Black hole bound states in five dimensions

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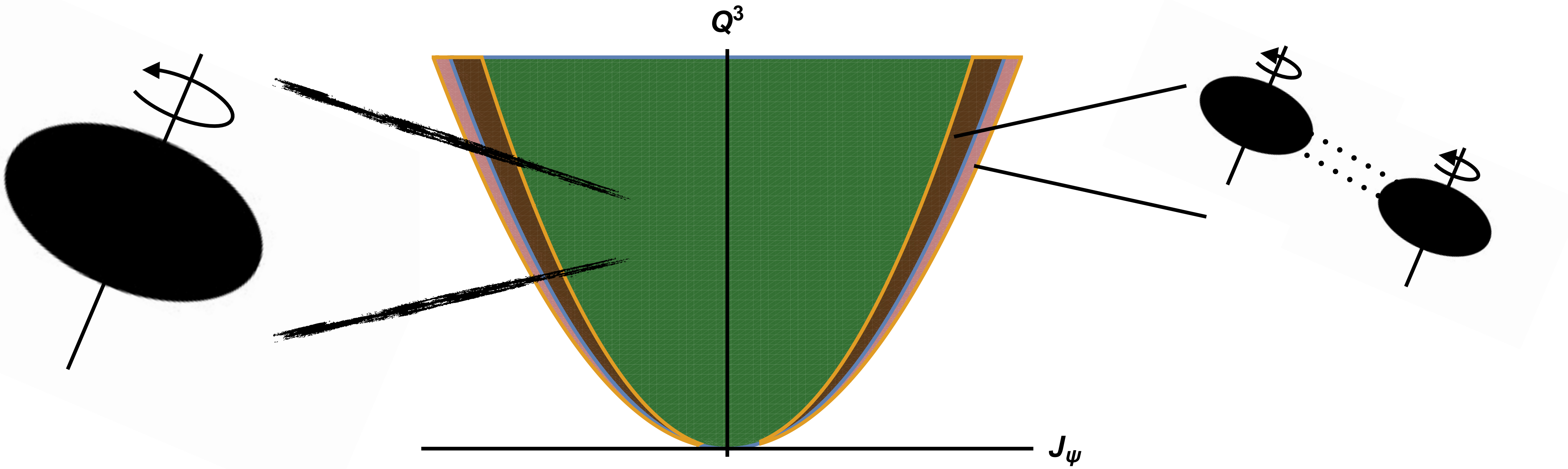
Based on arXiv:1603.09729 w/ **Marcos Crichigno, Stefan Vandoren**

Background & Motivations

- The zoo of 5d BPS black objects is far more populated than in 4d: rotating black holes, black rings, black lenses and bound states thereof…

- Can we give some order? E.g. pick the most entropic configuration for a given set of charges
- Can some other configuration 'dominate' over the single BH? Yes!

"Phase Diagram"



- 5d BPS black holes have a string theory (or M-theory or F-theory) realization.

- There is evidence for the existence of bound states of black holes with unusual entropy behavior coming from F-theory.

[Haghighat, Murthy, Vafa, Vandoren ’15]

- Are these states really there in classical, two-derivative, supergravity regime? Probably no!

How to build a multicenter BPS solution

Let's start from 6d, minimal supergravity

$$\mathrm{d}s_6^2 = -2H^{-1}(\mathrm{d}u+\beta)\big(\mathrm{d}v+\omega-\tfrac{1}{2}F(\mathrm{d}u+\beta)\big)+H\,\mathrm{d}s^2_{\mathrm{HK}_4}\,,$$

$$G_{(3)} = *G_{(3)} = \ldots$$

[Gutowski, Martelli, Reall '03]

- Nothing depends on 'v'. (BPS null Killing vector)
- H, F are functions of (u,x).
- β, ω are 1-forms on HK_4 (may depend on u)

u-independent solns are easily reduced to 5d:

$$\mathrm{d}s_5^2 = -f^2\,(\mathrm{d}t+\omega)^2+f^{-1}\mathrm{d}s^2_{\mathrm{HK}_4}\,,\qquad f^{-1}=\left(H^2F\right)^{1/3}\,,$$

$$e^{2\varphi} = H^{-1}F\,,$$

$$A = -F^{-1}\,(\mathrm{d}t+\omega)+\beta\,,$$

$$\widetilde{A} = -H^{-1}\,(\mathrm{d}t+\omega)+\gamma\,,$$

Nice class of solns when HK$_4$ = GH. (i.e., U(1) bundle over R^3.)

$$\mathrm{d}s^2_{\mathrm{GH}} = \frac{1}{H_2}\left(\mathrm{d}\psi + \chi\right)^2 + H_2\,\mathrm{d}s^2_{\mathbb{R}^3}\ ,$$

then the full soln can be written in terms of 6 harmonic functions on R^3, $H_{1,\ldots,6}$. They act as sources for the 1-forms

$$\omega = \hat{\omega}_i \mathrm{d}x^i + \omega_\psi(\mathrm{d}\psi + \chi)$$

$$*_3\mathrm{d}\chi = \mathrm{d}H_2 \qquad *_3\mathrm{d}\hat{\gamma} = -\mathrm{d}H_4$$

$$*_3\mathrm{d}\hat{\beta} = -\mathrm{d}H_3 \qquad *_3\mathrm{d}\hat{\omega} = \langle \mathbb{H}, \mathrm{d}\mathbb{H}\rangle$$

This defines a symplectic form on R^6.

These solutions have (at least) a $(U(1)_u\times)U(1)_\psi \times \mathbb{R}_t$ isometry.

The solution is completely specified by the residues and locations of the poles in the 6 harmonic functions

$$H_1 = \mu_\infty + \sum_a \frac{\mu_a}{|\vec{x}-\vec{x}_a|}\,, \qquad H_2 = m_\infty + \sum_a \frac{m_a}{|\vec{x}-\vec{x}_a|}\,, \qquad H_3 = q_\infty + \sum_a \frac{q_a}{|\vec{x}-\vec{x}_a|}\,,$$
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But the physical "gauge invariant" quantities are actually the following:

$$\tilde{Q}_a \equiv \mu_a + \frac{q_a p_a}{m_a}\,, \qquad Q_a \equiv n_a + \frac{p_a^2}{m_a}\,,$$
$$J_a^\psi \equiv j_a + \frac{q_a p_a^2}{m_a^2} + \frac{\mu_a p_a}{m_a} + \frac{q_a n_a}{2 m_a}$$

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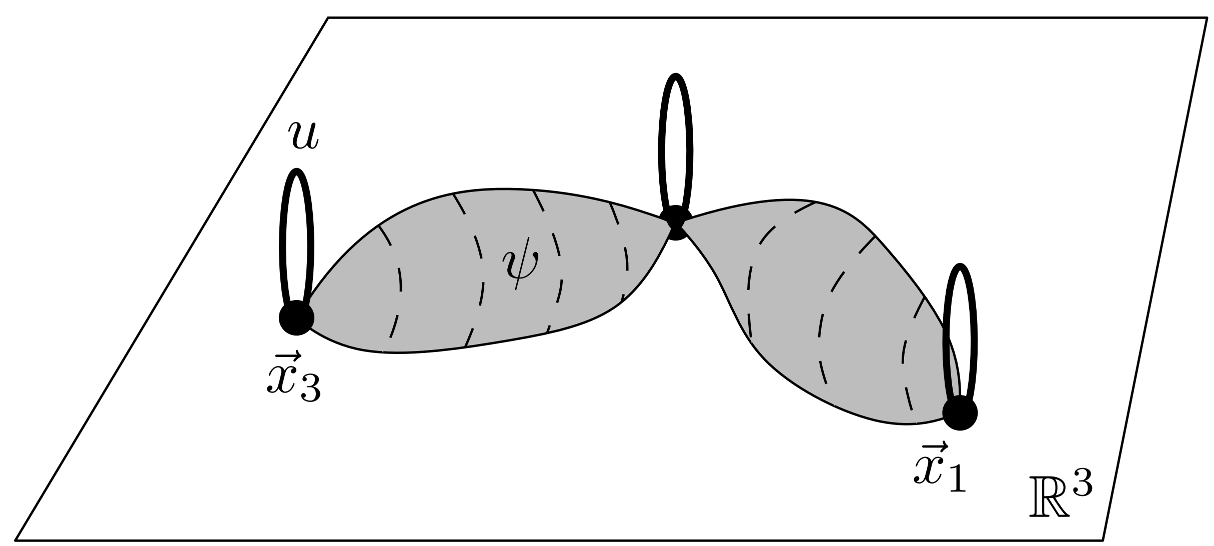
The 4d base has nontrivial homology if H_2 has more than one center.



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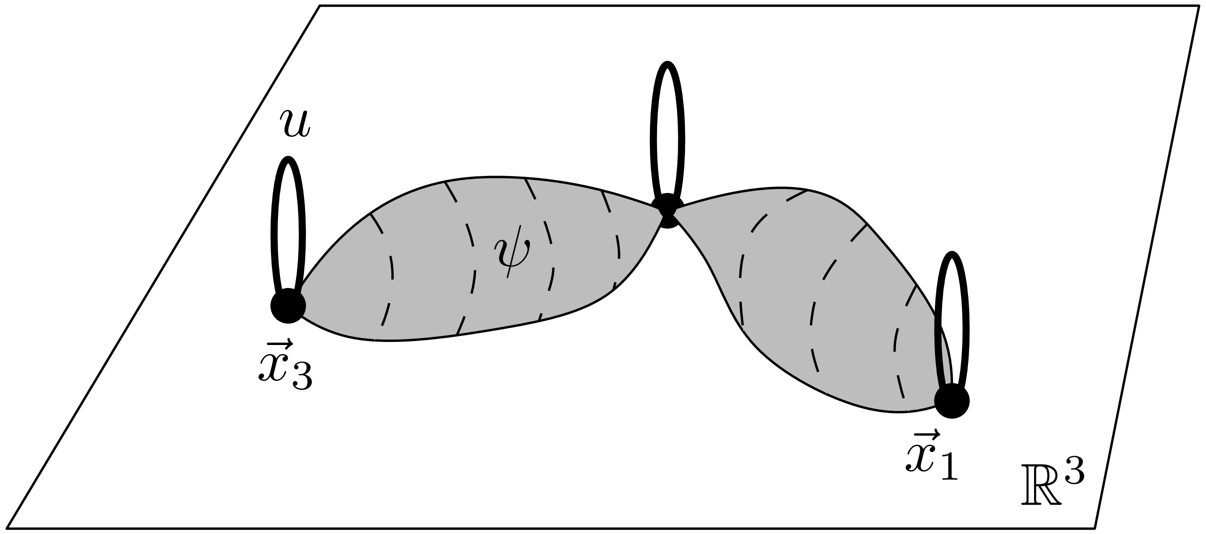

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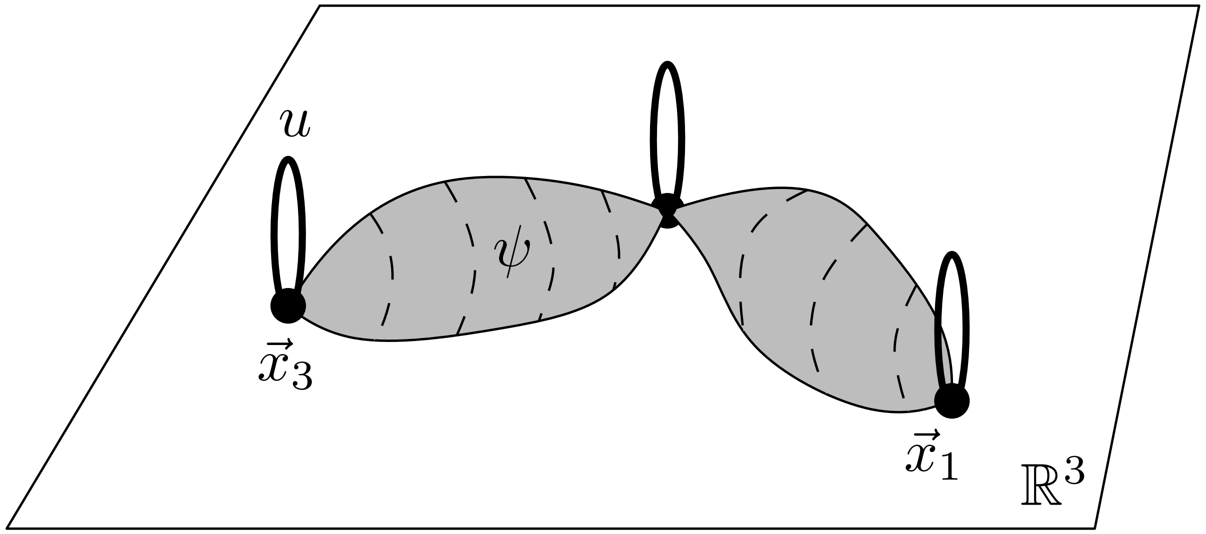

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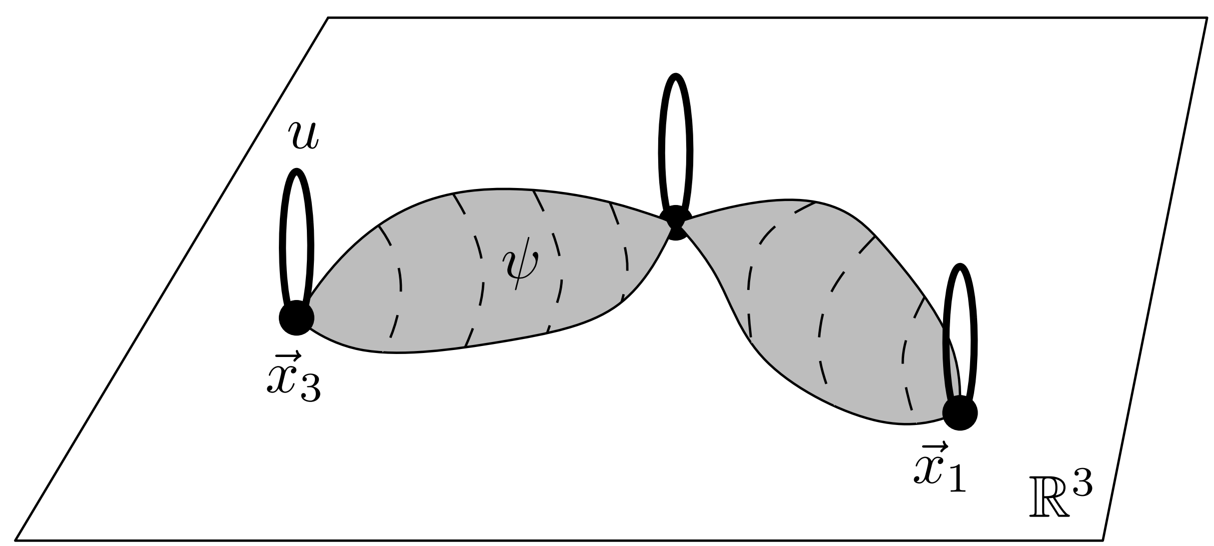

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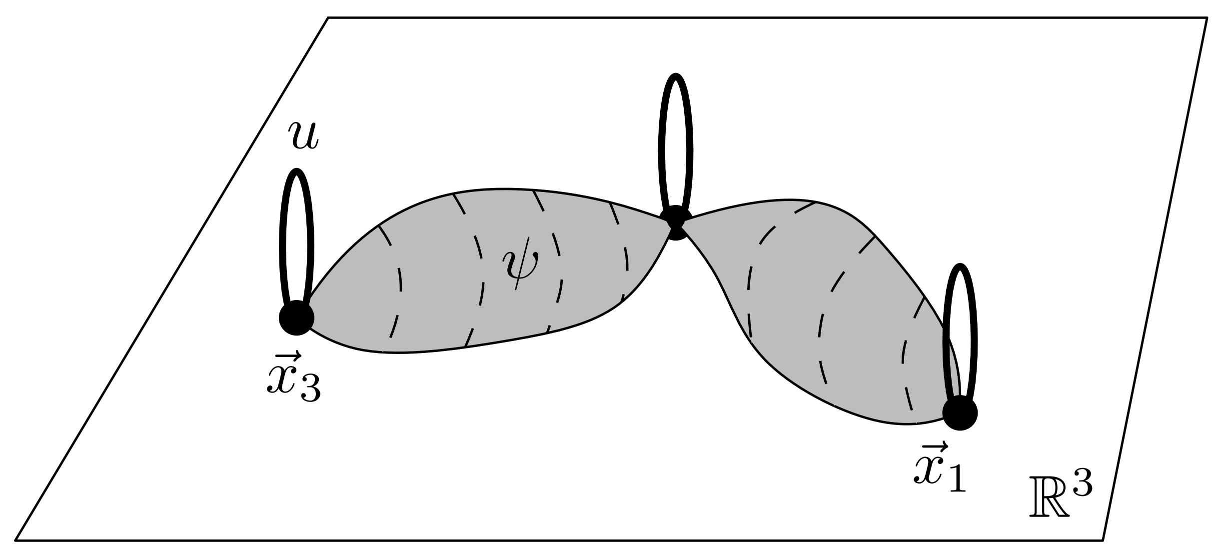

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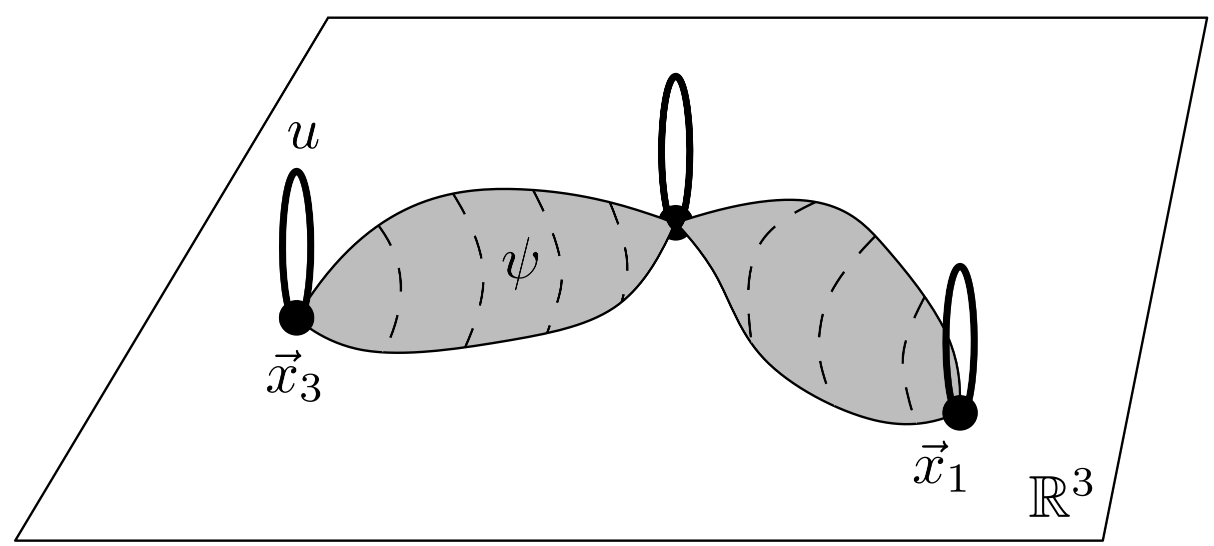

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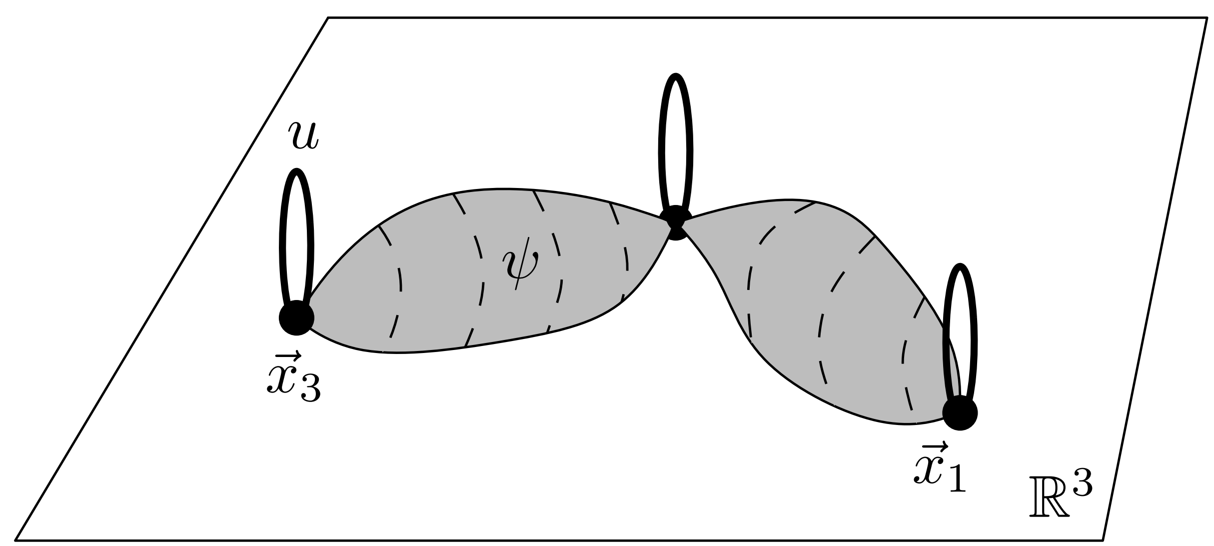


$$ds^2_{\rm GH} = \frac{1}{H_2}\left(d\psi + \chi\right)^2 + H_2\, ds^2_{\mathbb{R}^3}\,,$$

The data specifying the solution are contained in a bunch of charge vectors

$$\Gamma_\infty = \{\mu_\infty, m_\infty, q_\infty, p_\infty, n_\infty, j_\infty\} \qquad \Gamma_a = \{\mu_a, m_a, q_a, p_a, n_a, j_a\} \qquad \vec{x}_a$$

constrained by the following set of integrability conditions or "bubble equations"

$$*_3 \mathrm{d}\hat{\omega} = \langle \mathbb{H}, \mathrm{d}\mathbb{H} \rangle \qquad \sum_b \frac{\langle \Gamma_a, \Gamma_b \rangle}{|\vec{x}_a - \vec{x}_b|} = \langle \Gamma_\infty, \Gamma_a \rangle$$

Same symplectic form as before.

Fixes the distances in terms of the charges: when nontrivially satisfied it gives a bound state.

A pair of twin black holes

- This solution describes a bound state of two identical rotating black holes
- It secretly contains three centers. But the center in the middle is "smooth"

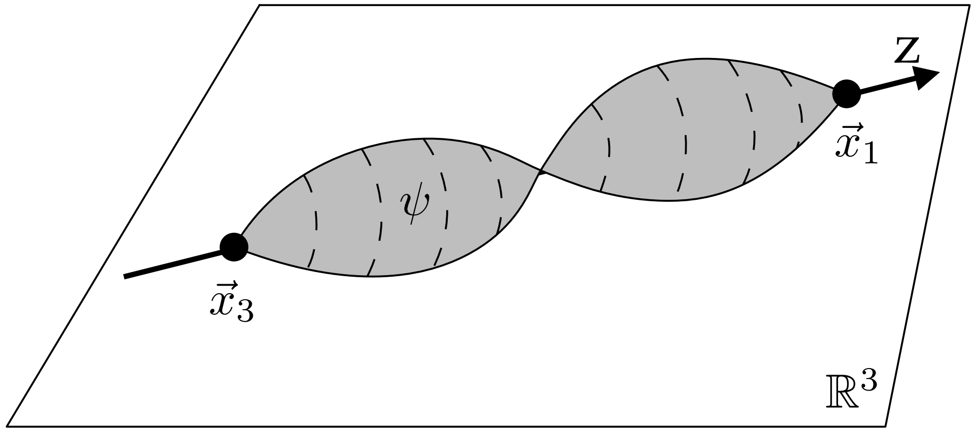
$$\vec{x}_a = \{0,0,-a\},\ \{0,0,0\},\ \{0,0,a\}$$

$$a = \frac{2j - nq + p^2 q - 2p\mu}{q + 2p}$$

$$\Gamma_1 = \Gamma_3 = \{\mu, 1, 0, 0, n, j\}$$

$$\Gamma_2 = \{qp, -1, q, p, p^2, \tfrac{qp^2}{2}\}$$

$$\Gamma_\infty = \{1, 0, 0, 0, 1, -(q+2p)\}$$



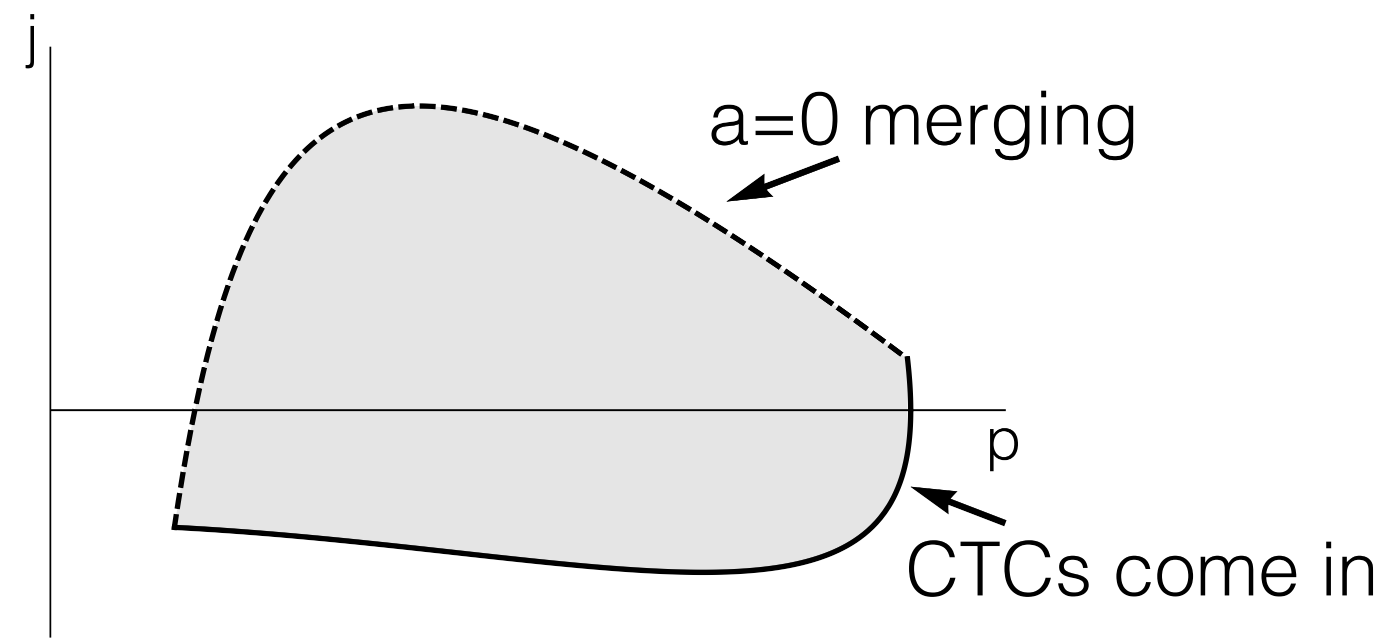
- The centers are aligned and the solution gains an extra $U(1)_\phi$ isometry

A 5 parameters family of solutions

$\{\mu, n, q, p, j\}$ or equivalently $\{Q, \tilde{Q}, J_\psi, p, j\}$

Restrictions on the parameter space come from absence of curvature singularities and closed timelike curves [CTCs].

Typical situation for fixed asymptotic charges.



Freaky facts about the Twin BH System:

- Bound by magnetic fluxes through nontrivial 2-cycles (balancing of "spin-spin" and "dipole-dipole" interactions).

- Exhibits frame dragging (also true for the single black hole).

- An evanescent ergosphere is present (not true for the single black hole) but neither of the black holes is enclosed by it!

Conclusions

- We found a family of regular BPS solutions describing a bound state of black holes in 5d.
- Its phase space overlaps with the single black hole one and in some subregion is even entropically dominant.
- The solution can be uplifted to 10d or 11d supergravity. The exact brane configuration is yet not known.
- This is not responsible for the exotic contributions to the elliptic genus I mentioned in the introduction.

Thanks!