac{dQ}{Q^2}\, dt$$

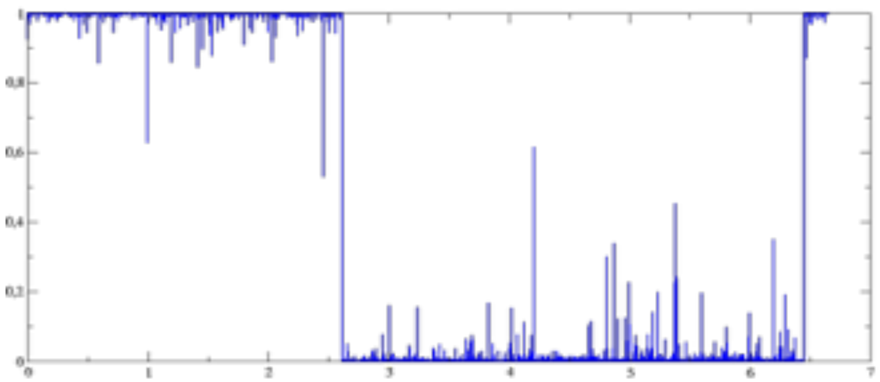
This is not in contradiction with the fact that all finite distributions of Qt converge to those of the jump Markov chain.

Internal structure of the spikes?...

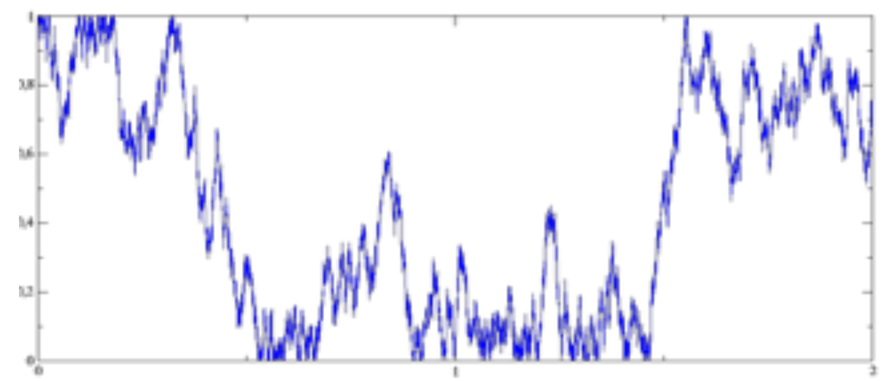
— Spikes are (almost) instantaneous (time scale of order γ^{-2})

—> Instead of the \`natural' time parametrization used a \`effective' time only sensible to state variation. Say:

$$\tau := \sum_n \mathrm{Tr}[(\delta\rho_n)^2] \quad \text{or} \quad \tau := \sum_n \mathrm{Tr}[(\delta\rho_n)^2_{\mathrm{diag}}]$$



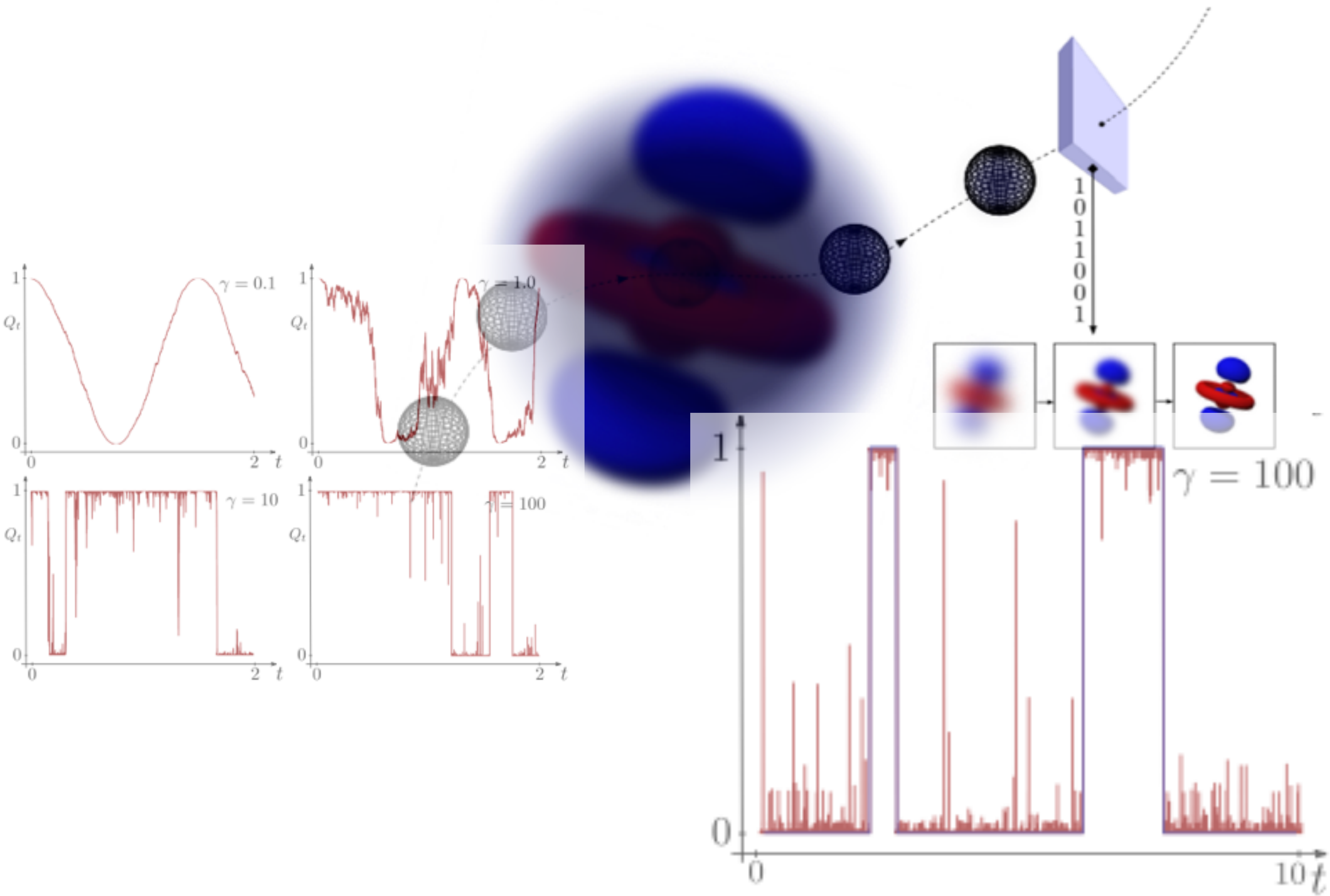
with natural time parametrization



parametrized by the quadratic variation

Claim: [Bauer-Bernard-Tilloy]
For a monitored Q-bit, quantum trajectories at strong measurement parametrized by their quadratic variation are **reflected Brownian samples**.

— **Can this be experimentally observed/verified ?......**



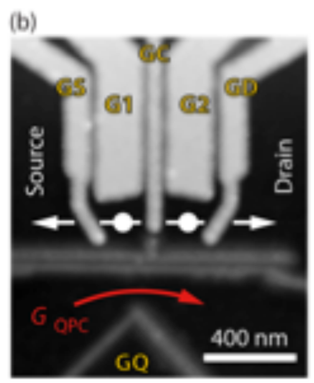
Thank you.

Application to control: a mesoscopic Maxwell deamon

— Possible **control of quantum system based on** the fact that dissipative and hamiltonian channels don't react the same to **the Zeno effect**:

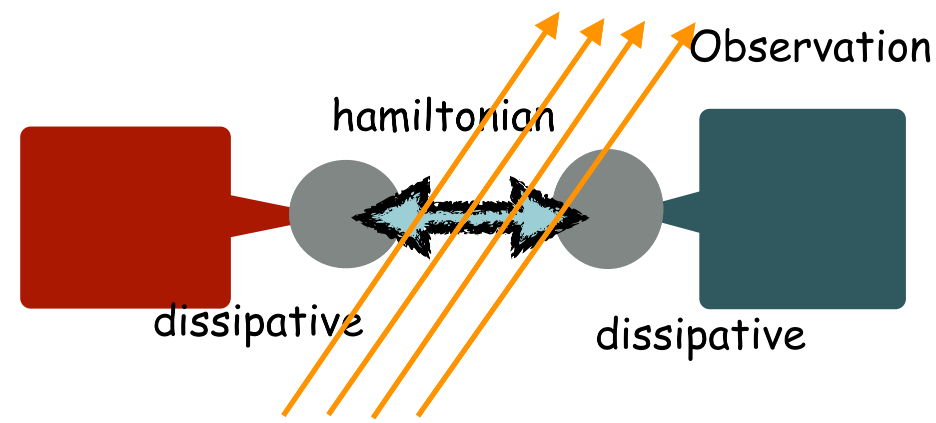
-> **adaptive measurement used to close hamiltonian channels but not dissipative ones.**

— Application to generate **quantum flux** in a double quantum dot geometry.



DQD occupation measurement via QPC conductivity.

<— a DQD,
its idealisation —>



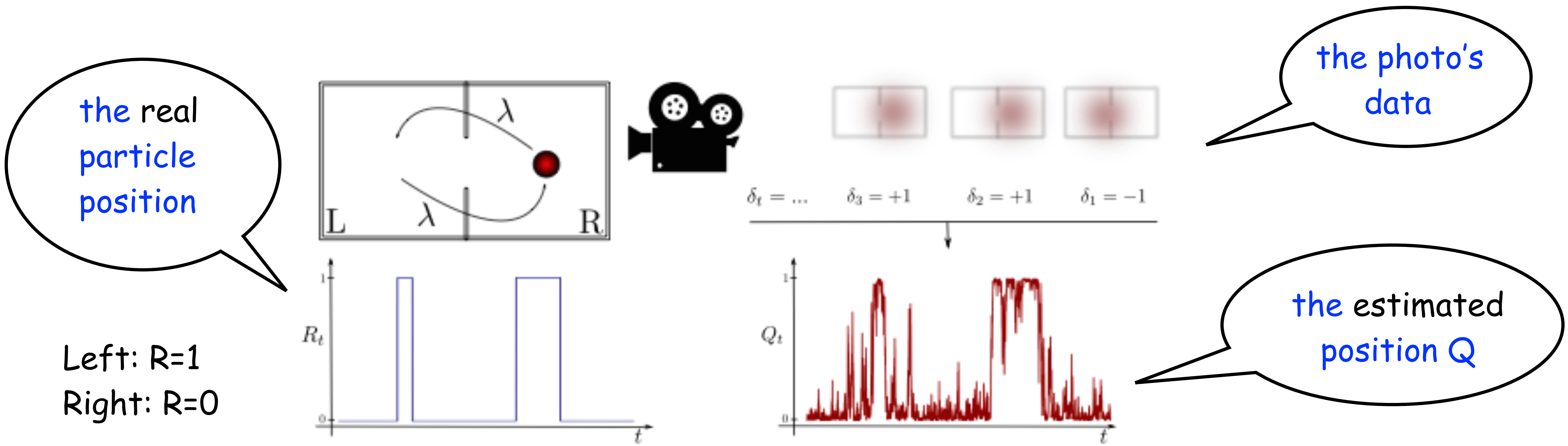
— By **changing the intensity of the observation depending on the information** on the electron position one has, we may control the electron flux.

For instance, we may measure more strongly when it is « known » that the electron is on the right dot and lightly when it is « believed » to be on the left —> **net flux from left to right**.

— Opening/closing the tunnel channel similarly as in **Maxwell daemon experiments**.

A Classical Toy Model: ‘Bayesian’ measurements

— Imagine a ‘classical’ particle in a box, with a probability to hope from left to right and back. One ‘**observes**’ the system by taking blurry photos and ‘**estimates**’ the particle position from the photos.



— Bad photos => some probability to have ‘(un)-correct’ information on the particle position:

$$\mathbb{P}(\delta = 1|\text{particle on the left}) = \frac{1+\epsilon}{2},\ \mathbb{P}(\delta = 1|\text{particle on the right}) = \frac{1-\epsilon}{2}$$

Epsilon codes how the value of delta is correlated to that of true position R.

— Estimated position (at time n given the past photo’s data):

$$Q_n := \mathbb{P}(\text{particle on the left at time } n \big| \text{pictures before } n)$$

How to code the jumps and spikes statistics? (II)

— The jumps and spike statistics can be computed from the SDE of the quantum trajectories, and these are not universal

$$d\rho_t = \big(i[H,\rho_t] + L_N(\rho_t)\big)dt + D_N(\rho_t)\,dB_t,$$

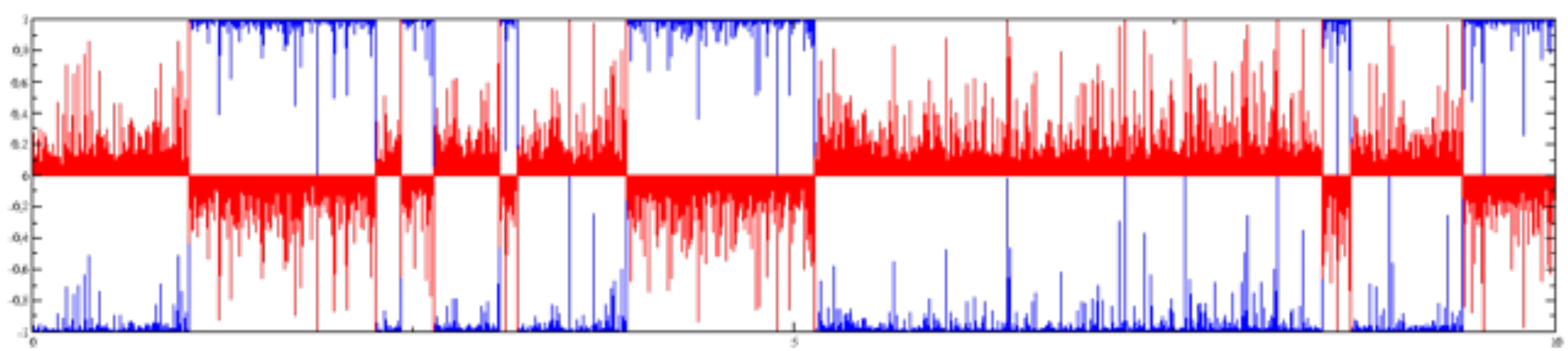
— But, asymptotically, the spike statistics is « **geometrical and universal** »:

> **Claim**: Let Q be the diagonal component of the density matrix.
> The **maximum (minima)** of Q on a **quantum trajectory** (at strong measurement) **form a point Poisson process with intensity**

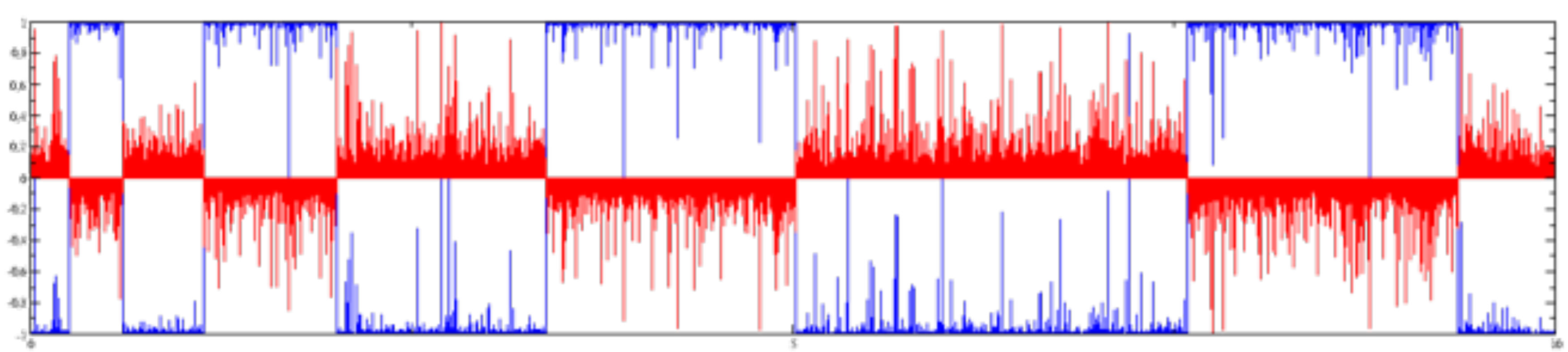
$$\boxed{d\nu = \lambda\,dt\,[\delta(1-Q)\,dQ + \frac{dQ}{Q^2}]}$$

— For spikes emerging from Q=0.

Reconstructed by solving the SDE



Reconstructed by using the Poisson point process



A finer structure: quantum spikes

Discrete:
system evolution + weak measurement at high frequency.

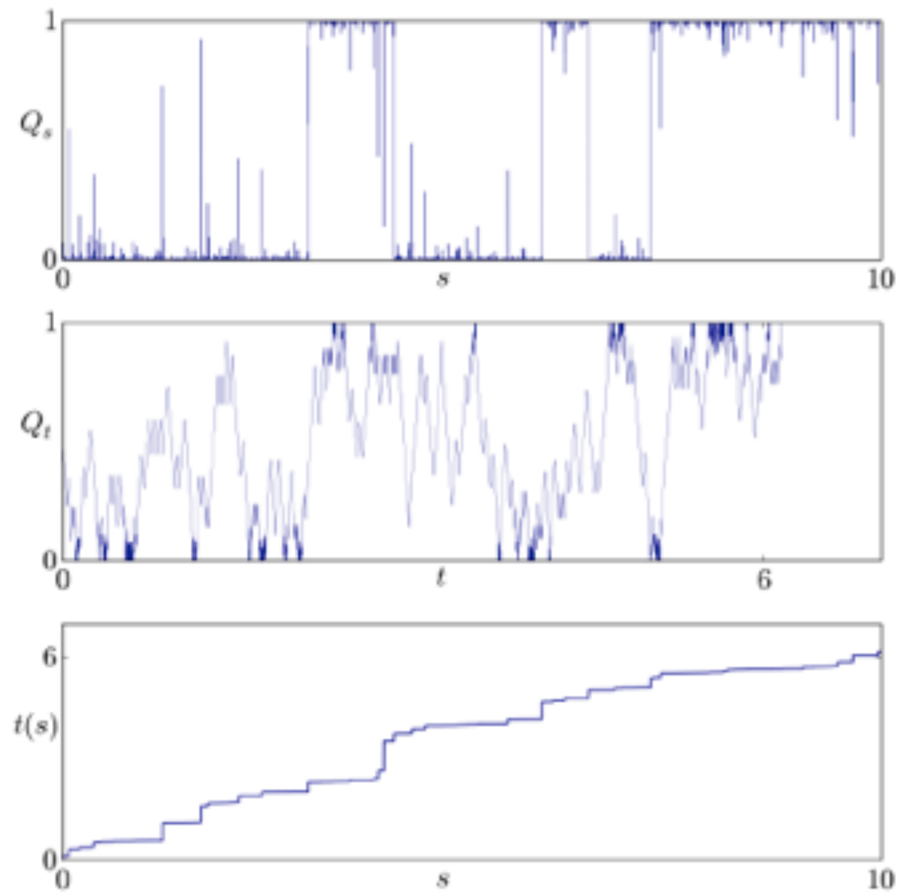


FIG. 1. *Discrete quantum trajectories in real and effective time.* Top: *The evolution of the ground state probability Q in real time s shows sharp jumps and spikes.* Center: *The evolution of Q in effective time t allows to resolve what happens outside the boundaries 0 and 1, the details are unfolded.* Bottom: *Effective time as a function of the real time. The plots are shown for the same realisation with $\varepsilon = 0.3$, $\Delta s = 10^{-5}$, $p = 0.5$ and $\lambda = 1$.*

Continuous:
system evolution + continuous monitoring at high rate.

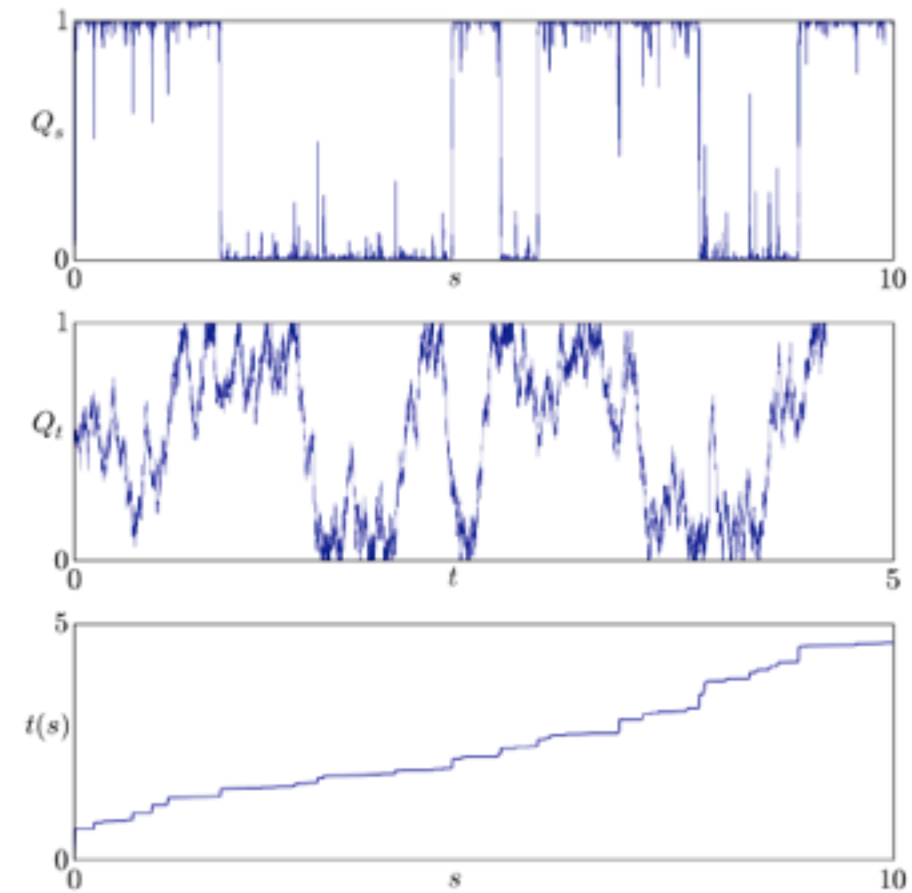
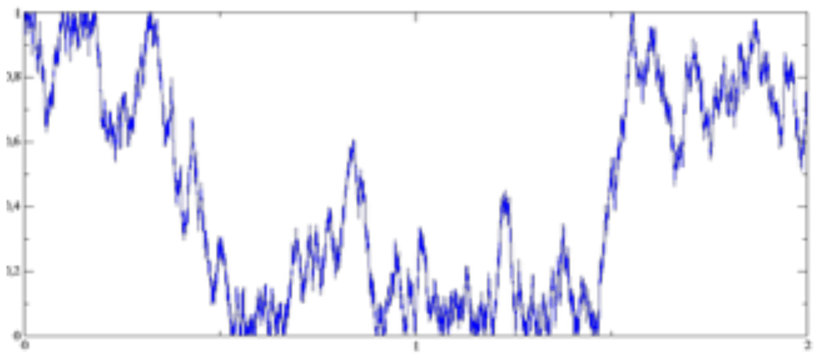


FIG. 2. *Continuous quantum trajectories in real and effective time.* Top: *The evolution of the ground state probability Q in real time s shows sharp jumps and spikes.* Center: *The evolution of Q in effective time t looks like a reflected Brownian motion without sharp transitions.* Bottom: *Effective time as a function of the real time. The plots are shown for the same realisation with $\gamma = 200$ (which looks like $\gamma \to +\infty$), $p = 0.5$ and $\lambda = 1$.*

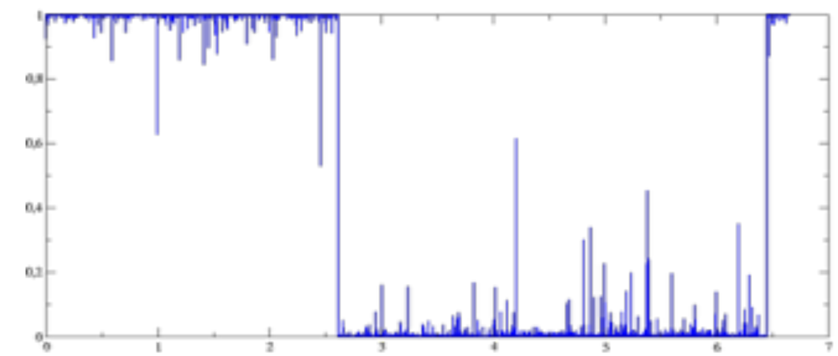
$$\tau := \sum_n \mathrm{Tr}[(\delta\rho_n)^2] \quad \text{or} \quad \tau := \sum_n \mathrm{Tr}[(\delta\rho_n)^2_{\mathrm{diag}}]$$

Internal structure of the spikes?...

— **A geometrical construction:**



A Brownian sample, reflected at 0 and 1, parametrized with its natural time.



The same example but parametrized by its local time spend at 0 and 1.

— **Claim:**

> *The quantum trajectories Q_t, for a Qu-bit in contact with a thermal bath, are equivalent in law to a Brownian motion $\hat{W}_\tau$ reflected at 0 and 1 but parametrized by its local times via the relation*
>
> $$t = \frac{1}{\lambda p} L_\tau^{(0)} + \frac{1}{\lambda(1-p)} L_\tau^{(1)},$$
>
> *with $L_\tau^{(0)}$ and $L_\tau^{(1)}$ the local times at 0 and 1 respectively.*

— **Hint for a proof:** Look again at the linear equation (close to Q=0): $dX_t = \lambda p dt + \gamma X_t dB_t$.

Change time (quadratic variation): new 'time' τ by $d\tau := \gamma^2 X_t^2 dt = (dX_t)^2$

Yield a new SDE $(Z_\tau = X_t)$ $$dZ_\tau = \frac{\lambda p}{\gamma^2 Z_\tau^2} d\tau + dW_\tau = \lambda p\, dt_\tau + dW_\tau$$

Matters only at Z=0-> reflection

a (new) Brownian motion

Z has to be a Brownian motion reflected a zero —> $$dZ_\tau = dL_\tau^{(0)} + dW_\tau$$ [Tanaka formula]

By identification: $dL_\tau^{(0)} = \lambda p\, dt$

Local time at 0