Symmetry Breaking by Topology and Energy Gap

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Scuola Normale Superiore, Pisa

based on

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Outline

1. Gap Generation and Symmetry Breaking
   - Particle on a Circle
   - Goldstone and Higgs Mechanisms
2. The Role of Topology
   - Particle on a Circle and Energy Gap
   - Particles on Manifolds and Energy Gap

Outline

Quantum Mechanical Model: Particle on a Circle

- Motivation for this talk is provided by simple quantum mechanical examples.
- **Particle on a circle**: the observables are *periodic functions* of the angle ϕ and generic functions of momentum p.
- Consider the momentum shift

$$\rho^{\alpha} : p \mapsto e^{i\alpha\phi} p e^{-i\alpha\phi} = p + \alpha,$$

  for $\alpha \in \mathbb{R}$.
- This symmetry transformation commutes with the free dynamics

$$e^{i\alpha\phi(t)} p e^{-i\alpha\phi(t)} = e^{i\alpha\phi} p e^{-i\alpha\phi}$$

  since $\phi(t) = \phi + tp$.
- And it is broken since $\langle p \rangle \mapsto \langle p \rangle + \alpha$ in ground state expectations.

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There is an Energy Gap.

- The free dynamics, with hamiltonian $H = p^2/2$, has eigenvalues

$$E_n = \frac{n^2}{2} \text{ for } n \in \mathbb{Z}.$$

- The energy spectrum exhibits an **energy gap**.
- A similar structure is displayed by the Bloch electron, *i.e.*, a quantum particle in a periodic potential.
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Symmetry Breaking vs Energy Gap

- How does this energy gap reconcile with the presence of symmetry breaking?
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Goldstone Symmetry Breaking.

- Goldstone theorem:

  *The spontaneous breaking of a continuous internal symmetry induces **gapless** excitations in the spectrum.*

- Crucial ingredients:
  - the presence of an **order parameter**, *i.e.* and observable A such that

    $$\langle \delta A \rangle \neq 0$$

    in ground state expectations $\langle \cdot \rangle$,
  - the generation of the symmetry transformation by a local charge (Ward identity)

    $$\langle \delta A \rangle = i \lim_{n \to \infty} \langle [Q_n(t), A] \rangle = i \lim_{n \to \infty} \langle [Q_n(0), A] \rangle$$

    in a *time-independent* fashion.
- Examples: crystals (lattice phonons), ferromagnets (spin waves), superfluids (Landau phonons).

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Higgs Symmetry Breaking.

The presence of **local gauge symmetry** allows to evade the conclusions of Goldstone's theorem:

- In local gauges (*e.g.* Feynman), the massless modes appear in the unphysical sector.
- In physical gauges (*e.g.* Coulomb), the **delocalization** induced by the dynamics spoils the time-independence of the symmetry breaking Ward identity.

These delocalization effects account for the presence of gap in:

- electroweak symmetry breaking,
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Particle on a Circle: Topology

- Rotations of 2π commute with any observable.
- Technically, $e^{i2\pi p}$ generates the **center** of the observable algebra and thus labels its irreducible representations π_θ via

$$\pi_\theta\left(e^{i2\pi p}\right) = e^{i\theta}, \text{ for } 0 \leq \theta < 2\pi,$$

  called θ sectors.
- However, ρ^α does not leave the θ sectors invariant:

$$\pi_\theta\left(\rho^\alpha\left(e^{i2\pi p}\right)\right) = e^{i(\theta+2\pi\alpha)}.$$

- Hence it is **broken** in each irreducible representation of the observable algebra.
- Also, the **spectrum** of the free hamiltonian in each θ sector is given by

$$E_{n,\theta} = \frac{1}{2}\left(n + \frac{\theta}{2\pi}\right)^2.$$

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Topology prohibits generators.

- In the particle on a circle example, the "**generator**" of the momentum shift ρ^α ought to be ϕ, since

$$\delta p = i[p, \phi]$$

  by canonical quantization.
- However ϕ is **not a legitimate operator**: for any observable A, applying a rotation of 2π

$$\langle A\phi \rangle_\theta = \langle A\phi \rangle_\theta + 2\pi \langle A \rangle,$$

  which is absurd.
- This shows that the charge associated to ρ^α has no meaning (only its exponential being well-defined).
- Then, the symmetry breaking Ward identity cannot be written down and **the Goldstone theorem does not apply.**

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Analogies with QCD

- The automorphism

$$\rho^\alpha(p) = e^{i\alpha\phi} p e^{-i\alpha\phi} = p + \alpha$$

  corresponds to **chiral transformations**;
- the topological invariants which generate the center

$$T_n = e^{i2\pi np}$$

  correspond to **large gauge transformations** with winding number n;
- the fact that only

$$e^{i\alpha\phi}, \text{ not } \phi,$$

  is a well-defined operator corresponds to the fact that chiral transformations are **not generated by a charge**, and hence the Goldstone theorem does not apply.
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General Framework.

This example allows to extract general features. Consider a quantum system with configuration manifold $\mathcal{M}$.

- The fundamental group $\pi_1(\mathcal{M})$ gives rise to **topological invariants**, *i.e.*, elements of the center of the observable algebra.
- Symmetries which do not commute with these invariants are **spontaneously broken** in each irreducible representation of the observable algebra.
- This is compatible with the presence of **energy gap** since the topology forbids the corresponding symmetry breaking Ward identity.

The corresponding gap is in fact furnished by the spectrum of the **first homology group** of the manifold.

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Summary

- Nontrivial topology gives rise to central elements of the observable algebra.
- Symmetries which do not leave these elements invariant are spontaneously broken in all irreps.
- The presence of an energy gap is allowed, and is in fact given by the homology group.
- Outlook
  - General strategy which abstracts from specific quantum mechanical models and does not involve the semiclassical approximation;
  - This mechanism plays a role in the discussion of QCD and might apply to other topologically non-trivial gauge groups (LQG?).

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