$Q\bar{Q}$ interactions in strong external magnetic field

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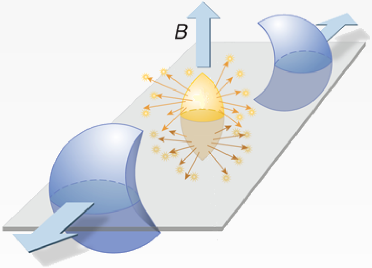
physical conditions

QCD in the presence of strong magnetic fields $eB \simeq m_\pi^2$ is relevant in many physical conditions (10^6T$\simeq 0.1$GeV^2)

- Non-central heavy ion collisions with eB $\sim 10^{15}$T [Skokov et al. '09]
- Possible production in early universe eB $\sim 10^{16}$T [Vachaspati '91]

in heavy ion collisions:
- expected eB $\simeq$ 0.3 GeV^2 at LHC in Pb+Pb at $\sqrt{s_{NN}}$=4.5TeV and b=4fm
- timescales depend on thermal medium properties (most pessimistic case: 0.1-0.5 fm/c)

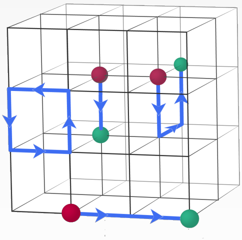
but spatial distribution of the field and lifetime are still debated!



QCD on the lattice

QCD +
path integral +
euclidean +
discretization +
finite volume =

Lattice QCD



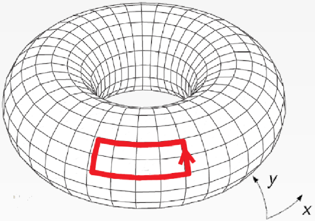
Lattice QCD is a useful approach to investigate non-perturbative properties of the strong interacting matter

Quark fields $\psi(n)$ and gluon links $U_\mu(n)$ (SU(3) parallel transports) discretized in a $N^3 \times N_t$ lattice with spacing a and temperature $T = 1/(aN_t)$

Monte-Carlo algorithms are used: physical observables are computed integrating over system configurations distributed as $\exp(-S_{QCD}[U, \psi, \bar{\psi}])$

intro **turning on the B field**

an external magnetic field B on the lattice can be introduced through abelian parallel transports $u_\mu(n)$ into the covariant derivative



- New abelian phases

$$U_\mu(n) \to U_\mu(n) u_\mu(n)$$

- External field is fixed (non-propagating fields, no kinetic term)
- Periodic boundary conditions lead to the quantization

$$|q_{\text{min}}| B = \frac{2\pi b}{a^2 N_x N_y} \quad b \in \mathbb{Z}$$

possibility to investigate the effects of a B field on the lattice

static potential

in the confining phase at low temperatures, the $Q\bar{Q}$ interaction is well described by the Cornell potential:

$$V_C(r) = -\frac{\alpha}{r} + \sigma r \qquad \sigma \simeq (420\text{MeV})^2 \quad \alpha \sim 0.4$$

on the lattice the static potential has been largely investigated and is extracted from ground state / free energy of a $Q\bar{Q}$ pair at distance R

- T=0: from Wilson loops

$$aV(R) = -\lim_{T\to\infty} \log\left(\frac{W(R,T+1)}{W(R,T)}\right)$$

with $W(R,T)$ a rectangular $R \times T$ loop made up by link variables $U_\mu(n)$

- T>0: from Polyakov correlators

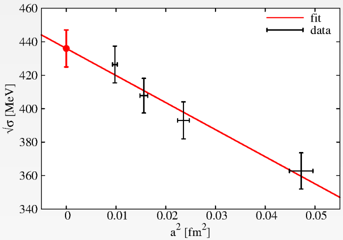
$$F(R,T) \simeq -\frac{1}{\beta}\log\langle \mathrm{Tr}L^\dagger(R+x)\mathrm{Tr}L(x)\rangle$$

where $L(R)$ is a loop winding in the compact imaginary time direction.

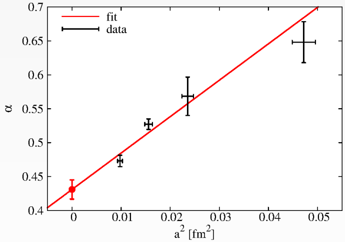
potential **parameters at B=0**

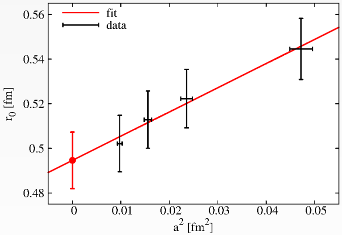
preliminary results for the continuum limit $\mathcal{O}(a) = \mathcal{O} + Ca^2$:

$$\sqrt{\sigma}=436(11)\text{MeV}$$
$$\alpha=0.431(14)$$
$$r_0=0.495(13)\text{fm}$$



where the scale r_0 is the Sommer parameter [Sommer '94]





some details: N_f=2+1 theory at T=0 and spacing from a≃0.1fm to a≃ 0.2fm for physical m_q

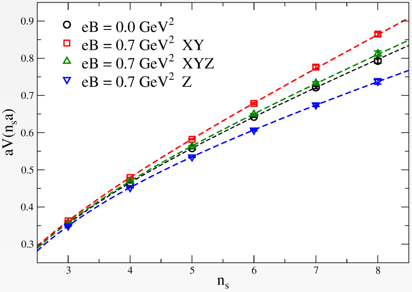
potential **effects of eB**

for non-vanishing eB the rotation symmetry is broken: $SO(3) \to SO(2)$

$$\Downarrow$$

two indipendent Wilson loops exist

results at T=0: the potential is weaker in the direction of the external field and stronger in the orthogonal plane



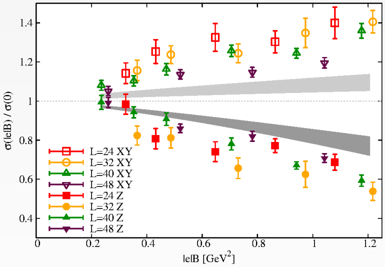
[Bonati et al. '14] Results from lattice 40^4 with N_f=2+1 at a=0.1249fm and B||z

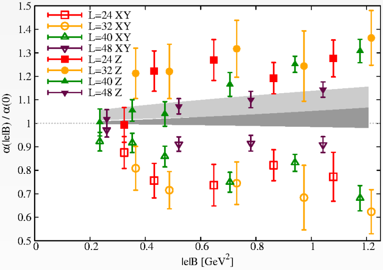
the interaction becomes anisotropic with $eB > 0$
what about the parameters σ and α?

potential **effects of eB**

a parametrization for the ratios of α and σ in the continuum

$$\frac{\mathcal{O}_d(eB)}{\mathcal{O}_d(0)} = 1 + A^{\mathcal{O}_d}(|e|B)^{D^{\mathcal{O}_d}} \qquad \mathcal{O}_d = \alpha_{XY,Z}, \sigma_{XY,Z}$$

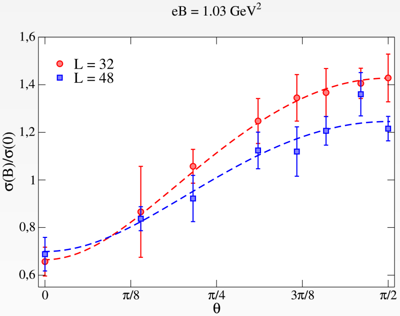




[preliminary results] N_f=2+1 theory at T=0 on lattices with spacing from a$\sim$ 0.1fm to 0.2fm; B||z

potential **eB anisotropy**

parameters acquire dependence on the magnetic field (both direction and strength)



[preliminary results]

N_f=2+1 at T=0 with a∼0.15fm and a∼0.1fm respectively for lattices 32^4 and 48^4

ansatz: the parametrization

$$\frac{\sigma(eB,\theta)}{\sigma(0)} = \epsilon_1^\sigma \sqrt{1+\epsilon_2^\sigma \sin^2\theta}$$

$$\frac{\alpha(eB,\theta)}{\alpha(0)} = \frac{1}{\epsilon_1^\alpha \sqrt{1+\epsilon_2^\alpha \sin^2\theta}}$$

where θ is the angle with respect to the eB field and $\epsilon_i^{\mathcal{O}} = \epsilon_i^{\mathcal{O}}(eB)$ carry the dependence to the field strength [Bonati et al. '15]

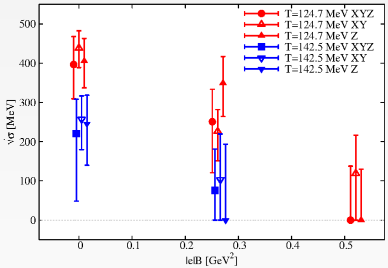
potential **high temperatures**

what about the anisotropy for T>0?

behaviour of the string tension at (not so) high temperatures:

- decreases near the deconfining transition at $T_C \sim 150$MeV
- anisotropy?
- decreases when magnetic field increases

possible explanation: the magnetic field slightly reduces T_C [Bruckmann et al. '13]



[preliminary results] lattice results from N_f=2+1 theory at a=0.0988fm, with eB||z at non zero temperatures

potential **heavy flavours**

anisotropic potential: effects on HF spectrum?

in the heavy ions collisions quarkonium formation takes place after $t_f \sim 0.5$fm/c

HF mesons are produced during initial stage

- low-momentum: strong interaction with the hot medium
- high-momentum: only low interaction with the hot medium

the latter can be used to probe the initial magnetic field

the description of HF bound states may be carried out in a non-relativistic framework

- using a (static) potential model
- turning on a magnetic field (also coupled to spins)
- tuning parameters to reproduce lattice and experimental data

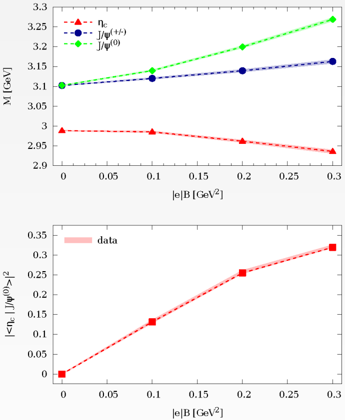
extract informations about the influence of B on $c\bar{c}$ and $b\bar{b}$ spectra

potential **heavy flavours**

the magnetic field affects the spectrum in several ways [Alford and Strickland '13, Bonati et al. '15]

- mass variations $\Delta m/m \sim 10\%$ for $c\bar{c}$ and $\sim 1\%$ for $b\bar{b}$ at $eB \sim 0.3\text{GeV}^2$
- spin state splittings and mixing
- possible experimental signature in dilepton decay channel contamination in 1S states [Alford and Strickland '13]

with the anisotropy greater mass variatios but no modifications on the mixings [Bonati et al. '15]



[Bonati et al. '15] Mass spectrum and mixing percentage in 1S $c\bar{c}$ states

conclusions

summary

- continuum extrapolations confirm the presence of an anisotropy in the static potential at $eB > 0$
- angular dependence of the parameters agrees with the simplest anisotropic description of the medium
- effects also at finite temperatures ($T < T_C$) but no evidence of anisotropy

still working and future studies

- improve the study of the potential at T>0
- investigation of the deconfined phase: effects of magnetic field on the chromo-electric and -magnetic screening masses

THANK YOU

backup **anisotropy in V_C**

from electromagnetism: potential in a medium with anisotropic dielectric constant

$$\frac{e}{r} \to \frac{e}{\sqrt{\epsilon_x x^2 + \epsilon_y y^2 + \epsilon_z z^2}}$$

then the ansatz

$$\frac{\alpha}{r} \to \frac{\alpha}{\sqrt{\epsilon^{\alpha}_{xy}(x^2+y^2) + \epsilon^{\alpha}_z z^2}} \qquad \sigma r \to \sigma\sqrt{\epsilon^{\sigma}_{xy}(x^2+y^2) + \epsilon^{\sigma}_z z^2}$$

Can be reformulated as

$$V_C \to V_C = -\frac{\alpha(eB,\theta)}{r} + \sigma(eB,\theta) r$$

$$\frac{\sigma(eB,\theta)}{\sigma(0)} = \epsilon^{\sigma}_1\sqrt{1+\epsilon^{\sigma}_2 \sin^2\theta} \qquad \frac{\alpha(eB,\theta)}{\alpha(0)} = \frac{1}{\epsilon^{\alpha}_1\sqrt{1+\epsilon^{\alpha}_2 \sin^2\theta}}$$

with θ azimutal angle and $\epsilon^{\mathcal{O}}_1 = \sqrt{\epsilon^{\mathcal{O}_z}}$, $\epsilon^{\mathcal{O}}_2 = \epsilon^{\mathcal{O}}_{xy}/\epsilon^{\mathcal{O}}_z - 1$

backup **NR bound-state model**

a $Q\bar{Q}$ bound state can be described by the Hamiltonian

$$\mathcal{H} = \sum_{i=1}^{2} \frac{1}{2m} \left[\vec{p}_i - q\vec{A}(\vec{x}_i)\right]^2 + V(\vec{x}_1, \vec{x}_2) - (\vec{\mu}_1 + \vec{\mu}_2) \cdot \vec{B}$$

where $-(\vec{\mu}_1 + \vec{\mu}_2) \cdot \vec{B} = -(gq/4m)(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \vec{B}$ acts mixing singlet and triplet states.

with an external field the rotational symmetry is broken and neither the kinetic nor the canonical momentums are conserved. In COM coordinates, symmetric gauge and definining the pseudomomentum $\vec{K}$ the model is recasted in the form

$$\hat{H} = \frac{\vec{K}^2}{2M} - \frac{q}{M}(\vec{K} \times \vec{B}) \cdot \vec{r} - \frac{\nabla^2}{2\mu} + \frac{q^2}{2\mu}(\vec{B} \times \vec{r})^2 + V(\vec{r}) - (\vec{\mu}_1 + \vec{\mu}_2) \cdot \vec{B}$$

$$M = 2m_q \qquad \mu = \frac{m_q}{2} \qquad \vec{r} = \vec{x}_1 - \vec{x}_2 \qquad \hat{K} = \sum_{i=1}^{2} \left(\hat{\vec{p}}_i + \frac{1}{2} q_i \vec{B} \times \vec{x}_i\right)$$

backup **solving NR model**

numerical approach [Bonati et al. '15]

• physical system enclosed in a Euclidean discretized volume

• eigenstates from evolution of test wavefunctions $\psi_T(\vec{r}, \tau)$ through

$$\left(\frac{\partial}{\partial \tau} + \hat{H}\right) \psi_t(\vec{r}, \tau) = 0$$

$$\psi_t(\vec{r}, \tau) = \sum_a c_a \Phi_a e^{-E_a \tau}$$

• spin part taken into account by costructing the $\mathcal{H}$ matrix

• observables obtained by diagonalizing the hamiltonian

the procedure is repeated for various spatial spacings

• physical results obtained through a continuum limit

• physical volume $V \sim$(6 fm)^3

• spacings from 0.250 GeV^{-1} to 0.625 GeV^{-1}

• simulations performed both in the presence and absence of the magnetic anisotropy in the static potential

• additional spin-spin term that takes into account observed 1S splitting at B=0 [Kawanai et al. '12]

backup **screening mass(es)**

above T$_c$ strong matter deconfines and color screening melts quarks bound states $\rightarrow$ chromo-electric and -magnetic screening masses

some previous studies on the lattice: [Maezawa et al. ’10, Borsany et al. ’15]

- choose a suitable gauge invariant correlator function: the Polyakov loop correlator $C_{LL^\dagger}(R)$

- use symmetries to separate magnetic ($\mathcal{T}$-even) and electric ($\mathcal{T}$-odd) contributions

$$C_{M+} = +\tfrac{1}{2}\mathrm{Re}\,(C_{LL} + C_{LL^\dagger}) - |\mathrm{Tr}L|^2$$
$$C_{E-} = -\tfrac{1}{2}\mathrm{Re}\,(C_{LL} - C_{LL^\dagger})$$

  where $\pm$ are $\mathcal{C}$ eigenvalues and $C_{M-} = C_{E+} = 0$

- extraction at large r according to [Nadkarni ’86, Braaten et al. ’95]

$$C_{E-,M+}(r,T)\big|_{r\to\infty} \simeq \frac{e^{-m_{E,M}(T)r}}{r}$$

backup **screening mass(es)**

separation of the Polyakov loop L magnetic and electric contributions through euclidean time reflection $\mathcal{T}$ and charge conjugation $\mathcal{C}$. Using

$$A_4(t,x) \overset{\mathcal{T}}{\to} A_4(-t,x) \qquad A_i(t,x) \overset{\mathcal{T}}{\to} A_i(t,x) \qquad A_\mu(t,x) \overset{\mathcal{C}}{\to} A^*_\mu(t,x)$$

then one can define the combinations with defined symmetries

$$L_M = \frac{1}{2}(L + L^\dagger) \quad L_E = \frac{1}{2}(L - L^\dagger) \quad L_{M\pm} = \frac{1}{2}(L_M \pm L_M^\dagger) \quad L_{E\pm} = \frac{1}{2}(L_E \pm L_E^\dagger)$$

because $\mathrm{Tr} L_{E+} = \mathrm{Tr} L_{M-} = 0$ one finds

$$C_{M^+} = +\frac{1}{2}\mathrm{Re}\left(C_{LL} + C_{LL^\dagger}\right) - |\mathrm{Tr}L|^2 \quad C_{E^-} = -\frac{1}{2}\mathrm{Re}\left(C_{LL} - C_{LL^\dagger}\right)$$

such that

$$C_{LL^\dagger} - C_{LL^\dagger}\big|_{r\to\infty} = C_{E^-} + C_{M^+}$$

backup **screening mass(es)**

expected results at eB=0:
$m_E > m_M$ and $m_{E,M} \propto T$

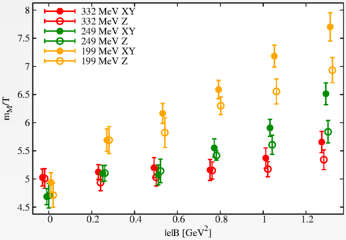
[Nadkarni ’86, Braaten et al. ’95, Borsanyi et al. ’15]

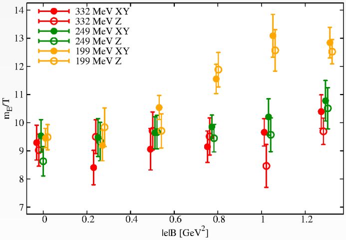
as in the case of the static potential we separated the contributions over the *xy* plane and B||z direction

- masses increase with *eB*
- anisotropy in the m_M?
- magnetic corrections decrease with *T*

good agreement of m_E and m_M values at $B = 0$ with previous works

[Borsanyi et al. ’15]





[preliminary results] N_f=2+1, a=0.0989fm

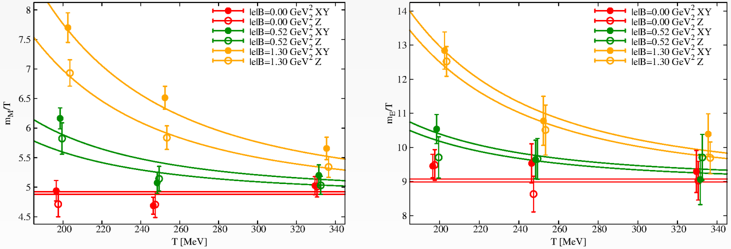
backup **screening mass(es)**

Ansatz on the functional dependence of the masses $m_{E,M}(eB,T)$:

$$\frac{m^d_{E,M}(eB,T)}{T} = A_{E,M}\left(1 + C_{E,M}\frac{eB}{T^3}\operatorname{atan}\frac{eB}{\lambda}\right)$$

with A, C constant parameters and λ a given scale. Why this form?

- no magnetic effects in the high temperature limit
- quadratic behaviour near the origin $eB \to 0$
- linear shape from $eB \gtrsim \lambda$ ($\sim$0.1-0.3 GeV from our data)



[preliminary results] N_f=2+1 theory with a=0.0989fm;