Ultraviolet and Infrared Divergences in Superstring Theory

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1. Summary of this talk

2. Review of ultraviolet (UV) and infrared (IR) divergences in quantum field theory (QFT)

3. Absence of UV divergence in superstring theory

4. Recent progress on understanding IR divergences in superstring theory

1. Summary of the talk

QFT's are the standard tools for describing the physics of elementary particles.

– a tool for computing physical quantities, e.g. scattering amplitudes of elementary particles.

But most QFT's suffer from UV and IR divergences

– infinities that appear in the expressions for various physical quantities – unless we are careful.

UV divergences arise from quantum fluctuations of small wavelength modes, and are 'bad'

– must be eliminated in order to get a sensible theory.

There is a class of QFT's where UV divergences can be removed by a standard procedure known as renormalization.

– renormalizable QFT.

We use only these kinds of QFT's for describing theories of elementary particles.

IR divergences arise from quantum fluctuations of long wavelength modes and have physical origin

– indicate that we are asking the wrong question.

e.g. they arise when we do not take into account the effect of change, due to interaction, of

quantum ground state and/or

masses of elementary particles.

$\Rightarrow$ tadpole divergences and mass renormalization divergences.

QFT's come with an in built mechanism that tells us how to ask the right questions and get rid of the IR divergences.

Gravity

General theory of relativity $\Rightarrow$ classical gravity.

Applying standard QFT techniques to general theory of relativity runs into difficulties with UV divergence.

The theory is not renormalizable.

Superstring theory resolves this problem by regarding the elementary constituents of matter as one dimensional objects – strings.

This theory contains gravity and no UV divergences!

There is no need for renormalization.

However superstring theory has IR divergences similar to those which appear in QFT's.

Since IR divergences in QFT's disappear once we ask the right questions, one might expect that the same may be true in superstring theory.

However conventional formulation of superstring theory does not tell us how to ask the right questions so that we get finite answers.

e.g. no systematic procedure for taking into account the effect of change of the quantum ground state and/or masses of 'elementary particles' due to interaction.

Various indirect methods have been suggested for dealing with this issue.

None of them lead to a fully systematic algorithm for dealing with all IR divergences.

In most computations in string theory this issue is avoided by working with

– ground states which are not changed by interactions

– elementary particles whose masses are not modified by interactions.

Recent progress

Construction of a quantum field theory whose scattering amplitudes agree with that of superstring theory.

– superstring field theory

This theory is free from UV divergences but has all the IR divergences of superstring theory.

However, since this is a QFT, there is a systematic procedure for removing IR divergences by 'asking the right questions'.

$\Rightarrow$ results free from IR divergences.

Conclusion

We now have a formulation of superstring theory that gives results free from UV and IR divergences.

2. UV and IR divergences in QFT

Most commonly used approach for studying scattering amplitude in QFT's is perturbation theory.

Take all the interaction effects to be small and carry out a Taylor series expansion in the parameters that label the interaction strengths.

The coefficients of the Taylor series expansion are given by sum of Feynman diagrams.

In d space-time dimensions, ‘g-loop contribution’ from a typical Feynman diagram looks like

$$\int d^d\ell_1 \cdots d^d\ell_g \prod_{j=1}^{r} (k_j^2 + m_j^2)^{-1} \mathcal{N}$$

each ℓ_i: a d-dimensional vector labelling loop momenta

each k_j: a d-dimensional vector given by appropriate linear combination of the ℓ_i's and $p_1, \cdots p_n$

$p_1, \cdots p_n$: the momenta carried by the incoming and outgoing particles whose scattering amplitude we are trying to calculate

m_j: the mass of one of the particles in the theory

$\mathcal{N}$: a polynomial in $\{\ell_i\}$ and $\{p_k\}$

$$\int \mathbf{d^d}\ell_1 \cdots \mathbf{d^d}\ell_\mathbf{g} \prod_{\mathbf{j=1}}^{\mathbf{r}} (\mathbf{k_j^2} + \mathbf{m_j^2})^{-1} \mathcal{N}$$

UV divergences: divergences from the region of integration where one or more of the $\ell_\mathbf{i}$'s become large

IR divergences: arise from the vanishing of one or more factors of $(\mathbf{k_j^2} + \mathbf{m_j^2})$

$$\int d^d\ell_1 \cdots d^d\ell_g \prod_{j=1}^{r} (k_j^2 + m_j^2)^{-1} \mathcal{N}$$

1. Use $(k_j^2 + m_j^2)^{-1} = \int_0^\infty ds_j \exp[-s_j(k_j^2 + m_j^2)]$

2. Carry out integration over ℓ_j's explicitly using rules of gaussian integration

Result

$$\int_0^\infty ds_1 \cdots \int_0^\infty ds_r \, F(\{s_i\})$$

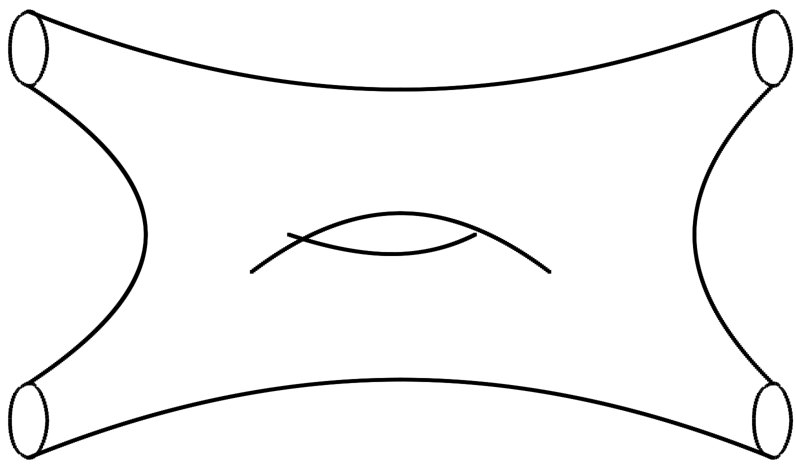
for some function $F(\{s_i\})$.

UV divergence: one or more $s_i \to 0$

IR divergence: one or more $s_i \to \infty$

3. Absence of UV divergence in superstring theory

Just as a particle trajectory gives a curve in space-time, the trajectory of a string gives a surface in space-time.



$\Rightarrow$ simple expression for scattering amplitudes g-loop scattering amplitude with n external states:

$$\int dm_1 \cdots dm_{6g-6+2n}\ \mathcal{I}_{g,n}$$

$\{m_i\}$: variables labelling different two dimensional Riemann surfaces of genus g with n marked points

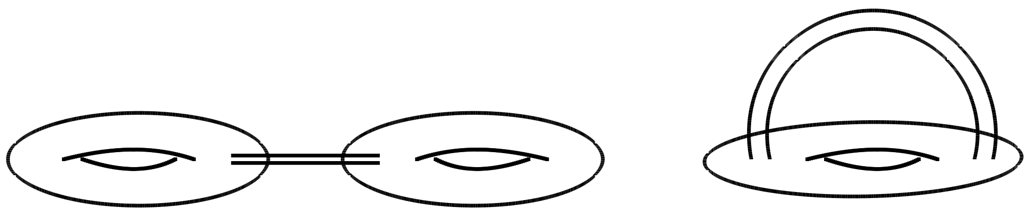
genus g: number of handles of the surface

Different values of $\{m_i\}$: Surfaces of different shape, each of genus g and with n marked points

Integrand $\mathcal{I}_{g,n}$: depends on the states that are being scattered and also the variables $\{m_i\}$

Possible divergences now come from divergences in the integration over $\{m_i\}$

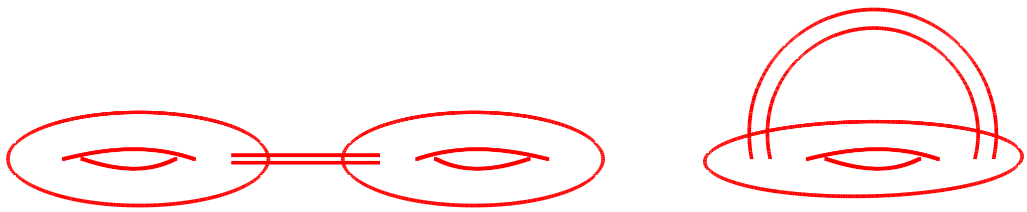
– arise from singular Riemann surfaces



(a) (b)

– the Riemann surface either becomes a pair of Riemann surfaces connected by an infinitely narrow tube (a)

or develops an infinitely narrow handle connecting two regions of a single Riemann surface (b)



(a) (b)

In this limit the integration over $\{m_i\}$ resembles integration over the parameters s_i in the QFT's with

$$s_i \sim 1 \, / \, \text{radius of the narrow tube}$$

In the singular limit, radius of the tube $\to 0$

$s_i \to \infty$

– IR divergence

This shows that all divergences in string theory are IR divergence and there are no UV divergences in the theory.

However unlike in a QFT, conventional superstring perturbation theory does not give us a systematic mechanism for removing IR divergences.

4. Recent progress on understanding IR divergences in superstring theory

If we could construct a QFT whose scattering amplitudes give us the amplitudes of superstring theory, then we would have a systematic procedure for removing IR divergences in string theory.

– had been attempted earlier

– successfully formulated for a cousin of superstring theory – the bosonic string theory. Witten; Zwiebach; · · ·

For superstrings there is an apparent no go theorem.

Low energy limit of a superstring theory gives type IIB supergravity for which we cannot write down a Lagrangian or an action.

Resolution

It is possible to construct a QFT that gives the correct scattering amplitudes of string theory, but contains an additional set of particles which are free.

These additional particles are unobservable since they do not scatter.

Scattering amplitude for the interacting part is given by a sum of Feynman diagrams as in conventional QFT's.

Each Feynman diagram gives integration over a part of the space spanned by $\{m_i\}$, and the sum of all contributions gives integral over the full space.

All IR divergences come from $s \to \infty$ limit for one or more propagators as in conventional QFT's.

On the other hand this theory has no UV divergence since its scattering amplitudes are the same as that of string theory.

With the help of this theory one can successfully remove the IR divergences of the theory following the usual procedure followed in a QFT

– gives a formulation of string theory free from all divergences.

Structure of the action

Two sets of string fields, ψ and ϕ

Each is an infinite component field, represented as a vector

Action takes the form

$$\mathbf{S} = \left[-\frac{1}{2}(\phi, \mathbf{QX}\phi) + (\phi, \mathbf{Q}\psi) + \mathbf{f}(\psi) \right]$$

Q, X: commuting linear operators (matrix with differential operators as entries)

(,): Lorentz invariant inner product

f(ψ): a functional of ψ describing interaction term.

$$\mathbf{S} = \left[-\frac{1}{2}(\phi, \mathbf{Q}\,\mathbf{X}\,\phi) + (\phi, \mathbf{Q}\,\psi) + \mathbf{f}(\psi)\right]$$

Equations of motion:

$$\mathbf{Q}(\psi - \mathbf{X}\,\phi) = \mathbf{0}$$

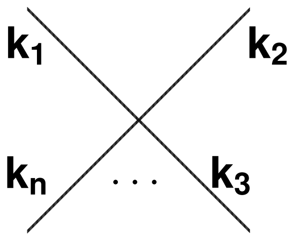
$$\mathbf{Q}\phi + \mathbf{f}'(\psi) = \mathbf{0}$$

first + X $\times$ second equation gives

$$\mathbf{Q}\psi + \mathbf{X}\,\mathbf{f}'(\psi) = \mathbf{0}$$

ψ: interacting fields, $\quad$ X$\phi - \psi$: free fields

Quantization of ψ gives the usual scattering amplitudes of string theory while quantization of X$\phi - \psi$ produces particles which do not scatter.



For Feynman rules, one finds that every vertex with external momentum $k_1, k_2, \cdots$ includes a factor proportional to

$$\exp\left[-C\sum_{i=1}^{n} k_i^2\right]$$

C: a positive constant

Due to this exponential suppression factor, integration over loop momenta never has any divergence from the region of large momentum.

– manifest UV finite theory.

All IR divergences can be treated using conventional quantum field theory methods.

Future prospects

A UV finite QFT description of string theory allows us to explore various properties of the scattering amplitudes.

1. Unitarity (conservation of probability)

2. Crossing symmetry, analyticity etc.